Selective mass scaling for thin structures discretized with multi-layered, solid-shell elements

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It is well known that explicit time integration algorithms for structural dynamics are conditionally stable according to the Courant-Friedrichs-Lewy (CFL) condition [1], stating that the critical time step coincides with the so called "traversal time", i.e. the time required by a dilatational stress wave to run across the shortest element dimension. Thin structures discretized with solid-shell elements [2,3] are, therefore, computationally expensive, because of their intrinsic small dimension in the thickness direction. A possible solution, that allows to enlarge the critical time step and, hence, to improve the computational efficiency without affecting the dynamical response, is to introduce a selective mass scaling procedure. The basic idea is to locally modify the solid-shell element mass matricx, artificially increasing the coefficients related to thickness eigenmodes, while those related to the translational rigid body motions are left unchanged. In this manner, the highest structural eigenfrequencies are reduced, without significantly alter the lowest ones. In [4], a mass scaling technique preserving the mass lumping, and based on a simple, computationally inexpensive, linear transformation of the element degrees of freedom, has been proposed for a single layer (solid-shell element) thin structure. In the present contribution, the approach is extended to a multi-layered thin structure (Figure 2).

Figure 1: solid-shell element

As in [4], the starting point is the definition of the *average* and *difference* accelerations of each element as a function of the corresponding upper or lower nodal degrees of freedom:

$$\boldsymbol{a}_i^{ave} = \frac{\boldsymbol{a}_i^{up} + \boldsymbol{a}_b^{low}}{2} \quad \text{and} \quad \boldsymbol{a}_i^{diff} = \frac{\boldsymbol{a}_i^{up} - \boldsymbol{a}_i^{low}}{2}, \qquad \text{i = 1,2, 3,4} \tag{1}$$

where *i* is the node number for the upper or lower face of the solid-shell element (see Fig. 1). Since the average degrees of freedom are related to the translational rigid body modes, the element maximum frequency can be reduced by scaling only the mass coefficients related to the *difference* accelerations. Thus, the elements mass matrix becomes:

$$\widehat{\boldsymbol{M}}^e = \begin{bmatrix} \boldsymbol{m}_b^{up} + \boldsymbol{m}_b^{low} & 0 \\ 0 & \alpha^e\left(\boldsymbol{m}_b^{up} + \boldsymbol{m}_b^{low}\right) \end{bmatrix} \tag{2}$$

Once the α^e is retrieved for each element, it is possible to increase the time step by scaling down the highest eigenfrequencies related to the rotational degrees of freedom, since the CFL condition states

that the time step must be $\Delta t \leq 2/\omega_{max}$ (with ω_{max} the maximum eigenfrequency of the system), and $\omega_{max} \leq \omega^e$ (maximum element eigenfrequency)

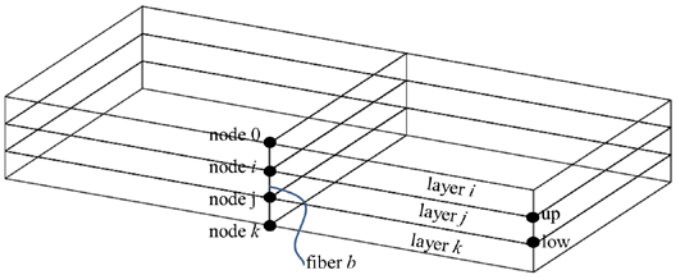

Figure 2: reference scheme for the discretization of a multi-layered thin structure with solid-shell elements.

It can be shown that, when a multi-layered structured is considered, the global mass matrix becomes a diagonal block matrix, each block corresponding to the degrees of freedom of a single fiber b. The overall solution can be computed simply solving a set of subsystems in the form:

$$\boldsymbol{M}_b \, \boldsymbol{a}_b = \boldsymbol{f}_b \tag{3}$$

being $\boldsymbol{M}_b = \sum_l \boldsymbol{M}_b^l$ a tridiagonal matrix, built by assembling the mass matrices arising for each layer *l* along the fiber, namely

$$\boldsymbol{M}_b^l = \frac{1}{4}\left(m_b^{up} + m_b^{low}\right)\begin{bmatrix} 1+\alpha^e & 1-\alpha^e \\ 1-\alpha^e & 1+\alpha^e \end{bmatrix} = \begin{bmatrix} M_{UU}^l & M_{UL}^l \\ M_{LU}^l & M_{LL}^l \end{bmatrix}$$

The resulting mass matrix is not diagonal. However, it is block diagonal, each block having a tri-diagonal structure and dimensions directly related to the number of layers, which are in general in a limited number through the shell thickness. Even though accelerations cannot be computed explicitly, the solution of the small linear system providing accelerations of nodes belonging to the same fiber is inexpensive and the small additional burden is by far compensated by the largest stable time step which can be used in the computation. This turns out to be a critical issue, as several other selective mass scaling techniques proposed in the literature provide a theoretically rigorous scaling, though at the cost of densely populated mass matrices, requiring an iterative solver for acceleration computation at each time step.
The accuracy of the mass scaling procedure and the computational gain are checked with the aid of numerical examples.

References

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