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Research Article

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Analysis of Monolithic and Sandwich Panels Subjected to Non-Uniform Thickness-Wise Boundary Conditions

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Abstract: The analysis of monolithic and sandwich plates is illustrated for those cases where the boundary conditions are not uniform along the thickness direction, and run at a given position along the thickness direction. For instance, a sandwich plate constrained at the bottom or top face can be considered. The approach relies upon a sublaminate formulation, which is applied here in the context of a Ritz-based approach. Due to the possibility of dividing the structure into smaller portions, viz. the sublaminates, the constraints can be applied at any given location, providing a high degree of flexibility in modeling the boundary conditions. Penalty functions and Lagrange multipliers are introduced for this scope. Results are presented for free-vibration and bending problems. The close matching with highly refined finite element analyses reveals the accuracy of the proposed formulation in determining the vibration frequencies, as well as the internal stress distribution. Reference results are provided for future benchmarking purposes.

Keywords: plates, sandwich plates, free vibrations, bending, sublaminate

1 Introduction

The modeling of two-dimensional layered structures has been the subject of several research efforts in the past. Indeed, their widespread use in many areas of engineering – including, but not limited to aerospace, marine and mechanical applications – stimulated the development

of analysis tools for accurately predicting their response, both in terms of static and dynamic behaviour. In this context, free-vibration, bending, and buckling analysis of composite structures have been successfully carried out referring to equivalent single layer theories (ESL) [1–10], layerwise approaches (LW) [1, 4, 11–14], as well as variable-kinematic strategies [15–25].

The wide majority of the mentioned studies considers boundary conditions of free, simply-supported and clamped edges. It is important to remark that these conditions reflect a modeling choice that inherently assigns null or infinite value to the stiffness of the restraint with respect to one or more directions of the displacement vector. In addition, the definition of the boundary conditions implies a choice regarding how the constraint is specified along the thickness direction: this definition is generally dictated by the degrees of freedom introduced by the kinematic model and/or the solution technique adopted, and does not necessarily reflect the actual modeling needs. For instance, FSDT models do naturally suggest the possibility of restraining the plate at its middle surface, as far as the generalized displacement components of the kinematic model are directly associated with displacements and rotations at the midsurface. On the contrary, this is less intuitive in the case of high-order ESL models, where the generalized displacement components are not directly associated with physical displacements, and restraining the overall edge displacements is then the simplest choice.

The mentioned simplifications can be in contrast with the real practice of engineering structures, where the possibility of obtaining accurate predictions is subordinated to the ability of properly specifying how the plate is constrained at its boundaries. In this context, improved modeling of the boundary conditions may imply the need for considering restraints of finite stiffnesses, as well as non-uniformity along the edge or the thickness direction.

The elasticity of the restraints has been considered, for instance, in Refs. [26, 27] with regard to the in-plane vibrations of isotropic plates in Ref. [26] using the superposition method. The case of non constant stiffness along the edges

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