

THREE DIMENSIONAL NONLINEAR DYNAMIC MODELING OF A VERTICALLY ISOLATED ANCIENT STATUE DISPLAYED IN A BASE ISOLATED MUSEUM BUILDING

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Abstract: *This research concerns with the question of how the seismic isolation methods can be applied efficiently to protect the ancient statues under both horizontal and vertical strong earthquake excitations. The results are achieved by carrying out nonlinear dynamical analyses under three dimensional earthquake ground motion data on a generic ancient statue model, considering that it is displayed in a non isolated and a horizontally isolated building. The model is developed with 8-node cubic finite elements, and placed on a rigid platform, which is modelled as an area element of rectangular shape. The isolation devices are modelled as non linear spring elements for the horizontal seismic isolation, and gap elements (compression only) for the vertical seismic isolation.*

1 INTRODUCTION

After the efficiency of seismic isolation technology has been proved for several types of structures such as bridges, buildings, power plants, etc., scientists and researchers in the field started to look for the possible ways to adapt the seismic isolation solutions and furthermore create innovative solutions for the cultural heritage of human history and art objects. The need to protect these objects against earthquakes was found inevitable since they are one of the most precious possessions of the human being on the earth.

Koumoussis et al. [1], in his work for the horizontal seismic isolation of the Statue of Hermes, has designed and proved the adequacy of the FPS™ (Friction pendulum system), after carrying out numerical analyses on the 3D model of the statue. Borri and Grazini [2] have carried out static and dynamic numerical analyses for the statue of Michelangelo's David Statue. They have shown that the horizontal accelerations that could be originated by even moderate earthquakes (0.10-0.15g) can cause very high stress levels on some particular parts of the marble statue. Grazini [3] has done rocking analysis for both the non isolated and horizontally isolated statue models. Borri [2] and Grazini [3] have both pointed out the benefits of seismic isolators for the seismic protection of statues. Tadahiro and Masahiko [4] have developed seismic isolation solutions that are capable of protecting The Gates of Hell statue, which was sculpted by Augustin Rodin and owned by National Museum of Western Art, against very strong earthquakes such as the Great Hanshin Earthquake that occurred, in Kobe in 1995. Yegian et al. [5], has carried out several tests and numerical analysis for the sculptures in Nagoya Museum, with mechanical horizontal seismic isolators. In this project, the effectiveness of the horizontal seismic isolation system is investigated for isolated and non-isolated museum buildings.

Once a rigid block starts rocking, its rotation continues smoothly from point O to O', as shown in the Figure 1. If it does not fail by overturning, its rotation will change direction, and rotate otherwise, and the motion will continue in this manner, and cease some time after the ground motion comes to an end. For the rigid block to sustain this rocking motion, the energy released during the impact should be dissipated. The amount of this energy dissipation increases with the decreasing slenderness of the block [6].

The minimum rocking impulse that causes rocking of an object has been expressed by Calio and Marletta [7]. The minimum rocking impulse I_u , has been derived by equating the overturning moment due to the inertia load caused by the maximum horizontal acceleration, to the resisting moment due to the gravity as;

$$\ddot{u}_{\max}(I_r) = \frac{gb}{h} = \frac{g}{\lambda} \quad (1)$$

Considering zero damping, the maximum acceleration is given by

$$\ddot{u}_{\max} = I\omega \quad (2)$$

Therefore the minimum impulse that starts the rocking motion of a rigid body is represented as;

$$I_u = \frac{g}{\omega\lambda} = \frac{g}{2\pi\lambda} T \quad (3)$$

Where,

g = gravity acceleration

h = height of the center of mass of the object

b = lateral distance between the overturning point and the center of mass

$\lambda = h/b$ (slenderness ratio)

ω = natural frequency of the object

T = natural period

Calio et. al [7] concluded that the object stays in full contact with the ground until the design impulse becomes equal to the minimum rocking impulse. When the minimum rocking impulse is surpassed, the object starts rocking. Calio et al. [7] have shown that the seismic isolation significantly contributes to the stability of the object under horizontal ground motions.

Oliveto, Calio and Greco [8], have evaluated the uplift conditions for the initiation of rocking motion, considering both horizontal and vertical ground motions. They introduced a dimensionless parameter γ , to take into account the vertical accelerations of the ground motion.

$$\gamma = 1 + \frac{\ddot{v}_g}{g} \quad (4)$$

Where

\ddot{v}_g = Vertical ground acceleration (positive, when directed downwards)

g = Gravity acceleration (m/s^2)

They pointed out the significance of the vertical accelerations on the minimum rocking impulse, and expressed it through modifying the minimum rocking impulse expression;

$$I_u = \frac{\gamma g}{\omega \lambda} = \frac{\gamma g}{2\pi \lambda} T \quad (5)$$

As it can be understood from the two expressions above, when the vertical accelerations are directed upwards (against gravity), the minimum rocking impulse decreases, therefore the initiation of rocking becomes easier for the object considered.

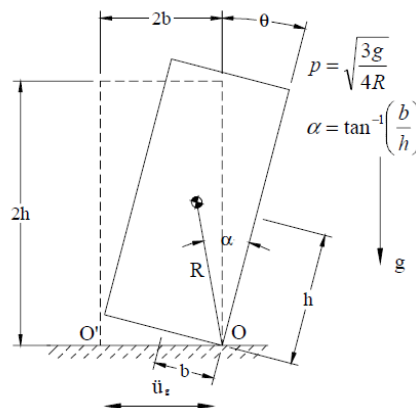


Figure 1 Rocking motion of a rigid body

This research concerns with the question of how the seismic isolation methods can be applied efficiently to protect the ancient statues subjected to both horizontal and vertical strong

earthquake excitations. The study seeks to classify the effects of conditions deriving from different building types and earthquake characteristics, to design an efficient isolation system for a particular ancient statue. To achieve the results, computer models under several earthquake ground motion data are developed and compared for a generic ancient statue, considering that it would have been placed either in a non isolated or horizontally isolated building.

2 ANALYSES UNDER HORIZONTAL AND VERTICAL EARTHQUAKE EXCITATIONS

The statues are analyzed under both horizontal and vertical earthquake ground excitations, and the properties of a three dimensional seismic isolation system are decided for the protection of the statues against the three dimensional earthquake excitations.

2.1 The methodology and Ground Motion Data Used for the Analyses

The nonlinear dynamic time history analyses are performed on the 3D finite element solid model of an ancient statue (Figure 2) [9]

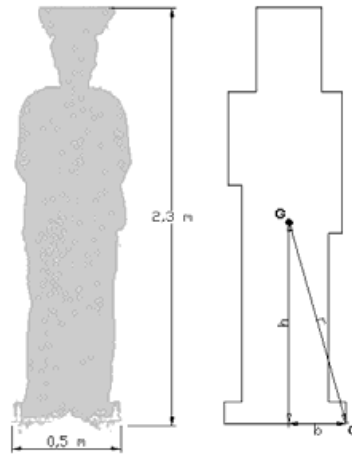


Figure 2 Ancient statue used in the numerical analysis

The model is analyzed under the three dimensional acceleration time history data of three different earthquakes which can be seen in Table 1.

Maximum Ground Accelerations (g)	Longitudinal Maximum Acceleration (g)	Translational Maximum Acceleration (g)	Vertical Maximum Acceleration (g)
Athens 1999	0,84	1,01	1,20
El Centro 1940	0,31	0,22	0,21
Aquila 2009	0,41	0,44	0,49

Table 1 Maximum accelerations used in the numerical analysis

The analyses are carried out in three stages. First, the acceleration time-history data of the earthquakes are applied to a 3 storey reinforced concrete frame which represents a generic museum building with a horizontal base isolation system. Then, the acceleration time-history data are obtained at each floor of the building model. Finally, the acceleration time-history data obtained from this model are given as the time history acceleration input to the ancient statue model. The analyses are carried out in two different parts;

1) The acceleration time-history data obtained from the non isolated building is given as the input time history data to the statues, in which case the statues are isolated against both horizontal and vertical excitations,

2) The acceleration time-history data obtained from the base isolated building is given as the input to the statues, in which case the statues are isolated against only vertical excitations.

2.2 Computer Model for the Statues with Three Dimensional Base Isolation System

The model to be analyzed under the earthquake ground motion data is developed with three dimensional 8-node finite elements, and placed on a rigid platform, which is modelled as an area element of rectangular shape (Figure 3).

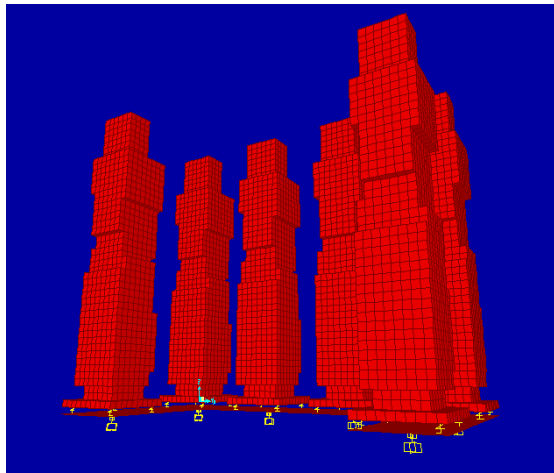


Figure 3 Three dimensional numerical model

The three dimensional seismic isolation devices are modelled as a combination of two types of elements;

- 1) Non linear spring elements for the horizontal seismic isolation
- 2) Gap elements (compression only) for the vertical seismic isolation

Horizontal seismic isolators are placed between the floor and the rigid platform on which the statues are placed. They are modelled as non-linear spring elements with initial stiffness k_e , post yield stiffness k_y and damping element with the damping coefficient c .

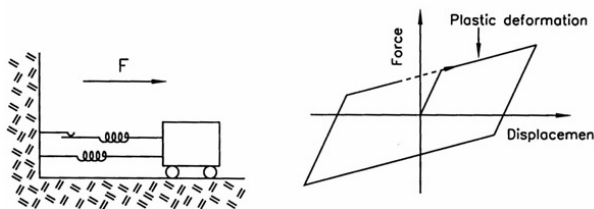


Figure 4 Horizontal isolator element

The hysteresis loop of a horizontal isolator element has a bilinear shape (Figure 4). The force deformation graph is composed of two slopes giving the 'initial' and 'yielded' stiffness respectively. The initial stiffness slope corresponds to the linear deformation; whereas the yielded stiffness slope corresponds to the plastic deformation that takes place in the isolator.

Vertical seismic isolators are placed between the rigid platform and the base of the statues. They are modeled as gap elements, which have positive stiffness values under compression, and zero stiffness under tension loads. Under compression, they show a linear behavior with spring constant k , and damping coefficient c .

2.3 Vertical Isolator Properties used in the Computer Model

The vertical isolation system consists of four springs with linear spring constant k and damping coefficient c . In the computer model, to simulate a vertical base isolation device, a gap (“compression only”) element is used (Figure 5). The force deformation relationship of this element is given by;

$$f(x) = \begin{cases} k(d + open) & \text{if } d + open < 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

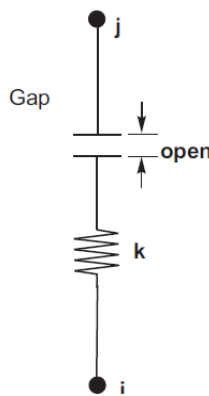


Figure 5 Vertical isolator element

As it can be seen in the Figure 6, the gap elements have a linear behavior under compression, and they can not carry any tensile load. In other words, they provide linear stiffness only under compressive loads.

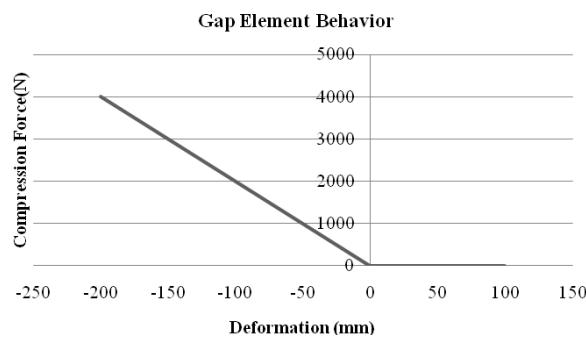


Figure 6 Gap element behaviour

In the model, thanks to the self weight of the statues, these elements are initially compressed. Therefore during the vertical earthquake excitations, they are expected to take action only in the compression range. The vertical isolator elements are constrained at the vertical (z) axis to not to permit differential displacements.

2.4 Behaviour of Three Dimensionally Isolated Statues Displayed in a Building without Horizontal Base Isolation

The analyses are carried out with different horizontal and vertical isolator elements, the properties of which can be seen in Table 2 and Table 3.

Vertical Seismic Isolation Device	Vertical Linear Stiffness(k_v) (N/mm)
Vertical Isolator 1	100
Vertical Isolator 2	50
Vertical Isolator 3	40
Vertical Isolator 4	30
Vertical Isolator 5	20

Table 2 Vertical isolator stiffnesses

Horizontal Seismic Isolation	Initial Stiffness(k_e) (N/mm)	Plastic Stiffness(k_y) (N/mm)	Yield Strength(f_y) (N/mm ²)
Isolator 4	1000	5	350

Table 3 Horizontal isolator properties

The analysis on the seismically isolated model of the statues are carried out under the acceleration time-history data obtained from the ground floor of a non-isolated building. In the Table 4 the reduced vertical accelerations can be seen for three different earthquakes and 5 different vertical isolators.

Maximum Vertical Accelerations on the Statues (Ground Floor)	Athens 1999 (g)	El Centro 1940 (g)	Aquila 2009 (g)
Earthquake Excitation:	<i>1,20g</i>	<i>0,21g</i>	<i>0,49g</i>
Vertical Linear Stiffness (N/mm)			
100	0,79	0,17	0,21
50	0,37	0,13	0,16
40	0,32	0,13	0,16
30	0,26	0,09	0,13
20	0,24	0,05	0,09

Table 4 Maximum vertical accelerations observed on the statues at the ground floor of the museum building

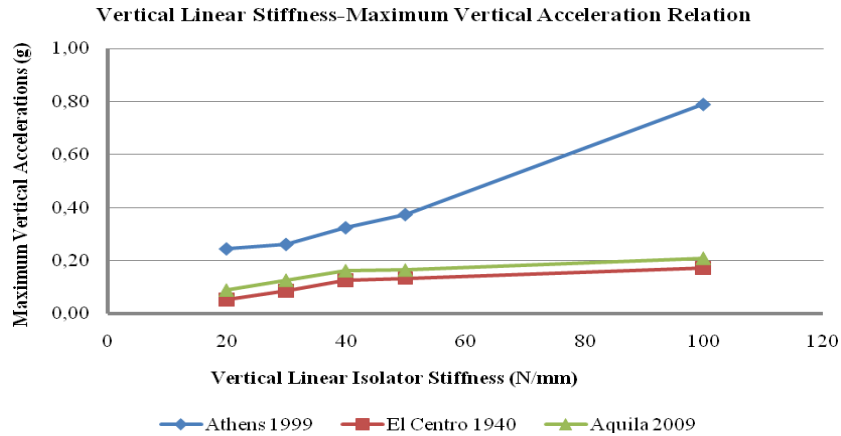


Figure 7 Vertical linear stiffness-maximum vertical acceleration relation

As it can be seen in the Figure 7 and Table 4, the vertical accelerations can be reduced significantly using vertical isolators with low stiffness values. Yet, the stiffness of the isolators must be tuned so that they must be capable of carrying the statue, under its self weight. In Table 5, the initial vertical displacements under the self weight of the statues can be seen.

Maximum Vertical Displacements at the Vertical Isolators	Displacements Under Self Weight (cm)
Vertical Linear Stiffness (N/mm)	
100	3,55
50	7,10
40	8,88
30	11,8
20	17,80

Table 5 Maximum vertical displacements observed at vertical isolators

Using four vertical isolator springs with the stiffness of 20 N/mm under the base of the statue, the initial vertical displacement under the self weight of the statue is 17.8 cm, whereas, it reaches up to 21.65 cm under the strongest vertical excitation (Athens 1999 Earthquake) These results show that under three earthquake excitations, the linear vertical isolator elements perform effectively, satisfying the two necessary requirements:

- They are always under compression
- Their vertical displacement values are within the limits that available products in the market can provide.

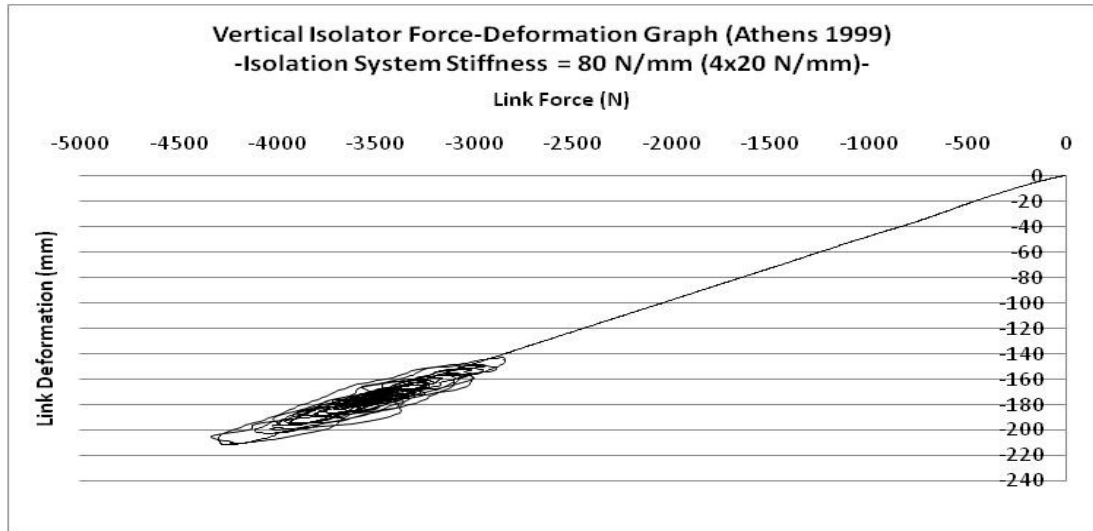


Figure 8 Vertical isolator force-deformation graph, Athens 1999 Earthquake

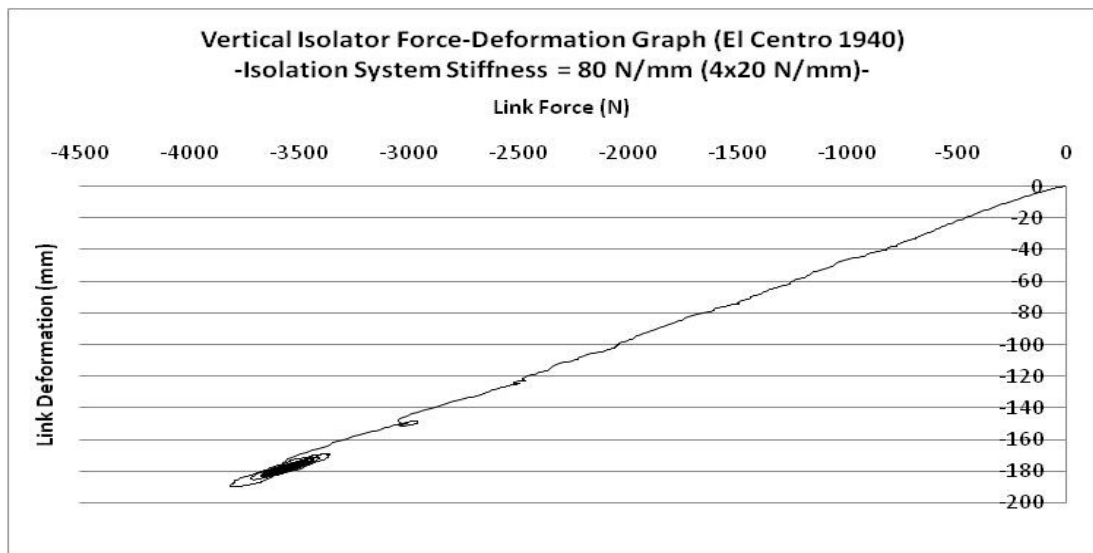


Figure 9 Vertical isolator force-deformation graph, El Centro 1940 Earthquake

From Figure 8 to Figure 10, the link-deformation graphs of the vertical isolator elements, obtained under three different earthquake ground motion can be seen. They represent the behavior of the vertical isolation system placed under each statue and composed of four isolator elements of 20 N/mm linear stiffness.

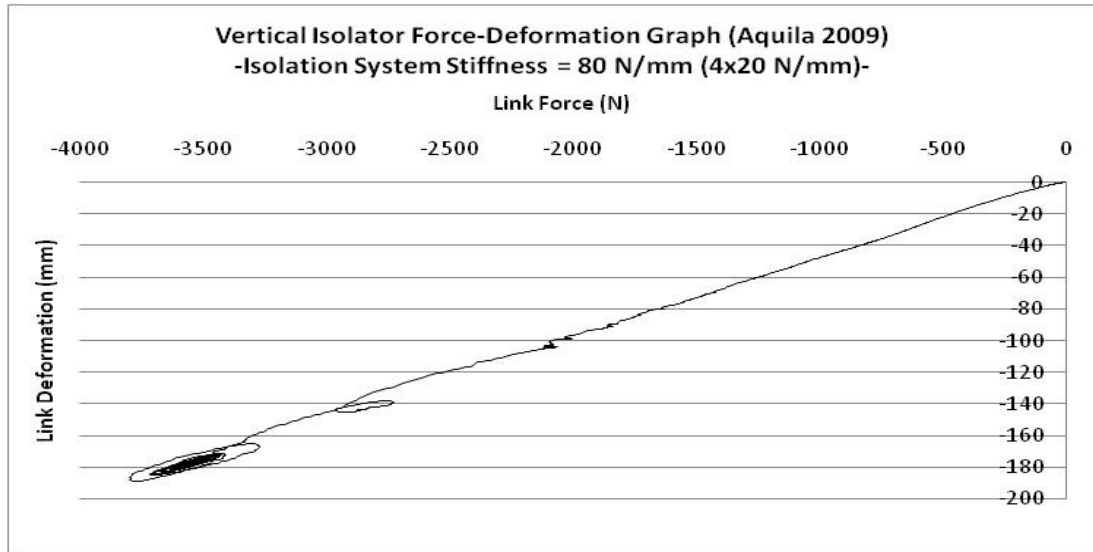


Figure 10 Vertical isolator force-deformation graph, Aquila 2009 Earthquake

In Figure 11, the displacements at the vertical isolators under the self weight of the statue obtained under three different earthquake ground motion can be seen. All the deformations observed refer to compression condition in the vertical isolation elements.

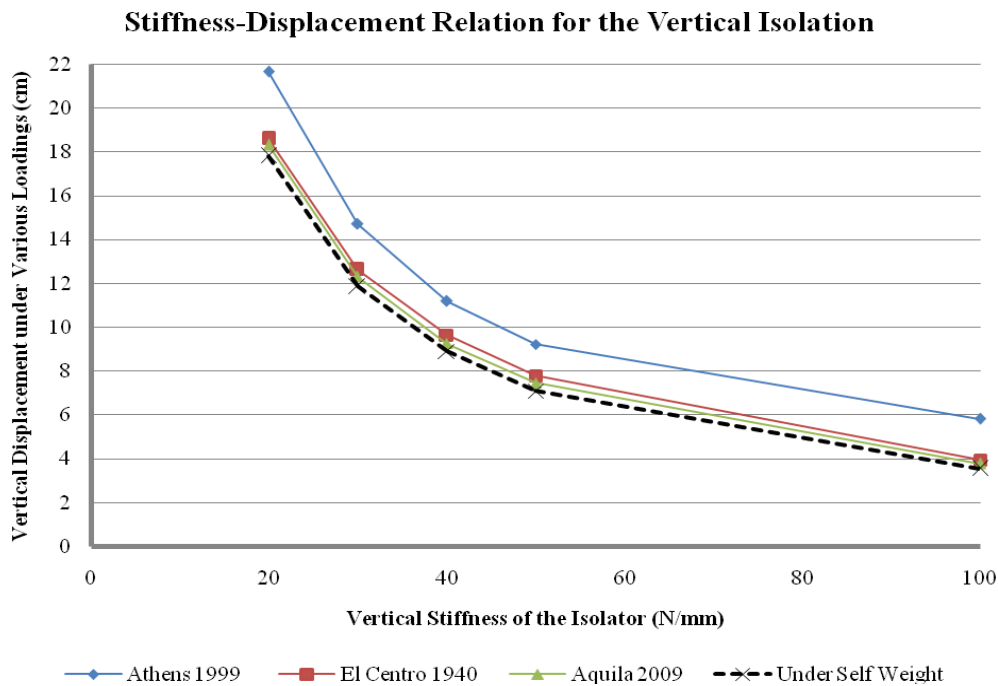


Figure 11 Stiffness-Displacement relation for the vertical isolation

From the results of the numerical analyses, it can be seen that the accelerations in any direction can be reduced significantly utilizing a three dimensional seismic isolation system. From Table 6, the amount of reduced accelerations and the percentage of reductions can be observed.

The Statues Isolated Three Dimensionally, Placed in a Non-Isolated Building	Longitudinal Maximum Acceleration	Translational Maximum Acceleration	Vertical Maximum Acceleration
	$k_h = 5 \text{ N/mm}$	$k_h = 5 \text{ N/mm}$	$k_v = 20 \text{ N/mm}$
Athens 1999 <i>reduced by:</i>	0,11g 87%	0,10g 90%	0,24g 80%
El Centro 1940 <i>reduced by:</i>	0,06g 81%	0,08g 65%	0,05g 75%
Aquila 2009 <i>reduced by:</i>	0,08g 79%	0,06g 86%	0,09g 82%

Table 6 Reductions in accelerations for the three dimensionally isolated statues placed in the non isolated museum building

The reduction of accelerations can also be seen in the Figure 12, where the peak ground and reduced vertical acceleration history data are drawn.

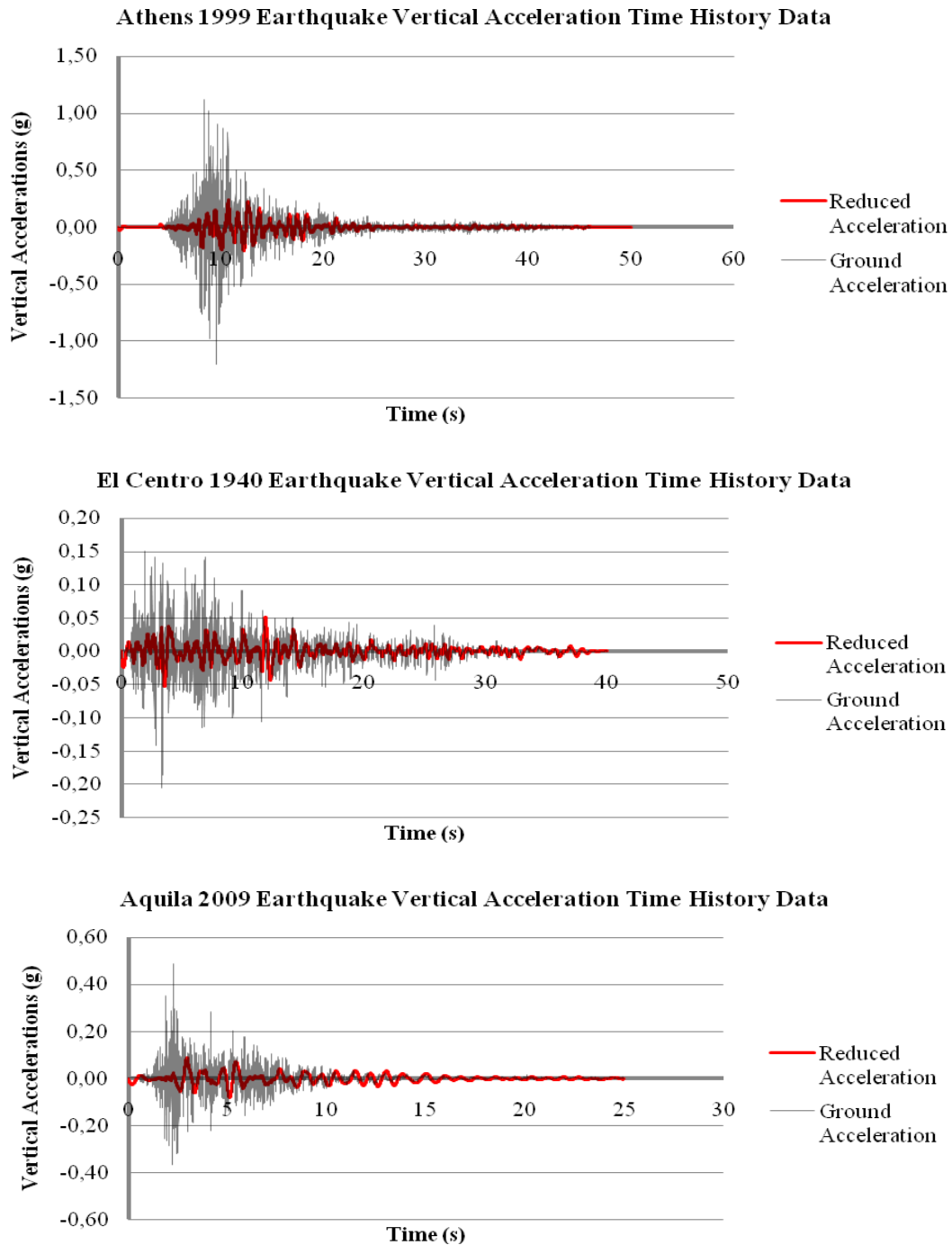


Figure 12 Reductions in Accelerations

2.5 Behaviour of Vertically Isolated Statues Displayed in a Building with Horizontal Base Isolation

In a base isolated museum building, placing the statues upon an additional local horizontal isolation system does not help reducing the accelerations, in contrary, since the frequency contents of the building and the isolated platform of statues become similar to each other, the seismic risk for the statues increase, because of a possible resonance problem. Therefore in order to achieve an effective three dimensional isolation system in a base isolated museum

building, the rigid platform on which the statues are placed is restrained to the ground horizontally, and vertical seismic isolators are mounted between the statue and the rigid platform (Figure 13).

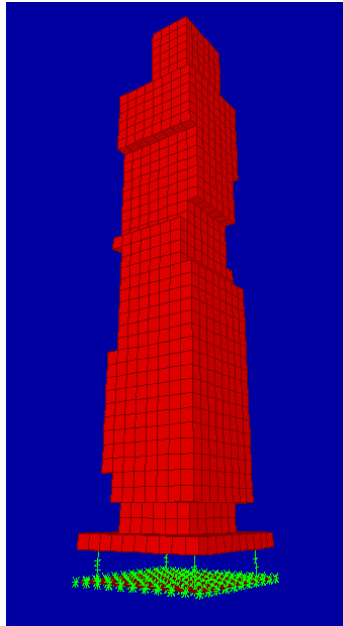


Figure 13 Vertical Isolation Model

For each statue, four vertical seismic isolator elements are used with linear stiffness values of 20 N/mm. From Table 7, the amount of reduced accelerations and the percentage of reductions in each direction can be seen. Note that in this table, the reductions in the horizontal accelerations are obtained thanks to the horizontal base isolation of the building.

Vertically Isolated Statue Placed in a Base Isolated Building	Longitudinal Maximum Acceleration	Translational Maximum Acceleration (g)	Vertical Maximum Acceleration (g)
Athens 1999 <i>reduced by:</i>	0,17g 80%	0,19g 81%	0,24g 80%
El Centro 1940 <i>reduced by:</i>	0,17g 46%	0,13g 41%	0,05g 75%
Aquila 2009 <i>reduced by:</i>	0,08g 81%	0,04g 91%	0,09g 82%

Table 7 Reductions in accelerations for the vertically isolated statues placed in the base isolated museum building

From Figure 14, the reductions of maximum vertical accelerations that the statues experience with decreasing spring stiffness can be seen.

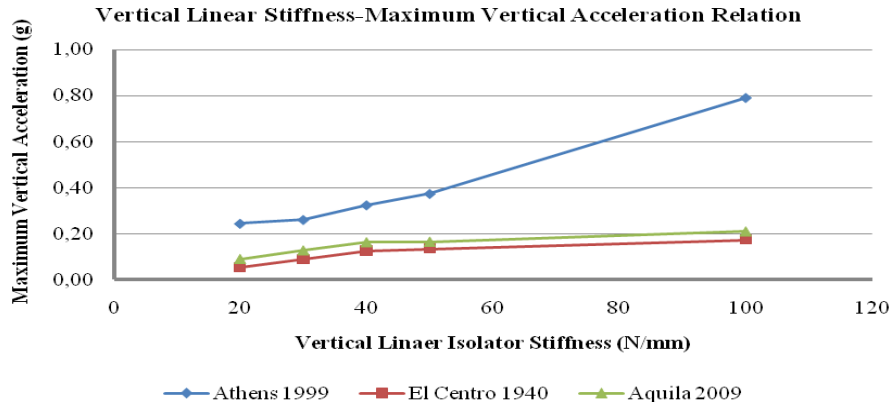


Figure 14 Vertical linear stiffness-maximum vertical acceleration relation

Therefore it can be concluded that in a base isolated museum building, adopting a vertical seismic isolation system, the accelerations that the statues experience in any direction can be lowered significantly.

3 ROCKING ANALYSES

If the vibrations are exceeded above a certain limit, the rigid blocks start “rocking”. Statically, when the overturning moment caused by earthquake forces reaches the resisting moment due to the self weight, the object starts to rock. This condition can be demonstrated with the following expression for a statue shown in Figure 15 :

$$M_{overturning} = M_{resisting} \quad (7)$$

$$h(ma_h) = b(mg \pm ma_v) \quad (8)$$

where,

h = Height of the center of mass of the object

b = Lateral distance between the overturning point and the center of mass of the object

m = Mass of the object

g = Gravity acceleration

a_h = Horizontal acceleration

a_v = Vertical acceleration

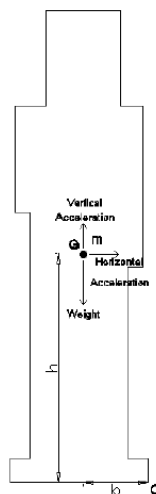


Figure 15 Ancient statue rigid body model

However, since earthquake forces are impulsive, the static approach may not give satisfying results. Therefore, the accelerations that can start the rocking of a rigid body is estimated using “Impulsive Loading Approach”, in which the minimum impulse that a specific rigid block can tolerate is compared with the impulse caused by the maximum accelerations of the earthquake excitations.

3.1 Initiation of Rocking Motion of a Rigid Block under Impulsive Loads

A rigid block, subjected to the horizontal and vertical accelerations, can either remain in full contact with the ground or it uplifts and starts rocking. During this motion, if the acceleration impulse caused by the earthquake forces becomes larger than the minimum overturning impulse of the system, the object overturns. Otherwise, it continues rocking until the ground motion comes to the end (Figure 16). The object’s full contact to the ground can be kept, if the magnitude of the impulse remains smaller than the minimum impulse required for uplift for the object [8].

While adopting an isolation solution for the protection of ancient statues, attention must be paid to the initiation of uplift because, once the object begins rocking, it starts to have impacts while rotating from one of its corners to the other, and this action may cause serious damage to an ancient statue. In this study, assessment of the horizontal and vertical accelerations that allow the statue to remain in full contact with the ground is carried out in presence of suitable vertical isolation elements.

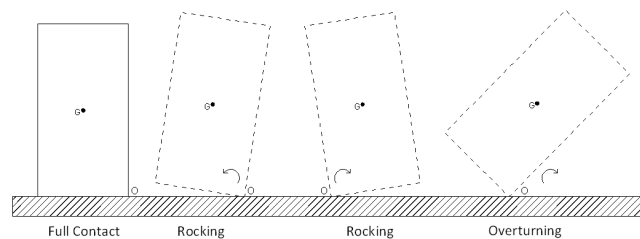


Figure 16 Rocking and overturning

For the tuning of accelerations through seismic isolation devices, the minimum impulse that can start the rocking motion of the statue is calculated and compared with the impulse caused by maximum earthquake accelerations, which is called as “design impulse”. Finally, interaction diagrams between horizontal and vertical accelerations for the specific ground motions are presented. Examining these diagrams, the allowable horizontal and vertical accelerations that can keep the statue model in full contact with the ground can be decided.

The minimum impulse that initiates the rocking motion of an object is called as the minimum uplift impulse [8]. It depends on the object’s geometric properties (slenderness), natural frequency, (stiffness and mass), and the vertical ground motions. Comparing the minimum impulse of a particular object with the impulse caused by the earthquake excitations, the probability for the initiation of rocking can be foreseen.

In the absence of damping, the minimum uplift impulse of a structure can be found according to the equation (5) in section 1, which shows that, the minimum uplift impulse decreases with the increasing natural frequency and slenderness. Also, the vertical accelerations have a direct effect on the motion. From the equation (5), it can be concluded that,

- More rigid structures (with higher natural frequency) are easier to uplift, than the flexible ones.

- More slender structures (higher h/b ratio) are easier to uplift than the less slender ones
- The upward vertical accelerations reduce the minimum uplift impulse, which results in a lower resistance to the uplift.

Thus, it can be understood that vertical accelerations of the earthquakes are very important for the uplift resistance of the object. The higher values the vertical accelerations have in the upward direction, the easier is the initiation of the uplift.

For instance, if the maximum vertical accelerations of an earthquake equals to $-g$, the vertical excitation parameter, γ (see equation 4) becomes zero, so does the minimum uplift impulse, I_u . This practically means “zero” resistance to uplift, therefore in such a case, even a slight horizontal impulse would be enough to start rocking of the object.

Therefore the effect of the magnitude of maximum vertical accelerations to the uplift resistance of the statue is investigated. To compare with the minimum uplift impulse of the statue, the design impulse caused by the horizontal accelerations of the earthquake excitation must be calculated. After making a comparison between these two values, the probability for the initiation of rocking for the particular object can be estimated.

3.2 Design Impulse

The maximum impulse caused by the horizontal earthquake excitation considered for the analysis is called the design impulse. To estimate the magnitude of the design impulse, the time integral of the earthquake force $p(t)$ should be calculated.

$$\text{Earthquake Force: } p(t) = m \ddot{u}_h(t) \quad (9)$$

$$\text{Magnitude of Design Impulse: } \int_{t_1}^{t_2} p(t) dt \quad (10)$$

This integral can be estimated as the multiplication of the mass of the statue and the area under the acceleration time history graph between two successive time instants, corresponding to the maximum horizontal acceleration. As a reasonable estimation, the triangular area that is drawn in the acceleration time history diagram in Figure 17 gives the magnitude of impulse for unit mass.

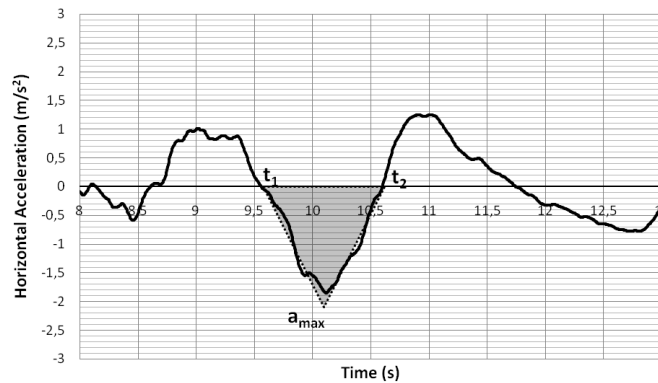


Figure 17 Impulse calculation graph

$$I_d = a_{\max} \cdot \frac{1}{2} (t_2 - t_1) \text{ (for unit mass)}$$

In the analyses, the comparison between the design impulse and the minimum rocking impulse of the statue is made by considering three different earthquake data, the properties of which are presented in the previous sections.

The comparison between the magnitude of the design impulse and the minimum uplift impulse shows if the object begins to rock or stays in full contact with the ground.

- If the design impulse is smaller than the minimum uplift impulse, the statue stays in full contact with the ground

$$I_d < I_u \rightarrow \text{Full Contact}$$

- If the design impulse is larger than the minimum uplift impulse, the object starts to rocking

$$I_d > I_u \rightarrow \text{Rocking}$$

To generalize the results for all three of the earthquake data, the same calculation procedure is applied with particular parameters of each analyses, and the results are presented as interaction graphs between the horizontal and vertical accelerations. The interaction graphs are shown in the figures 19,20 and 21. The trend line shows the intersection of the magnitudes of maximum horizontal and vertical accelerations below which the statue stays in full contact with the ground, during the specific earthquake motion. The points above the trend line represent the acceleration values that are sufficient to start the rocking motion.

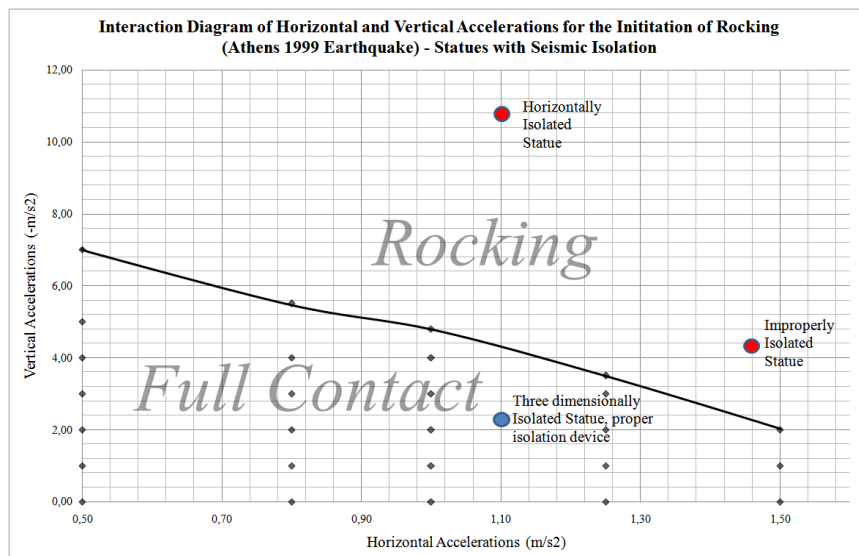


Figure 18 Interaction diagram of horizontal and vertical accelerations for the initiation of rocking

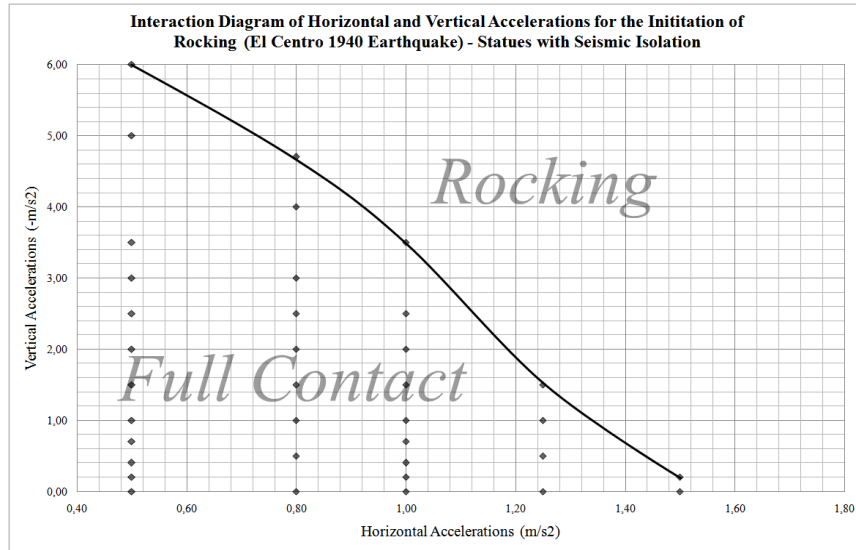


Figure 19 Interaction diagram of horizontal and vertical accelerations for the initiation of rocking

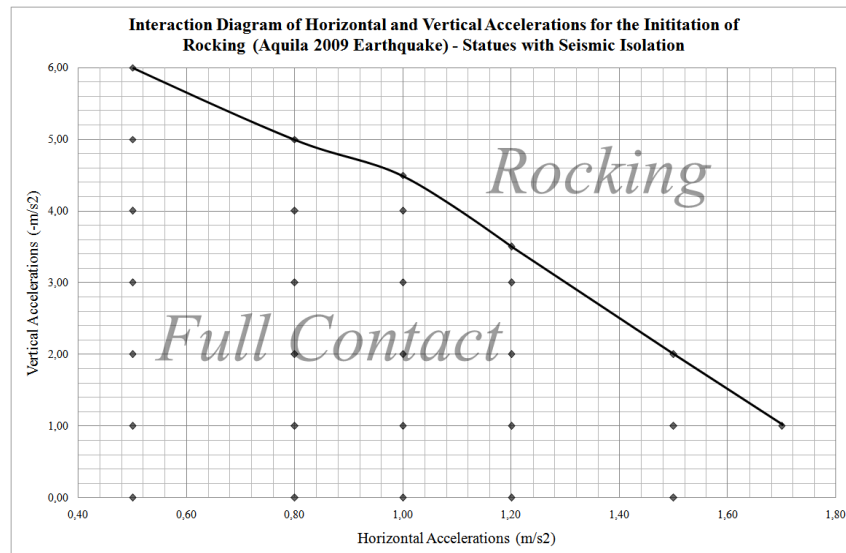


Figure 20 Interaction diagram of horizontal and vertical accelerations for the initiation of rocking

4 CONCLUSIONS

In this study, the effective ways to protect the ancient statues against three dimensional earthquake ground motions are investigated through seismic isolation methods. Several numerical analyses are carried out to understand which type of isolation system is suitable for each case. It is seen that both the horizontal and vertical accelerations transmitted to the statues can be reduced significantly, with a combination of vertical isolators at the base of the statues and horizontal base isolation system of the building. If the museum building in which the statues are to be displayed is not base isolated, then a three dimensional isolation system must be adopted for the statues.

Rocking analyses are carried out with the results obtained in each case. The results of the rocking calculations for each earthquake data are presented as interaction graphs to show the allowable accelerations under which the statues remain in full contact with the ground, and

suffers no damage due to the impacts during rocking. Principally, it is understood that the vertical earthquake excitations play a very important role on the initiation of uplift of the statues.

It is shown that ancient statues can be protected against three dimensional earthquake excitations effectively, when a seismic isolation system is designed according to their particular material and geometrical properties and the characteristics of the building in which they are to be displayed.

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