A CONVEX OPTIMIZATION APPROACH FOR AUTOMATED WATER AND ENERGY END USE DISAGGREGATION

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ABSTRACT

A detailed knowledge of water consumption at an end-use level is an essential requirement to design and evaluate the efficiency of water saving policies. In the last years, this has led to the development of automated tools to disaggregate high resolution water consumption data at the household level into end use categories. In this work, a new disaggregation algorithm is presented. The proposed algorithm is based on the assumption that the disaggregated signals to be identified are piecewise constant over the time and it exploits the information on the time-of-day probability in which a specific water use event might occur. The disaggregation problem is formulated as a convex optimization problem, whose solution can be efficiently computed through numerical solvers. Specifically, the disaggregation problem is treated as a least-square error minimization problem, with an additional (convex) penalty term aiming at enforcing the disaggregate signals to be piece-wise constant over the time. The proposed disaggregation algorithm has been initially tested against household electricity data available in the literature. The obtained results look promising and similar results are expected to be obtained for water data.

Keywords: Disaggregation, water and energy end-use characterization, convex optimization.

1. INTRODUCTION

Individual and collective behavioural responses to different water conservation policies acting on the demand side of residential water consumption (the so called Water Demand Management Strategies, WDMS) might significantly vary within the same urban context depending on economic drivers as well as socio-psychological determinants. Therefore, in order to design and to assess the effectiveness of alternative WDM policies, it is essential to build models that quantitatively describe how the water demand is influenced and varies in relation to exogenous determinants (e.g., climate conditions), socio-psychographic features (e.g., age, income, household features), social pressure, water restrictions, water tariffs, and reciprocal influence.

High spatial (household) and temporal (up to few seconds) resolution water consumption data gathered by smart meters provide a detailed user consumption profile. This enables an accurate characterization of the water consumption share and patterns of end-uses. Water end-use characterization, which aims at decomposing the aggregate (i.e., whole household) water consumption data collected from a single measurement point into water end use categories, plays an important role in modeling the users' behaviours and in assessing the effectiveness of WDMS. As a matter of fact, a detailed water end-use characterization allows both the users and the water utilities to understand how, when and where water is used. Beside using this information for building mathematical models of the user behavior, the generated knowledge can be also directly provided to customers, municipalities and water utilities, so that:

- household's components have a detailed knowledge on their water usage. For instance, they can see their hourly consumption, as well as charts on their water end-use patterns across major end-use categories (e.g., washing machine, toilet, shower, irrigation) and they can be alerted of occurring consumption anomalies (e.g., leak events). Furthermore, personalized hints for reducing water consumption can be directly delivered by the municipality and the water utilities;
- customers can be informed on potential savings in differing the usage of some water using appliances (e.g., washing machine and dishwasher) to peak-off hours, or in replacing low-efficient appliances into high-efficient ones, and personalized rewards schemes might be then proposed to stimulate customers to adopt water saving actions.

These challenges have motivated researchers to develop disaggregation algorithms to decompose water flow data collected from high-resolution smart meters into water end use categories. Basically, four different water disaggregation tools have developed, namely: *Trace Wizard®* (a commercial flow trace analysis toolkit developed by Aquacraft, Inc., see Mayer and DeOreo, 1999); *Identiflow®* (a tool developed by WRc, a research organization based in United Kingdom); *HydroSense* (a water disaggregation approach originally proposed in Froehlich et al., 2009) and the SEQREUS approach (a method based on *Hidden Markov Models* and developed in the SEQREUS project, see Beal and Stewart, 2011). Unfortunately, none of the aforementioned disaggregation algorithms is completely automatic, but all do require some level of interaction with the user (intrusive monitoring). Nevertheless, in the field of electric energy, there is a rich literature on automatic disaggregation methods (known as *Non Intrusive Appliance Load Monitoring* (NIALM) algorithms) aiming at decomposing the aggregate household energy consumption data collected from a single measurement point into device-level consumption data through a limited interaction with the user. The first algorithm for NIALM was proposed by (Hart, 1992). Hart's approach is based on the segmentation of the aggregate power signal into successive steps, which are then matched to the appliance signatures. However, this method is not able to detect multistate appliances and it is neither able to decompose power signals made of simultaneous on/off events on multiple appliances. Since Hart's contribution, the problem of Nonintrusive Appliance Load Monitoring has been extensively studied in the literature. The survey papers (Zeifman and Roth, 2011; Zoha et al., 2012) give a complete review on the state-of-the-art of NIALM methods, which can be classified into two main categories: optimization based and machine learning based approaches. The methods based on sparse coding (Figueiredo et al., 2013) and integer programming (Suzuki et al. 2008, Camier et al. 2013) belong the first category, while the approaches discussed in (Srinivasan et al. 2006), (Zia et al., 2011; Parson et al., 2012; Johnson and Willsky, 2013), which make use of *Hidden Markov Models* and *Artificial Neural Networks* belong to the second category.

As already mentioned, the energy disaggregation algorithms require a limited interaction with the user (i.e., a monitoring period less intrusive w.r.t. the one required by the methods for water end-use characterization). However, based on the authors' experience, these algorithms are not able to accurately reconstruct the power consumption trajectories over the time, but they have shown good performance only in estimating the fraction of energy consumed by each appliance. This represents a serious drawback, since: (i) no information on the time of use of each appliance can be derived, and so feedback on potential savings in differing the usage of some devices to peak-off hours cannot be provided; (ii) functioning anomalies can be barely detected; (iii) it is not evident if the accuracy in the estimate of the fraction of energy consumed by each appliance is due to fortuitous balancing mechanisms.

In this paper, we present a novel algorithm based on sparse optimization which can be used to disaggregate both water and energy consumption data. The proposed approach is based on the assumption that the power/water consumption profiles of each appliance are piecewise constant over the time (as it is typical for energy and water use patterns of household appliances), and it exploits the information on the time-of-day probability in which a specific appliance/fixture is likely to be used. The disaggregation problem is treated as a least-square error minimization problem, with an additional (convex) penalty term aiming at enforcing the disaggregate signals to be piece-wise constant over the time. The proposed algorithm is able to reconstruct the consumption trajectories over time, thus overcoming the main drawback of the disaggregation methods available in the literature. The proposed disaggregation algorithm has been initially tested against household electricity data available in the literature. The obtained results look promising and similar results are expected to be obtained for water data.

The paper is organized as follows. In Section 2, the disaggregation problem is formally defined. The developed disaggregation algorithm is discussed in Section 3, and suggestions for its practical implementation are given in Section 4. Although the developed method is expected to be used for disaggregating high resolution water flow data into water end-use categories, a public database with high resolution water data is not available in the literature. Therefore, the AMPds dataset (Makonin et al., 2013), which contains the energy consumption readings of a single house located in the Vancouver region in British Columbia, Canada, has been used to show the effectiveness of the developed algorithms for energy disaggregation. The obtained results are described in Section 5. Concluding remarks are given in Section 6, together with potential directions for future works.

2. PROBLEM DESCRIPTION

Consider the situation where *N* different water-using appliances/fixtures $(\mathrm{L_1}, \dots, \mathrm{L_N})$ are available in a house. Each appliance $\mathrm{L_i}$ has $\mathrm{C_i}$ operating modes and let $B_i^{(j)}$ be the water demand of the *i*-th appliance at the *j*-th operating mode (with j=1,…,$\mathrm{C_i}$). The water consumption $y_i(t)$ of the *i*-th appliance/fixture at time *t* is then given by:

$$\mathrm{y_i(t)} = \begin{bmatrix} \mathrm{B_i^{(1)}} & \mathrm{B_i^{(2)}} & \dots & \mathrm{B_i^{(C_i)}} \end{bmatrix} \begin{bmatrix} \mathrm{x_i^{(1)}(t)} \\ \mathrm{x_i^{(2)}(t)} \\ \vdots \\ \mathrm{x_i^{(C_i)}(t)} \end{bmatrix} + e_i(t),$$

with $e_i(t)$ being a modeling error. The time-varying variables $\mathrm{x_i^{(1)}(t)}, \dots, \mathrm{x_i^{(C_i)}(t)}$ can be either 0 or 1, and they satisfy the equality constraint $\sum_j^{\mathrm{C_i}} \mathrm{x_i^{(j)}(t)} = 1$ (i.e., each appliance can operate at a single mode at each time instant *t*).

Let $\mathrm{y(t)}$ be the aggregate water consumption measured by the smart meter at time t, which is given by:

$$\mathrm{y(t)} = \sum_{\mathrm{i}}^{\mathrm{N}} \mathrm{y_i(t)} + \mathrm{e(t)},$$

where $\mathrm{e(t)}$ is a measurement noise. Given a sequence $\mathrm{D_T}$ of T observations of the aggregate water consumption readings $\mathrm{y(t)}$ (with t=1,…, T), our goal is to reconstruct the actual water consumptions $\mathrm{y_i(t)}$ (with t=1,…, T) of each appliance/fixture based on the household aggregate water flow data $\mathrm{D_T}$.

A training dataset $\mathrm{D_{T_E}}$is assumed to be available. The training set consists of the observations of the water consumption profiles of each appliance/fixture available in the house. An intrusive period is needed to construct the set $\mathrm{D_{T_E}}$. During this period, the patterns of the water consumption of each appliance are observed, and information on time-of-day probability characterizing the usage of each appliance/fixture can be also gathered.

3. DISAGGREGATION ALGORITHM

In this section, the main ideas behind the proposed disaggregation algorithm are presented. Suggestions for its practical implementation are given in Section 4. The developed algorithm makes use of the following assumptions:

A1. A rough knowledge of the water consumption of each appliance/fixture at each operating mode (i.e., the terms $B_i^{(j)}$) is supposed to be available. For instance, the terms $B_i^{(j)}$ can be evaluated from the training dataset D_{T_E} through k-means clustering (Likas et al., 2003);

A2. The water consumption profiles of each appliance/fixture are piecewise constant over time (as it is typical for many residential water-using appliances/fixtures).

The ideas underlying the developed disaggregation algorithms are now described.

3.1 Standard Least-Squares estimate

In order to estimate the water consumption $y_i(t)$ of each appliance/fixture at the time sample *t*, the time varying parameters $x_i^{(j)}(t)$ might be computed by solving the standard least-squares problem:

$$\min_{\substack{x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)\\ t=1,\dots,T\\ i=1,\dots,N}} \sum_{t=1}^{T}\left(y(t)-\sum_{i=1}^{N}\hat{y}_i(t,x_i)\right)^2, \qquad [1]$$

where $\hat{y}_i(t,x_i)$ denotes the model of the water usage of the i-th appliance at time t, i.e.,

$$\hat{y}_i(t,x_i)=\begin{bmatrix}B_i^{(1)} & B_i^{(2)} & \dots & B_i^{(C_i)}\end{bmatrix}\begin{bmatrix}x_i^{(1)}(t)\\ x_i^{(2)}(t)\\ \vdots\\ x_i^{(C_i)}(t)\end{bmatrix}$$

Unfortunately, the least-squares optimization problem in Eq. [1] is an overparametrized problem, since it involves more unknown parameters than measurements. As a consequence, overfitting occurs in computing the time varying parameters $x_i^{(j)}(t)$. A possible solution to overcome this problem is to introduce regularization terms (or equivalently penalty terms) terms in [1] in order to:

- Enforce each appliance at operating at a single mode at each time instant;
- Enforce water usage patterns $\hat{y}_i(t,x_i)$ to be piecewise constant over the time, according to assumption **A2**.

3.2 Adding regularization

In order to exploit the information that: (i) the parameters $x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)$ can be either 0 or 1; (ii) each appliance/fixture can only operate at a single mode at each time instant, the following regularized problem can be solved instead of (1):

$$\min_{\substack{x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)\\ t=1,\dots,T\\ i=1,\dots,N}} \sum_{t=1}^{T}\left(y(t)-\sum_{i=1}^{N}\hat{y}_i(t,x_i)\right)^2+\gamma_1\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|\begin{bmatrix}x_i^{(1)}(t)\\ x_i^{(2)}(t)\\ \vdots\\ x_i^{(C_i)}(t)\end{bmatrix}\right\|_0, \qquad [2]$$

$$\text{s.t.}\quad x_i^{(j)}(t)\geq 0,\qquad \sum_{j=1}^{C_i}x_i^{(j)}(t)=1,\quad i=1,\dots,N;\quad t=1,\dots,T,$$

where $\|\cdot\|_0$ denotes the 0-norm of a vector (i.e., number of nonzero elements). Note that, on one hand, the second term in the objective function of Problem [2] aims at minimizing the number of nonzero elements in the vector $\left[x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)\right]$. On the other hand, because of the constraint $\sum_{j=1}^{C_i}x_i^{(j)}(t)=1$, the vector $\left[x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)\right]$ is guaranteed to have at least a nonzero element. The parameter $\gamma_1\geq 0$ is tuned by the user (for instance through cross validation, see Section 4.3) for balancing the tradeoff between minimizing the fitting error (by decreasing the value of γ_1) and minimizing number of the nonzero elements in the vector $\left[x_i^{(1)}(t),\dots,x_i^{(C_i)}(t)\right]$ (by increasing the value of γ_1). Because of the 0-norm, Problem [2] is nonconvex, and thus difficult to be solved through numerical optimization solvers available in the literature. Nevertheless, an approximate solution of Problem [2] can be obtained by replacing the 0-norm with the (convex) 1-norm (i.e., sum of the absolute value of the elements of the vector). Furthermore, the final estimate can be improved by scaling the parameters

$\left[x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\right]$ with nonnegative weights $\left[w_i^{(1)}(t),\ldots,w_i^{(C_i)}(t)\right]$. This leads to the following approximation of Problem [2]:

$$\min_{\substack{x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\\ t=1,\ldots,T\\ i=1,\ldots,N}} \sum_{t=1}^{T}\left(y(t)-\sum_{i=1}^{N}\hat{y}_i(t,x_i)\right)^2+\gamma_1\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|\begin{bmatrix}w_i^{(1)}(t)\\ w_i^{(2)}(t)\\ \vdots\\ w_i^{(C_i)}(t)\end{bmatrix}*\begin{bmatrix}x_i^{(1)}(t)\\ x_i^{(2)}(t)\\ \vdots\\ x_i^{(C_i)}(t)\end{bmatrix}\right\|_1, \qquad [3]$$

$$\text{s.t.}\quad x_i^{(j)}(t)\geq 0,\qquad \sum_{j=1}^{C_i}x_i^{(j)}(t)=1,\quad i=1,\ldots,N;\quad t=1,\ldots,T,$$

where $*$ denotes the element-wise multiplication. An appropriate choice of the weights $w_i^{(j)}(t)$ is discussed in Section 4.1.

3.3 Adding regularization to enforce piecewise constant water consumption profiles

In order to improve the estimate given by Eq. [3], we might exploit the additional information that the patterns of water consumption are piece-wise constant over time (Assumption **A2**). In order to enforce the estimated water consumption profiles to be piecewise constant, a new regularization term can be added to Problem [3], i.e.,

$$\min_{\substack{x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\\ t=1,\ldots,T\\ i=1,\ldots,N}} \sum_{t=1}^{T}\left(y(t)-\sum_{i=1}^{N}\hat{y}_i(t,x_i)\right)^2+\gamma_1\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|\begin{bmatrix}w_i^{(1)}(t)\\ w_i^{(2)}(t)\\ \vdots\\ w_i^{(C_i)}(t)\end{bmatrix}*\begin{bmatrix}x_i^{(1)}(t)\\ x_i^{(2)}(t)\\ \vdots\\ x_i^{(C_i)}(t)\end{bmatrix}\right\|_1 + \qquad [4]$$

$$+\gamma_2\sum_{i=1}^{N}\sum_{t=2}^{T}\left\|k_i\begin{bmatrix}x_i^{(1)}(t)-x_i^{(1)}(t-1)\\ x_i^{(2)}(t)-x_i^{(2)}(t-1)\\ \vdots\\ x_i^{(C_i)}(t)-x_i^{(C_i)}(t-1)\end{bmatrix}\right\|_\infty$$

$$\text{s.t.}\quad x_i^{(j)}(t)\geq 0,\qquad \sum_{j=1}^{C_i}x_i^{(j)}(t)=1,\quad i=1,\ldots,N;\quad t=1,\ldots,T,$$

with γ_2 being a tuning parameter playing a role similar to γ_1. The terms k_i (with i=1,…,N) are a-priori specified nonnegative weights which can be chosen through the method described in Section 4.2. Note that the infinity norm of a vector (i.e., maximum absolute value among the elements of the vector) is considered in Eq. [4]. In this way, if one of the parameters $\left[x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\right]$ changes from time t-1 to time t, a variation of the other parameters does not change the cost function. Specifically, only the largest time variation among the elements of the vector $\left[x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\right]$ affects the cost function.

Summarizing, the time-varying parameters $\left[x_i^{(1)}(t),\ldots,x_i^{(C_i)}(t)\right]$ describing the water consumption of each appliance/fixture are computed by solving the regularized (convex) optimization Problem [4].

4. PRACTICAL IMPLEMENTATION

In this section, the choice of the weighting parameters (appearing in Problem [4]) from the training dataset D_{T_E} is discussed. Furthermore, some advices for reducing the computational load of the developed disaggregation algorithm are given.

4.1 On the choice of the weights $w_i^{(j)}(t)$

The main idea behind the choice of the weights $w_i^{(1)}(t),\ldots,w_i^{(C_i)}(t)$ is the following: if the i-th water-using appliance/fixture is likely to operate at mode j at time t, then the parameter $x_i^{(j)}(t)$ is likely to be equal to 1, while the other parameters $x_i^{(g)}(t)$ (with $g\neq j$) are likely to be equal to 0. In terms of the optimization problem [4], the parameters $x_i^{(g)}(t)$ (with $g\neq j$) should be more penalized than $x_i^{(j)}(t)$, or equivalently, the scaling weights $w_i^{(g)}(t)$ (with $g\neq j$) should be higher than $w_i^{(j)}(t)$. The information on time-of-day probability of the usage of each appliance/fixture can be inferred from the training dataset D_{T_E}. Specifically, for given i and t, the weights $w_i^{(1)}(t),\ldots,w_i^{(C_i)}(t)$ can be chosen as follows:

- Given the training dataset D_{T_E}, for each time sample t compute the number of times the i-th fixture/appliance is operating at mode j at the time samples t+k24h, where k=0,1,-1,2,-2,… Denote the computed number as $q_i^{(j)}(t)$;
- If $q_i^{(j)}(t)\neq 0$, the weight $w_i^{(j)}(t)$ is given by the inverse of $q_i^{(j)}(t)$, i.e., $q_i^{(j)}(t)=\frac{1}{w_i^{(j)}(t)}$. Otherwise, set the parameter $x_i^{(j)}(t)$ equal to 0.

4.2 On the choice of the weights k_i

The weights k_i (with i=1,…,N) can be chosen as follows: if the i-th appliance/fixture changes its operating mode rarely over the time, than the time variation of the parameters $x_i^{(j)}(t)$ should be more penalized w.r.t. the time variation of the parameters characterizing another appliance/fixture which frequently changes its operating mode. The weight k_i can be then inversely proportional to the number of mode changes observed in the training dataset for the i-th appliance.

4.3 On the choice of the tuning parameters γ_1 and γ_2

In order to tune the parameters γ_1 and γ_2, a subset D_{T_C} of length T_C is extracted from the original training dataset D_{T_E}. The D_{T_C} is referred as calibration dataset. The values of γ_1 and γ_2 are then chosen through a cross-validation procedure, that is by minimizing (with a grid search) the *Total Relative Square Error* (TRSE) over the calibration dataset D_{T_C}, where the TRSE is defined as

$$\text{TRSE} = \sum_{i=1}^{N} \frac{\sum_{t=1}^{T} \left(y_i(t) - \hat{y}_i(t)\right)^2}{\sum_{t=1}^{T} y_i^2(t)}$$

The values of γ_1 and γ_2 leading to the minimum TRSE are chosen.

4.4 Reducing the computational complexity

It is worth remarking that the number of optimization variables in Problem [4] grows linearly with the length T of the signal y(t) to be disaggregated. As a consequence, the applicability of the proposed approach might be limited to small/medium values of T. In order to overcome this problem, sub-optimal solutions of Problem [4] can be simply computed by splitting the dataset D_T into *M* disjoint subsets $D^{(h)}$ of length T_h (with *h = 1, . . . ,M*). Problem [4] is then solved only for the subsets $D^{(h)}$.

5. APPLICATION ON REAL DATA

Although the algorithm is expected to be mainly used for disaggregating water flow data into end use categories, a public database with high-resolution water consumption data is not available in the literature for research purposes. Therefore, in order to assess the performance of the developed algorithm, the proposed method have been initially tested against electric energy data available in the literature.

5.1 Dataset description

The AMPds dataset (Makonin et al., 2013) is used to test the performance of the developed algorithm. The AMPds dataset is available online and it contains the energy consumption readings of a single house located in the Vancouver region in British Columbia, Canada. Specifically, 21 breakers/loads have been sub-metered for an entire year (from April 1, 2012 to March 31, 2013) at one minute read intervals.

For the sake of analysis, we considered only the aggregate power consumption given by the sum of the power consumption readings of the following four electric appliances:

- Washing machine;
- Fridge;
- Dishwasher;
- Heat Pump.

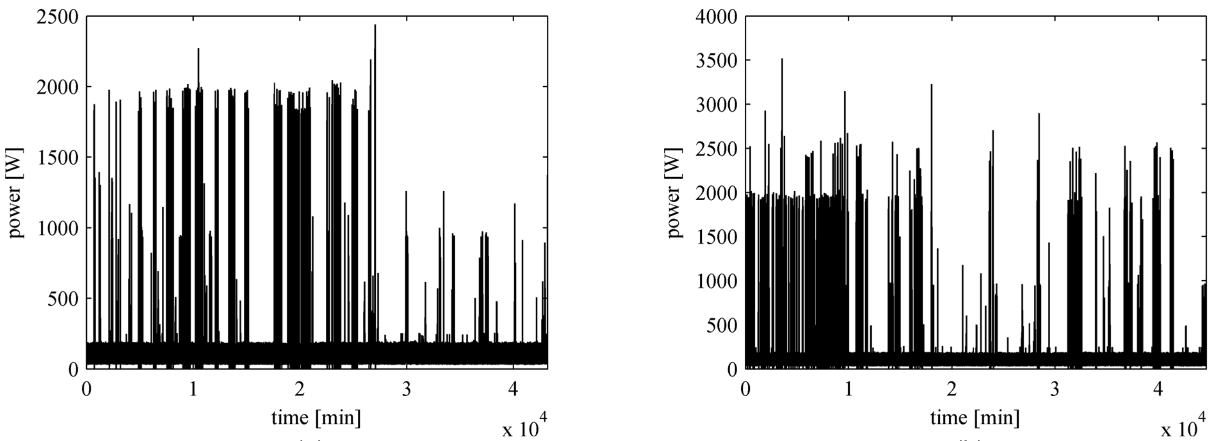
These four appliances share the largest contribution of the total energy consumption both in Summer and in Winter, and they contribute at least for the 5% (Summer period) and 3% (Winter period) of the total energy consumption.

Furthermore, in order to assess the robustness of the disaggregation algorithm w.r.t. a measurement noise which might corrupt the power readings, the aggregate power consumption signal $y(t)$ has been corrupted by an additive zero-mean random Gaussian noise $e(t)$ with standard deviation $\sigma = 4$ W. Note that, because of the added fictitious noise, the aggregate power consumption signal can become negative. At the time samples when this happens, the power consumption signal is set to 0 W.

The available AMPds dataset has been divided into two disjoint datasets:

- A training dataset D_{T_E} containing the power readings from April 1, 2012 to May 31, 2012; and from October 1, 2012 to November 30, 2012. The training set is used to estimate the power demand of each appliance at each operating mode (i.e., the terms $B_i^{(j)}$) as well as the weights $w_i(t)$ and k_i through the procedure discussed in Sections 4.1 and 4.2. Furthermore, in order to tune the parameters γ_1 and γ_2 used in the optimization algorithm (see Section 4.3), a calibration dataset D_{T_C} has been extracted from the original training dataset D_{T_E}. Such a calibration dataset consists of the data for the days 16-31 May 2012 and 16-30 November 2012. Note that the sub-metered power consumptions of each appliance are supposed to be available in the training and calibration phase;

- Since seasonality is expected to have an impact on the consumption pattern of the different end uses, the algorithm has been used to disaggregate a portion of data extracted from the Summer period and a portion from the Winter period. Specifically, the dataset D_T to be disaggregated consists of the data for the days 1-30 June 2012 (plotted in Figure 1a) and 1-31 December 2012 (plotted in Figure 1b).



(a) (b)

Figure 1. Electric power consumption: (a) June 2012; (b) December 2012.

5.2 Performance metrics

The following metrics have been used to assess the performance of the developed disaggregation method:

- The *Estimated Energy Fraction Index* (EEFI), defined as:

$$\hat{h}_i = \frac{\sum_{t=1}^{T} \hat{y}_i(t),}{\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{y}_i(t)}$$

  The index $\hat{h}_i$ provides the fraction of energy assigned to the i-th appliance, and it should be compared to the *Actual Energy Fraction Index* (AEFI), defined as

$$h_i = \frac{\sum_{t=1}^{T} y_i(t),}{\sum_{i=1}^{N} \sum_{t=1}^{T} y_i(t)}$$

  which in turn provides the actual fraction of energy consumed by the i-th appliance. The *Estimated Energy Fraction Index* $\hat{h}_i$ gives the users the information on how much energy each appliance is consuming, and so personalized hints for reducing their energy consumption can be provided.

- The *Relative Square Error* (RSE), defined, for the *i*-th appliance, as:

$$RSE_i = \frac{\sum_{t=1}^{T} (y_i(t) - \hat{y}_i(t))^2}{\sum_{t=1}^{T} y_i^2(t)}$$

  The RSE provides a normalized measure of the difference between the actual and the estimated power consumption of the *i*-th appliance.
- The R^2 coefficient, defined for the *i*-th appliance as:

$$R_i^2 = 1 - \frac{\sum_{t=1}^{T} (y_i(t) - \hat{y}_i(t))^2}{\sum_{t=1}^{T} (y_i(t) - \bar{y}_i)^2},$$

  with $\bar{y}_i$ denoting the mean of the power consumption, i.e.,

$$\bar{y}_i = \frac{1}{T} \sum_{t=1}^{T} y_i(t)$$

Both the R^2 coefficient and the RSE measure how well the estimated power profiles match the actual power profiles over time. This information is essential to detect functioning anomalies or to inform consumers on potential savings in deferring the use of some appliances to peak-off hours. Obviously, high value of the R^2 coefficients (or equivalently low values of the RSE) lead also to an accurate estimate of the *Estimated Energy Fraction Index* $\hat{h}_i$.

5.3 Numerical results

The performance metrics introduced in the previous section and the estimated disaggregate power profiles are computed in order to assess the performance of the presented algorithm. Specifically:

- Table 1 shows the *Estimated Energy Fraction Index* $\hat{h}_i$ for each appliance, along with the Actual *Energy Fraction Index* h_i;
- Table 2 shows the Relative Square Errors and the R^2 coefficients for each appliance. It is worth remarking that the Relative Square Errors and the R^2 coefficients, as well as the indexes $\hat{h}_i$ and h_i, are referred to the portion of the dataset to be disaggregated (i.e., June and December).
- Figure 2 and Figure 3 show the estimated power consumption profiles for each appliance. For the sake of visualization, only the power profiles at June 2, 2012 and December 3, 2012 are plotted.

Table 1 Fraction of energy assigned to each appliance ($\hat{\mathbf{h}}_i$) and actual fraction of energy consumed by each appliance ($\mathbf{h}_i$)

	June 2012		December 2012	
	$\hat{h}_i$	h_i	$\hat{h}_i$	h_i
Washing machine	2.7 %	2.8 %	1.1 %	1.1 %
Fridge	25.1 %	25.7%	10.5 %	10.7 %
Dishwasher	6.5 %	6.7 %	4.6 %	4.2 %
Heat pump	65.7 %	64.8 %	83.8 %	84.0 %

Table 2 Relative Square Errors and $\mathbf{R}^2$ coefficients

	June 2012		December 2012	
	RSE_i	R_i^2	RSE_i	R_i^2
Washing machine	0.10	89.7 %	0.04	95.6 %
Fridge	0.68	89.0 %	0.15	78.2 %
Dishwasher	0.62	93.4 %	0.09	90.1 %
Heat pump	0.09	90.2 %	0.06	91.1 %

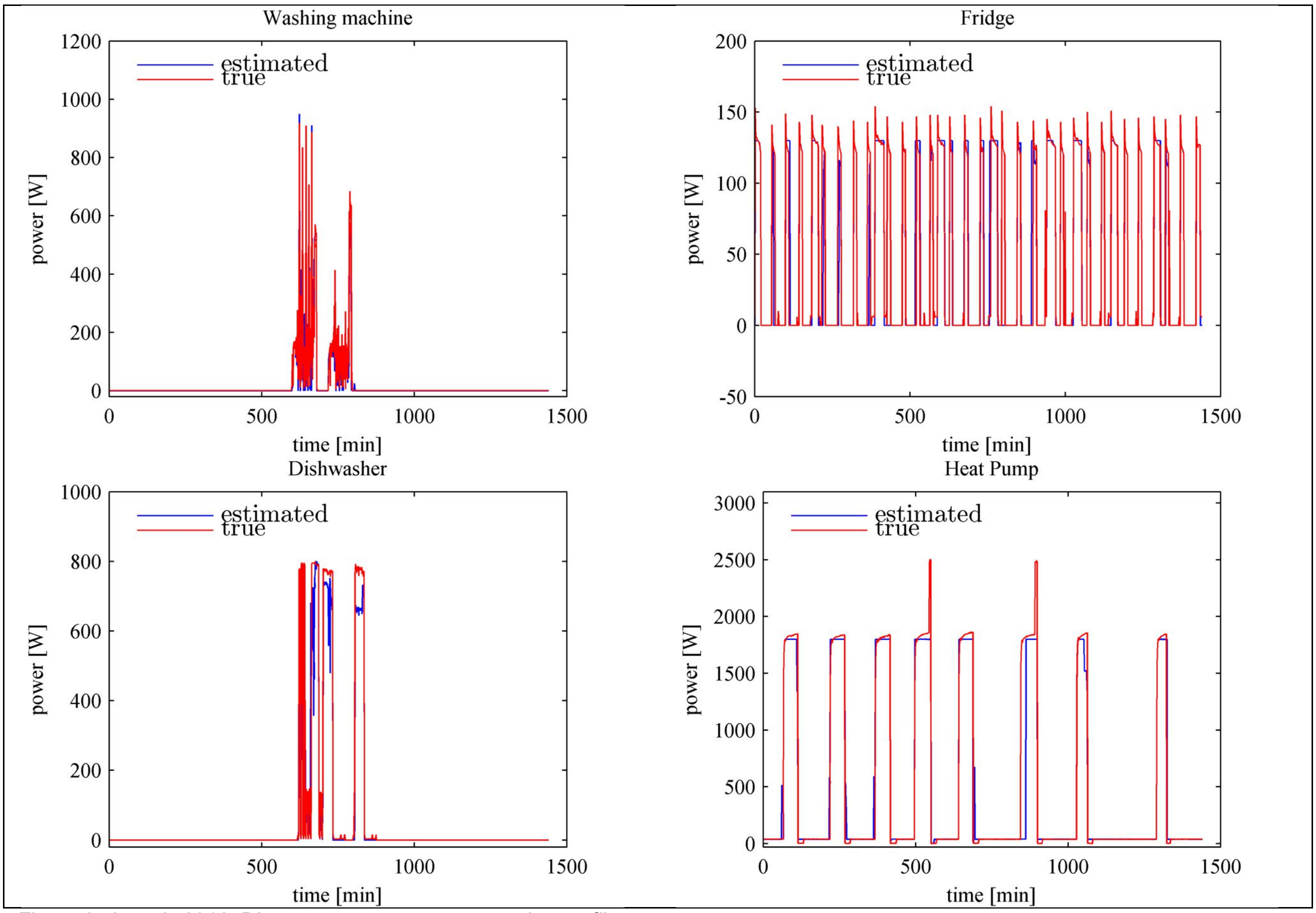


Figure 2. June 2, 2012. Disaggregate power consumption profiles

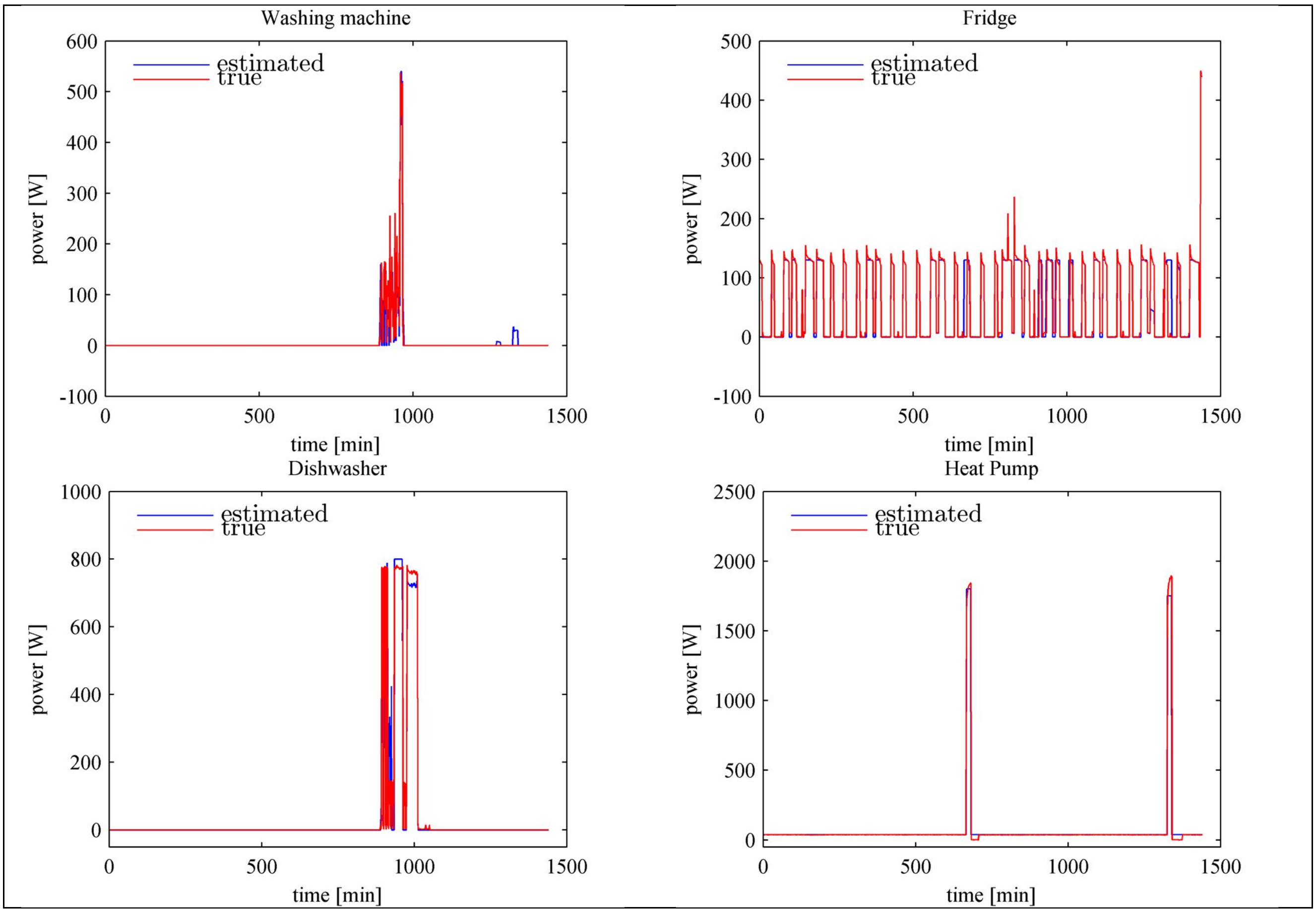


Figure 3. December 3, 2012. Disaggregate power consumption profiles

The obtained results show that the developed algorithm is able to accurately estimate the fraction of energy consumed by each appliance in the household (see Table 1). As a matter fact, for each appliance, the *Estimated Energy Fraction Index* $\hat{h}_i$ is very similar (and in some cases equal) to the *Actual Energy Fraction Index* h_i. This good performance is manly due to an accurate estimate of the disaggregated consumption trajectories over the time. This can be clearly seen from the time trajectories plotted in Figure 2 and Figure 3, as well as from the obtained values of the RSE and R^2 coefficients (Table 2). Note that the R^2 coefficients are close (and in some cases higher than) 90%, with the exception of the estimate of the power consumption of the fridge in December (associated R^2 coefficient: 78.2%). An interesting property of the proposed method is that it is not able to detect consumption spikes followed by a period of constant power consumption (see, e.g., Figure 2, *Fridge* and *Heat Pump* plots). This behaviour can be explained as follows: two different criteria are minimized in Problem [4], namely: the (aggregate) fitting error and a penalty term aiming at enforcing the disaggregated power consumption profiles to be piecewise constant. In order to minimize the latter penalty term, spikes are not reconstructed (at the cost of slightly increasing the fitting error). In other words, in terms of Problem [4], it is better not to detect a spike (thus slightly increasing the fitting error) than allowing a time variation of the power consumption profiles.

6. CONCLUSIONS

In this paper, we presented a novel algorithm which can be used both for water and energy end use characterization. The disaggregation problem is treated as a least-square error minimization problem, with an additional penalty term aiming at enforcing the disaggregate consumption signals to be piece-wise constant over time. Unlike many disaggregation algorithms available in the literature, the proposed method is able to handle situations where multiple appliances are operating simultaneously, and also to accurately estimate the disaggregate consumption profiles over time. Ongoing research activities are focused on:

- Extensive testing of the algorithm's generalization potential across different data sampling (i.e., 2s, 15 min, 1 h);
- Application of the proposed method to water consumption data.

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