Abstract—This work considers the strategic situation which arises when Mobile Network Operators (MNOs) coexisting in a given geographical area have to decide whether to invest in new radio access technology and whether to share the investment (and the infrastructure) with other operators. We focus on heterogeneous networks (HetNet) where MNOs add a layer of small cells to their existing macro cells. We address such a strategic scenario by proposing a Mixed Integer Linear Programming formulation of the infrastructure sharing problem which takes as input techno-economic parameters as the achievable throughput in different sharing configurations, the pricing models for the service offered to the end users and the expectations on the return on investment for the mobile operators, and returns as output the "best" infrastructure/investment sharing options for the MNOs. The proposed formulation is finally leveraged to analyze the dynamics involved in the infrastructure sharing process under different techno-economic conditions in realistic network scenarios.

I. INTRODUCTION

The last two decades have witnessed several technological upgrades in mobile network technology, starting with the introduction of 3G to the recent efforts on LTE-Advanced and 5G networks. Such rapid evolution in the mobile networks ecosystem has led to two major facts: on one hand, it has made possible for Over The Top (OTT) service providers to enrich their set of offered services, and, on the other hand, it has made the end users more and more bandwidth "eager". As a consequence, the traffic to be delivered through mobile networks is exponentially growing and the end users are more and more concerned about the Quality of Experience of the connectivity service.

This scenario often forces the Mobile Network Operators (MNOs) to "chase" technology developments and users demands by frequently upgrading their network architectures. In this context, the conventional model according to which each mobile operator retains complete control and ownership of its network shows some limitations due to the large and frequent investments which are requested on the network infrastructure. As a result, migrating to new generations of mobile systems may lead to marginal profits or even being unprofitable for the MNOs due to the mismatch between, on one side, the growth rate and required quality of the mobile traffic and, on the other side, the expected return of investment (ROI) [1].

Network infrastructure sharing among MNOs has been proposed as a way to cope with the aforementioned shortcomings. In this work, we focus on Radio Access Network (RAN) sharing scheme in which MNOs may share at some level their radio infrastructure while maintaining separation and full control over the back hauling and core network infrastructures. In details, we consider a scenario where multiple MNOs with a consolidated network infrastructure, consisting of macro cells, and market shares coexist in a given geographical area; the MNOs have then to decide if it is profitable to upgrade their RAN technology by deploying additional LTE small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators.

We address such strategic problem by providing a Mixed Integer Linear Programming (MILP) formulation for the RAN infrastructure sharing problem which returns the "best" infrastructure sharing configurations among operators when varying techno-economic parameters as the achievable throughput in different sharing configurations, the pricing models for the service offered to the users and the expectations on the return on investment for the mobile operators. The proposed formulation is then leveraged to analyze the impact of the aforementioned parameters/input in a realistic mobile network environment.

The manuscript is organized as follows: Sec. II reviews the mainstream literature in the field of infrastructure sharing highlighting the main novelties of the proposed approach. In Sec. III, we introduce the reference scenario describing the techno-economic parameters involved in the infrastructure sharing problem which is then represented in a MILP formulation. Sec. IV applies the proposed optimization model to derive insights in the strategic behavior of MNOs in realistic network scenarios. Our concluding remarks are given in Sec. V.

II. RELATED WORK

Broadly speaking, there are two major research tracks in the field of infrastructure sharing: (i) work dealing with techno-economic modeling of network sharing and (ii) work on practical algorithms on management and allocation of shared network resources.

The first track includes mostly qualitative and quantitative study of different sharing scenarios and models for estimating capital and operational expenditures. Meddour et al. [2] suggest a classification of sharing scenarios; this work also assesses technical constraints, suggests guidelines for MNO involved in the sharing process and emphasizes the need for subsidization and assistance from regulatory entities. Similarly
Beckman et. al [3] show that the role of regulatory entities is crucial to avoid the decline of market competition; moreover, the authors of [4] model the capital and operational expenditures for different levels of sharing and suggest outsourcing as the solution to the challenges posed by network sharing. The work in [5] provides a benchmark-based model that delivers high-quality cost estimates for alternative delivery options of the MNO processes such as “regionalization”, “centralization” and “outsourcing”. Vaz et al. propose a framework to evaluate the performance of heterogeneous network deployment patterns in terms of net present value, capacity, coverage, and carbon footprint [6].

In the field of strategic modeling of infrastructure/resource sharing situation, it is worth mentioning the work resorting to game theory. Malanchini et al. [7] resort to non-cooperative games to model the problems of network selection, when users can choose among multiple heterogeneous wireless access, and of resource allocation in which mobile network operators compete to capture users by properly allocating their radio resources. The work in [8] uses a non-cooperative game to model the strategic decision of an MNO regarding sharing of its LTE infrastructure in a non–monopolistic telecom market. Another example of 4G infrastructure sharing is given in [9] which considers sharing LTE access network femtocells with other access technologies such as Wi–Fi. Cooperative game theory is used in [10] and [11]; in [10], the resource allocation problem in a shared network is formalized in a two step problem: resource sharing among the operators; and resource bargaining among the users and MVNOs of each operator; the work in [11] considers not only sharing among MNO but also among operators of different wireless access technology.

The research track on practical aspect of resource/infrastructure sharing focuses on algorithms and architectures for managing shared resources. The work in [12] suggests that radio resource management is handled by a third-party service provider or an inter-connection provider to preserve competition and reduce exposure. The authors in [13] introduce the Network without Borders (NWoB) concept as a pool of virtualized wireless resources with a shared radio resource manager. Along the same lines, Rahman et al. introduce a novel architecture based on wireless access network virtualization, where the key tenet is to offload the baseband process from physical base station to backend devices; in this way, the physical base stations can be sliced into virtual base stations [14].

The aforementioned literature work either abstracts away technical aspects related to the mobile network performance to focus on more economic-oriented analysis and modeling, or, the other way around. To the best of our knowledge, ours is one of the first attempts to strike a better balance between these two aspects of the sharing problem, by quantitatively modeling the relation between technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues) in mobile network infrastructure sharing.

### III. Problem Formulation

We consider a set $\mathcal{O}$ of MNOs who have up and running 3G/4G networks with the respective customer sets in a dense urban area. Parameter $\sigma_i$ gives the share of users of $i \in \mathcal{O}$ out of the total $N$ users in the area. The MNOs may consider investing to deploy additional LTE small-cells (HetNets). A MNO can either invest by itself or share the investment (and the deployed infrastructure) with a subset (or all) of the other MNOs. $\mathcal{S}$ is the set of all possible coalitions that can be created for the given set of MNOs (thus, $|\mathcal{S}|$ is equal to $2^{|\mathcal{O}|} − 1$). If a MNO invests by itself, the coalition is referred to as singleton. $\mathcal{O}_s$ is the set of MNOs of a coalition $s \in \mathcal{S}$, whereas $\mathcal{S}_i$ the set of coalitions $i \in \mathcal{O}$ can be part of. Each MNO inherits the customer base from its current network, assuming that users will not change their MNO but will potentially subscribe to a new (LTE) data plan. The problem consists in determining the set of coalitions which are formed up by the MNOs and the number of new small-cell base stations (BSs) activated by each coalition. A maximum number of BSs $U_{max}$ can be activated by all coalitions in the given area.

The decision of the MNOs on whether to invest or not is affected by the users’ response to the improved service. Users are characterized by their willingness to pay for 1 Mbps of LTE rate on a monthly basis, $\delta$, which alternatively represents the monthly price of 1 Mbps. Moreover, such decision is strongly related to the MNOs’ financial targets, i.e., a minimum expected return on investment at the end of the investment lifetime; if it is not achievable, MNOs do not invest. We model such expectations through parameter $\gamma$ which represents the minimum monthly revenue expected out of a single user in the return on investment. Since these targets are set for the investment lifetime $D$, the investment costs are calculated over the same period. Both capital ($e.g.$, site and BS acquisition) and operational ($e.g.$, hardware and software maintenance, land renting and power supply) expenditure contribute to the overall costs of the infrastructure [2]. The BS cost $g$ is estimated according to Equations (1), with reference to the pricing model in [15]: $g_{\text{capex}}$ is the fixed CAPEX component, whereas $g_{\text{opex}}$ is the annual OPEX component expressed as a fixed percentage ($\xi$) of $g_{\text{capex}}$. The cost of a single BS for the investment lifetime $D$ is the sum of the fixed initial CAPEX and the accumulated OPEX during $D$.

The BSs installation cost of a coalition is then divided among the coalition members.

$$g_{\text{opex}} = \xi g_{\text{capex}}$$
$$g_{\text{opex}} = D g_{\text{opex}}$$

$$g = g_{\text{capex}} + g_{\text{opex}}$$

The decision of a MNO $i$ to join coalition $s$ or not is captured by a binary variable $x_{is}$, which equals zero if $i$ selects any other coalition in $\mathcal{S}_i$ but $s$ or when all MNOs in $s$ do not invest at all, as financial targets are not satisfied. Binary variables $y_s$ to keep track of the selected coalitions: $y_s$ equals one if all the MNOs in $s$ select $s$ and invest, and thus $s$ is activated. The coalition selection is modeled through Constraints (2), which force each MNO to join at most one
coalition. Constraints (3) make sure that a coalition exists only if all of its members agree to collaborate.

\[ \sum_{s \in \sigma_i} x_{is} \leq 1, \forall i \in \mathcal{O} \quad (2) \]

\[ x_{is} = y_s, \forall s \in \mathcal{S}, \forall i \in \mathcal{O}_s \quad (3) \]

If coalition \( s \) is created, a certain number of BSs, represented by a non-negative integer variables \( u_s \), is installed. If coalition \( s \) is not selected or there is no investment \( (y_s = 0) \), the corresponding variable \( u_s \) is forced to zero by means of Constraints (4). Constraint (5) limits the overall number of BSs activated by all coalitions.

\[ u_s \leq U_{\text{max}} y_s, \forall s \in \mathcal{S} \quad (4) \]

\[ \sum_{s \in \mathcal{S}} u_s \leq U_{\text{max}} \quad (5) \]

We assess the quality of service provided by MNOs through the average rate perceived by the users, being the latter an important indicator of the users’ level of satisfaction. There are three types of LTE user rate in our proposed model: nominal user rate, coalition user rate and user rate per MNO. The nominal user rate is the maximum achievable LTE rate for a certain level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth\(^1\) that a user perceives when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR is a function of the number of activated BSs by the coalition the user belongs to, since a larger number of BSs results in the user being on the average closer to the interfering ones\(^2\). Thus, also the nominal user rate of coalition \( s \), represented by a non-negative continuous variables \( \rho_s^{\text{nom}} \), is a function of the number of activated BSs \( u_s \).

The average rate perceived by a user in coalition \( s \), \( \rho_s \), depends on the nominal user rate and on the load of its serving BS and can be modeled by the following Equations (6):

\[ \rho_s = \rho_s^{\text{nom}} (1 - \eta) \frac{\sum_{i \in \mathcal{O}_s} \sigma_i N}{u_s}, \forall s \in \mathcal{S} \quad (6) \]

where parameter \( \eta \) is the user activity factor, that is, the probability that a user is actually active in his/her serving BS, the \( \sum_{i \in \mathcal{O}_s} \sigma_i N \) is the total number of users that belong to coalition \( s \), and the ratio \( \frac{\sum_{i \in \mathcal{O}_s} \sigma_i N}{u_s} \) is the average number of users served by one BS. As a result, the nominal rate is scaled down by the factor \((1 - \eta)\)\(^3\) which accounts for the average congestion level at a serving BS.

Assuming that a user that belongs to a MNO in a coalition \( s \) can be served by any of the BSs activated by \( s \), the average user rate per MNO \( i \), represented by continuous non-negative variable \( q_i \), is equal to the average user rate of the coalition joined by the MNO, that is,

\[ q_i = \sum_{s \in \mathcal{S}_i} \rho_s, \forall i \in \mathcal{O} \quad (7) \]

\(^1\)We consider a 10 MHz bandwidth in our simulations whether the BS is shared or not.

\(^2\)Since we are considering nominal rate, any other BS transmission will use a subset or all the resource blocks and therefore unavoidably interfere.

As for the investment cost and revenues for the MNOs, it is reasonable to model the revenue per MNO \( i \) as a continuous non-negative variable \( r_i \) which is linearly dependent on the MNO’s user rate \( q_i \), being the proportionality constant the product of the monthly price of 1 Mbps \( \delta \), the investment lifetime \( D \) and the number of users of MNO \( i \), as shown in the following:

\[ r_i = \delta D \sigma_i N q_i, \forall i \in \mathcal{O} \quad (8) \]

The cost incurred by MNO \( i \), represented by non-negative continuous variable \( c_i \), is a linear function of the number of installed BSs, divided among the coalition’s members proportionally to their number of users. Equations (9) define the costs for each MNO.

\[ c_i = \sum_{s \in \mathcal{S}_i} \sum_{j \in \mathcal{O}_s} \sigma_j u_{sj}, \forall i \in \mathcal{O} \quad (9) \]

The following constraints (10) force the return on investment of a MNO \( i \) to be greater or equal than the product of the monthly return on investment expected from the single user \( \gamma \), the investment lifetime \( D \) and the MNO’s user share \( \sigma_i N \), if \( i \) joins one coalition and thus invests.

\[ r_i - c_i \geq \gamma \sigma_i N D \sum_{s \in \mathcal{S}_i} x_{is}, \forall i \in \mathcal{O} \quad (10) \]

We investigate the optimal solution of the proposed model (Equations 1-10) for 5 objective functions.

\[ \max \sum_{i \in \mathcal{O}} q_i \quad (11a) \]

\[ \max \min_{i \in \mathcal{O}} q_i \quad (11b) \]

\[ \max (\min_{i \in \mathcal{O}} q_i + \sum_{i \in \mathcal{O}} q_i) \quad (11c) \]

\[ \max \left( \sum_{i \in \mathcal{O}} (r_i - c_i) + \min_{i \in \mathcal{O}} (r_i - c_i) \right) \quad (11d) \]

Objectives (11a), (11b), (11c) are user-oriented, since they focus on the quality observed by the user, whereas (11d) and (11e) are operator-oriented, as they focus on the return on investment. However, even though the user-oriented objectives prioritize the offered quality, by means of Constraints (10), the MNOs financial targets for the investment are still taken into account. Objective (11a) maximizes the sum of user rate over all MNOs, whereas (11b) maximizes the smallest user rate, so as to provide more fair solutions. Even though it is against the realistic selfish behavior of a MNO to improve the quality offered by another MNO at the cost of degrading its own, such objective could be enforced by an external regulatory entity that strikes a balance between the shared investment and the quality perceived by the worst-served users. Objective (11c) maximizes both terms, trying to avoid solutions that advantage a subset of MNOs by disadvantaging the others, while still maximizing the sum term. Similarly for two operator-oriented objectives, (11d) maximizes the sum of the return on investment over all MNOs, whereas (11e) maximizes the sum of both factors.
It is important to notice the non-linearity of the right side of Equations (6), due both to the behavior of $\rho_i^{nom}$ with respect to $u_s$ and the non-linearity of the load factor with respect to $u_s$. The behavior of the nominal/coalition user rate with respect to $u_s$ is investigated by simulating the small-cell BS deployment (see next section). Objectives (11b) and (11e) are also nonlinear due to the max min terms. A MILP formulation can be obtained by applying suitable linearization/approximation (see [16] for a detailed description the linearization). Tables I and II recap parameters and variables.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>Set of MNOs</td>
<td>${A,B,C}$</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of coalitions</td>
<td>${A,B,C,AB,AC,BC,ABC}$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>User rate for MNO</td>
<td>${0.001}$</td>
</tr>
<tr>
<td>$O_i$</td>
<td>Set of MNOs in coalition $i \in S$</td>
<td>${s \in S</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Number of MNOs in coalition</td>
<td>${15}$</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Number of BSs installed by coalition</td>
<td>${4km^2}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Cost of MNO</td>
<td>${{0.05,0.1,0.2,0.8,1.2}}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Cost of MNO</td>
<td>${{0.001}}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>OPEX annual %</td>
<td>${15%}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>CAPEX of BS cost</td>
<td>${3000}$</td>
</tr>
<tr>
<td>$g$</td>
<td>BS cost normalized for $D$</td>
<td>${7500}$</td>
</tr>
</tbody>
</table>

### TABLE I: Sets and parameters, and corresponding values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{is}$</td>
<td>If MNO $i \in O$ joins coalition $s \in S$, 0 otherwise</td>
</tr>
<tr>
<td>$y_{is}$</td>
<td>If coalition $s \in S$ is created, 0 otherwise</td>
</tr>
<tr>
<td>$\rho_i^{nom}$</td>
<td>Nominal user rate for coalition $s \in S$</td>
</tr>
<tr>
<td>$\rho_i^c$</td>
<td>User rate for coalition $s \in S$</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>Costs of MNO $i \in O$</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>Revenues of MNO $i \in O$</td>
</tr>
</tbody>
</table>

### IV. RESULTS

The MILP model has been implemented in AMPL [19]. It has been tested on 24 instances, using CPLEX V12.6.0 as a MILP solver [20]. All tests were run on an Intel Xeon dual socket quad core CPUs @2GHz. The average computational time is negligible for the considered instances.

A simulation environment was set up to derive the coalition user rate, $\rho_s$, as a function of each number of activated BSs by coalition $s (u_s)$, i.e., from 1 up to $U_{max}$. In details, the entire set of $U_{max}$ BSs is uniformly distributed in a pseudo-random fashion on the considered square area. 10 sample users are also randomly distributed over the area; the downlink SINR of each sample user is calculated according to Equations (12) for each value of $u_s$:

$$
SINR_s = \frac{P_i}{\sigma_i + \sum_{j=1..u_s|j \neq i}^P_j + P_{i,noise}}, \forall s \in S, \quad (12)
$$

which provide the SINR of the sample user served by BS $i$ (from which it receives the strongest signal) from the $u_s$ BSs activated by coalition $s$. $P_i$ is the signal power the sample user receives from its serving BS, whereas $\sum_{j=1..u_s|j \neq i}^P_j$ is the one received from the interfering (non-serving) BSs. The received signal power is determined according to the three-parameter path loss model (transmitted signal power $P_{tx}$, fixed path loss $C_{pl}$ and path loss exponent $\Gamma$) defined within the GreenTouch Consortium [21]:

$$
P_{rx}[dBm] = P_{tx}[dBm] - C_{pl}[dB] - 10\Gamma \log(d[km]), \quad (13)
$$

where $d$ is the sample user–BS distance. The captured interference is then scaled down by the load of coalition $s (l_s = (1 - (\eta_j \sum_{s \in C_s} C_j))$ since users are characterized by an activity factor $\eta$. $P_{noise}$ is the white gaussian noise power for the considered system bandwidth.

The calculated SINR is finally mapped to LTE nominal rate ($\rho_i^{nom}$) according to a multilevel SINR–to–rate scheme [21]. A single value for $\rho_i^{nom}$ is obtained by averaging over the 10 sample users. An additional averaging is obtained by applying 100 iterations for each value of $u_s$. $\rho_s$ is then obtained from $\rho_i^{nom}$ according to Equations (6).

The instances consider a $4 km^2$ square area ($A$) populated by 20000 users. In this work we considered instances with three MNOs: A, B and C, which is quite reasonable for the Italian (also European) telecom playground [22]. The users are distributed among MNOs in a non-uniform way: B has 50% of them, while A and C have 25% each. The upper bound on the overall number of new BSs $U_{max}$ is set to 1000, being this the number of BSs for which rate saturation is reached; deploying more BSs neither increases the user rate, nor improves the MNOs’ return on investment. These features are common to all the 24 instances, which differ for the key economic parameters, $\delta$ and $\gamma$. The values of the user’s willingness to pay for 1 Mbps of service on a monthly basis $\delta$ were deduced from the current pricing models of Italian MNOs: values $\{0.05, 0.1, 0.2, 0.8, 1.2\} \in Euro/Mbps$ are considered. The values of the monthly expected contribution from a single user in the return on investment were chosen intuitively to create reasonable lower bounds on the return on investment: values $\{0, 1, 5, 10\} \in Euro$ are considered.

Table I provides the value for all the parameters, whereas Table III maps the notation used throughout the remaining part of this section for the objective functions with their exact definition.

### TABLE III: Notation of the different objective functions

<table>
<thead>
<tr>
<th>Notation</th>
<th>Objective functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\max \sum_{i \in O} q_i$</td>
<td>$\max \sum \min_{i \in O \cap q_i}$</td>
</tr>
<tr>
<td>$\max \min_{i \in O} q_i$</td>
<td>$\max \min \sum_{i \in O} q_i + \sum_{i \in O} q_i$</td>
</tr>
<tr>
<td>$\max \sum \min_{i \in O}(r_i - c_i)$</td>
<td>$\max \sum \min_{i \in O}(r_i - c_i) + \min \sum_{i \in O}(r_i - c_i)$</td>
</tr>
</tbody>
</table>

Figures 1, 2 and 3 illustrate the selected coalitions and the corresponding number of activated BSs for each tested instance and for each objective. Although there are five possible coalitions, only three of them occur in the results: a coalition between A and C and B going alone (B,AC), denoted by two numbers in the cell, the first one being the number of BSs activated by B, the second being the one activated by the coalition of A and C; the global coalition (ABC), for which the
total number of BSs activated by the big coalition is reported, and the no-investment case, represented by blank cells.

<table>
<thead>
<tr>
<th>γ</th>
<th>δ</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>627</td>
<td>348652</td>
<td>348652</td>
<td>348652</td>
<td>348652</td>
<td>348652</td>
<td>348652</td>
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<tr>
<td>1</td>
<td>1000</td>
<td>348652</td>
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<td>348652</td>
<td>348652</td>
<td>348652</td>
<td>348652</td>
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<tr>
<td>5</td>
<td>411589</td>
<td>348652</td>
<td>348652</td>
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<tr>
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<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Fig. 1:** Coalitions and nr. of BSs - $TOT_Q$

<table>
<thead>
<tr>
<th>γ</th>
<th>δ</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>186</td>
<td>427</td>
<td>762</td>
<td>1000</td>
<td>1000</td>
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<td>1000</td>
</tr>
<tr>
<td>1</td>
<td>427</td>
<td>762</td>
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<td>5</td>
<td>762</td>
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</tr>
<tr>
<td>10</td>
<td>186</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Fig. 2:** Coalitions and nr. of BSs - $MIN_Q$ and $TOT_Q + MIN_Q$

A vertical reading of these tables shows how the greedier the MNOs, the smaller the chances that the investment is actually performed (higher $γ$, fixed $δ$). Instead, by reading them horizontally, it can be observed that when users are more willing to pay for the new service (higher $δ$, fixed $γ$), more BSs are activated: all the 1000 BS for $δ ≥ 0.8$ for the operator-oriented objectives and for $δ ≥ 0.1$ for the user-oriented ones. Thus MNOs improve the offered quality and therefore "reward" their users. For smaller values of $δ$ (0.05, 0.1 and 0.2) more BSs are activated for the user-oriented objectives compared to operator-oriented (e.g., for $δ=0.05$ and $γ=0$, 627 BSs are activated by the big coalition under the user-oriented objectives but only 186 under the operator oriented): when there is not much revenue from users, MNOs limit their investment, in order to maximize their profit. However, for higher values of $δ$, solutions associated to different objectives tend to behave similarly since higher revenues compensate the costs of activating more BSs.

It is important to notice how for $TOT_Q$ the small MNOs (A and C) collaborate, whilst B invests by itself. $MIN_Q$ and $TOT_Q + MIN_Q$ force the big coalition (Figure 2), so that all MNOs provide the same quality to their users (thus, the solution is fair to all MNOs with respect to the user rate).

The optimal solution for $TOT_P + MIN_P$ and $TOT_P$ leads to the big coalition for any instance (Figure 3). This result, even though rather counter-intuitive, shows how despite varying $δ$ and $γ$, when the financial targets are met, it is economically more beneficial for MNOs to invest together. It also shows how the sum term $(\sum_{i \in O}(r_i - c_i))$ of $TOT_P + MIN_P$ dominates the min term $(\min_{i \in O}(r_i - c_i))$ since the outcomes for both objectives are identical. This is due to the non-uniform user distribution which unavoidably leads to different revenues per MNO and therefore different return on investment. Thus, any solution that attempts to average out the revenue of a single MNO would significantly reduce the overall return on investment.

Figures 4 and 5 show the behavior of the average user rate ($Q_{ave} = \frac{\sum_{O \in \mathbb{O}} Q_O}{|O|}$) and the average return on investment ($P_{ave} = \frac{\sum_{O \in \mathbb{O}} P_{O,J}}{|O|}$) as a function of $δ$ for each considered objective and for two values of $γ$ (0 and 5). When MNOs have high expectations from their investment ($γ=5$) but users are willing to pay very little for the new service ($δ=0.05$ and 0.1), MNOs do not invest (represented by both $Q_{ave}$ and $P_{ave}$ being equal to zero). Instead, if MNOs lower their expectations, e.g., $γ=0$, they invest even if users are willing to pay very little for the new service. The two families of objectives behave differently for small values of $δ$ (0.05, 0.1 and 0.02). The $Q_{ave}$ gap between the two decreases with the increase of $δ$ (it is approximately 30% for $δ=0.05$, 16% for $δ=0.1$ and only 4% for $δ=0.2$). Similarly for $P_{ave}$: the operator-oriented objectives provide higher revenues with respect to the user-oriented ones (99.7% of gap for $δ=0.05$, 51% for $δ=0.1$ and only 9.5% for $δ=0.2$). As observed in Figures 1, 2 and 3 when users are willing to pay more ($δ$ equal to 0.8, 0.1 and 0.2) the two families of objectives behave similarly, which is also verified in terms of having the same $Q_{ave}/P_{ave}$.

Figures 6 illustrates the behavior of the user quality ($Q$) with respect to the return on investment ($P$) for each MNO and each objective when $δ=0.2$ and $γ=5$. 
It can be easily observed that, for the selected instance, the family of user-oriented objectives lead to higher user rate but lower return on investment with respect to the operator-oriented ones. Moreover, $MNQ$, imposed by the regulatory entity, forces the MNOs to collaborate all together (outcome ABC) guaranteeing the same rate to all users. In comparison to $TOT$ (outcome (B,AC)), $MNQ$ improves the user rate for MNO B (4.2% increase) at the cost of slightly reducing the one of A and C (2.4% decrease). However, even though by joining the big coalition MNOs offer the same quality to their users, they can still differentiate their return on investment according to their user share (i.e., MNO B has twice the return on investment with respect to A and C for the same user rate).

V. CONCLUSIONS

This work analyzes the strategic situation in which MNOs have to decide whether to invest in LTE picocell BSs and whether to share the investment with other MNOs. We introduced reasonable cost functions for the infrastructure sharing problem, a pricing model for the new service and MNOs’ financial targets. We formalized the infrastructure sharing problem through a MILP, which allowed us to investigate the infrastructure sharing problem for different objectives and varying techno-economic parameters concerning the user and the MNOs. In details, two sets of objective functions were considered: user-oriented ones, that prioritize users by aiming to maximize the offered quality, and operator-oriented, that prioritize the MNOs’ return on investment instead. The main results of this study are the following: The MNOs’ decision on whether to upgrade their RAN or not is simultaneously affected by their expectations on the return of investment and by how much users are willing to pay for the improved service. The combination of very greedy MNOs with users that are very little inclined to pay for new services leads to no investment. If MNOs lower their expectations, it is possible to invest and in almost all cases it is in both MNOs’ and users’ best interest that all MNOs collaborate, that is, share the infrastructure. The optimal solution behaves differently for the two families of objectives when users are willing to pay little; the user-oriented objectives, by focusing on the user rate, merely satisfy the return on investment requirement, providing higher quality for users, whereas operator-oriented ones limit the investment, that is, fewer BSs are activated compared to the user-oriented objectives, in order to maximize the return on investment instead. However, when users are more eager for new services, both families of objectives tend to behave similarly. The proposed model can be further extended to the case in which MNOs plan to simultaneously upgrade their RAN technology in multiple dense areas. Moreover, spectrum sharing can be naturally incorporated in the formulation.

REFERENCES