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Combining aesthetics and engineering specifications for fashion-driven product design: a case study on spectacle frames

Abstract

The successful combination of aesthetic and engineering specifications is a long-standing issue. The literature reports some examples where this problem was achieved developing tools to support the automatic generation of new product shapes, embedding and linking predefined rule-sets. Notwithstanding, these kinds of tools are effective if and only if the relations among these specifications are known. Other complementary strategies act upstream by building a common ground: they aid in the formalisation of these specifications, fostering the use of a shared language and the same level of detail. This paper lies in between the previous approaches, since its purpose is the description of a strategy to formalise the relations among aesthetic and engineering specifications and whose validities are not affected by the product variability. Indeed, fashion-driven products are subject to continuous innovations and changes. Therefore the identification of these predefined rule-sets is challenging. In detail, the paper objective is to build a high-level and long-lasting formalisation of these relations, based on topological and functional rules. To demonstrate the effectiveness of this approach, we developed a case study in the eyewear industry. We started considering the spectacle-frame functionality and derived the high-level formulation linking aesthetic and engineering specifications. We used this formulation to generate an abstraction of the frame geometry, i.e., an archetype, to be used as a reference for the design of new collections. We implemented the archetype through a MATLAB script and we translated it into a design tool, to wit an Excel spreadsheet. The validity of both the archetype and the tool has been tested, in collaboration with an eyewear manufacturer, designing and manufacturing two new models of frames.

Keywords:
Design specifications; Product variability; Design methods; Design tool; Archetype; Eyewear industry

1 Introduction

Nowadays spectacles are much more than devices for patients needing them for vision correction (e.g., corrective glasses) or protection (e.g., sun glasses):
in years, fashion trends and eyewear industry have transformed eyeglasses from being a medical device to a fashionable accessory. Indeed, nowadays it is not so uncommon to find people wearing spectacles without prescription lenses (e.g., see [1]).

This transformation has undoubtedly led to positive consequences for the acceptability of spectacles in the society, as also underlined in [2]. On the other hand, this strong focus on the aesthetic characteristics of the product is constantly pushing the limit in search of new style-lines through the generation of innovative shapes, and the introduction of new materials and surface textures. This innovation strategy, combined with the short (seasonal) time-to-market [3] as well as the hand-crafting essence which still characterises this industry (as an added value for the product), has made the work of engineers more challenging.

First, each spectacles — to be released as high-end wearable medical device — must fulfil a wide range of ISO standards (e.g., ISO 10685, ISO 8624, ISO 12870) and quality control requirements. The design process is thus characterised by a continuous search for the optimal shape, simultaneously fulfilling aesthetic, functional, and manufacturing requirements. However, if the fulfillment of the aesthetic demands represents a priority especially for certain collections, finding the optimum is not an easy task. It is emblematic, for example, that not all sunglasses of luxury collections can accommodate prescription lenses. Despite high-brand spectacles are usually characterised by a high level of design innovations, some models require lenses with base-curve radii or geometries, incompatible with non-neutral lenses due to thicknesses and shapes. The design process is thus characterised by many redesign and aesthetic approval cycles, since there are no clear rules and indications which engineers can implement to correlate aesthetic-and-engineering specifications during the spectacle-frame design. This situation is even more stressed by a further element of complexity: the high variability of materials, colours, and shapes to be managed by the R&D departments in a short period, due to the seasonality of the market [4]. All these elements of variability are matched in almost unlimited combinations, to guarantee brand uniqueness.

To support the design of spectacle frames, in this paper we describe a strategy to conciliate aesthetics and engineering specifications. We considered fundamental to make explicit the hidden topological-and-functional rules that control the spectacle shape: the implementation of these rules should guarantee to the designers — on the one hand — the control over the product functionality and the manufacturing constraints, while — on the other one — the possibility to freely modify this shape in order to fulfil the aesthetic requirements. These relations among the spectacle design parameters were used to extrapolate a universal representation — to wit a formalisation — of the frame geometry to be used as a reference whenever a new collection is designed. This reference frame, which we name archetype, can be used to master variability issues. Through this formalisation, we also aimed to generate new knowledge and to develop a dedicated tool to support the design of spectacle frames. We used the MATLAB® R2016a environment for a preliminary mathematical verification of the topological and functional relationships among spectacle components. Thereafter, a Microsoft® Excel® 2016 spreadsheet was used to transform a simplified version of these
relations into design rules, and to provide design inputs to the engineers. We performed a first geometrical validation of the archetype using the CAD system PTC Creo® 3.0. Finally, we validated the correctness of the topological rules as well as the effectiveness of the Excel® spreadsheet through a case study developed in collaboration with an eyewear manufacturer.

The contribution of the paper is thus twofold. First, from the application point of view, the proposed formalisation strategy represents a first attempt of this kind within the eyewear industry. The high product variability which characterises this industrial field — as a consequence of the continuous search for uniqueness and originality — makes this one a relevant research context for testing new effective strategies for conciliating aesthetic and engineering specifications. Indeed, considering how much aesthetics is playing a positive and relevant role in transforming the design of products, this conciliation is particularly crucial to change the social perception of those products used by people having impairments, as already done for spectacles (see [2]). Second, from the theoretical point of view, it demonstrates the effectiveness and the importance of combining aesthetics and engineering specifications, starting from the identification of the functional and topological rules. This kind of formalisation of the relations among product specification transcends the product variability and — in a broader sense — also the specific field. Hence, despite our formalisation strategy has been conceived for eyewear, it could be potentially implemented even for those products where fashion and aesthetics play a leading role in the design process.

The paper is structured as follow. In Section 2, we analyse the strategies already discussed in the literature to conciliate aesthetics and engineering requirements and we also discuss the importance of looking for a reference model, when designing, acting as the link among different perspectives. Section 3 concerns the description of the research context, to better clarify the challenges that we had to address to support the design of spectacle frames. Section 4 is focused on the description of the formalisation strategy adopted to generate this reference model. In addition, in this section we also describe how this model (i.e., the archetype) has been transformed into a design tool. In Section 5, we discuss how both the model and the design tool have been validated. Finally, conclusions are drawn in Section 6.

2 Background

The need of conciliating aesthetics and functionality when designing, is something well-known both in industry and in academia. Reaching such a holistic target is challenging due to, for example, the necessity to integrate all the different competencies and perspectives driving the development of the product (see [5]). In addition, design decisions should be taken not only considering the perspective of the end-user but also how much they will impact on the next phases of the development process and on all the stakeholders involved (e.g., see [6]). Actually, interesting attempts have been already discussed in the literature. Some of them are discussed in this paper, clustered into two complementary groups.
On the one hand, several tools have been developed to guide product developers towards the automatic fulfilment of all the settled design specifications. Some of them are software tools conceived to suggest solutions or propose alternatives on the basis of predefined rules and constraints; others are conceived to stimulate the interaction and an active involvement of the various experts/stakeholders, facilitating the convergence towards a jointly approved solution. On the other hand, effort has been also put on the importance of using a shared vocabulary and the same level of detail when describing these specifications. These approaches make easier the discussion about the relevance and the weight of the same specifications and the settled design targets.

Concerning the first group, for example, as underlined by [7], since separating form and function could lead to unexpected manufacturing issues, in their work the authors describe a computer-aided design tool which use rules (ergonomic, aesthetic and manufacturing) to guide the generation of preliminary concepts of the new product. Being their generation “rules-driven”, the settled requirements are automatically fulfilled. Another approach is discussed in [8], where an archetype is used as a starting point for generating new shapes through an evolution process conceived to take into account product requirements. In [9] the authors have embedded a model describing user’s shape preference into a design optimisation algorithm. The intent is to provide a design tool able to suggest to the engineer the optimal shape which is the one satisfying both users’ and engineering requirements. A different strategy to make easily converging the work of the stylist and the one of the engineer is described in [10]. In this work, the authors discuss a prototype of a desktop system able to automatically acquire in real-time the surface of the physical model elaborated by the stylist so as the engineer can immediately evaluated it and thus ask for changes when necessary.

Concerning the second group, as underlined, it is not only a matter of using the same terms, but also the same level of detail when setting these specifications. Indeed, aesthetic requirements are usually expressed in qualitative terms. They are also difficult to catch since they depend on humans’ needs and preferences. For this reason, several researchers are exploring strategies to help designers to translate these needs and preferences into quantitative indications. For example, in [11] a methodology based on the use of the Repertory Grid Technique is proposed to extrapolate and translate, into measurable requirements, the latent needs of prosthetic users. This information should then guide the design of the aesthetic of prostheses. In [12] it is demonstrated how the quantification of Gestalt principles could be another effective strategy for deriving indications about the aesthetic appraisal of a product. Extending this analysis to the quantification of the product experience, in [13] it is argued that virtual and mixed reality prototypes could be of great help to this aim. Indeed, their introduction into a specific step of the product development process makes possible the translation of part of the multisensory product experience into design specifications.

Summarising, we can note that the first group of approaches can help once both the specifications and their mutual relations are known (see the third step mentioned in Figure 1). Rule-based systems can be built to guide experts’ decisions together with tools able to stimulate collaborative design-strategies. The second group of approaches acts upstream, since they help
Step 1
Identification and formalisation of design specifications

Step 2
Identification and mapping of the relations among these specifications

Step 3
Implementation of this map into design tools (e.g., rule-based systems, collaborative design tools)

Figure 1: The three main research actions analysed in Section 2 concerning the identification, formalisation, and use of the design specifications and of the relations among them. The teal boxes highlights the aspects discussed in this paper.

to catch and describe these specifications (see the first step in Figure 1).

Our research contribution is in between the two cited groups and also complementary to them. Our purpose is defining a strategy to identify and map the relations among these specifications. In addition, this formalisation has not to be affected by the product variability (see the second step in Figure 1). The fashion products are emblematic from this point of view, because their continuous variability makes challenging to build a network of relations. In this sense, a proper formalisation strategy allows us to build a model of the product that can be used as a reference during the whole design process and to generate new design tools. That is the reason why also the third step is highlighted in Figure 1. In particular, the product variations can be free since the formalisation always guarantees the coherence of the all specifications.

Obviously, the use of models to drive the design process, is a recognised strategy, especially in the engineering field to support the concept design phase (e.g., see [14, 15]). For example, models driven by rules are at the basis of Knowledge Based Engineering (KBE) Systems (e.g., see [16, 17]), since they are used as a tool for the integration of the product knowledge coming from different engineering disciplines and sources. They are not a static representation of the information since KBE systems allow these models to evolve to certain implemented rules [17]. Given the high relevance that product models are playing, several research activities are thus focused on elaborating new knowledge representation tools and strategies (e.g., see [18] for an interesting literature review). The final aim is to enable a synthesis among different perspectives and design targets as well as to make tacit knowledge explicit (e.g., see [19]). For example, in [20] the authors describe an evolution of the Design Structure Matrix (DSM), discussing a methodology for the generation of a semantic representations of the product to control the behaviour of mechanical products upon certain conditions. In [21], a graph-based approach is described to guide the ideation of efficient and technical feasible
mechanisms taking into account design constraints. In [16], an approach based on the definition of a knowledge-based master model is defined in order to allow experts — in multiple engineering fields — to work concurrently on the same design problem. In other works (e.g., see [22]), this reference model is represented by the CAD model, which is seen as the connecting element among different engineering disciplines. In the garment industry, mannequins — to wit the 3D topological model of the human body — are used as reference when designing clothes (e.g., see [23, 24]). In this case, the reference model is not the product but rather of the “end-user”.

With respect to this body of knowledge, our research is a further and novel demonstration of the importance of using models during design. As already underlined in Section 1, no similar approaches in the same industrial field (i.e., the eyewear industry) have been already described in the literature therefore. This paper is also a novel example, since the building of the model is neither tailored to a specific phase of the design process (e.g., concept or detailed design phase) nor based on an existing, or defined on purpose, formalisation language or tool. We focused on the functional-and-topological aspects that characterise the main features of the product geometry, since the objective is to guarantee that the validity of the model would not be affected by the continuous variability of the product. In addition, the resulting model is thought to be used as reference by the various experts involved in the development of a new product. In case of fashion-driven products, the required competencies are highly multidisciplinary and not only focused on the engineering field.

However, we need to better explain the motivations that have driven our research activity. For this reason, it is important to firstly provide a brief description of the main challenges to be faced when designing spectacle frames, so as to clarify the research context. They are discussed in the next Section 3.

3 Research context: variability issues in the development of spectacle frames

Stylists of fashion brands often consider eyeglasses as an integral part of seasonal collections; therefore, their shape, colour, texture, decorations, and style must be aligned to ones selected for the other elements of the collection (e.g., accessories, clothes). Furthermore, each visage has peculiarities (e.g., shape, dimensions, nose shape and size, eye distance, cheekbone conformation, head shape, etc.) and each person has personal preferences (e.g., desired style, tastes, different usage needs, fashion influences, etc.). Spectacles should fit all these requests. The logical consequence is the high variability that needs to be faced when designing frames. Also, eyeglasses can be medical devices. Hence, in this case, the visual correction must be feasible. All these aspects impact seriously on the frame geometry, thicknesses, materials, type of usable lenses, weight distribution, etc. For all these reasons, the choice of a new frame is very often guided by specialists in optical stores which facilitate the match of the needs and fashion preferences of the customers.
The geometry of a spectacle frame is characterised by three important angles: face-form, pantoscopic and opening-temple angle (Figure 2). The face-form and the pantoscopic angles influence both the aesthetics and the functionality of the frame. Precisely, these two angles radically affect the frame envelopment over the user's face — an important stylistic feature — and also control the orientation of the lens with respect to the position of the eye. The last aspect represents the main function of a spectacle frame, and it is a critical characteristic in the case of prescription lenses. The opening-temple angle influences, instead, the wearability of the frame because it directly controls the temples orientation with respect to head and ears. Consequently, wearability can be considered as another functional characteristic of the product, to be carefully taken into account during the frame design.

The values of these angles are set at the beginning of the design phase by both stylists, market experts, and engineers. Then the stylists develop a concept of the new frame, releasing pictorial drafts and fixing some key dimensional characteristics of the frame. The concept is then actualised in a physical prototype, establishing also the preliminary geometric dimensions of the frame (i.e., lens shape, lens curvature radius, horizontal boxed size, angles). The engineers are consequently able to generate a detailed CAD model of the frame, using the preliminary dimensions, the pictorial drafts, and the physical prototype.

To guarantee the observance of the value set for the angles, this design workflow undergoes several re-design and re-manufacturing cycles, due to the impossibility to predict the resulting shapes of the final product. A con-
sistent number of issues often occur, altering the angles measured in the produced frames. Delays in the entire production chain could, thus, occur. These issues will be discussed in Sections 3.1 and 3.2.

It is worth mentioning here, that this discussion is focused on cellulose acetate frames. Indeed historically, acetate frames represent one of the main market segments of the eyewear industry, thanks to the wide customisation possibilities — in terms of hues, textures, and achievable geometries — offered by this material.

### 3.1 Variability due to the frame geometry

The shape and the size of the lenses are certainly the first class of parameters to be considered when discussing about the variability of the frame geometry. Indeed, some eyewear manufacturers and opticians classify the different frame shapes on the basis of the lens shape. The main lens shapes are conventionally defined as: rounded, oval, rectangular, squared, pilot, butterfly, cat, etc. (Figure 3). In any case, this classification may vary according to the manufacturer/opticians.

The second class of parameters is related to the rim characteristics, to wit the part of the frame that surrounds the lenses. Indeed, also this one could vary: the material can wrap completely both the lenses creating a monolithic structure (full rim), any lens can be alone and the assembling is made through a metal bridge (cerchietti type), the lower (or the upper) part of the front frame can be missed (semi rim), or the lateral part can miss (fork type). For example, an overview of the main frame shapes is available here [26].

Other features concern the geometrical dimensions of the frame and the presence of further components. For example, a third class deals with the...
frame front characteristics in terms of: thickness (fix or variable), width, decorations (e.g., incisions, combination of different materials, insertion of beautifiers). The fourth class of parameters are related to the bridge (i.e., the beam above the nose and between the lenses, see Figure 2). The bridge could vary: in shape (e.g., thin, large, curved), in position (the height respect to the nose), and in the way it is connected to the lenses. The fifth class of parameters deals with the lugs, which are the lateral parts of the front that are joined with the temples (Figure 2). The lugs can also vary both from the geometric point of view (e.g., their height, thickness, and their vertical position with respect to the lens) and in the way they are assembled with the frame front (e.g., lateral, frontal, bent, etc.). Finally, another fundamental geometric characteristic of the front is the lens enveloping on the visage. As anticipated in Section 3, this feature is obtained associating conveniently the lens shape and size with the base curve radius of the lenses, the face-form, and the pantoscopic angles for a specific model of spectacles.

Further categorisations of the frame variability could be added to this description. However, we believe that the information provided until now is sufficient to demonstrate the wide amount of possibilities that stylists have for innovating the frame shapes. This high geometric variability significantly impacts the design process in terms of re-design cycles: these new shapes must not only properly fulfil aesthetic requirements but also — once all the elements of the frame are assembled — satisfy the values set for the angles. However, since up to know there are no explicit rules explaining how the geometry of the frame and the way this one is manufactured influence spectacle functionalities (i.e., the values of the face-form, pantoscopic, and opening-temple angles) a number of physical prototypes need to be built. They are used for tuning the shape geometries so as to get the desired angles. Such tuning is made by the engineers, mainly on the basis of their expertise. Any change of the frame geometry needs to be consequently validated, and the entire process is inevitably affected by inefficiencies.

### 3.2 Variability due to the frame material and the manufacturing process

Together with the variability of the frame geometry, we have also to consider the variability given by the kind of material used, as well as the ones introduced by the manufacturing process. Indeed, the values of the face-form, pantoscopic, and opening-temple angles are not only influenced by the frame geometry, but also by the bending phase (i.e., the so-called meniscatura phase) that each acetate frame undergoes in order to take on the desired front curvature: each flat pieces of material is first milled, then heated up, and finally curved through a conformational press. Here, issues are of two kinds. The first is related to the high variability of the material used for manufacturing the frame. Indeed, by simply changing some characteristics of the cellulose acetate (e.g., different additives) in order to get, for example, different surface textures or colours, the thermoforming behaviour of the material can vary. The second issue deals with the fact that the meniscatura is a manual process that involves the thermo-mechanical behaviour of the materials. Hence, engineers have also to take into account this aspect when
designing the frame geometry in order to counterbalance any curvature variations.

Usually, a high number of different frame-models are concurrently manufactured. In order to optimise production times, for models whose geometries have a similar face-form angle, it is not uncommon that the same mould-surface is used. This aspect introduces a further element of variability to take into account.

Finally, another source of distortion of the frame curvature is represented by the lens insertion phase during the assembly process. This effect is especially evident in thin frames. Lenses are generally more rigid than frames, consequently thin frames will be conspicuously distorted, while thicker frames will oppose more resistance even if — in this case — an undesirable lens distortion could occur.

It can be noted that a manual adjusting phase of the frame geometry is regularly planned at the end of the manufacturing process. This phase helps the work of the engineers since it guaranties high quality standards for the product and it eliminates any geometric error. However, even if all the ISO and quality goals are always satisfied, overall the whole process lacks of efficiency.

4 The generation of the frame archetype

As already stated in Section 3, the control of the values of the angles is difficult to get due to multiple issues. First, the design and manufacturing processes are both affected by a continuous variability of: shapes/geometries of the frames; angles values; materials. This variability makes difficult the adoption of standard procedures and thus foster the elaboration of ad-hoc solutions/approaches for each specific model, leading to inefficient operating strategies.

As discussed, the final front curvatures depends on: the frame geometry, the frame material, the mould surface, and the meniscatura settings (e.g., heating temperature, time into the press). Considering as feasible the proper tuning of the meniscatura settings in order to control any unexpected behaviour due to the material, the curvature of the front is consequently the result of the meniscatura mould curvature and of the lens insertion process. Hence, in order to support the work of the engineers in the frame-curvature control, the first obstacle that needed to be faced is to understand the relations existing among the frame elements, as already introduced in Section 2. Specifically, we decided to make explicit the functional and topological relations existing among these elements through mathematical equations. This mathematical approach allowed us to define a general and abstract model of the frame (i.e., the archetype), whose validity should not be restricted to a specific model/geometry/aesthetic characteristic, but to any possible frame/collection. To build this model, we started from the analysis of the main functionality of the frame (i.e., keep in the proper position the two lenses) and, basing on that, we derived the mathematical relations which link the frame geometry with its functionality and its manufacturing process (see Section 4.1). Albeit the archetype is a generalised mathematical
model, its formulation can be considerably simplified in some specific cases. We chose one of these cases and we developed an easy-to-use tool (i.e., a spreadsheet), which can help engineers to take advantage of the archetype during the spectacle design phase (see Section 4.2). This spreadsheet embeds the simplified archetype and makes the necessary calculations in real-time to get the data for starting the design of a frame whose geometry fulfil all the requirements.

4.1 The topological relations at the basis of the frame archetype

Although spectacles are nowadays fashion products, the main purpose of a frame is to keep in a exact position a couple of lenses. Since lenses have the main role, the generation of the reference frame (i.e., the archetype) should be focused on them. This model of the frame should thus be constituted by a set of instructions and components/elements of the frame, linked together through rules both topological and functional. Furthermore, the model should be uniquely focused on those parts of the frame, which are directly involved in the generation of the requested frame angles (i.e., face-form, pantoscopic, and opening-temples), in order to simplify the archetype as much as possible. This means to exclude from the analysis any add-on of the frame irrelevant to the purpose (e.g., we omitted beautifiers, nose-pads, etc.).

4.1.1 Controlling variability issues due to lenses insertion

The distortion of the frame curvature during the lens insertion is emblematic of the incompatibility existing between frame-and-lens geometry. Lenses are portions of spherical caps, whose radii and thicknesses depend on the aesthetic requirements and are set for each model (see Section 3.1). In Figure 4, the physical constraint existing between the frame and the lens is represented. The lenses have bevels on their borders, which wedge on the grooves of the frames, fastening the assembly. The bevel runs tangentially to the side edge of the lens, passing through its median surface (Figure 4).

In order to avoid distortion of both lenses and frame curvature, the groove geometry must coincide with the bevels during the assembly. Essentially, the groove must have the same shape of the bevel at the end of the meniscatura phase. In this way, the lenses would not alter the frame curvature during the insertion. That is a first radical difference with respect to the current modelling procedure of the frame, where the groove geometry is automatically drawn starting from the medial plane of the frame shape, instead of considering the real spatial-evolution of the bevel.

The bevel shape $f_b$ can be derived considering that it belongs to the spherical surface of the lens with radius $R$, that is

$$f_b = \begin{cases} x = x(u), \\ y = y(u), \\ z = \sqrt{R^2 - x(u)^2 - y(u)^2} \end{cases}$$

where $u \in [\min_b, \max_b]$. 

11
where \( u \) is a parameter of the parametric shape-functions of the lens \( x(u) \) and \( y(u) \), respectively along \( x \) and \( y \) axes (Figure 5). However, that is not sufficient to determine the exact bevel shape, since we have also to take into account the lens orientations as consequence of the face-form angle of the frame. According to the ISO 7998 [25], this angle could be seen as the rotation of the box lens (see [27]) around its vertical centreline (see Figure 6). Hence, to get the real bevel function \( f_b \), the function \( f_b \) has to be rotated using the rotation matrix,

\[
\hat{R}_y = \begin{bmatrix}
\cos(\alpha_f) & 0 & \sin(\alpha_f) \\
0 & 1 & 0 \\
-\sin(\alpha_f) & 0 & \cos(\alpha_f)
\end{bmatrix},
\]

where \( \alpha_f \) is the face-form angle to impose. From the multiplication, we get

\[
\hat{f}_b = \hat{R}_y \cdot f_g.
\]

The function \( \hat{f}_b \) is a three-dimensional curve which represents the bevel evolution of a lenses inserted in a frame, having \( \alpha_f \) as the face-form angle. Hence, the geometry of the frame groove \( f_g \) should coincide with \( \hat{f}_b \), that is

\[
f_g = \hat{f}_b.
\]

It can be noted that the condition set by Equation (2) has to be always satisfied. This means that engineers should check whether the selected curvature and the thickness of a frame allow for the modelling of a groove having a shape described by the 3D function \( f_g \).

### 4.1.2 Controlling variability issues due to the meniscatura process

The next step is to derive the topological relations to obtain a frame with the requested pantoscopic and opening-temple angles. In this case, the elements of the frame involved in the angle definitions are: temples and frame lugs. Specifically, the implementation of the pantoscopic and the opening-temple angles in a frame entails the rotations of the joint surfaces of each lug respectively around the \( x \) and \( y \) axes (see Figure 2 and Figure 5). Similarly, also the temples have to rotate of the same angles and around the same axes.

Temples and front are assembled trough a hinge, designed to make the joint surfaces of the front, parallel with the temple tips when the spectacle frame is donned (see Figure 2). The joint is consequently a constraint, which
Figure 5: Front view of half of a spectacles frame (flat, before the *meniscatura* bending). The generic parametric-functions $x(u)$ and $y(u)$ represent the shape of the lens (and consequently the bevel), enclosed by the boxed lens $b$. The box centre $O$ represents the centre of the lens, while the $z$-axis (outgoing and perpendicular to the X-Y plane) is the optical axis of the lens. The point $L$ is the centre of the joint surface and $x_L$ and $y_L$ coordinates represent its position.

Figure 6: During the *meniscatura* phase, the frame is bent to obtain the face-form angle ($\alpha_f$). In this image the lens box of Figure 5 is represented as a top view (i.e., as an edge) where the $z$-axis is the optical axes before the *meniscatura*. However, in this image the crimson edge represents the boxed lens before the *meniscatura* phase, while the teal one is the top view of the boxed lens after the bending phase; during this process the lens undergoes a rotation around its vertical centreline, represented by the $y$-axis in the figure (ingoing and perpendicular to the X-Z plane).
binds the orientation of a temple to the orientation of the joint surface. Both
the pantoscopic and opening-temple angles involve the temple orientation
(Figure 2), consequently imposing these two angles implies a modification
of the orientation of the joint surface of the front.

Hence, the final orientation/position of the joint surfaces (Figure 2) is de-
termined by the front curvature, given through the meniscatura mould. How-
ever, the selected mould curvature could not allow a proper orientation of
the joint surface to get the desired pantoscopic and opening-temple angles.
Engineers used to modify the joint-surface orientations basing uniquely on
their experience and the prototype measurements. In order to support this
task, we explored the mathematical relations among the frame curvature
and the lug orientation, taking into account the requested pantoscopic and
opening-temple angles.

Before detailing the topological relations, some hypotheses and consid-
erations about what occurs during the frame bending phase should be high-
lighted. Since the radius of curvature of the frame (about $1 \cdot 10^{-1}$ m) is much
greater than the frame thickness (up to $5 \cdot 10^{-3}$ m), we can assume that the
frame will locally undergo small deformations during the meniscatura phase.
This assumption implies that:

- the meniscatura is mainly a permanent flexural deformation;
- the two frame surfaces of the front (internal and external) are parallel
  also after the meniscatura phase;
- the neutral-surface of the frame does not change its dimensions during
  the meniscatura;
- the external surface of the frame corresponds exactly with the surface
  of the meniscatura mould.

The previous assumptions led us to an essential consequence: given a
datum point of the undeformed frame $P(x, y, z)$, we can determine the cor-
responding point on the deformed frame $P'(x, y, z)$, basing uniquely on the
meniscatura mould shape. In addition, the curvature in the point $P'$ is the
same of the corresponding point of the mould. In this way we can correlate
the position of the joint surface of the flat front with its orientation after the
meniscatura phase, using the mould surface as input.

The surface of a meniscatura mould can be mathematically described
through a parametric surface $\Pi_m$

$$\Pi_m = \begin{cases} x = s, \\ y = t, \\ z = z(s, t) \end{cases}, \quad (s, t) \in D, \quad (3)$$

where $s$ and $t$ are the parameters used to define the mould surface through
the generic function $z(s, t)$, into the domain $D$. Referring to Figures 2 and 5,
the joint-surface centre $L$ of the undeformed frame has coordinate $x_L$ and $y_L$.
Through the meniscatura phase, the frame is deformed and the joint centre
will move to the new position $L'(x_L', y_L')$ (see Figure 7).

On the hypothesis of small deformation — and the consequential non-
deformation of the neutral plane — the length $OL'$ is equal to $OL$ (see Figure 5
Figure 7: The effect of the meniscatura phase on the opening-temple angle. In this image, a simplified representation of the top view of a flat front (in crimson) and of a bent frame (in teal) is represented. The point $L$ is the position of joint-surface centre on the flat front (in crimson), i.e. before the meniscatura (see also Figure 2). The bent frame assumes the configuration of the mould surface and $\ell_x$ is the projection of the deformation on the $xz$ plane. Despite $OL = OL'$, the meniscatura mould moves $L$ in $L'$ and re-orient the joint-surface of $\alpha_m$ around the $y$-axis. If a temple is assembled on this bent frame without a correction of the joint-surface orientation, the opening-temple angle instead of being $\alpha_t$ (see Figure 2), would be $\alpha_{out}$.

and Figure 7). Therefore, also their projection along the $x$ and $y$ axes should be equal. This means that

$$\int_{\ell_x} z(s,t) \, d\ell = \|x_L\| \quad \text{and} \quad \int_{\ell_y} z(s,t) \, d\ell = \|y_L\|, \quad (4)$$

in which $\ell_x$ and $\ell_y$ are respectively the path laying on the mould surface $\Pi_{im}$, orthogonal to the $x$ and $y$ axes (see Figure 7). Solving the two Equations (4), the values $x_L$ and $y_L$ are identified. They represent the new position of the joint-surfaces centre once the front of the frame is bent.

Finally, we can evaluate the local orientation of the mould and consequently also the orientation assumed by the joint surface after the meniscatura. An additional counterbalanced orientation is required to obtain the settled pantoscopic angle $\alpha_p$ and an $\alpha_t$ opening-temple angle (see Figure 2). In Figure 7, the effect of the absence of this counterbalancing is shown, obtaining an incorrect orientation of the joint surface (and consequently also of the temple) in case of opening-temple angles; analogous effects occur for the pantoscopic angle. The final orientation of the joint surfaces — conveniently counterbalanced — are

$$\beta_x = \arctan\left(\frac{\partial z(s,t)}{\partial s} \big|_{L'}\right) - \alpha_p \quad \text{and} \quad \beta_y = \arctan\left(\frac{\partial z(s,t)}{\partial t} \big|_{L'}\right) - \alpha_t. \quad (5)$$

The two angles $\beta_x$ and $\beta_y$ represent the orientations that the joint surfaces of the lugs (and thus of the temples tips) must undergo, in order to get a frame with the requested pantoscopic and opening-temple angles after the meniscatura phase. If distortions due to the lens insertion during
Figure 8: Plot of the Matlab®-script output. This output represents the archetype of a frame having a spherical curvature, oval lenses and the settled pantoscopic, face-form and opening-temple angles. Red lines are used to highlight the joint-surfaces (see also Figure 2).

the assembly phase are avoided, by implementing the topological relations discussed in Section 4.1.1, the frame shape satisfies all the values set for the three angles (i.e., pantoscopic, face-form, and opening-temple). Indeed, once the geometry of the mould surface is known — which is set in order to fulfil the aesthetic requirements —, engineers can use $\beta_x$ and $\beta_y$ to conveniently tune the orientation of the joint surfaces of the front.

4.2 The generation of the archetype and its translation into a design tool

In Section 4.1, we have mathematically described the functional-and-topological relations which link the frame geometry with the other components of the spectacle frame (i.e., lenses and temples). The purpose was to derive the rules through which the frame geometry is correlated with the requested face-form, pantoscopic, and opening-temple angles. Through this analysis we realised that the shape of the lens leads the entire design-and-manufacturing workflow. Indeed, the shape of the front groove must correspond to the one of the lens bevel, to avoid the distortion of the front or of the lenses during the assembly. Then, the meniscatura mould is in charge of not only bending the front, but also of ensuring the adequate orientation of the groove according to the face-form angle (see Figure 2). However, we have seen that this meniscatura phase does not correctly orient the joint-surface of the front. Hence, the engineers have to overcome this negative effect applying a correction in order to get the required pantoscopic and opening-temple angles. To validate the correctness of these considerations we built a model of the frame (i.e., the archetype) on MathWorks® MATLAB® R2016a, embedding the mathematical relations described in Section 4.1.

The necessary input information to create the model is the same of the
one usually used by the engineers when starting the design of the frame, such as: an image of the lens shape selected for the model, the lens curvature radius, the horizontal boxed lens size, the values of the desired frame angles (i.e., pantoscopic, frontal, opening-temple). In addition, the following inputs are also needed: frame thickness, desired bridge-and-lug dimensions and positions with respect to the lenses, the temple sizes.

According to what discussed in Section 4.1.2, as input it is also fundamental to have the geometry of the selected mould surface. As an example, we implemented the case in which the meniscatura mould is made up of two spherical surfaces, whose centres coincide with the ones of the two lenses. This choice entails a groove perfectly centred and coincident with the neutral plane of the frame. The steps carried out to build the model are hereby discussed and reflect the mathematical method introduced in Section 4.1.

The bevel shape is automatically extracted by the lens image and projected on a spherical surface. The \(R_y\) rotation matrix is applied (see Equation (1)) to the bevel shape in order to apply the face-form angle. The new position of the centres of the lenses are calculated since the \(f_b\) function is now known (see Section 4.1.1). Using these new centres, the lens surfaces as well as the rims are generated (taking into account their thicknesses which are inputs of the model). Then the bridge is modelled taking into account its input data (i.e., its position and dimensions). The lugs — and particularly their joint surfaces — are automatically drawn using as input the results obtained from Equations (5).

A representation of the final output of the MATLAB® script is shown in Figure 8. This image represents the archetype of a frame having a spherical curvature, an oval lens shape (see also Figure 3) and the settled face-form, pantoscopic and opening-temple angles. Starting from this archetype model, not only some manufacturing information is already set at the design phase, but the engineer can use this model to fine-tuning the geometry of the frame in order to add stylistic changes.

The mathematical formulation of the archetype allows in addition to evaluate the orientation corrections for the joint surfaces. The solution of the equations could not be easy to find — especially with complex geometries — but in some cases the entire problem can be simplified and solved through basic geometric formulations. For example, considering the same case as before, i.e., a mould having a spherical surface, as already underlined, the groove coincides with the median surface of the frame; in this case the conditions set by the Equation (2) and by the imposition of the face-form angle are automatically satisfied moving conveniently the spherical centres of the mould. In addition, integrals of Equations (4) can be solved evaluating arcs of circumferences and the orientation angles of Equations (5) can be evaluated through goniometric calculations.

Hence, we decided to explicit these simplified equations into a Microsoft® Excel® 2016 spreadsheet. In this way, instead of using the whole mathematical formulation — when designing — engineers can make use of this tool to get the output data necessary to generate the 3D model of frame that has to be curved using a spherical meniscatura mould.

Once inserted the requested angles (i.e., face-form, pantoscopic, and opening-temple) and some geometric information related to the frame (i.e., horizontal lens dimension, bridge length, and lug position) into the
Figure 9: A screenshot of the Microsoft® Excel® spreadsheet with embedded the archetype equations for the case of spherical mould. Once inserted the necessary input information, the spreadsheet evaluates in real-time the orientation of the required joint-surface and main architectural data related to the meniscatura mould.

spreadsheet, engineers can instantly get, as outputs, the values of the angles of the joint surface together with the main architectural data related to the mould shape. In Figure 9, the spreadsheet tool is shown with the visual guide to help the filling of the input fields.

Using a different mould surface (e.g., planar or cylindrical), the groove-bevel condition is no more satisfied, as well as the median surface condition. Hence, different mould surfaces (e.g., planar or cylindrical surfaces) require also different spreadsheets. In case of more complex or irregular surfaces, due to the complexity of the mathematics to be implemented, a spreadsheet is no more sufficient and in this case e.g. the MATLAB® environment could be used.

5 Validation of the archetype and of the design tool

5.1 Geometrical validation

Both the mathematical model (i.e., the archetype) and the design tool (i.e., the Excel® tool) required to be validated. We carried out both geometrical and industrial validations, which are described in the following sections.

The geometrical validation of the archetype and the spreadsheet was performed creating the 3D model of the frame described in Section 4.2 by means of the CAD system PTC Creo® 3.0. Indeed, fixing the proper orientation for the temples (which is determined by the pantoscopic and opening-temple
angles, see Figure 2) we got that the change in the orientation of the joint-surfaces is equal to the values calculated by the MATLAB® script and confirmed by the spreadsheet. A detail of a joint-surface orientation is shown in Figure 10. Varying the input parameters (i.e., lens shape, horizontal box size, size and position of the lugs, requested angles, etc.), the obtained $\beta_x$ and $\beta_y$ angles match the ones calculated through the Equation (5).

This 3D model of the frame can be considered as the implementation of the archetype: once the key geometric and functional requirements are fixed and satisfied, variations can be applied on it to fulfil the aesthetic requirements, without affecting the product functionality and the manufacturing constraints. This model can be used as a reference by stylists and engineers at the beginning of the design process, when discussing about the input data to be set for the new spectacle collection.

### 5.2 Industrial validation

For the industrial validation we used the same case study described in Sections 4.2 and 5.1, i.e. the one obtained from a spherical mould surface.

A 3D-model of a real frame was built using the data provided by the spreadsheet as output. The 3D model of the flat frame was then bent through a virtual meniscatura mould, whose radii and centre positions were suggested by the spreadsheet. Then — starting from this model — two variations (i.e., two new models of frames) were designed and manufactured. During the meniscatura phase, the operators carefully bent the frame, in order to exclude any variability related to unexpected behaviours of the material (see Section 4). Two batches — consisting of 40 samples each — of frames were produced also in different material hues, then the angles were measured after the lens insertion phase. The average value measured for each angle (i.e., pantoscopic, face-front and opening-temple), coincided with the desired ones showing an acceptable standard deviation equals to 0.4° for the face-form angles (negligible alterations of the values are due to the natural variability of the manufacturing process). These results confirmed once more the correctness of the new design approach. In Figure 11, an image of a
Figure 11: Measurement process of the spectacle-frames angles. Pictures of the lateral and front view were analysed through an algorithm (described in [4]) designed to automatically measure the pantoscopic ($\alpha_p$), the face-form ($\alpha_f$), and the opening-temple ($\alpha_t$) angles. The white lines are graphical representations of the measured angles.

manufactured frame is shown during the measurement process performed by means of the inspection system described in [4].

6 Conclusions

In this paper, we describe a new design approach tested within the eyewear industry in order to support engineers in fulfilling all the aesthetics, functional, and manufacturing specifications set for the product. In years the need for a continuous and rapid innovation of spectacles, in terms of new shapes and materials, has made the design process more challenging. Pursuing such innovation trend is fundamental both from the market and the social point of view. Indeed, as discussed in [2] this strong focus on trends and fashion has positively transformed spectacles from medical devices to fashion accessories. Eyewear manufacturers are nowadays able to guarantee high-quality standards for their products, but reaching this target often implies inefficient re-design and re-manufacturing cycles, due to the high number and the high variability of aspects that need to be simultaneously taken into account.

The product functionality, which is keeping the two lenses in the right position, is determined by two angles (i.e., the pantoscopic and face-form angle) that set the orientation of the frame front with respect to the user visage; the product wearability is determined by the opening-temple angle,
which controls the orientation of the temples. However, these design specifications have to be combined with: the requirements set by stylists, who are in charge to define the main characteristics of the frame geometry; the requirements set by the production engineers, so as to guarantee the efficiency of the manufacturing process. The combination of all these aspects is in charge of the engineers, who have to find the geometry able to satisfy all these specifications. Even if the re-design and re-manufacturing strategy is effective, it is also time-consuming, since there are no explicit indications that the designer can use to quickly correlated them.

Starting from this industrial need, in this paper we propose a strategy to combine aesthetics and engineering specifications, for the design of those products characterised by a high variability. The objective is to build a formalisation of the relations existing among these specifications, whose validity would not be affected by the continuous product changes. This formalisation would act as a solid and long-lasting ground, upon which additional tools and strategies can be built to further speed-up and make more and more efficient the design process. To build this kind of formalisation, we started from the identification of the functional-and-topological relations that link the main features of the frame (the product) geometry to its aesthetic and manufacturing constraints.

This formalisation allowed us to build an archetype, which is a reference model that can be used as a starting point both by engineers and stylists when reasoning about new spectacles collections. Indeed, these ones could be seen as one of the specific developments of this reference frame. This formalisation has been implemented into a MATLAB® script, which allowed us to generate a reference model for a cluster of spectacles.

From the MATLAB® script we derived an Excel® spreadsheet, to be directly used as a design tool. Indeed, through this spreadsheet, engineers can derive the necessary inputs for starting the design of the geometry of the reference frame, upon which all the necessary stylistic changes can be applied. We tested the validity of our approach twice: initially through a model on PTC Creo® environment and then in collaboration with an eyewear manufacturer. Starting from the archetype of a spherical frame surface, we manufactured 40 samples of two new spherical frame models. All the manufactured spectacles fully satisfied the functional, aesthetic, and manufacturing requirements set, demonstrating the correctness and the effectiveness of the proposed formalisation strategy and of the design tool.

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