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USE OF ENERGETIC METHODS FOR THE OPTIMISED DESIGN OF HYBRID STEPPING MOTORS

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Abstract. The modelling used in the study of stepping motors is largely concentrated on the analysis of the motor operation. Effectively, the complex electromagnetic structure of these machines, particularly of hybrid motors, has hitherto limited the development of modelling methodologies with direct design orientation.

The present paper shows the application of an energy-based method to the synthetic evaluation of the components of the holding torque. The use of this method is found to a be a real help in the design stage.

Some examples of the application of this method are shown, demonstrating its suitability for employment in the optimised design.

1. Introduction.

In the design of stepping motors use is made, above all, of past construction data, in terms of results on both prototypes and production units. The desired operating characteristics are thus the result of general dimensioning critieria, coupled with experimental data and taking into account technical and manufacturing constraints.

The determination of performance is usually entrusted to checking calculations. The simulation of operation in time domain makes possible, with a fair approximation, the evaluation of the main characteristics, such as the holding torque, pull-in and pull-out curves and machine dynamics.

The complex electromagnetic structure of these machines, and particularly of hybrid motors, has effectively discouraged up till now the development of design methods and/or dimensioning formulas which indicate suitably and directly the influence of design parameters.

A previous paper [1] has presented the basic elements of a new method of calculation, called the "energy variation" method. This is based on the law of energy balance, applied to a finite rotation under constant-current conditions and taking magnetic saturation into account. The present work shows the application of this method to machine design. Using this approach, it is possible to calculate, directly and synthetically, the holding torque harmonic components, once the motor structure and the value of the supply current are defined.

As the holding torque is a good index of motor performance, the method appears suitable for employment in design optimisation.

2. Fundamentals of the energy variation method.

The energy variation method is of general applicability. In what follows, it will be described with reference to a hybrid type two-phase motor, provided with a permanent magnet (axially magnetized and contained in the toothed rotor) and a stator with eight wound poles (whose shoes are also toothed towards the gap). For a motor of this type, the classical energy balance equation can be written thus:

$$T\cdot d\theta = i_1\cdot d\Psi_1 + i_2\cdot d\Psi_2 - dW_{mag} \quad , \qquad (1)$$

where T, electromagnetic torque, is a function of the current and of the rotor angular position θ; Ψ_h (h = 1,2) is the flux linkage of the h-th phase winding (consisting of 4 coils, each of N turns); W_{mag} is the total magnetic energy stored.

The method is based on the integration of eq.(1) between two suitably chosen initial and final positions θ_i and θ_f; given that the currents are constant as far as holding torque is concerned, we have:

$$W_T = \int_{\theta_i}^{\theta_f} T(\theta)\cdot d\theta = \sum_{1}^{8}{}_k N\cdot i_{ck}\cdot\int_{\Phi_k(\theta_i)}^{\Phi_k(\theta_f)} d\Phi_k - \int_{\theta_i}^{\theta_f} dW_{mag} \quad . \qquad (2)$$

where i_{ck} is the current in the k-th coil: $i_{ck} = i_1$ for odd coils (K = 1,3,5,7); $i_{ck} = i_2$ for even coils (K = 2,4,6,8).

Eq.(2) applies in the real conditions of machine saturation, on which depend both the fluxes in the magnetic circuit branches and the values of magnetic energy W_{mag}. It is useful to express this energy in the following explicit form:

$$W_{mag} = W_{mv} + W_{mf\ell} + W_{mfs} + W_{mfm} \quad . \qquad (3)$$

where:

- W_{mv} is the energy in the reluctances at the gap $R_{\delta k}(\theta)$, which are linear and of variable geometry:

$$W_{mv} = \sum_{1}^{8}{}_k \frac{1}{2}\cdot R_{\delta k}(\theta)\cdot\Phi_{\delta k}^2 \quad ; \qquad (4)$$

- $W_{mf\ell}$ is the energy in the linear reluctances of fixed geometry (leakage and lamination):

$$W_{mf\ell} = \sum_j \frac{1}{2}\cdot R_{\ell j}\cdot\Phi_j^2 \quad ; \qquad (5)$$

- W_{mfs} is the energy in the fixed geometry branches made of saturable magnetic material:

$$W_{mfs} = \int_{\mathcal{V}ol}\int H\cdot dB\, d\mathcal{V} \quad ; \qquad (6)$$

- W_{mfm} is the energy associated with the magnet; indicating with F_m and R_m the equivalent MMF and reluctance of the magnet recoil line, we have:

$$W_{mfm} = \frac{1}{2}\cdot R_m\cdot\Phi_m^2 - F_m\cdot\Phi_m . \qquad (7)$$

Thus, the work W_T, performed by the holding torque $T(\theta)$ during the finite rotation $\Delta\theta = \theta_f - \theta_i$, can be expressed by:

$$\int_{\theta_i}^{\theta_f} T(\theta)\cdot d\theta = \sum_{1k}^{8} F_k \cdot \Big[\Phi_k(\theta)\Big]_{\theta_i}^{\theta_f} + F_m \cdot \Big[\Phi_m(\theta)\Big]_{\theta_i}^{\theta_f} - \Big[\frac{1}{2}\cdot R_m \cdot \Phi_m^2\Big]_{\theta_i}^{\theta_f} + $$
$$ - \Big[\sum_{1\,k}^{8} \frac{1}{2}\cdot R_{\delta k}(\theta)\cdot \Phi_{\delta k}^2 + \sum_j \frac{1}{2}\cdot R_{\ell j}\cdot \Phi_j^2 + \int\!\!\int_{\mathcal{V}_{\sigma\ell}} H\cdot dB\, d\mathcal{V}\Big]_{\theta_i}^{\theta_f} . \qquad (8)$$

Using eq.(8), the calculation of work W_T reduces to establishing the magnetic circuit fluxes and the energy in the saturated parts, at positions corresponding to the extremes of rotation only.

To obtain the most general case, it has been assumed in eq.(8) that both phase windings carry current: the corresponding holding torque can be denoted by $T_{II}(\theta)$. However, eq.(8) applies also in the case with one phase on (this torque can be denoted $T_I(\theta)$) and in the case when no power is fed to the machine (this torque, called detent torque, is due to the magnet only and can be denoted $T_m(\theta)$).

3. Synthetic calculation of holding torque harmonics.

Let us for the moment consider one phase only being fed; the torque $T_I(\theta)$ is a periodic function of period $4\cdot\theta_s$ (θ_s = step angle), just as the reluctance at the gap corresponding to each stator pole.

Measuring the angle θ starting from any stable equilibrium position, it can be seen that the reluctance at the gap is the same at both positive and negative values of the same angle θ ($\pm\theta$); the fluxes and magnetic energies stored are also equal. It can be concluded that the holding torque is an odd periodic function, expressed thus:

$$T_I(\theta) = -\,T_I(-\theta) \quad . \qquad (9)$$

Because of this, the torque can be expressed as a Fourier series containing sine functions only:

$$T_I(\theta) = -\sum_{1\,n}^{\infty} T_{IMn}\cdot \sin\left[n\cdot\frac{\pi\cdot\theta}{2\cdot\theta_s}\right] \quad . \qquad (10)$$

Substituting eq.(10) into the integral defining the work done by the torque, we obtain:

$$W_I = \int_{\theta_i}^{\theta_f} T_I(\theta)\cdot d\theta = \sum_{1\,n}^{\infty} T_{IMn}\cdot\frac{2\cdot\theta_s}{\pi\cdot n}\cdot\left[\cos\left(\frac{n\cdot\pi\cdot\theta}{2\cdot\theta_s}\right)\right]_{\theta_i}^{\theta_f} = \sum_{1\,n}^{\infty} W_{In} . \qquad (11)$$

In eq.(11), it is assumed that the work W_I is known from eq.(8), after the resolution of the magnetic circuit at the positions θ_i and θ_f. Eq.(11) thus contains an infinite number (∞^1) of unknown factors T_{IMn}, that are the torque harmonic amplitudes.

At any given value of the current, the calculation of the harmonics of the torque is theoretically possible by solving a system of infinite linear algebraic equations of type (11), associated with different pairs of positions θ_i and θ_f. In practice, it is permissible to stop the series at a comparatively low harmonic number, for the following reasons:

- generally speaking, the amplitude of the torque harmonics decreases with increasing harmonic number; this can also be checked experimentally;
- in eq.(11) each harmonic of the torque is divided by the corresponding harmonic number n.

Accordingly, the contribution to the overall work W_I made by harmonic work components W_{In} decreases rapidly with increasing n.

The choice of the number of harmonics and position pairs is a problem, where arbitrary decisions must be taken. In the present analysis, it has been judged convenient to proceed as follows:

- calculations have been limited to the 4th harmonic of the holding torque;
- the extreme positions of each finite rotation have been chosen so as to diminish (or reduce to zero) the work done by some of the torque harmonics, thus simplifying the calculation.

Denoting the 4 position pairs (equal to the number of harmonics considered) with a, b, c, d, the set of 4 type (11) equations can be written:

$$\begin{bmatrix} \Delta\theta_{a1} & \Delta\theta_{a2} & \Delta\theta_{a3} & \Delta\theta_{a4} \\ \Delta\theta_{b1} & \Delta\theta_{b2} & \Delta\theta_{b3} & \Delta\theta_{b4} \\ \Delta\theta_{c1} & \Delta\theta_{c2} & \Delta\theta_{c3} & \Delta\theta_{c4} \\ \Delta\theta_{d1} & \Delta\theta_{d2} & \Delta\theta_{d3} & \Delta\theta_{d4} \end{bmatrix} \cdot \begin{bmatrix} T_{IM1} \\ T_{IM2} \\ T_{IM3} \\ T_{IM4} \end{bmatrix} \approx \begin{bmatrix} W_{Ia} \\ W_{Ib} \\ W_{Ic} \\ W_{Id} \end{bmatrix} , \qquad (12)$$

where

$$\Delta\theta_{\nu n} = \frac{2\cdot\theta_s}{n\cdot\pi}\cdot\left\{\cos\left[\frac{n\cdot\pi\cdot\theta_{f\nu}}{2\cdot\theta_s}\right] - \cos\left[\frac{n\cdot\pi\cdot\theta_{i\nu}}{2\cdot\theta_s}\right]\right\} . \qquad (13)$$

In order to calculate the coefficients $\Delta\theta_{\nu n}$ (where ν = a, b, c, d and n = 1, 2, 3, 4), the positions shown below were chosen.

- Rotation a: $\theta_{ia} = (1/3)\cdot\theta_s$; $\theta_{fa} = (7/3)\cdot\theta_s$.

The insertion of these values into eq.(13) shows that harmonics N° 2, 3 and 4 give a zero contribution to the work W_{Ia}. Thus, the fundamental of the torque can be found directly from the first of eq.s (12):

$$T_{IM1} \approx -\frac{\pi}{2\cdot\sqrt{3}\cdot\theta_s}\cdot W_{Ia} \quad . \qquad (14)$$

- Rotation b: $\theta_{ib} = (1/6)\cdot\theta_s$; $\theta_{fb} = (7/6)\cdot\theta_s$

These values reduce to zero the works done by the 3rd and 4th harmonics. Substituting expression (14) in the second of equations (12), we obtain:

$$T_{IM2} \approx \frac{\pi}{\sqrt{6}\cdot\theta_s}\cdot W_{Ia} - \frac{\pi}{\sqrt{3}\cdot\theta_s}\cdot W_{Ib} \quad . \qquad (15)$$

- Rotation c: $\theta_{ic} = 0$; $\theta_{fc} = 2\cdot\theta_s$.

These values reduce to zero the work done by the 2nd and 4th harmonics. Thus, by substituting expression (14) into the third of eq.s (12), we obtain the amplitude of the third harmonic thus:

$$T_{IM3} \approx \frac{\pi\cdot\sqrt{3}}{2\cdot\theta_s}\cdot W_{Ia} - \frac{3\cdot\pi}{4\cdot\theta_s}\cdot W_{Ic} \quad . \qquad (16)$$

- Rotation d: $\theta_{id} = 0$; $\theta_{fd} = (4/3)\cdot\theta_s$.

This choice reduces to zero the work done by the 3rd harmonic. From the fourth of eq.s (12) we obtain:

$$T_{IM4} \approx \frac{2\cdot\pi}{\theta_s}\cdot\left[\frac{\sqrt{2}-1}{\sqrt{6}}\right]\cdot W_{Ia} + \frac{2\cdot\pi\cdot W_{Ib}}{\theta_s\cdot\sqrt{3}} - \frac{4\cdot\pi\cdot W_{Id}}{3\cdot\theta_s}. \quad (17)$$

Knowing the values of the rotation work W_{Ia}, W_{Ib}, W_{Ic} and W_{Id}, eq.s (14)÷(17) make it possible to calculate the first 4 harmonics of the holding torque.

In practice, the fact that harmonics higher than the fourth one have been ignored produces an error in the calculation of those considered above: those particularly affected are the 3rd and 4th harmonic. On the other hand, the extension of the system (12) to an order higher than 4 requires a very accurate magnetic model, particularly in terms of the reluctance at the gap which is difficult to determine accurately.

Considering now the case where both windings are fed with current, it is possible to repeat an analysis similar to the one above, as the holding torque is still a periodic function with a period $4\cdot\theta_s$. The new angular origin θ_{II} is placed in the old angular position $\theta = (1/2)\theta_s$, so that:

$$\theta_{II} = \theta - (1/2)\cdot\theta_s \quad ; \quad (18)$$

with both windings fed with power, this position is one of the stable equilibrium.

The properties of construction symmetry, deriving from the reciprocal position of stator and rotor teeth, apply also to this position. Hence all the relations (9) to (17) above are valid in this case too, once θ_{II} is substituted for θ.

4. Example of application.

Using the energy variation method, the holding torque of a hybrid 2-phase stepping motor has been calculated. The principal data of the motor are shown in Table 1. The calculation has been carried out for the cases of one and both phases being fed, over a range of currents $0 \le I \le I_n$

Table 1 - Test motor principal data.

Motor designation	MAE HY 200 2220 100 N8
steps per revolution	200
rated current	1 A
number of turns of a coil	67
material of the magnet	ALNICO
residual flux density (B_r)	1.28 T
coercitive force (H_c)	57300 A/m
recoil line intercepts:	$B_o \simeq$ 0.98 T; $H_o \simeq$ 612500 A/m
magnet dimensions	ℓ_m = 10 mm; D_{ext} = 23.6 mm
stator length / int. diam.	ℓ_s = 26.4 mm; D_{is} = 30 mm
lamination type	TERNI 1550
number of teeth: stator shoe / rotor	5 / 50
stator tooth width at the air gap	0.898 mm
rotor tooth width at the air gap	0.763 mm
air gap	0.065 mm

Figures 1 and 2 show the calculated and measured values of the holding torque, for one and both phases fed respectively. As can be seen, good agreement exists between the calculated and measured values, as long as the values of the MMF are not too high.

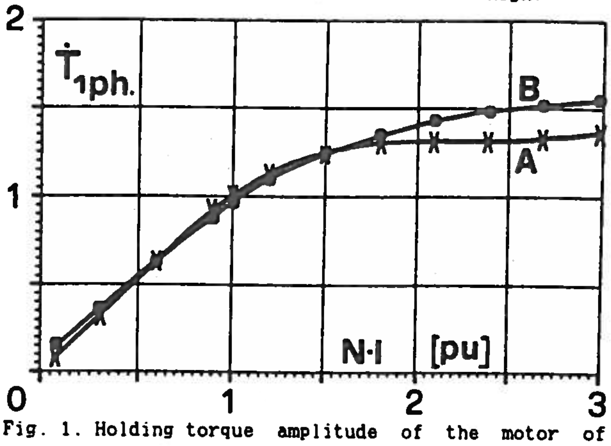

Fig. 1. Holding torque amplitude of the motor of Tab.1: 1 phase on; A = calculated; B = experimental; torque reference: rated $T_{1ph.(calc.)}$ = 390 mNm .

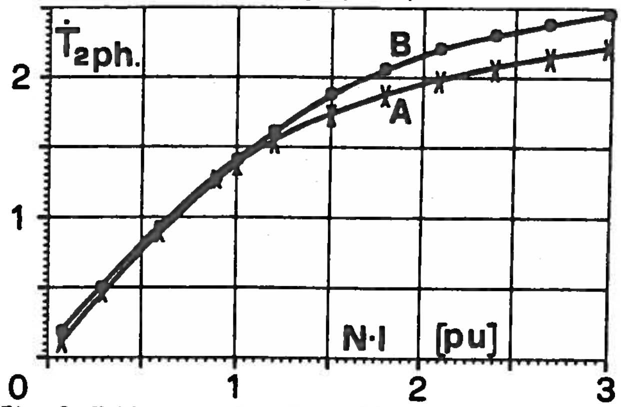

Fig. 2. Holding torque amplitude of the motor of Tab.1: 2 phases on; A = calculated; B = experimental; torque reference: rated $T_{1ph.(calc.)}$ = 390 mNm .

At high MMF values (over 1.5÷2 times the rated value) it can be seen that the calculated values tend to be underestimated. This is caused by the following:

- the contribution due to higher harmonics of the torque is ignored;
- as the MMF increases, the increased saturation makes difficult the correct evaluation of the saturated reluctances, particularly at the teeth;
- the calculation of the reluctances at the gap (carried out here using the approximate analytical method of representing the field by straight and circular segments [1, 2]) becomes increasingly imprecise as the tooth saturation increases. In this case, in fact, validity of the basic assumption that the toothed surfaces are equipotential is weakened.

One can, however, maintain that the results presented should encourage the use of the energy variation method at the design stage, because:

- the method permits a rapid and reliable determination of the torque, up and beyond the nominal current;
- it permits the calculation of torque components by means of the study of the magnetic circuit, at a

limited number of angular positions. This property makes it particularly suitable as an interface with more sophisticated algorithms for the study of the magnetic structure, such as the finite element method.

5. Parametric sensitivity during the design.

Some results obtained by varying the main design parameters with respect to the reference data of Table 1 are shown below. Bearing in mind that parameter variation represents the basic criterion for any design optimisation algorithm, the energy variation method lends itself to form its central core.

5.1. Variation of the air gap.

Fig.3 shows the effect of varying the air gap while all the other parameters are kept at their nominal values. The variation considered is limited, in the lower zone because of construction considerations, and in the upper zone by the need to avoid irreversible demagnetisation of the magnet.

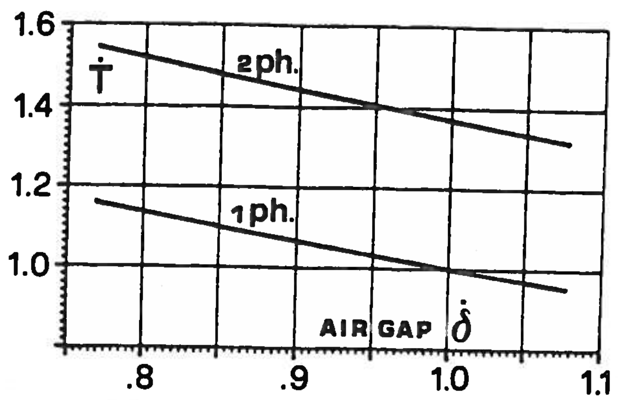

Fig. 3. Sensitivity of the holding torque to the variation of the air gap.

It can be seen that an increase in the air gap produces a reduction in the holding torque, in both cases of single-phase and two-phase feed: the two resulting curves are substantially parallel. This is due mainly to the reduction of the variation in the air gap reluctance during the rotation.

5.2. Variation of the magnet length.

Table 2 shows the influence of the axial length of the magnet. The use of ALNICO limits the downward variation in this case too, with demagnetisation as the danger. Clearly, this problem does not arise when using materials such as Sm-Co or Nd-Fe-Bo.

It can be seen that there is no appreciable variation in the torque as the length of the magnet is changed. This behaviour can be explained by noting that when this length increases, both the reluctance and the equivalent MMF of the magnet increase, and that the overall value of the magnet flux depends, as a first approximation, on those two quantities only.

Table 2 - Sensitivity of the holding torque to the variation of the magnet lenght.

$\dot{l}_m$ [pu]	0.93	1.00	1.20	1.40
$\dot{T}_{1ph}$ [pu]	0.997	1.000	1.008	1.013
$\dot{T}_{2ph}$ [pu]	1.366	1.371	1.381	1.388

On the other hand, the variation in the magnet length affects considerably its operating conditions. Referring to the extreme cases in Table 2 (with 2 phases fed) the following heavier operating conditions for the magnet occur:

$$\dot{l}_{m\,min} = 0.93 : \quad B_m = 0.898\ T \ ; \quad H_m = 51.04\ kA/m$$
$$\dot{l}_{m\,max} = 1.40 : \quad B_m = 0.924\ T \ ; \quad H_m = 34.84\ kA/m \ ;$$

because of this, the lengthening of the magnet produces more favourable working conditions, with a higher margin against the danger of irreversible demagnetisation.

5.3. Variation of the magnet external diameter.

The influence on the holding torque of varying the external diameter of the magnet is illustrated by fig.4. In this case too, dimensional variations are limited by construction and operational problems.

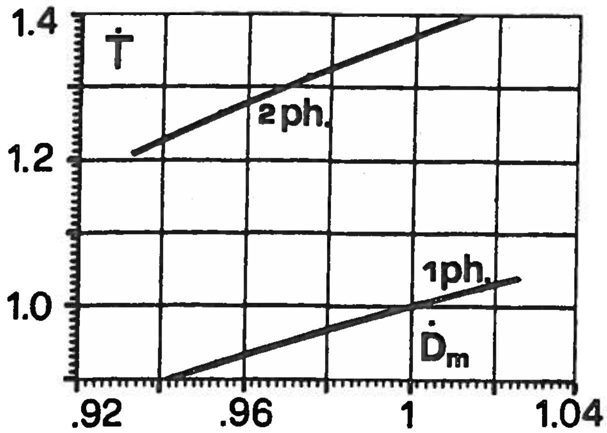

Fig. 4. Sensitivity of the holding torque to the variation of the magnet external diameter.

The figure shows a high degree of sensitivity to changes in external diameter. Effectively, an increase in the magnet cross-section produces a reduction in its equivalent reluctance, with the equivalent MMF remaining constant. This results in an increase of the magnet flux and, hence, of the torque.

5.4. Variation of the tooth width at the air-gap.

Figures 5 and 6 show the influence of the stator and rotor tooth widths (b_{ts} and b_{tr} respectively). The two variations are independent of each other.

In both cases, the holding torque can be seen to decrease as the tooth width at the gap increases. This can be explained by the observation that with the increase in tooth width (at equal tooth pitch) the air-gap configuration tends to the situation of two smoothed structures, which would correspond to a constant value of air-gap reluctance during rotation, hence to a zero torque.

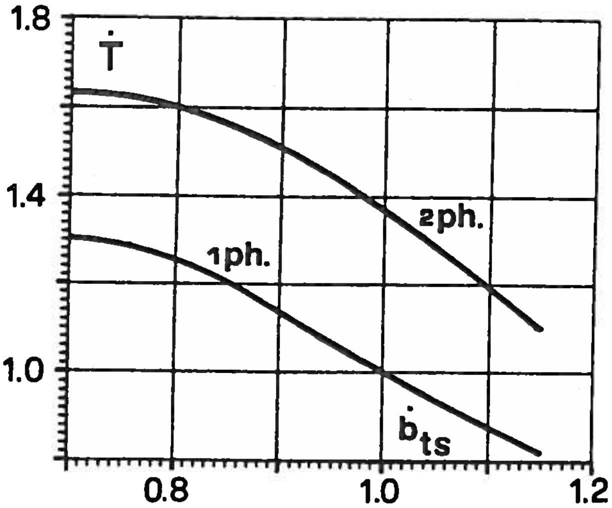

Fig. 5. Sensitivity of the holding torque to the variation of the stator tooth width at the air-gap.

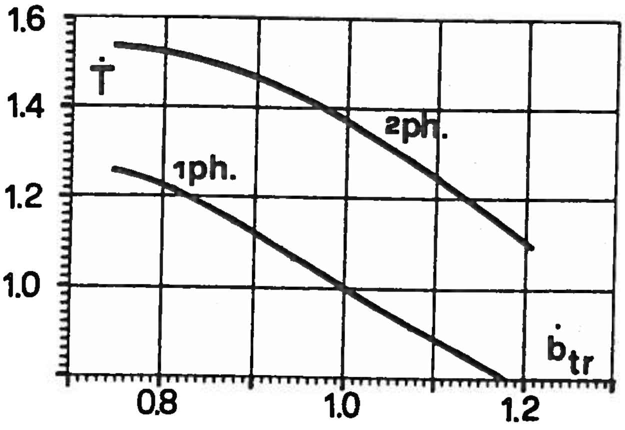

Fig. 6. Sensitivity of the holding torque to the variation of the rotor tooth width at the air-gap.

5.5. Choice of magnetic materials.

An analysis has been carried out of the influence which the type of lamination used has on the holding torque. The results, pertaining to the torque when only one phase is fed, are shown in Table 3. In this table, H and L denote respectively a lamination with high permeability (TERNI 1550) and a lamination with low permeability (ordinary steel); S and R refer to their use in the stator and rotor respectively.

The results in Table 3 show that, always referring to laminated materials, the holding torque is significantly affected by the use of high-permeability material in the magnetic circuit of the stator, while the choice of the rotor material is relatively unimportant. This can be explained as follows:

- the reluctance caused by the magnetic material used for the rotor is negligible when compared with the equivalent reluctance of the magnet, which, in practice, dominates the overall value of the flux produced by the magnet;
- conversely, the reluctances due to the magnetic material used for the stator (toothed zone, pole pieces, stator yoke) play an important part, insofar as they enter into the division of the fluxes between the various stator poles. Because of this, they influence the value of the torque.

Table 3 - Sensitivity of the holding torque (1 phase on) to the permeability characteristics (H=high, L=low) of the lamination for stator (S) and rotor (R)

N·I [pu]	$\dot{T}_{(HS-HR)}$	$\dot{T}_{(HS-LR)}$	$\dot{T}_{(LS-HR)}$	$\dot{T}_{(LS-LR)}$
0.6	0.631	0.627	0.571	0.567
1.0	1.000	0.990	0.900	0.894
1.8	1.307	1.305	1.266	1.262

It should finally be noted that the data in Table 3 demonstrate a positive aspect of the energy variation method, namely that it accounts correctly for saturation effects in the ferromagnetic material. This characteristic is very relevant from the design point of view.

6. Conclusions.

The present work has demonstrated the application of the energy variation method to the design of a hybrid 2-phase stepping motor.

Using this method, it is possible to determine the harmonic contributions to the holding torque: this knowledge permits a good estimate of the performance at the design stage. In addition, the method is suitable for the evaluation of sensitivity to parameter variation and lends itself well to being employed in design optimisation algorithms.

Further studies will be carried out in this area, particularly regarding the number and location of extreme positions in finite rotations and the refinement of the machine model.

Acknowledgement

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