Tracking Control via Switched Integral Sliding Mode with Application to Robot Manipulators

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Abstract

This paper presents a switching structure scheme for motion control of industrial robot manipulators. To overcome the issues deriving from choosing a priori a specific control scheme, which can result in limited performances when the operating condition of the system varies, the scheme implements both a decentralized approach, suited for lower performance requirements and high transmission ratios, and the inverse dynamics based centralized approach, suited for higher performances in terms of velocity and acceleration. In both cases, the Integral Sliding Mode algorithm is used to compensate matched disturbances and to estimate the unmodeled dynamics used for the switching decision mechanism.

Keywords: Sliding mode control, robotics, integral action, switching control, switched systems, uncertain systems.

1. Introduction

One of the classical problems when studying robotic systems is motion planning and control\textsuperscript{[1,2]}. It consists in defining the desired trajectories in terms of positions, velocities and accelerations as well as a suitable control structure, able to produce the required control inputs (typically torques) that ensure the motion of the robot in the desired manner. When controlling a robot to perform motion, several aspects must be taken into account during the design phase of the control structure.

In the case of industrial robotic manipulators (i.e., mechanical systems composed by a series of joints connected by rigid links), which are the main focus of this paper, two approaches can be adopted depending on the type of operation required, the desired performances and the type of actuators employed: a decentralized control structure or a centralized one. The main difference between the two methodologies is, respectively, whether the robotic system is regarded as a composition of linear and decoupled Single-Input-Single-Output (SISO) systems (one for each actuator), or as Multi-Input-Multi-Output (MIMO) plant.

The decentralized approach is generally employed in applications where low performances in terms of velocity and acceleration are required and the joints present high transmission ratios, i.e., they are naturally decoupled; in this case, non-linearities and coupling effects are regarded as disturbances acting on each joint\textsuperscript{[3]}. The centralized approach, instead, is adopted when the manipulator joints do not have gear boxes (i.e., no transmissions) and there is a higher demand in terms of velocity and acceleration performances. Such a control structure usually relies on the inverse dynamics control approach, that allows to eliminate the nonlinear dynamics acting on the system, which is not negligible in this case and must be taken into account during the design of the controllers. Therefore, inverse dynamics allows to perform a global feedback linearization resulting in a decoupling of the considered MIMO plant\textsuperscript{[2]}.

Although the decentralized approach presents several advantages due to its simple structure and light computational weight in contrast with the centralized approach, which instead requires extensive online computation of the inverse dynamics, it is not always suitable when high precision and high velocity performances during the operation are needed. On the other hand, it is not always possible to have an accurate enough model of the system to perform the inverse dynamics exactly, thus resulting in unmodeled disturbances further affecting the robot performance. This might be the case when the robot presents Variable Gear-ratios Actuators (VGA) to achieve a wider range of speed, impedance and forces\textsuperscript{[4,5,6]}.

Because of the above reasons, an a priori choice of the con-
The main extension with respect to [25] is the applicability of the approach to systems presenting VGA besides a wider range of trajectory variations. Moreover, a more detailed stability discussion of the proposed approach is reported together with the robustness analysis due to the presence of the ISM component. An extensive simulation campaign has been performed relying on the model of a 6-axis manipulator Comau Smart3-S2, whose dynamics has been identified on the basis of real data [20]. The proposed strategy has been finally compared with an active disturbance rejection method in order to highlight its advantages.

The paper is organized as follows. In Section 2 the main notation used in the paper is introduced. In Section 3 background on SMC is briefly recalled together with the model of the robot dynamics and the problem formulation. In Section 4 the control scheme is described, while in Section 5 the ISM controller employed in the structure is discussed and theoretically analyzed. An example application on a planar robot implementing VQA and simulation results obtained relying on the model of a Comau Smart3-S2 manipulator are illustrated in Section 6. Some conclusions are finally gathered in Section 7.

### 2. Notation

The notation used in the paper is mostly standard. Let $\mathbb{N}$ denote the set of natural numbers, while $\mathbb{R}$ be the set of real numbers. Let $|.|$ be the absolute value while $\|\|_{\infty}$ is the infinity norm. Let $x$ be a vector and $x_i$ its entry and $x^T$ its transpose. Given a signal $w$ belonging to the set $W$, then its upperbound is $\|w\|_{\infty}$ which is the state vector with $\|x\|_{\infty}$.

Let $G(t)$ be a switching function. A switching function $\sigma(t)$ is defined as:

$$\sigma(t) = \begin{cases} 1 & \text{if } g(t) = 0, \\ -1 & \text{if } g(t) < 0, \\ 0 & \text{if } g(t) > 0. \end{cases}$$

### 3. Preliminaries and Problem Formulation

In this section, background elements on SMC and the model of the robot manipulator are introduced. Canonical form for the state model of the plant, which enables to describe the dynamics of the tracking error in a suitable form for control design is presented. Finally, the control problem is formulated.

### 3.1. Preliminary on Sliding Mode Control

A canonical form frequently used in the development of SMC laws is introduced. To this end, assume a SISO system, written as a perturbed double integrator as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = u(t) + \eta(t) \\ y(t) = s(x(t)) \end{cases}$$

where $x \in \Omega \subset \mathbb{R}^2$ is the state vector with $x(t_0) = x_0$, $u(t) \in \mathbb{R}$ is the input and $\eta(t) \in \mathcal{H}$ is a bounded matched uncertainty.
with \( \mathcal{H} \) being a compact set containing the origin, and \( \mathcal{H}^{\text{sup}} \)
known. The output function \( s(x) : \Omega \to \mathbb{R} \) is of class \( C(\Omega) \).
This function is the so-called “sliding variable” and, according to the sliding mode control theory \([17,9]\), it has to be steered to zero in finite time. Furthermore, \( s(x) \) has to be selected such that if \( u(t) \) is designed so that, in a finite time \( t_t \) (ideal reaching time), \( s(x(t)) = 0 \ \forall x_0 \in \Omega \) and \( s(x(t)) = 0 \ \forall t > t_t \), then \( \forall t \geq t_t \) the origin is an asymptotically stable equilibrium point of \( \dot{x} = f(x) \) constrained to \( s(x(t)) = 0 \).

3.2. The Robot Model

The dynamics of a \( n \)-joints robot can be described as the following MIMO nonlinear coupled model

\[
B(q)\ddot{q} + n(q,q) = \tau
\]

\[
m(q,q) = C(q,q)\dot{q} + F(q) + g(q)
\]

where \( B(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q,q) \in \mathbb{R}^{n \times n} \) represents centripetal and Coriolis torques, \( F(q) \in \mathbb{R}^{n \times n} \) is the viscous friction matrix, \( g(q) \in \mathbb{R}^{n} \) is the vector of gravitational torques, and \( \tau \in \mathbb{R}^{n} \) represents the motor torques. In the following, the time dependence of the joint variables \( q(t) \) and \( \dot{q}(t) \) has been omitted for the sake of simplicity.

3.3. Problem Formulation

Given the robot manipulator model in (2)-(3), assume that \( \dot{q}^* \) and \( \dot{q}^* \in \mathbb{R}^{n} \) are pre-specified reference signals for the joint variables and their first time derivative, respectively. It is assumed that the components of \( \dot{q}^* \) are bounded and \( \dot{q}^* \) is Lipschitz continuous. Now, define the tracking errors as follows

\[
e(t) = \dot{q}^* - \dot{q}, \quad e(t) = q^* - q.
\]

letting \( e_1 = e \) and \( e_2 = \dot{e} \) be the position error and the velocity error. Note that, for the sake of simplicity, without any loss of generality, motion control problems without any interaction with the environment have been considered. It is worth noting that the proposed strategy is still valid even in case of interaction control.

4. Switched Structure Control Scheme

The control structure used in this work is illustrated in Figure 1. The first loop implements the decentralized control approach described in Subsection 4.1, the second loop implements instead the inverse dynamics based centralized control structure, illustrated in Subsection 4.2. For each structure, an ISM controller, described in detail in Section 5, is designed to perform a perturbation estimation, thus producing the signals used for the switching criteria.

4.1. Decentralized Control Structure

When using a decentralized approach, the robot manipulator is regarded as the composition of \( n \) linear and decoupled SISO systems. Assume that a motor acts on each joint of the manipulator. A simplified way to account for the dynamics of such motors is to only consider the effect related to the spinning

of the motor around its own axis. Consider a single joint and let \( J_m \) be the motor inertia, \( D_m \) be the coefficient of viscous friction of the motor, \( \tau_m \) be the load torque at the axis of the \( j \)th motor such that

\[
\tau_m = \frac{\tau_j}{k_{ij}}
\]

with \( k_{ij} \) being the reduction ratio defined as

\[
k_{ij} = \frac{q_{mj}}{\dot{q}_j} = \frac{\dot{q}_{mj}}{\dot{q}_j}.
\]

The equation of the \( j \)th motor is

\[
J_m \ddot{\theta}_j + D_m \dot{\theta}_j = \tau_m - \tau_{mj},
\]

where \( \tau_{mj} \) is the torque exerted by motor \( j \). Substituting (2)-(3) to (7) for each joint, considering \( D_m \) negligible for any \( j \) and posing

\[
J_m \ddot{\theta}_m = \tau_m - K_1^{-1}(B(q)\dot{q} + C(q,q)\dot{q} + F(q) + g(q)).
\]

one obtains

\[
J_m \dot{q}_m = \tau_m - K_1^{-1}(B(q)\dot{q} + C(q,q)\dot{q} + F(q) + g(q)).
\]

Equivalently, one can write

\[
(J_m + K_1^{-1}B(q)K_1^{-1})\dot{q}_m = \tau_m - K_1^{-1}C(q,q)K_1^{-1}q_m - K_1^{-1}F(q)K_1^{-1}q_m - K_1^{-1}g(q).
\]

The obtained equations can be interpreted as those of a linear and completely decoupled system, subjected to a disturbance deriving from the nonlinear and coupled terms of the dynamic model, i.e.,

\[
M(q)\ddot{q}_m = \tau_m - d,
\]

where \( M(q) = J_m + K_1^{-1}B(q)K_1^{-1} \) and \( d = K_1^{-1}C(q,q)K_1^{-1}q_m - K_1^{-1}F(q)K_1^{-1}q_m - K_1^{-1}g(q) \). The larger is the reduction ratios \( k_{ij} \),
the less relevant is the disturbance term. It is now possible to achieve the control torque in (2) as follows

$$\tau = K_M q K_P u_1$$

(12)

such that system (11) becomes

$$\dot{q} = u_1 - K^{-1}_M M^{-1}(q) \ddot{q} = u_1 - \eta_1 .$$

(13)

where \(u_1\) is the control law to be designed. Letting \(x_1 = e_1\) and \(x_2 = e_2\), consider now a Proportional-Derivative (PD) controller defined as

$$u_1 = \dot{q}^* + K_{D1} x_2 + K_{P1} x_1 ,$$

(14)

with \(K_{D1}\) and \(K_{P1}\) positive definite diagonal matrices (see Chapter 6 for further details). Combining the previous equations it holds

$$x_2 + K_{D1} x_2 + K_{P1} x_1 - \eta_1 = 0 .$$

(15)

Finally, the state space representation of the closed-loop system becomes

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -K_{D1} x_2 - K_{P1} x_1 + \eta_1 ,
\end{align*}$$

(16)

which is characterized by the matched uncertain terms \(\eta_1\) such that the following assumption holds.

**Assumption 1.** The uncertainty \(\eta_1\) is such that

$$\|\eta_1\|_\infty \leq \beta_1 .$$

(17)

with \(\beta_1\) being a positive constant.

In order to reject \(\eta_1\), a decentralized ISM (ISMd) control of the type

$$u_1 = \dot{q}^* + K_{D1} x_2 + K_{P1} x_1 + u_{11} ,$$

(18)

is designed, where \(u_{11}\) is typically a discontinuous control action. As a consequence, the equivalent controlled dynamics becomes

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -K_{P1} & -K_{D1} \end{bmatrix} x(t)$$

(19)

or in a compact way

$$\dot{x}(t) = A_1 x(t), \ x(0) = x_0$$

(20)

with \(A_1\) being the closed-loop dynamics matrix. Then, one can conclude that the error is governed by an asymptotically stable second order dynamics that can be arbitrarily assigned, on each joint, by suitably selecting the gains in the diagonal matrices \(K_{D1}\) and \(K_{P1}\).

**Remark 1.** Note that the choice of the PD controller is not mandatory in the sense that any other stabilizing control law can be used as high level controller. Yet, the use of PD controllers is quite common in robotics. As for the ISM control component, it is preferred to other sliding mode control solutions for its capability to make the controlled system insensitive to the matched uncertain terms since the initial time instant, thus eliminating the so-called reaching phase [22].

### 4.2. Centralized Control Structure

In the absence of decoupling effects given by the high reduction ratios (for instance in case of direct drive motors), the use of a centralized control strategy might turn out to be the only viable solution. In this work, the centralized approach relies on the so-called inverse dynamics controller. Assume again to exactly estimate the inertia matrix \(B(q)\) and to have a quite accurate replica of the vector \(\dot{m}(q, \dot{q})\), such that \(\dot{\hat{m}}(q, \dot{q}) \neq \dot{m}(q, \dot{q})\). Moreover, let \(u_2\) be an auxiliary control vector such that the control torque is selected as

$$\tau = B(q) u_2 + \dot{\hat{m}}(q, \dot{q}) .$$

(21)

Substituting (21) into model (2)-(3), one has

$$B(q) \dot{q} + \dot{\hat{m}}(q, \dot{q}) = B(q) u_2 + \dot{\hat{m}}(q, \dot{q}) ,$$

(22)

which is a chain of \(n\) decoupled double integrator plants of the type

$$\dot{q} = u_2 - \eta_2 .$$

(23)

Letting again \(x_1 = e_1\) and \(x_2 = e_2\), consider a PD controller defined as

$$u_2 = \dot{q}^* + K_{D2} x_2 + K_{P2} x_1 ,$$

(24)

with \(K_{D2}\) and \(K_{P2}\) positive definite diagonal matrices, which substituted to the previous equation gives

$$\dot{x}_2 + K_{D2} x_2 + K_{P2} x_1 - \eta_2 = 0 .$$

(25)

The state space representation of the closed-loop system is

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -K_{D2} x_2 - K_{P2} x_1 + \eta_2 .
\end{align*}$$

(26)

This system is characterized by the matched uncertain terms \(\eta_2\) such that the following assumption holds.

**Assumption 2.** The uncertainty \(\eta_2\) is such that

$$\|\eta_2\|_\infty \leq \beta_2 .$$

(27)

with \(\beta_2\) being a positive constant.

In order to reject \(\eta_2\), a centralized ISM (ISMc) control of the type

$$u_2 = K_{D2} x_2 + K_{P2} x_1 + u_{12}$$

(28)

designed, where \(u_{12}\) is a discontinuous control action. As a consequence, the equivalent controlled dynamics becomes

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -K_{P2} & -K_{D2} \end{bmatrix} x(t)$$

(29)

or in a compact way

$$\dot{x}(t) = A_2 x(t), \ x(0) = x_0$$

(30)

with \(A_2\) being the closed-loop dynamics matrix. Then, one can conclude again that the error is governed by an asymptotically stable second order dynamics that can be arbitrarily assigned, on each joint, by suitably selecting the gains in the diagonal matrices \(K_{D2}\) and \(K_{P2}\).
4.3. Switching Block

In order to switch between the considered structures, the block (SWr) contains a switching rule based on a performance index \( I_{\sigma(t)} \), where \( \sigma \in \{1, 2\} \) is the so-called switching signal related to the decentralized control and the centralized one, respectively. Since the dynamics of the controlled system is affected by hard uncertain terms which depend on the velocity and acceleration required to the robot, a good choice for \( I_{\sigma(t)} \) can be a function of the matched unknown terms \( \eta_1 \) and \( \eta_2 \), i.e.,

\[
I_{\sigma(t)} = \begin{cases} I(\eta_1), & \text{if } \sigma(t) = 1 \\ I(\eta_2), & \text{if } \sigma(t) = 2 \end{cases}.
\]

Specifically, the switching rule is chosen as

\[
I_{\sigma[1, 2]} \geq P,
\]

with \( P \) being a suitable selected threshold. The latter can be determined, for instance, via a trial and error procedure or by the knowledge of the physical quantities involved in the system and provided by the robot data sheet. The logic of the switching rule is: if, while using a decentralized approach, the coupling terms are greater than \( P \) (i.e., high velocity and acceleration performance are required), switch to the centralized control. Vice versa, if the centralized control is active and the uncertain terms are less than \( P \) (i.e., high velocity and acceleration performance are not required) switch to the decentralized control.

Remark 2. A possible choice of the index for the considered application is the Root Mean Square (RMS) value of \( \eta_1 \) and \( \eta_2 \), since it represents the deviation of the input signal from a given baseline, which corresponds to the case without uncertain terms.

What is left to be determined is the estimation of the matched uncertainties. In the following these estimates, namely \( \hat{\eta}_1 \) or \( \hat{\eta}_2 \), are obtained by exploiting the properties of ISM control in \[22\]. Finally, for the sake of stability, as discussed in the Subsection 5.1, a dwell-time \( T_{dw} \) has been also taken into account.

5. Integral Sliding Mode Control Component

Now the aim is to design the ISM control laws to be used in the decentralized and centralized case, respectively. ISM is typically characterized by a control variable \( u_{\sigma} \), \( \sigma \in \{1, 2\} \) depending on the used structure, split into two parts, i.e.,

\[
u_{\sigma}(t) = u_{0\sigma}(t) + u_{1\sigma}(t)
\]

where \( u_{0\sigma} \) is generated by a suitable PD controller designed relying on the nominal model (i.e., the model of the plant assuming that no uncertainty is present), and \( u_{1\sigma} \) is the sliding mode control action in order to reject the uncertainties affecting the system.

The \( u_{1\sigma} \) component has to be designed relying on the errors \( x_1, x_2 \) previously defined. The so-called integral sliding manifold is defined as

\[
\Sigma(t) = s(t) + \varphi(t) = 0
\]

where \( \Sigma \) is a vector of the auxiliary sliding variables, while \( s = Sx \), with \( S \) being a matrix of positive coefficients, is the actual sliding variable. Furthermore, the integral term \( \varphi \) is given by

\[
\varphi(t) = -s(0) - \int_{0}^{t} S[x_2(z), u_{0\sigma}(z)]' dz
\]

with the initial condition \( \varphi(0) = -s(0) \). Then, the discontinuous control law is defined as

\[
u_{1\sigma}(t) = -K_{\sigma} \text{sgn}(\Sigma(t)),
\]

with \( K_{\sigma} \) being the ISM control gain.

5.1. Stability Analysis

In the following some useful theoretical results are reported to explain the capability of “perturbation estimation” provided by the ISM component and needed for the previously described switching mechanism. Note that the whole system, controlled via PD and ISM control, can be captured by the following autonomous dynamics

\[
x(t) = A_{\sigma(t)} x(t), \quad x(t_0) = x_0
\]

where \( \sigma(t) \) is admissible, meaning that in finite time only a finite number of switchings can occur, by virtue of the presence of the dwell-time. Necessary condition for stability under the switching signal \( \sigma(t) \) is that all matrices \( A_i \), \( i = 1, 2 \) are Hurwitz \[27\]. Moreover, the dwell-time \( T_{dw} \) is such that the equilibrium point \( x = 0 \) of the system (37) is asymptotically stable with the switching signal \( \sigma(t) = i \in \{1, 2\} \), \( t \in [t_k, t_{k+1}] \) where \( t_k \) and \( t_{k+1} \) are successive switching times with \( t_{k+1} - t_k \geq T_{dw} \), for all \( k \in \mathbb{N} \) and index \( i \in \{1, 2\} \). What is left to prove is that the ISM component allows to achieve autonomous systems (37) where each matrix \( A_i \) is Hurwitz in spite of the uncertainties.

Theorem 1. Given the error system \[16\] (or \[26\]) such that Assumption \[2\] (or \[2\]) hold, controlled via \[33\] with \[36\] and sliding variable as in \[34\] - \[35\], if \( \|K_1\|_\infty > \beta_1 \) (or \( \|K_2\|_\infty > \beta_2 \)), then a sliding mode \( \Sigma = 0 \) is enforced \( \forall t \geq t_0 \).

Proof 1. For the sake of simplicity, let \( \sigma = 1 \) (\( \sigma = 2 \) is the same). Consider system \[16\] expressed as

\[
x(t) = A_1 x(t) + B(u_{11} + \eta_1), \quad x(t_0) = x_0 .
\]

Define the Lyapunov function candidate as

\[
V = \frac{1}{2} \Sigma' \Sigma.
\]

Compute its first time derivative, that is

\[
\Sigma' \dot{\Sigma} = \Sigma' S(A_1 x + B(u_{11} + \eta_1) - A_1 x) = S' SB(u_{11} + \eta_1)
\]

\[
= - \Sigma' (K_1 \text{sgn}(\Sigma) - \eta_1) < - (\|K_1\|_\infty - \beta_1) \|\Sigma\|
\]

(40)
Hence, $V$ is negative definite if and only if $\|K_1\|_\infty > \beta_1$, thus implying that there exists a reaching time $t_\diamond > 0$ such that $\Sigma(t) = 0$, $\forall t \geq t_\diamond$. Since, by construction in $\eqref{14}$, $\Sigma(0) = 0$, then $\Sigma(t) = 0$, $\forall t \geq 0$, which concludes the proof.

By virtue of Theorem 1, it is possible to formulate the result assessing the perturbation estimation capability of the algorithm.

\textbf{Theorem 2.} Given the error system $\eqref{16}$ (or $\eqref{26}$) such that Assumption 2 (or 2) hold, controlled via $\eqref{35}$ and sliding variable as in $\eqref{33}$-$\eqref{35}$, if $\Sigma = 0$ is enforced $\forall t \geq 0$ solving $u_{11}$ (or $u_{12}$) from $\Sigma = 0$, then the equivalent control is such that $u_{11eq} = -\eta_1$ (or $u_{12eq} = -\eta_2$). Moreover, the equivalent system is Hurwitz and invariant with respect to the uncertainties.

\textbf{Proof 2.} For the sake of simplicity, let $\sigma = 1$ ($\sigma = 2$ is the same) and consider system $\eqref{16}$ expressed as

$$\dot{x}(t) = A_1x(t) + B(u_{11} + \eta_1), \quad x(t_0) = x_0.$$  \hfill \eqref{40}

By virtue of Theorem 2 with $\|K_1\|_\infty > \beta_1$ an integral sliding mode is enforced since the initial time instant $t = 0$, i.e., $\Sigma(t) = 0$, $\forall t \geq 0$. The latter implies that

$$\Sigma = \dot{s} + \phi = S(A_1x(t) + B(u_{11} + \eta_1)) = 0 \Rightarrow u_{11} = -\eta_1.$$  \hfill \eqref{41}

By definition of equivalent control, solving $u_{11}$ from $\eqref{41}$ allows one to obtain $u_{11eq}$, i.e.,

$$u_{11eq}(t) = -\eta_1(t), \quad \forall t \geq 0.$$  \hfill \eqref{42}

Finally, by substituting the equivalent control $u_{11eq}(t)$ in $\eqref{16}$, one directly achieves system $\eqref{20}$ which is Hurwitz and invariant with respect to the uncertainties, concluding the proof.

### 5.2. Perturbation Estimator and Chattering Alleviation

The previous results imply that the ISM controller is able to estimate the uncertain and coupling terms if the equivalent control is available. In fact, it cannot be computed in practice because of the dependence on the uncertain terms. However, as claimed in $\cite{22}$, an approximation of the equivalent control can be obtained via a first order linear filter with the real discontinuous control $\eqref{36}$ as input signal, i.e.,

$$\tilde{u}_{1req}(t) = \frac{1}{\mu} \int_0^t e^{-\frac{z}{\mu}} \tilde{u}_{1eq}(z) dz$$  \hfill \eqref{43}

where $\mu$ is the time constant of the filter. It should be set such that the linear filter does not distort the slow component of the switching action. Furthermore, the integral term in $\eqref{35}$ has to be redesigned as

$$\varphi(t) = -s(0) - \int_0^t S[x_2(z), u_{eq}(z) - u_{1eq}(z)] dz$$  \hfill \eqref{44}

with initial condition $\varphi(0) = -s(0)$. By virtue of Theorem 2 one has that $\tilde{u}_{1req}(t)$ is a good estimate of $\eta_{\sigma}$. This quantity can be used to compute the performance index $I(\eta_{\sigma})$ in $\eqref{42}$, which allows to realize the switching mechanism illustrated in Subsection 4.3. Moreover, by virtue of the presence of the filter, the control law in $\eqref{44}$ is continuous and can be fed into the plant, thus implying a chattering alleviation feature. Note that, having in mind a robotic application, in the following the equivalent control computed as in $\eqref{43}$ with the integral term designed as $\eqref{44}$ will be used.

### 6. Case study

In the following subsections, two different examples will be illustrated. The first one is an academic example (see $\cite{2}$) where the case with a robot implementing VGA is presented and the proposed strategy is successfully applied. The second one is carried out on a robot Comau Smart3-S2, the model of which was realized on the basis of real data and the considered scenario was made realistic adding uncertain terms retrieved from the real robot (see Figure 2 where the simulation environment is illustrated).

#### 6.1. Academic example

In the following section, a concept example of the use of the presented switched scheme for a 2-joints planar manipulator (see $\cite{2}$ Chapter 7) is presented. The example is intended to show the behavior of the strategy in presence of VGA at both joints, which are not identified by the inverse dynamics. Indeed, the feedback linearization takes into account the dynamics of the robot at the nominal value of reduction ratios. The reference trajectory is a step signal in the joint space equal to 10 rad, and the reduction ratios evolve in time from 1 to their nominal value as a sinusoidal signal. In this case, the switching rule introduced in Subsection 4.3 is defined so that $P = 5$. A dwell time of $T_{dw} = 2$ s has been imposed, to avoid high frequency switching.

In Figure 3 the profile of the reduction ratio variation and the torques exerted at joint 1 are reported, alongside the performance index $I$ and the corresponding switching signal $\sigma$. 

![Virtual replica of the Comau Smart3-S2 anthropomorphic manipulator](Image)}
Figure 3. Joint 1: variation of reduction ratios in time (top left); performance index $I(\dot{\theta}_c)$ (top right); control torques $\tau$ (bottom left); switching signal $\sigma$: 1 for the decentralized approach, 2 for centralized approach (bottom right)

Figure 4. Position and velocity with references in the joint space for joint 1

Figure 5. Time evolution of the auxiliary sliding variable $\Sigma$, the sliding variable $s_1$ and the integral term $\varphi_1$ for joint 1

Table 1. Control Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dec. ($\sigma = 1$)</th>
<th>Cen. ($\sigma = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{1\sigma}$</td>
<td>1000</td>
<td>2</td>
</tr>
<tr>
<td>$K_{2\sigma}$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$K_{P2\sigma}$</td>
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<td>200</td>
</tr>
<tr>
<td>$K_{P3\sigma}$</td>
<td>$2.5 \times 10^4$</td>
<td>200</td>
</tr>
<tr>
<td>$K_{D1\sigma}$</td>
<td>$5 \times 10^3$</td>
<td>100</td>
</tr>
<tr>
<td>$K_{D2\sigma}$</td>
<td>$5 \times 10^3$</td>
<td>100</td>
</tr>
<tr>
<td>$K_{D3\sigma}$</td>
<td>$5 \times 10^3$</td>
<td>100</td>
</tr>
</tbody>
</table>

Indeed, by virtue of the ISM component, the coupling terms are rejected making the two joints decoupled and behave in the same way.

The control architecture is initialized as the centralized one ($\sigma = 2$) and after $T_{dw} = 2$ seconds, since the performance index $I_2$ is below $P$, it switches to the decentralized structure ($\sigma = 1$). After 5 seconds the index increases up to the threshold and, since the dwell-time is last, $\sigma$ switches again to 2. Before the time instant 10 s, the index $I_2$ is below $P$ and $\sigma$ becomes 1, but the index $I_1$ again abruptly increases up to the threshold. Although the best solution would be the centralized one, the switching signal $\sigma$ remains equal to 1 due to the dwell-time. After 2 seconds $\sigma$ becomes 2 but the index decreases below $P$, causing, after the dwell-time is last, the switch again to the decentralized architecture, as expected. As it can be noted in Figure 5, sliding mode is always enforced, despite switches in the control structure. Note that, because of the presence of a first order filter to generate the equivalent control, an overshoot is present at the beginning in the time interval when the system is still sensitive to the uncertainties and [44] has to converge.
6.2. Tests on Comau Smart3-S2

In this section, simulation results carried out relying on a realistic model of a robot manipulator Comau Smart3-S2 are presented. The Smart3-S2 robot consists of six links and six rotational joints driven by brushless electric motors. To acquire the joints positions, resolvers are fastened on the three motors.

For the sake of simplicity, in the simulation tests, only vertical planar motions of the robot have been enabled. Furthermore, a comparative analysis with the so-called Active Disturbance Rejection Control (ADRC) in [28] has been performed.

Simulations have been run using a model of the actual robot, identified on the basis of real data. The control structure has been applied to a motion control problem that requires the tracking of pick-and-place trajectories in the operative space. The control structure has been suitably adapted. Specifically, in practical case it was possible to estimate the uncertain terms acting on the torques for the decentralized case and on the acceleration for the centralized one.

For this reason two different thresholds were specified: \( P_r \) for the torques and \( P_q \) for the acceleration. When the performance index overcomes \( P_r \), the centralized architecture is activated; when the performance index is instead below \( P_q \), the decentralized architecture is activated. Note that, the value of the thresholds, which are related to the uncertain terms, have been suitably selected on the basis of the data collected from the real plant.

The Proportional-Derivative parameters used for the nominal control in the regulators and the control gains for the sliding mode components are reported in Table 1. The performance index \( I_{\sigma} \) (see (15)) has been chosen as the RMS value of the output of the perturbation estimator provided by the ISM controller. Thresholds for the switching logic have been set equal to \( P_r = 150 \) and \( P_q = 0.08 \). The dwell-time has been set equal to \( T_{dw} = 2 \text{s} \).

The evolution of the sliding variables \( s_j, j = 1, 2, 3 \) is shown in Figure 6, where it can be seen that a sliding mode is always ensured despite changes in the control scheme. In Figure 7 the corresponding reference signal \( p_{r,j}^* \) and the actual trajectory \( p_j \) of the end-effector in the operative space are represented. The torques exerted by each joint are also reported in the same picture in order to highlight their smoothness due to the chattering alleviation features provided by the ISM component. On the right, the performance index \( I_{\sigma} \) is illustrated. Specifically, the first part of the picture is a closed-up of the performance index when the decentralized architecture is used (switching signal \( \sigma = 1 \)). As it can be noticed, the system runs on the decentralized loop while following a “slow” trajectory, and changes when a sudden variation occurs at 13.125 s, requiring higher velocity performances that can be guaranteed by the centralized approach (switching signal \( \sigma = 2 \)). Finally, velocity and acceleration profiles with the corresponding errors are reported in Figure 8. The joints follow the reference signals in the joint space represented in Figure 9. Another example is given by the trajectory reported in Figure 11, where the use of the centralized approach can be avoided entirely due to the low demanding nature of the trajectory given as reference. The control is then maintained on the decentralized approach (see Figure 9).

Results of the realistic simulations using all three approaches (decentralized only, centralized only, and switched structure) are reported in Table 2. The RMS values of the position error \( \epsilon_{\text{RMS}} \), of the torque \( \tau_{\text{RMS}} \), and of the estimated uncertainties \( \eta_{\text{RMS}} \) and \( \eta_{\text{RMS}} \) have been computed for each joint. The RMS values obtained through the switching scheme are comparable with the other ones. The proposed approach is then suitable for motion operations and enables the robot to track a wider range of varying trajectories, which require both high and low performances in terms of velocity and acceleration. Nevertheless, it allows the use of lower control gains when possi-

![Figure 6. Time evolution of the sliding variables \( s_j, j = 1, 2, 3 \)](image-url)

![Figure 7. The corresponding reference signal \( p_{r,j}^* \) and the actual trajectory \( p_j \) of the end-effector in the operative space](image-url)

![Figure 8. The joints follow the reference signals in the joint space](image-url)

![Figure 9. Another example is given by the trajectory reported in Figure 11](image-url)

![Figure 10. Results of the realistic simulations using all three approaches](image-url)

![Figure 11. The use of the centralized approach can be avoided entirely due to the low demanding nature of the trajectory given as reference](image-url)

Table 2. Result comparison with ISM control

<table>
<thead>
<tr>
<th>Joint</th>
<th>( \epsilon_{\text{RMS}} )</th>
<th>( \tau_{\text{RMS}} )</th>
<th>( \eta_{\text{RMS}} )</th>
<th>( \eta_{\text{RMS}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sw.</td>
<td>1 0.0177 206.30</td>
<td>141.65 0.2375</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 0.0007 122.92</td>
<td>126.95 0.2615</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 0.0006 9.2403</td>
<td>6.917 0.2346</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec.</td>
<td>1 0.0176 208.031</td>
<td>146.22 –</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 0.00036 123.00</td>
<td>120.47 –</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 0.00003 9.2396</td>
<td>7.988 –</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cen.</td>
<td>1 0.0270 171.76</td>
<td>– 0.407</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 0.00085 122.48</td>
<td>– 0.336</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 0.00085 8.167</td>
<td>– 0.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Figure 7. Trajectory tracking in the operative space (top left); performance index $I$ and switching thresholds $P_{\tau}$ and $P_{\dot{q}}$ with closed-up when the switching occur at 13.125 s (top right); control torques $\tau$ for each joint (bottom left); switching signal $\sigma$: 1 for the decentralized approach, 2 for the centralized approach (bottom right). In this case, switching is required towards the end of the simulation due to a rapid variation of the reference trajectory.

Figure 8. Profiles of velocity $\dot{q}$ and acceleration $\ddot{q}$ (black lines, left axis) and corresponding errors $\dot{e}$, $\ddot{e}$ (gray lines, right axis) for each joint.

Figure 9. Trajectory tracking in the operative space (top left); performance index $J$ and switching thresholds $P_{\tau}$ and $P_{\dot{q}}$ (top right); control torques $\tau$ for each joint (bottom left); switching signal $\sigma$: 1 for the decentralized approach, 2 for the centralized approach (bottom right). In this case, no switching is required.
Table 3. Percentage improvements obtained with the proposed switched approach for different velocity profiles

<table>
<thead>
<tr>
<th>$\dot{q}_{\text{max}}$</th>
<th>$\eta_{1\text{RMS Dec.}}$</th>
<th>$\eta_{1\text{RMS Sw.}}$</th>
<th>$\Delta \eta_{1\text{RMS}}%$</th>
<th>$\eta_{2\text{RMS Cen.}}$</th>
<th>$\eta_{2\text{RMS Sw.}}$</th>
<th>$\Delta \eta_{2\text{RMS}}%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0475</td>
<td>206.0</td>
<td>190.0</td>
<td>7.86</td>
<td>0.291</td>
<td>0.169</td>
<td>42.1</td>
</tr>
<tr>
<td>0.0845</td>
<td>236.0</td>
<td>218.0</td>
<td>7.88</td>
<td>0.339</td>
<td>0.168</td>
<td>50.5</td>
</tr>
<tr>
<td>0.19</td>
<td>286.0</td>
<td>264.0</td>
<td>7.85</td>
<td>0.42</td>
<td>0.167</td>
<td>60.4</td>
</tr>
<tr>
<td>0.601</td>
<td>84.3</td>
<td>64.7</td>
<td>23.2</td>
<td>0.206</td>
<td>0.157</td>
<td>23.7</td>
</tr>
<tr>
<td>0.76</td>
<td>133.0</td>
<td>92.6</td>
<td>30.5</td>
<td>0.357</td>
<td>0.166</td>
<td>53.4</td>
</tr>
<tr>
<td>1.35</td>
<td>101.0</td>
<td>79.5</td>
<td>21.7</td>
<td>0.374</td>
<td>0.146</td>
<td>61.0</td>
</tr>
<tr>
<td>3.04</td>
<td>134.0</td>
<td>121.0</td>
<td>9.98</td>
<td>0.871</td>
<td>0.787</td>
<td>9.6</td>
</tr>
<tr>
<td>5.41</td>
<td>142.0</td>
<td>113.0</td>
<td>20.4</td>
<td>0.849</td>
<td>0.819</td>
<td>3.48</td>
</tr>
<tr>
<td>12.2</td>
<td>128.0</td>
<td>117.0</td>
<td>9.07</td>
<td>0.532</td>
<td>0.399</td>
<td>24.9</td>
</tr>
<tr>
<td>48.7</td>
<td>211.0</td>
<td>190.0</td>
<td>10.4</td>
<td>0.744</td>
<td>0.17</td>
<td>77.1</td>
</tr>
</tbody>
</table>

Figure 10. Trajectory profiles in the joint space. Joints must reach $\pi/2$, $\pi/2$ and 0 until $7/8$ of the simulation duration, and then revert the motion.

Figure 11. Trajectory profiles in the joint space. Joints must reach $\pi/2$, $\pi/2$ and 0 until $1/2$ of the simulation duration, and then revert the motion.

Finally, the proposed ISM control approach has been compared with the ADRC method based on an External State Observer (ESO) as discussed in [25]. Results are summarized in Table 4 where it can be noticed that the performance are similar with the ISM control allowing to achieve a slightly better precision, as denoted by the RMS error. Yet, as discussed in this paper, the ISM control does not require the use of any ESO to achieve this satisfactory performance. This makes the proposal...
appealing for practical applications where the use of a low complexity but effective control scheme is required.

7. Conclusions

Thanks to the perturbation estimation property of the ISM control law, it is possible to design a switching architecture for a robot manipulator performing a motion task. Suitably defining a performance index, such an information can be then used to decide whether a decentralized or a centralized approach is best suitable for the control of the system, and it is possible to define a switching rule adapting the control scheme to the required performances. This enables the system to operate under a wider range of conditions without the need of an a priori choice. This can be an advantage in case it is not possible to exactly estimate the inverse dynamics (for example when in presence of VGA) or when abrupt changes in the performance requirements arise. The proposed ISM based switching structure approach has been validated on an academic example of a robot with unmodeled dynamics and on the realistic model of a Comau Smart3-S2 anthropomorphic robot manipulator identified on the basis of real data.

Conflict of interest - none declared.

References


