Lecture Notes in Networks and Systems

Volume 88

Series Editor
Janusz Kacprzyk, Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland

Advisory Editors
Fernando Gomide, Department of Computer Engineering and Automation—DCA, School of Electrical and Computer Engineering—FEEC, University of Campinas—UNICAMP, São Paulo, Brazil
Okyay Kaynak, Department of Electrical and Electronic Engineering, Bogazici University, Istanbul, Turkey
Derong Liu, Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, USA; Institute of Automation, Chinese Academy of Sciences, Beijing, China
Witold Pedrycz, Department of Electrical and Computer Engineering, University of Alberta, Alberta, Canada; Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland
Marios M. Polycarpou, Department of Electrical and Computer Engineering, KIOS Research Center for Intelligent Systems and Networks, University of Cyprus, Nicosia, Cyprus
Imre J. Rudas, Óbuda University, Budapest, Hungary
Jun Wang, Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong

giorgio.buratti@polimi.it
The series “Lecture Notes in Networks and Systems” publishes the latest developments in Networks and Systems—quickly, informally and with high quality. Original research reported in proceedings and post-proceedings represents the core of LNNS.

Volumes published in LNNS embrace all aspects and subfields of, as well as new challenges in, Networks and Systems.


The series covers the theory, applications, and perspectives on the state of the art and future developments relevant to systems and networks, decision making, control, complex processes and related areas, as embedded in the fields of interdisciplinary and applied sciences, engineering, computer science, physics, economics, social, and life sciences, as well as the paradigms and methodologies behind them.

** Indexing: The books of this series are submitted to ISI Proceedings, SCOPUS, Google Scholar and Springerlink **

More information about this series at [http://www.springer.com/series/15179](http://www.springer.com/series/15179)
Indeed, I am fully convinced that in this age... every Woman ought to exert herself, and endeavor to promote the glory of her sex, and to contribute her utmost to increase that luster...

Committees

Conference Organizing Board

Alessandra Capanna, Università di Roma “La Sapienza”, Roma, Italy
Luigi Cocchiarella, Dipartimento DASTU del Politecnico di Milano, Milano, Italy
Paola Magnaghi-Delfino, Dipartimento di Matematica del Politecnico di Milano, Milano, Italy
Giampiero Mele, Università degli Studi eCampus, Novedrate, Italy
Tullia Norando, Dipartimento di Matematica del Politecnico di Milano, Milano, Italy

Conference International Scientific Committee

Giuseppe Amoruso, Politecnico di Milano, Milano, Italy
Alessandra Capanna, Università di Roma “La Sapienza”, Roma, Italy
Stefano Chiarenza, Università Telematica San Raffaele, Roma, Italy
Luigi Cocchiarella, Politecnico di Milano, Milano, Italy
Marie Francoise Roy, Université de Rennes 1, Rennes, France
Kay Bea Jones, Knowlton School Ohio State University, Columbus, OH, USA
Cornelie Leopold, TU Kaiserslautern, Germany
Marcella Giulia Lorenzi, Università della Calabria, Italy
Giulio Magli, Politecnico di Milano, Milano, Italy
Paola Magnaghi-Delfino, Politecnico di Milano, Milano, Italy
Giampiero Mele, Università degli Studi eCampus, Novedrate, Italy
Tullia Norando, Politecnico di Milano, Milano, Italy
Mine Özkar Kabakçıoğlu, Istanbul Technical University, Turkey
Giulia Pellegrini, Università degli Studi di Genova, Genova, Italy
Michela Rossi, Politecnico di Milano, Milano, Italy

vii
Rossella Salerno, Politecnico di Milano, Milano, Italy
Laura Tedeschini Lalli, Università Roma Tre, Roma, Italy
Daniela Velichova, Slovak University of Technology in Bratislava, KMDG Bratislava, Slovakia
Chiara Vernizzi, Università degli Studi di Parma, Parma, Italy
Maria Zdimalova, Slovak University of Technology in Bratislava, KMDG Bratislava, Slovakia
Preface

Some introductory remarks about the reasons that motivated the choice of the topics of the Conference *Faces of Geometry. From Agnesi to Mirzakhani*.

We have two purposes, equally important.

First, we have the intent of promoting interdisciplinary discussions and connections between theoretical researches and practical studies on geometric structures and its applications in architecture, arts, design, education, engineering, mathematics.

Indeed, we believe that these related fields of study might enrich each other and renew common interests on these topics through networks of common inspirations.

We invite researchers, teachers and students to share their ideas, to discuss their scientific opinions in teaching these disciplines, in order to enhance the quality of geometry and graphics education.

Second, but not less important. We are sure that the scientific community and mathematics, in particular, needs the contribution of women.

Women have made significant contributions to science from the earliest times. Historians with an interest in gender and science have illuminated the scientific endeavours and accomplishments of women, the barriers they have faced, and the strategies they have implemented to have their work peer-reviewed and accepted in major scientific journals and other publications. The historical, critical and socio- logical study of these issues has become an academic discipline in its own right.

In 2018, we celebrated, in Politecnico di Milano, the anniversary of Maria Gaetana Agnesi, Milanese mathematician, the first woman to write the first vernacular handbook of mathematics for learners.

Nowadays, we celebrate the first Women in Mathematics Day, dedicated to Maryam Mirzakhani, the first woman that wins the Fields Medal.

The Turkish mathematician Betul Tanbay, in her tribute to Mirzakhani, recalled the classic geometric problem, called illumination problem, and compared Maryam Mirzakhani to the candle lighting the path for others to follow. Quoting, she said “Maryam showed forever that excellence is not a matter of gender or geography. Maths is a universal truth that is available to us all”.

giorgio.buratti@polimi.it
During the conference, we commemorate Giuseppina Biggiogero, the first woman that taught Descriptive Geometry in the Faculty of Architecture at Politecnico di Milano.

The Organizing Board of the Conference announced the birth of The International Association in Mathematics and Art—Italy (IAMAI), promoted by Italian scholars from various academic, disciplinary and cultural backgrounds.

The Mission of the Association is the promotion of researches and the dissemination of results in the various application fields, in reference to national and international contexts, enhancing the plots and convergences between areas that link Mathematics to Art, opened to forms of collaboration and involvement of other subjects, institutions and organizations.

Mathematics is the fruit of the thought both creative and logical, inspired and deeply linked to the Beauty, recognizable in various expressions of Art, from Architecture to Design and Fashion, from Painting to Sculpture, from Music to Dance and Theatre, including their digital and virtual expressions. For centuries, Italy has been a land of promotion and encounter between Art and Science and our country is full of signs of the Italian Cultural Heritage. The aim of the association is to give the maximum sharing to these witness through the appropriate communication and publishing channels.

Milan, Italy
Novedrate, Italy
Milan, Italy

Paola Magnaghi-Delfino
Giampiero Mele
Tullia Norando

giorgio.buratti@polimi.it
Contents

Unexpected Geometries Exploring the Design of the Gothic City........ 1
Maria Teresa Bartoli

Federico Alberto Brunetti

Computational Process and Code-Form Definition in Design .......... 31
Giorgio Buratti

To Observe, to Deduce, to Reconstruct, to Know ...................... 41
Franca Caliò and Elena Marchetti

The Role of Geometry in the Architecture of Louis Kahn and Anne Tyng ............................................... 57
Cristina Cândito

Thinking Architecture in Four Dimensions .............................. 67
Alessandra Capanna

Reuleaux Triangle in Architecture and Applications .................... 79
Giuseppe Conti and Raffaella Paoletti

Interplays of Geometry and Music: How to Use Geometry to Analyze an Artwork in Order to Compose a Musical Piece ........ 91
Chiara de Fabritiis

Harmony in Space ................................................................ 101
Biagio Di Carlo

Caterina Marcenaro + Franco Albini for the Love of Art ............... 105
Kay Bea Jones

How to Solve Second Degree Algebraic Equations Using Geometry... 121
Paola Magnaghi-Delfino and Tullia Norando
Teatro Comunale, Ferrara: The Question of the Curve.
From the Debate to the Geometric Analysis ................. 131
Giampiero Mele and Susanna Clemente

Women and Descriptive Geometry in Italian University .......... 141
Barbara Messina

Witch of Agnesi: The True Story .................................. 155
Tullia Norando and Paola Magnaghi-Delfino

The Dividing of the Sphere in Domes of Medieval Anatolia .......... 165
Sibel Yasemin Özgan and Mine Özkar

Organic Reference in Design. The Shape Between Invention and Imitation ............................................. 177
Michela Rossi

Inverse Formulas: From Elementary Geometry to Differential Calculus .................................................. 187
Anna Salvadori and Primo Brandi

A Concrete Approach to Geometry ................................. 217
Emanuela Ughi

Tessellation ................................................................. 225
Mária Ždímalová
Computational Process and Code-Form Definition in Design

Giorgio Buratti

In design process, drawing has always preceded the construction phase. The act of drawing, based on basic geometric elements such as lines, curves, surfaces and solid, allows to organize one’s ideas, manage resources and predict results. In recent years, the increased level of digital literacy has led to a new kind of draw generated through the creation of algorithms. Form is not a priori defined, but it is consequence of a discrete rules set resulting from a refinement process of conceptual, communicative, structural and geometric instances, leading to the outcome that best meets the project hypotheses. This approach requires the adoption of theoretical analysis and understanding tools capable of managing a high level of complexity. In an age where the digital model can directly inform a machine able to manufacture it, the role of geometry is fundamental not only in understanding reality, but also in controlling the act of shaping matter. The paper analyses some experiences in design field where form is described and constructed by computational process.

1 Introduction

Computational design is a multidisciplinary area of study which, in general terms, can be defined as the application of computational strategies to design process and whose relevant aspect relates to the logical-creative nature of “calculation”. In computation the real world’s complexity is translated into elementary steps subsequently elaborated as algorithm, a systematic procedure based on a series of unambiguous instructions that explain how to achieve a specific objective. Combining computa-
tional principles with design practice configures a new multidisciplinary area where the conscious use of IT tools is translated into procedures and rules for the project. In design context, computational project strategies are instantiated in the ability to manage complexity, understood both as a set of structures and relationships and as amount of data and information.

2 Computational Design and Complex Systems

The term “complex” does not simply mean “complicated”; rather, it is a precise definition that refers to the science of complexity, a field of research that has not yet been completely formalised but which is equipped with theoretical tools suitable for the new context. The term ‘complicated’ in fact, comes from the Latin *cum plicum*, which means “paper crease”. A complicated problem can therefore be solved by explaining, or rather “smoothing the creases”. ‘Complex’, meanwhile, derives from *cum plexum* which means “knot” or “weave”. The solution to a “complex” problem lies in the intricate weave created by the knots, i.e. the relations among the elements. The study of complex systems implies the analysis of phenomena composed by a large number of elements, also diverse, that interact to create a dynamic that is not predictable when observing the behaviour of the individual elements.

This systems, apparently chaotic, can be described by non-linear and non-additive dynamics. In a linear system the effect of a group of elements is the sum of the effects considered separately. In the group there are no new properties that are not already present in the individual elements.

Meanwhile, in a non-linear system the whole may be greater than the sum of its parts as it is the connections between the various elements that determine the structure and organisation of the system. Collective properties that are not foreseeable a priori emerge as a result of the multiple interactions between the various agents that make up the system. These dynamics disappear as soon as the system is separated, materially and theoretically, into isolated elements.

In the systemic vision the units are relationship patterns, inserted within a broader network of connections. In design, for example, form may be considered the result of the interaction of precise formalisable and quantifiable conditions (formal aspect, materials, physical and temporal constraints, pre-established goal, interaction with the user, economic and production factors) and a creative instance that must be implemented. These determining factors interact reciprocally to achieve a common goal and so the design process has all of the typical characteristics of a complex system.

The revolution inherent computational design is the possibility to represent relations and processes. In this new dimension the various design instances can be organised in emerging relational structures that transfer typical characteristics of living systems to the design process, such as the ability to adapt and transform, and self-organisation. This behaviour cannot be controlled according to the classic linear method (topdown), which seeks to predict all possible situations and subsequently
prescribes the solution for dealing with them. Only by defining the behaviour of entities on the basis of the design (bottom-up) and leaving the task of simulating the collective effect of the interactions to the calculating power of the computer is it possible to check the validity of the design hypothesis [1].

If we view design as a complex system based on the interconnection of various factors, computational design is the device capable of integrating the interaction of the various components, fostering the interaction of the physical context, cultural characteristics, social aspects and construction system [2].

### 3 Managing the Complexity: Triply Periodic Minimal Surfaces

This paragraph presents an experimentation that shows the potential of algorithmic generative modelling and check the veracity of the theoretical deductions.

These tests were carried out using Rhinoceros, a McNeel CAD software, which combines a powerful NURBS engine, ideal for creating and managing complex forms, with a complete programming environment based on Visual Basic language.

The use of the programming code was simplified thanks to the use of the Grasshopper plug-in. The application is based on already-compiled functions which, without requiring specific knowledge of the programming language, can be assembled directly in the graphic interface, inspired conceptually by flow diagrams.

The potential of the tool was applied to the study of minimal surfaces, geometric objects with very interesting characteristics, not only in design terms. In recent years, many scientific disciplines have been turning with great interest to the study of minimal surfaces. This focus is justified both by the problems of a mathematical nature that have been revealed by the research and by the discovery of a number of properties (mechanical, structural and associated with electrical conductivity) that are distinctive of them. Configurations of minimal surfaces have been found in a wide variety of different systems: from the arrangement of calcite crystals that form the exoskeleton of certain organisms and the composition of human tissue to the basic structure of synthetic foams and the theories that explain the nature of astronomical phenomena.

A minimal surface is a surface whose mean curvature is always zero. This definition is closely related to the Plateau Problem, also known as the first law: if a closed polygon or oblique plane (similar to a closed frame of any shape) is assigned, then there is always a system of surfaces, including all possible surfaces that touch the frame, which are able to minimise the area. In other words, the problem is to identify the shape which covers the largest surface with the same perimeter [3].

To make this principle clear it is necessary to gain a deeper understanding of the concept of the mean curvature of a surface: consider point \( P \) of a surface and the perpendicular to the surface at point \( N \), which therefore intersects the surface with the plane \( \pi \) on which \( N \) lies (Fig. 1) [4].
In the intersection curve obtained consider the curvature at point $P$. Even at an intuitive level the curvature provides information on the behaviour of the curve: if a straight line is taken as the example of the curve, there is no inflection and the curvature in this case will be zero, whereas in the particular case where the curve is a circumference its inflection will be constant at every point. For a generic curve, the curvature, which varies from point to point, is defined by the construction of the osculating circle, i.e. the circle tangent to the curve that best approximates it, and will therefore be defined by the relation $C_{P} = 1/r$ where $r$ is the radius of the osculating circle.

If the plane $\pi$ is rotated around the perpendicular $N$, then for each of the positions of the plane section curves are obtained that are characterised by a different value for the curvature at point $P$ (obviously if the surface considered is not a sphere, in which case they will all be of equal value). In the case of a generic surface between the different curvatures the one whose value is the largest and the one whose value is the smallest are preferred, which are designated as the principal curvatures of the surface and indicated with $H_1$ and $H_2$. 

**Fig. 1** Perpendicular plane, tangent to the point $P$ and the principal curvatures of a saddle minimal surface
The mean curvature $H$ is the algebraic sum of the two principal curvatures defined by the equation:

$$H = H_1 + H_2/2$$

It follows that the equation that characterises minimal surfaces (also called Lagrange’s theorem), expressed in terms of the principal curvatures, becomes:

$$H_1 + H_2 = 0$$

This condition can be obtained either because both values are zero, as in the case of the plane which is therefore a minimal surface, or because:

$$H_1 = -H_2$$

That is, at any point one of the principal curvatures is concave and one is convex, as in the case of a saddle surface.

4 Description and Genesis of Minimal Surfaces: Implicit Method

Minimal surfaces can be described in different way, in this work we will only talk about the formulation we used: the implicit method. The implicit form is appropriate to the digital description because it allows the handling of the large number of elements that characterize TPMS, without overload the calculation process and also does not allow self-intersections. Typically, an implicit surface is defined by an equation of the form:

$$f(x, y, z) = 0$$

The implicit surfaces divide the space into three regions, where:

$f(x, y, z) < 0$ for points outside the surface
$f(x, y, z) > 0$ for points inside the surface
$f(x, y, z) = 0$ for points on the edge

Some minimal surfaces (Fig. 2 P, D and G surfaces) can be described implicitly to a good degree of approximation by the following equations:

P: $\cos(x) + \cos(y) + \cos(z) = 0$;
D: $\sin(x) \sin(y) \sin(z) + \sin(x) \cos(y) \cos(z)$
$+ \cos(x) \sin(y) \cos(z) + \cos(x) \cos(y) \sin(z) = 0$;
G: $\cos(x) \sin(y) + \cos(y) \sin(z) + \cos(z) \sin(x) = 0$;
Ideal as it is for the rapid display of certain surfaces in digital representation, the implicit method provides no information on the topology.

5 Triply Periodic Minimal Surfaces

Triply Periodic Minimal Surfaces (Tpms) have interesting characteristics for project purposes. They are called periodic because they consist of a base unit that can be replicated in Cartesian space in three dimensions (triply), thus creating a new surface seamlessly and without intersections [5].

A uniform minimal surface is, usually, characterised by different curvatures; in other words, some surfaces are flatter than others. It follows that not all points of the surface support any concentrated loads equally well.

If the same surface is, however, associated with a periodic distribution, i.e. the individual units are repeated next to each other, the physical iteration between the modules causes a compensatory effect that greatly increases their structural efficiency.

This is achieved, by the definition of minimal surface, through the use of as little material as possible.

The advantages mentioned above are real when the surface obtained is a system under voltage or the material with which it is constructed is able to withstand tensile stresses and compression.

In summary:

1. Tpms have natural geometric rigidity
2. Allow optimum use of materials
3. Configure stable and resistant structures.
There is a large number of known embedded triply periodic minimal surfaces. Moreover, it seems that the examples come in 5-dimensional families, most of which are only partially understood [6]. Our lack of knowledge of these surfaces makes it very hard to put them into categories. For the moment, we use the genus of the quotient by the largest lattice of orientation preserving translations as a guide. In this thesis we study three–periodic minimal surfaces that have three lattice vectors, i.e., they are invariant under translation along three independent directions. Numerous examples are known with cubic, tetragonal, rhombohedral, and orthorhombic symmetries. The symmetries of a TPMS allow the surface to be constructed from a single asymmetric surface patch, which extends to the entire surface under the action of the symmetry group (Fig. 3).

The most important local symmetries of minimal surfaces are Euclidean reflections (in mirror planes) and two–fold rotations. Many triply periodic minimal surfaces can best be understood and constructed in terms of fundamental regions bounded by mirror symmetry planes. According to H. S. M. Coxeter\(^1\) there are exactly seven types of such regions of finite size. Many triply periodic minimal surfaces have embedded straight lines, which of necessity must be C2 symmetry axes (180° rota-

---

\(^1\)Harold Scott MacDonald Coxeter, (1907–2003) was a British-born Canadian geometer. He was most noted for his work on regular polytopes and higher-dimensional geometries. He was a champion of the classical approach to geometry, in a period when the tendency was to approach geometry more and more via algebra.
tional symmetry). Possible C2 axes are shown in color below. There are two classes of kaleidoscopic cells: the prisms and the tetrahedra. A prism in the general sense is a plane polygon extended at right angles in the third dimension. A tetrahedron is a polyhedron with four flat faces. The relations of symmetry previously described structure the different Tpms (Fig. 4).

Now we have to find a way to generate and control the TPMS in digital environment. The computation played an essential role in the simulation and modelling process of such complex phenomena [7]. It was used Grasshopper, a graphical algorithm editor tightly integrated with Rhino’s 3-D modelling tools in order to create an algorithm able to describe and to control various types of TPMS.

Using Grasshopper it’s possible to define algorithms that are able to describe with good approximation any minimal surfaces directly from its implicit formulation. The algorithm translates the algebraic equation into a finished form that can be studied, manipulated and replicated. The process can be conceptually simplified imagining that, in the domain of Cartesian space, the equation “selects” points, belonging to the surface you decide to represent. The next algorithm’s instruction connects them by triangulation creating the surface. It is now possible to exploit the symmetry characteristics of the single unit by replicating it in a symmetrical cell, which is suitable to further replication in a modular lattice and to study the processes of adaptation to any required morphology (Fig. 5).

Fig. 4 The cube has 13 axes of symmetry: 6C2 (axes joining midpoints of opposite edges), 4C3 (space diagonals), and 3C4 (axes joining opposite face centroids). It can be divided into 24 Trirectangular Tetrahedron or 48 Quadrirectangular Tetrahedron
Fig. 5  Step of the algorithm: (1) Definition of points in the fundamental cell; (2) Triangulation creates the surface; (3) Gyroid surface; (4) Invariant translation to create a TPMS based on Gyroid; (5) Discretization of the Hemispherical dome to obtain a surface composed by Gyroid. Below Experimentation on a sphere with a Diamond and a Gyroid surface and their complementary

References


giorgio.buratti@polimi.it