ACTIVE BALANCING SYSTEMS FOR ROTATING ORBITAL DEVICES

S. Meraglia[1*], D. Invernizzi[1], M. Lovera[1]

Dipartimento di Scienze e Tecnologie Aerospaziali, Politecnico di Milano
via La Masa 34, 20156, Milano, Italy
*salvatore.meraglia@mail.polimi.it, davide.invernizzi@polimi.it, marco.lovera@polimi.it

ABSTRACT

In this paper we study active balancing systems tailored to space applications which involve rotating devices mounted on spacecraft. The behavior of these systems can be severely affected by the force and torque arising at the interface point from unpredictable inertial asymmetries of the rotating device. Therefore, a suitable mechanism to counteract unbalance effects must be envisaged: we suggest using a set of actuated movable masses and sensors and a control law assigning the position of the masses to cancel the rotor unbalance. The control design that we propose is based on tools of harmonic control and provides satisfactory performance even in the presence of parameters uncertainties.

Keywords: Rotating orbital device, Rotational unbalance, Adaptive control

1 INTRODUCTION

Future space missions will increasingly rely on payloads the operation of which will require them to rotate with respect to the platform. These systems need a careful design, because the unbalanced force and moment connected to inertial asymmetries can lead to the reduction of accuracy and stability of the satellite's attitude and the aroused vibration in the satellite will directly affect the quality of the collected data. These problems are magnified when the payload is large and, as a consequence, the working life could become shorter and operational reliability could degrade. Even worse, the attitude control system may fail to stabilize the spacecraft or the unbalanced loads may damage the motor sustaining the spin motion, thereby undermining the outcome of the mission. Accommodation restrictions, whether due to launcher fairing envelopes or limitations and constraints by the spacecraft, make it necessary to stow for launch and deploy in-orbit the payloads that are exceeding these restrictions due to their required operational dimensions [1]. For this reason the inertial properties cannot be predicted with high accuracy, so that the resulting unbalances might have to be corrected at commissioning by means of a dedicated balancing system [2]. In this paper a concept for such a system, inspired by previous works [3] on the design of active balancing systems (short, ABS) is presented and discussed. More precisely, a detailed analysis of the balancing problem is carried out by referring to an

ABS concept based on actuated movable masses and a set of sensors. Furthermore, an analytical framework for the design of proposed ABS is briefly outlined. In the last part of the paper, a simplified benchmark model of the proposed ABS is presented together with dedicated solutions to the active control problem, based on ideas borrowed from Harmonic Control (short, HC) [7, 4, 6]. The performance level achieved by the proposed control design is assessed in a simulation scenario based on a multibody model of the system. We provide results also for the case in which parametric uncertainties are accounted for in the simulation model and we show the performance improvement gained when a preliminary identification step is carried out before applying the control law.

2 PROBLEM STATEMENT

This section is devoted to presenting the problem addressed in this work. We consider a system made by a rigid body, with uncertain inertial properties, which is rotating about a fixed axis at constant angular velocity. As mentioned in the Introduction, this case is representative of space applications in which a rotating device (henceforth called "rotor") is mounted on a spacecraft. To tackle the undesired effects associated with rotor unbalance, we suggest using an active balancing system made by a set of M movable masses mounted on the rotor and sensors capable of measuring the inertial components of the joint force and torque to be canceled. We assume that the rotor is spinning at constant angular velocity about the third axis of the inertial frame[1]. In this setting, the rotor configuration is described by $R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where θ is the angle of rotation. By definition, R is a rotation matrix that transforms vector components from the inertial frame $(O_I, \{i_1, i_2, i_3\})$ to the body frame $(O_B, \{b_1, b_2, b_3\})$. Due to the joint constraint, we have the following relationships: $O_I = O_B =: O$ and $i_3 = b_3$. The position of the *i-th* balancing mass, resolved in the inertial frame, is a function of the corresponding relative displacement $s_i \in \mathbb{R}$ as follows:

$$r_i = R^T (\bar{r}_i + s_i n_i) \tag{1}$$

where $\bar{r}_i = \begin{bmatrix} \bar{x}_i & \bar{y}_i & \bar{z}_i \end{bmatrix}^T \in \mathbb{R}^3$ is the zero location $(s_i = 0)$ of the *i-th* balancing mass and $n_i \in \mathbb{R}^3$ is the unit vector assigning the corresponding displacement direction. The velocity of the *i-th* mass, resolved in the inertial frame, is given by

$$\dot{r}_i = R^T \left(\dot{s}_i n_i + \omega \times (\bar{r}_i + s_i n_i) \right), \tag{2}$$

where $\omega = \dot{\theta} e_3 = \begin{bmatrix} 0 & 0 & \dot{\theta} \end{bmatrix}^T$ is the (constant) angular velocity of the rotor. In the following, we use the compact matrix notation $S(\omega)x = \omega \times x$ to represent the cross product operation.
The objective of the ABS is to guarantee that the in-plane components of the reaction force and torque at joint O, measured by sensors, are ideally canceled. To this end, we first compute the inertial components of the force and torque (f_O, τ_O) at joint O by applying Newton's law:

$$\tau_O^I = \frac{dh}{dt}, \qquad f_O^I = \frac{dQ}{dt} \tag{3}$$

where h and q are the angular and linear momentum, respectively, whose expressions are not reported for space limitations. Herein, $J^R \in \mathbb{R}^{3\times 3}$ and $m^R \in \mathbb{R}_{>0}$ are the inertia matrix with

[1] Equivalently, we assume that the attitude control system is capable of instantaneously rejecting the disturbances associated with rotor unbalance. Future work will address the rotor-ABS coupling problem.

respect to O and the mass of the rotor, respectively, $m_i \in \mathbb{R}_{>0}$ is the *i-th* balancing mass, $r_G^R \in \mathbb{R}^3$ is the location of the rotor center of mass, resolved in the body frame. Then, the in-plane components of f_O^I and τ_O^I can be derived from (3) to obtain

$$\begin{bmatrix} f_{O_1}^I & f_{O_2}^I & \tau_{O_1}^I & \tau_{O_2}^I \end{bmatrix}^T = \begin{bmatrix} \bar{R}(\theta(t)) & 0 \\ 0 & \bar{R}(\theta(t)) \end{bmatrix} \begin{bmatrix} f_{O_1}^B & f_{O_2}^B & \tau_{O_1}^B & \tau_{O_2}^B \end{bmatrix}^T, \quad (4)$$

where $\bar{R}(\theta(t)) = \begin{bmatrix} \cos(\theta(t)) & -\sin(\theta(t)) \\ \sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix}$ and, given $e_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ and $e_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$,

$$f_{O_1}^B = -\dot{\theta}^2 \left(m^R x_G^R + \sum_{i=1}^{M} m_i(\bar{x}_i + s_i e_1^T n_i) \right) + 2\dot{\theta} \sum_{i=1}^{M} m_i e_2^T n_i \dot{s}_i + \sum_{i=1}^{M} m_i e_1^T n_i \ddot{s}_i f_O \quad (5)$$

$$f_{O_2}^B = -\dot{\theta}^2 \left(m^R y_G^R + \sum_{i=1}^{M} m_i(\bar{y}_i + s_i e_2^T n_i) \right) - 2\dot{\theta} \sum_{i=1}^{M} m_i e_1^T n_i \dot{s}_i + \sum_{i=1}^{M} m_i e_2^T n_i \ddot{s}_i f_O \quad (6)$$

$$\begin{aligned} \tau_{O_1}^B = & -\dot{\theta}^2 \left(J_{23}^R - \sum_{i=1}^{M} m_i(\bar{y}_i + s_i e_2^T n_i)(\bar{z}_i + s_i e_3^T n_i) \right) + \sum_{i=1}^{M} m_i \begin{bmatrix} 0 & -\bar{z}_i & \bar{y}_i \end{bmatrix} n_i \ddot{s}_i \\ & + \dot{\theta} \sum_{i=1}^{M} m_i \left(\begin{bmatrix} -\bar{z}_i & 0 & \bar{x}_i \end{bmatrix} n_i + e_1^T S(n_i) \begin{bmatrix} -\bar{y}_i - s_i e_2^T n_i & \bar{x}_i + s_i e_1^T n_i & 0 \end{bmatrix}^T + \begin{bmatrix} 0 & -\bar{z}_i - s_i e_3^T n_i & \bar{y}_i + s_i e_2^T n_i \end{bmatrix} S^T(n_i) e_3 \right) \dot{s}_i \end{aligned} \quad (7)$$

$$\begin{aligned} \tau_{O_2}^B = & \dot{\theta}^2 \left(J_{13}^R - \sum_{i=1}^{M} m_i(\bar{x}_i + s_i e_1^T n_i)(\bar{z}_i + s_i e_3^T n_i) \right) + \sum_{i=1}^{M} m_i \begin{bmatrix} \bar{z}_i & 0 & -\bar{x}_i \end{bmatrix} n_i \ddot{s}_i \\ & + \dot{\theta} \sum_{i=1}^{M} m_i \left(\begin{bmatrix} 0 & -\bar{z}_i & \bar{y}_i \end{bmatrix} n_i + e_2^T S(n_i) \begin{bmatrix} -\bar{y}_i - s_i e_2^T n_i & \bar{x}_i + s_i e_1^T n_i & 0 \end{bmatrix}^T + \begin{bmatrix} \bar{z}_i + s_i e_3^T n_i & 0 & -\bar{x}_i - s_i e_1^T n_i \end{bmatrix} S^T(n_i) e_3 \right) \dot{s}_i. \end{aligned} \quad (8)$$

Without loss of generality, we assume that the rotor and the balancing system can be split in a nominal balanced configuration and perturbation terms as follows:

$$\begin{aligned} m^R x_G^R + \sum_{i=1}^{M} m_i \bar{x}_i = \bar{S}_1 + \Delta S_1 = \Delta S_1 \qquad & m^R x_G^R + \sum_{i=1}^{M} m_i \bar{y}_i = \bar{S}_2 + \Delta S_2 = \Delta S_2 \\ J_{13}^R - \sum_{i=1}^{3} \bar{x}_i \bar{z}_i = \bar{J}_{13} + \Delta J_{13} = \Delta J_{13} \qquad & J_{23}^R - \sum_{i=1}^{3} \bar{y}_i \bar{z}_i = \bar{J}_{23} + \Delta J_{23} = \Delta J_{23} \end{aligned} \quad (9)$$

where, given $i = 2, 3$, $\bar{S}_i$ and ΔS_i denote the static moment and the corresponding perturbation and similarly $\bar{J}_{i3}$ and ΔJ_{i3} denote the nominal inertia moment and the corresponding perturbation. For constant perturbations, balanced equilibrium conditions ($\dot{s}_i = \ddot{s}_i = f_{O_1}^B = f_{O_2}^B = \tau_{O_1}^B = \tau_{O_2}^B = 0$) can be obtained provided that there is a sufficient number of (suitably) placed balancing masses. Specifically, a basic requirement is that the system

$$\begin{aligned} \sum_{i=1}^{M} m_i e_1^T n_i s_i = -\Delta S_1 \qquad & \sum_{i=1}^{M} m_i \left((e_2^T n_i)(e_3^T n_i) s_i^2 \bar{y}_i (e_3^T n_i) + \bar{z}_i (e_2^T n_i) s_i \right) = \Delta J_{23} \\ \sum_{i=1}^{M} m_i e_2^T n_i s_i = -\Delta S_2 \qquad & \sum_{i=1}^{M} m_i \left((e_1^T n_i)(e_3^T n_i) s_i^2 \bar{x}_i (e_3^T n_i) + \bar{z}_i (e_1^T n_i) s_i \right) = \Delta J_{13}, \end{aligned} \quad (10)$$

derived from equations (25)-(26), admits at least one solution. By defining $w = [f^B_{O_1}\ f^B_{O_2}\ \tau^B_{O_1}\ \tau^B_{O_2}]^T$ and by means of (9), equations (25)-(26) can be compactly written as:

$$w = \sum_{i=1}^{M} C^i_{abs} y^i_a + D_{abs} d, \tag{11}$$

with $y^i_a = [s_i\ \dot{s}_i\ \ddot{s}_i]^T$, $d = [\Delta S_1\ \Delta S_2\ \Delta J_{13}\ \Delta J_{23}]^T$. The exact expressions of C^i_{abs} and D_{abs} can be derived from (5)-(8) but is omitted here for space reasons. The ABS system includes position-controlled linear actuators to assign the motion of the balancing masses. Assuming a linear behavior, we can compactly write the actuators dynamics as

$$\dot{x}_a = A_a x_a + B_a u, \qquad y_a = C_a x_a + D_a u \tag{12}$$

where $x_a = [x^1_a\ \cdots\ x^M_a]^T \in \mathbb{R}^{Mn_a}$ is a vector including all the states of the M actuators, $y_a = [y^i_a\ \cdots\ y^M_a]^T \in \mathbb{R}^{Mn_a}$ is a vector collecting the outputs defined in (11) and $u = [u_1\ \cdots\ u_M] \in \mathbb{R}^M$ is the vector of control variables, *i.e.*, the desired positions of the masses. Finally, $A_a = \text{blkdiag}(A^i_a)$, $B_a = \text{blkdiag}(B^i_a)$, $C_a = \text{blkdiag}(C^i_a)$ and $D_a = \text{blkdiag}(D^i_a)$ are block diagonal matrices formed from the quadruples $(A^i_a, D^i_a, C^i_a, D^i_a)$ characterizing the *i-th* actuator dynamics, which has order n_a. The inertial in-plane torque and force, *i.e.*, the components of vector $\bar{R}(\theta(t))w$, are assumed to be measured by suitable sensors, for which we assume again a linear behavior, described by:

$$\dot{x}_s = A_s x_s + B_s \bar{R}(\theta(t))w = A_s x_s + B_s \bar{R}(\theta(t))\left(C_{abs} C_a x_a + C_{abs} D_a u + D_{abs} d\right), \quad y_s = C_s x_s \tag{13}$$

where $C_{abs} = [C^1_{abs}\ \cdots\ C^M_{abs}]$. By defining $x = [x_a\ x_s]^T$ and $y = y_s$, the overall system can be written in state-space form as follows:

$$\dot{x} = A(t)x + B_u(t)u + B_d(t)d \qquad y = \begin{bmatrix} 0 & C_s \end{bmatrix} x \tag{14}$$

where

$$A(t) = \begin{bmatrix} A_a & 0 \\ B_s \bar{R}(\theta(t)) C_{abs} C_a & A_s \end{bmatrix} \quad B_u(t) = \begin{bmatrix} B_a \\ B_s \bar{R}(\theta(t)) C_{abs} D_a \end{bmatrix} \quad B_d(t) = \begin{bmatrix} 0 \\ B_s \bar{R}(\theta(t)) D_{abs} \end{bmatrix}. \tag{15}$$

3 CONTROL LAW DESIGN

3.1 Overview of Harmonic Control

A typical non-adaptive Harmonic Control (HC) system is based on a discrete time mathematical model describing the response of the system to harmonic inputs with the general form

$$y_N(k) = Tu(k) + d_N(k), \tag{16}$$

where k is the rotor revolution index, $y_N \in \mathbb{R}^{2n_y}$ is a vector of N/rev harmonics of measured outputs

$$y_N(k) \quad = \quad \begin{bmatrix} y_{Nc}(k) \\ y_{Ns}(k) \end{bmatrix} = \begin{bmatrix} \frac{1}{\pi}\int_{k\pi}^{(k+1)\pi} y(\psi)\cos(N\psi)\, d\psi \\ \frac{1}{\pi}\int_{k\pi}^{(k+1)\pi} y(\psi)\sin(N\psi)\, d\psi \end{bmatrix} \tag{17}$$

$u \in \mathbb{R}^N$ is a vector of control inputs, and T is a $2n_y \times N$ constant matrix. The vector $d_N \in \mathbb{R}^{2n_y}$ contains the N/rev harmonics of the "baseline" vibrations, *i.e.*, the vibrations in the absence of

HC. The HC inputs are generally updated at discrete time intervals, for example, once per rotor revolution. The conventional HC control law is derived by minimizing at each discrete-time step k the cost function

$$J(k) = y_N(k)^T Q y_N(k) + \Delta u_N(k)^T R \Delta u_N(k) \tag{18}$$

where $Q = Q^T \geq 0$, $R > 0$ and $\Delta u(k) = u(k) - u(k-1)$ is the increment of the control variable at time k. Differentiating equation (18) with respect to $\Delta u_N(k)$ yields the T-matrix control law

$$u(k+1) = u(k) - K y_N(k) \tag{19}$$

where $K = (T^T Q T + R)^{-1} T^T Q$.

3.2 Harmonic transfer function and T-matrix computation

This section summarizes the main aspects of the development of the Harmonic Transfer Function (HTF) (see [7] and the references therein). Consider a continuous-time linear periodic system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \qquad y(t) = C(t)x(t) + D(t)u(t). \tag{20}$$

By deriving Fourier expansions for $A(t)$, $B(t)$, $C(t)$ and $D(t)$, it is possible to prove that the EMP (Exponentially Modulated Periodic) steady-state response of the system can be expressed as the infinite dimensional matrix equation with *constant* elements

$$s\mathcal{X} = (\mathcal{A} - \mathcal{N})\mathcal{X} + \mathcal{B}\mathcal{U}, \qquad \mathcal{Y} = \mathcal{C}\mathcal{X} + \mathcal{D}\mathcal{U} \tag{21}$$

where $\mathcal{X}$, $\mathcal{U}$ and $\mathcal{Y}$ are doubly infinite vectors formed with the harmonics of x, u and y respectively, organized as $\mathcal{X}^T = \begin{bmatrix} \cdots & x_{-2}^T & x_{-1}^T & x_0^T & x_1^T & x_2^T & \cdots \end{bmatrix}$ and similarly for $\mathcal{U}$ and $\mathcal{Y}$. $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are doubly infinite Toeplitz matrices formed with the harmonics of $A(\cdot)$, $B(\cdot)$, $C(\cdot)$ and $D(\cdot)$ respectively (see [7]). From equation (21), one can define the HTF operator:

$$\mathcal{G}(s) = \mathcal{C}[s\mathcal{I} - (\mathcal{A} - \mathcal{N})]^{-1}\mathcal{B} + \mathcal{D} \tag{22}$$

which relates the input harmonics and the output harmonics (contained in the infinite vectors $\mathcal{U}$ and $\mathcal{Y}$ respectively). The T-matrix used in the formulation of HC algorithms can be related to the elements of the HTF of the model, following the procedure presented in [7]:

$$T = 2\begin{bmatrix} \text{Real}[G_{N,0}] \\ \text{Imag}[G_{N,0}] \end{bmatrix} \tag{23}$$

where $G_{N,0}$ is obtained from the steady state response $\mathcal{Y} = [\cdots\ y_{-2N}\ y_{-N}\ y_0\ y_N\ y_{2N} \cdots]$, given by $\mathcal{Y} = \mathcal{G}(s)|_{s=0}\mathcal{U}$, for a constant input $u(t) = u_0$, *i.e.*, $\mathcal{U}^T = \begin{bmatrix} \cdots & 0 & 0 & u_0^T & 0 & 0 & \cdots \end{bmatrix}$.

3.3 Robustness analysis

The implementation of HC control requires the knowledge of matrix T: an erroneous model of such matrix could result in deteriorated performance and even instability of the closed-loop system. When an estimate $\hat{T} = T + \Delta T$ is given, closed-loop stability is guaranteed [6] if

$$\sigma_{max}(\Delta T) < -\frac{\sigma_{max}(T)}{2} + \frac{1}{2}\sqrt{\sigma_{max}(T)^2 + 4\frac{\sigma_{min}(R)}{\sigma_{max}(Q)}} \tag{24}$$

where $\sigma_{min}(\cdot), \sigma_{max}(\cdot)$ denote the minimum and maximum singular values, respectively. Equation (24) points out that there is an unavoidable trade-off between robustness and performance properties of the closed-loop system. As shown in Figure 2 for the simulation scenario reported in Section 4, the higher the ratio $\rho = \frac{\sigma_{max}(Q)}{\sigma_{min}(R)}$ (usually set as a performance indicator) the lower is the robustness degree, meaning that a smaller relative uncertainty can be tolerated by the system [5].

4 SIMULATION RESULTS

4.1 Simulation model: single-plane balancing system

By referring to equations (5)-(8), one sees that the balancing problem can be decoupled in two sub-problems, one for the xz plane and one for the yz plane, provided that the ABS is made by a suitable set of strokes directed along the coordinate axes. Therefore, to simplify the testing of the proposed control design, we refer to an ABS for the xz plane alone consisting of three movable masses in which the first mass can be moved along the x-axis $(n_1 = e_1)$ while the other two along the z-axis $n_{2,3} = e_3)$ (see Figure 1). The two red masses shown in the figure are used to replicate the effects of inertial asymmetries in the rotor.

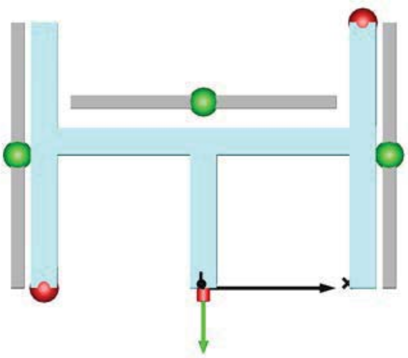

Figure 1: Multibody model of the breadboard.

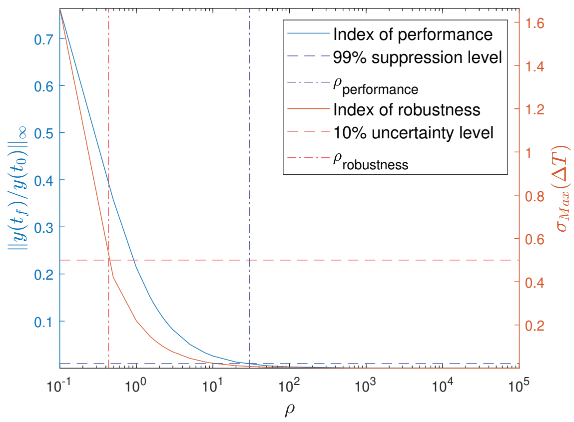

Figure 2: Robustness-performance trade-off.

For this setup, the reaction force $f^B_{O_1}$ (5) and torque $\tau^B_{O_2}$ (8) are given by the following expressions:

$$f^B_{O_1} = -\dot\theta^2 (\Delta S_1 + m_1 s_1) + m_1 \ddot s_1 \tag{25}$$

$$\tau^B_{O_2} = \dot\theta^2 \left(\Delta J^R_{13} - m_1 \bar z_1 s_1 - m_2 \bar x_2 s_2 - m_3 \bar x_3 s_3\right) + \sum_{i=1}^{3} m_i \left[\bar z_i \;\; 0 \;\; -\bar x_i\right] n_i \ddot s_i \tag{26}$$

where $\Delta S_1 = m_4 \bar x_4 + m_5 \bar x_5$ and $\Delta J^R_{13} = -m_4 \bar x_4 \bar z_4 - m_5 \bar x_5 \bar z_5$. The proposed ABS is capable of balancing the rotor for any perturbation ΔS_1 and ΔJ_{13} since the system

$$\begin{bmatrix} m_1 & 0 & 0 \\ m_1 \bar z_1 & m_2 \bar x_2 & m_3 \bar x_3 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} -\Delta S_3 \\ \Delta J_{13} \end{bmatrix}, \tag{27}$$

derived from equations (25)-(26), admits ∞^1 solutions provided that $\bar{x}_2$ and $\bar{x}_3$ are different from zero. Moreover, the matrices in equation (11) are given by:

$$C^1_{abs} = m_1 \begin{bmatrix} -\dot{\theta}^2 & 0 & 1 \\ -\dot{\theta}^2 \bar{z}_1 & 0 & \bar{z}_1 \end{bmatrix}, \quad C^2_{abs} = -m_2 \begin{bmatrix} 0 & 0 & 1 \\ \dot{\theta}^2 \bar{x}_2 & 0 & \bar{x}_2 \end{bmatrix}, \quad C^2_{abs} = -m_3 \begin{bmatrix} 0 & 0 & 0 \\ \dot{\theta}^2 \bar{x}_3 & 0 & \bar{x}_3 \end{bmatrix}. \tag{28}$$

For simplicity, we consider second order systems for both the actuators and the sensors dynamics with unit DC-gain. Matrix $A(t)$, computed according to (15), can be expanded in a complex Fourier series $A(t) = \sum_{m=-\infty}^{\infty} A_m e^{jm\Omega t}$: it can be easily seen from its definition that only the terms A_0, A_1 and A_{-1} are different from the zero matrix for the considered case. Expanding in the same fashion $B(t)$ and C, the Toeplitz matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ are given by:

$$\mathcal{A} = \begin{bmatrix} A_0 & A_{-1} & 0 \\ A_1 & A_0 & A_{-1} \\ 0 & A_1 & A_0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B_0 & B_{-1} & 0 \\ B_1 & B_0 & B_{-1} \\ 0 & B_1 & B_0 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}. \tag{29}$$

Finally, it is possible to derive the T-matrix to apply HC by means of equations (22), (23).

4.2 Control law tuning

As explained in Section 3.3, the selection of the weighting matrices Q, R in the T-matrix algorithm (19) is based on the trade-off between performance and robustness. Indeed, Figure 2 shows that it is impossible to achieve a high level of performance (measured by the ratio $\|y(t_f)/y(t_0)\|_\infty$) together with a high degree of robustness(10% uncertainty on the T-matrix). For this reason, an on-board identification is needed in order to have a better estimate of the system dynamics. The Recursive Least Squares (RLS) algorithm seems the natural choice for this task, given the linear nature of the model [5].

4.3 Numerical results

In this section a numerical example is reported to illustrate the performance of the proposed ABS combined with HC. For this purpose, a multibody model written in the Modelica modeling language has been developed (see Figure 1). The performance of the control law has been analyzed when the T-matrix is exactly known and when it is $\tilde{T} = 0.9T$. The results obtained for the former (ideal) case are plotted in Figure 3: as expected, a satisfactory vibration suppression is achieved after one update of the control law. The residual oscillations shown at steady state are related to the selected value of $\rho = 3$, which guarantees a 99% suppression of the loads (see Figure 2). In the latter case, the results of the standard HC based on $\tilde{T}$ (erroneous model) are compared with those obtained when an open-loop identification step is performed to estimate T prior to the application of HC. In Figure 4 one can appreciate the benefits of the identification step in achieving a faster and better rejection of the unbalance loads.

5 CONCLUSIONS

In this work we presented preliminary results on the problem of rotor balancing with focus to spacecraft applications: to mitigate the effect associated with inertial asymmetries, we proposed an active system made by actuated movable masses and suitable sensors. We showed that such a concept, combined with a controller based on harmonic control ideas, is capable of significantly reducing the force and torque induced by the unbalance at the interface between the fixed and the rotating part, even in the presence of imperfect knowledge of the system parameters.

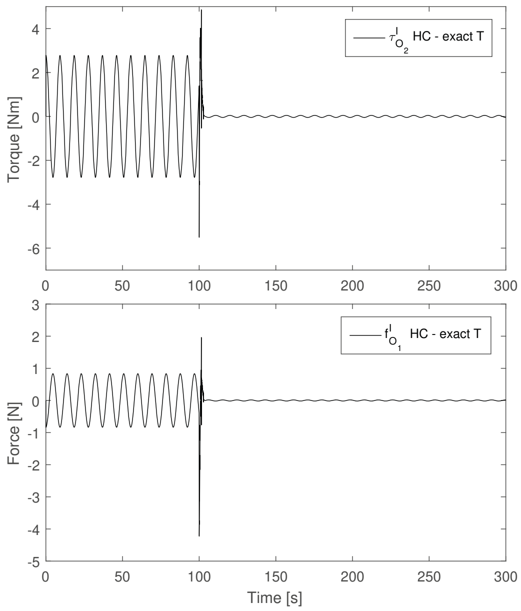

Figure 3: Force and torque suppression - ideal case.

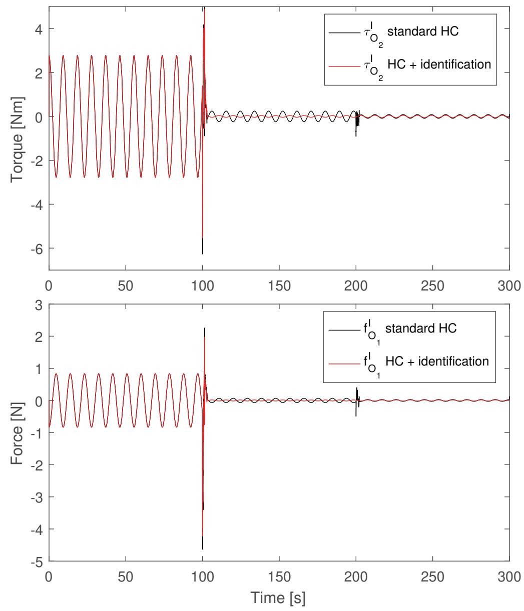

Figure 4: Force and torque suppression - HC with/without the identification step.

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