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Abstracts

## VIRTUAL ELEMENT METHOD AND TOPOLOGY OPTIMIZATION ON POLYGONAL MESHES

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Topology optimization is a fertile area of research that is mainly concerned with the automatic generation of optimal layouts to solve design problems in Engineering. The classical formulation addresses the problem of finding the best distribution of an isotropic material that minimizes the work of the external loads at equilibrium, while respecting a constraint on the assigned amount of volume. This is the so-called minimum compliance formulation that can be conveniently employed to achieve stiff truss-like layout within a two-dimensional domain. A classical implementation resorts to the adoption of four node displacement-based finite elements that are coupled with an elementwise discretization of the (unknown) density field. When regular meshes made of square elements are used, well-known numerical instabilities arise, see in particular the so-called checkerboard patterns. On the other hand, when unstructured meshes are needed to cope with geometry of any shape, additional instabilities can steer the optimizer towards local minima instead of the expected global one. Unstructured meshes approximate the strain energy of truss-like members with an accuracy that is strictly related to the geometrical features of the discretization, thus remarkably affecting the achieved layouts. In this talk we will consider several benchmarks of truss design and explore the performance of the Virtual Element Method (VEM) in driving the topology optimization procedure. In particular, we will show how the capability of VEM of efficiently approximating elasticity equations on very general polygonal meshes can contribute to overcome the aforementioned mesh-dependent instabilities exhibited by classical finite element based discretization techniques.