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WAYPOINT-OPTIMIZED CLOSED-LOOP GUIDANCE FOR SPACECRAFT RENDEZVOUS IN RELATIVE MOTION

Roberto Furfaro*, Roberto Ruggiero†, Francesco Topputo‡, Marco Lovera§, Richard Linares**

The design of a closed-loop guidance algorithm for autonomous relative motion is an important issue within the field of orbital dynamics. In this paper, we develop a closed-loop, waypoint-based, quasi-optimal algorithm that can be employed to execute autonomous rendezvous in relative motion. Specifically, the deputy spacecraft is executing an autonomous rendezvous with the chief spacecraft via a modified version of the zero-effort-miss/zero-effort-velocity (ZEM/ZEV) feedback guidance. Here, the concept of waypoints-based guidance is introduced; they are defined as intermediate position and velocity targets between the departure point and the real final rendezvous. The position and velocity guidance is therefore divided in intervals. The ZEM/ZEV guidance parameters, represented by the coordinates of the final desired position, the components of the final required velocity and the time needed to reach these targets, will be different depending on the time interval. To determine the guidance parameters, referred to as waypoints parameters, different strategies are analyzed. Specifically, a series of optimization problems, based on the minimization of the fuel consumption constrained by the need to achieve high level of position and velocity accuracy, are formulated and solved. The first the case analyzed is the one in which the position trajectory of the spacecraft is unconstrained. The dynamical models considered for this case are the Clohessy-Wiltshire-Hills (CWH) model (circular orbit) and the Linearized equations of relative motion (LERM) model (elliptic orbit). Then, a more challenging case is studied: some nonlinear constraints related to the entire position trajectory are introduced in the optimization problem formulation. It is demonstrated that in all scenarios, the performances are satisfactory both from the point of view of the mass propellant expenditure and of the final position and velocity errors. Finally, the robustness of the waypoint-based ZEM/ZEM guidance is tested by simulating the closed-loop guidance in a higher fidelity dynamical model comprising the Restricted-two-body-problem (R2BP) nonlinear model with perturbations, expressed in form of acceleration. In addition to disturbances,

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a Monte Carlo analysis is conducted to test the system under off-nominal conditions. The results show that the waypoint-based ZEM/ZEV feedback guidance is able to execute not only precise but also quasi-optimal rendezvous maneuvers in perturbed working conditions.

INTRODUCTION

Designing comprehensive and integrated closed-loop guidance and control algorithms for autonomous relative motion is an important problem in space flight mechanics. Indeed, over the past twenty years, a large variety of control and guidance approaches for controlling the chief-deputy relative motion in circular and elliptical motion have been proposed and studied. Most of the proposed algorithms included impulsive and continuous control formulated in both Cartesian and orbital element formulations. Schaub et al. [1,2] provided the basis for devising impulsive feedback control algorithms using mean orbital elements. More recently, following a similar line of though, Anderson and Schaub[3], devised a N-impulse control schema for formation flight in geostationary orbit using non-singular element description. On the continuous control side, both linear and non-linear feedback approaches have been considered. Naasz et al.[4] solved the relative motion control problem via the H_2/H_∞approach using the Clohessy-Wiltshire equations. Massari and Zamaro[5] proposed a control algorithm based on the solution of the state-dependent Riccati Equation. Queiroz et al.[6] presented a non-linear Lyapunov based approach to devise an adaptive controller for multiple spacecraft in formation flight. More recently, Sherrill et al.[7] proposed a method for continuous control of spacecraft formation flight in elliptical orbits based on Lyapunov-Floquet theory. The proposed controller featured a set of time-varying feedback gained to guide the deputy spacecraft toward the rendezvous with the chief satellite.

However, generating closed-loop feedback trajectories that are rooted in optimal control theory is not an easy task. Recently, generalized Zero-Effort-Miss/Zero-Effort-Velocity (ZEM/ZEV) feedback guidance[8] and its robustified version known as Optimal Sliding Guidance[9] have been developed and applied for both planetary landing and general space guidance. The ZEM/ZEV feedback guidance has been studied extensively and can be found in the literature for intercept, rendezvous, terminal guidance and landing applications. Such analytical closed-loop guidance has been originally conceived by Battin[10] who devised an energy optimal, feedback acceleration command for powered planetary descent. Ebrahimi et al.[11] introduced the ZEV concept, as a partner for the well-known ZEM and integrated it with a sliding surface for missile guidance with fixed-time propulsive maneuvers. Furfaro et al.[12] extended the idea to the problem of lunar landing guidance and set the basis for the theoretical development of a robust closed-loop algorithm for precision landing. ZEM/ZEM feedback guidance is attractive because of its analytical simplicity as well as potential for quasi-optimal fuel performance for constant gravitational field. When robustified by a time-dependent sliding term, the resulting OSG can be proven to be Globally Finite-Time Stable (GFTS) in spite of perturbation with known upper bound[9].

More recently, ZEM/ZEV feedback algorithm has been explored as possible closed-loop guidance algorithm for relative motion guidance[13]. More specifically, we studied the guided relative motion of two spacecraft for which one of them is executing an autonomous rendezvous via the ZEM/ZEV feedback guidance as well as its robustifed OSG counterpart. Starting from the classical Clohessy-Wiltshire (CW) model, Furfaro et al.[13] analyzed the ability of the ZEM/ZEV feedback guidance to execute closed-loop maneuvers as well as its ability to correct disturbances for precision guidance. Comparison with numerically-based, open-loop, fuel efficient solution provided an assessment of the algorithm in terms of accuracy and fuel efficiency. It was demonstrated that whereas the algorithm is very accurate in terms of targeting a desired final position and velocity, it performs sub-optimally in terms of fuel consumption. The theoretical optimal guidance gains and the computed time-to-go did not yield good fuel performance. A parametric study demonstrated that such parameters can be changed to achieve better fuel-efficiency, although quasi-optimality was never achieved. Most of the problem is related to the fact that the ZEM/ZEV algorithm was derived under fairly strict conditions, that assumed that the gravitational field is constant or at most time-dependent[9]. In relative motion, such assumption is violated as the mathematical model comprises a "generalized" gravitational field $\boldsymbol{g}(\boldsymbol{r}, \boldsymbol{v})$ that includes spatially and velocity varying terms.

In this paper, we explore a way-point guidance approach as a method to improve fuel performances of the original ZEM/ZEV closed-loop algorithm. More specifically, we devised and tested an algorithm that selects a set of intermediate target points on the way to the final target point and employs the ZEM/ZEV feedback algorithm to sequentially target the intermediate states until the final target is achieved. The problem of how many intermediate target points, its position and velocity as well as the time-of-flight between points is approached using a set of local and global optimization methods. Here, the ZEM/ZEV analytical solution has been embedded in both genetic and particle swarm optimization routines to optimize intermediate states and time of flight. Results are compared with numerically computed open-loop solutions and demonstrate that quasi-optimality is achievable with a limited number of way-points. Importantly, the way-point method is implemented to demonstrate that feasible solutions can be computed for case where constraints are imposed to implement collision-avoidance, i.e. avoid the so-called keep-out zones. Finally, the results show that the waypoint-based ZEM/ZEV feedback guidance is able to execute not only precise but also quasi-optimal rendezvous maneuvers in perturbed working conditions.

RELATIVE MOTION GUIDANCE MODEL

The relative motion of a deputy spacecraft with respect to the chief satellite is generally described in the Local-Vertical Local-Horizontal (LVLH) coordinate frame. The LVLH frame is attached to the chief satellite. In the usual representation of the LVLH coordinate frame, x is directed as the chief satellite radial direction, z is oriented in the direction of the chief's angular momentum (orbital), and y is consequently oriented such that the LVLH frame is right orthogonal and right-handed. Within this framework, the $x - y$ coordinates describe the deputy in-plane motion and the z coordinate describes the out-of-plane motion. For highly eccentric orbits, the equations of relative motion can be described using a linearized model, also known as Linearized Equations of Relative Motion (LERM):

$$\ddot{x} - 2\dot{f}\dot{y} - \left(\dot{f}^2 + 2\frac{\mu}{r^3}\right)x - \ddot{f}y = a_{Cx} + a_{px} \tag{1}$$

$$\ddot{y} + 2\dot{f}\dot{x} + \ddot{f}x - \left(\dot{f}^2 - 2\frac{\mu}{r^3}\right)y = a_{Cy} + a_{py} \tag{2}$$

$$\ddot{z} + \frac{\mu}{r^3}z = a_{Cz} + a_{pz} \tag{3}$$

where f is the true anomaly of the chief orbit, μ is the gravitational parameter of the central body, r is the orbital radius of the chief, a_{Cx}, a_{Cy}, a_{Cz} are the components in the LVLH framework of acceleration command (feedback) and a_{px}, a_{py}, a_{pz} are the components of the perturbing acceleration. The latter may include higher-order terms not considered in the linear dynamics and additional modelled perturbing acceleration different than the two-body Newtonian term (e.g. higher-order gravitational harmonics, solar radiation pressure, third-body perturbation, etc.). The equations can be rewritten in a more compact form using a state-space formulation:

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{4}$$

$$\dot{\boldsymbol{v}} = \begin{bmatrix} \left(\dot{f}^2 + 2\frac{\mu}{r^3}\right) & \ddot{f} & 0 \\ -\ddot{f} & \left(\dot{f}^2 - 2\frac{\mu}{r^3}\right) & \mathbf{0} \\ 0 & 0 & -\frac{\mu}{r^3} \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} 0 & 2\dot{f} & 0 \\ -2\dot{f} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{v} + \boldsymbol{a}_C + \boldsymbol{a}_p \tag{5}$$

Here, we set $\boldsymbol{r} = [x, y, z]^T$, $\boldsymbol{v} = [\dot{x}, \dot{y}, \dot{z}]^T$, $\boldsymbol{a}_C = \left[a_{Cx}, a_{Cy}, a_{Cz}\right]^T$ and $\boldsymbol{a}_p = \left[a_{px}, a_{py}, a_{pz}\right]^T$. The CW equations are customarily obtained by setting the chief eccentricity equal to zero, resulting in the following:

$$\ddot{x} - 2n\dot{y} - 3n^2x = a_{Cx} + a_{px} \tag{6}$$

$$\ddot{y} + 2n\dot{x} = a_{Cy} + a_{py} \tag{7}$$

$$\ddot{z} + 3n^2z = a_{Cz} + a_{pz} \tag{8}$$

or

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{9}$$

$$\dot{\boldsymbol{v}} = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{v} + \boldsymbol{a}_C + \boldsymbol{a}_p. \tag{10}$$

GENERALIZED ZEM/ZEV: THEORETICAL ALGORITHM

The ZEM/ZEV feedback guidance algorithm has its roots in optimal control theory. The waypoint-based feedback guidance is determined analytically by finding the acceleration command that satisfies the unconstrained energy-optimal control problem in a constant (or time-varying) gravitational field and assuming no perturbation is acting on the spacecraft. The following definitions hold true:

Definition 1: We define Zero-Effort-Miss (**ZEM**) as the distance (vector) the spacecraft misses the target if no acceleration command is executed after time t. Formally:

$$\boldsymbol{ZEM}(t) = \boldsymbol{r}_f - \boldsymbol{r}(t_f), \boldsymbol{a}_C(\tau) = \boldsymbol{0},\ \tau \in [t, t_f] \tag{11}$$

Definition 2: We define Zero-Effort-Velocity (**ZEV**) as the velocity (vector) the spacecraft misses the target velocity if no acceleration command is executed after time t. Formally:

$$\boldsymbol{ZEV}(t) = \boldsymbol{v}_f - \boldsymbol{v}(t_f), \boldsymbol{a}_C(\tau) = \boldsymbol{0},\ \tau \in [t, t_f] \tag{12}$$

With ZEM and ZEM formally defined, one can formally solve the following energy optimal guidance problem:

(Energy Optimal Guidance Problem): *find the acceleration command $\boldsymbol{a}_C$ as function of $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ that minimizes the energy-optimal cost (quadratic control effort):*

$$J(\boldsymbol{a}_C) = \int_t^{t_f} \boldsymbol{a}_C(\tau)^T \boldsymbol{a}_C(\tau) d\tau \tag{13}$$

Subject to the dynamical equations of motion as physical constraints

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{14}$$

$$\dot{\boldsymbol{v}} = \boldsymbol{g}(t) + \boldsymbol{a}_C \tag{15}$$

with initial conditions at time t $\boldsymbol{r}(t), \boldsymbol{v}(t)$ and final conditions $\boldsymbol{r}_f, \boldsymbol{v}_f$.

Here, the acceleration command is assumed to be unbounded, that is, no constraints in the acceleration (thrust) magnitude is enforced. The problem can be solved by a straightforward application of the Pontryagin's Minimum Principle (PMP) to determine a set of necessary conditions for the existence of an optimal solution. For this specific case, the resulting Two-Point Boundary Value Problem (TPBVP) has an analytical solution. Indeed the acceleration command can be expressed as linear function of $\boldsymbol{ZEM}(t)$ and $\boldsymbol{ZEV}(t)$ and t_{go} as follows:

$$\boldsymbol{a}_C = \frac{k_R}{t_{go}^2} \boldsymbol{ZEM}(t) + \frac{k_V}{t_{go}} \boldsymbol{ZEV}(t) \tag{16}$$

Here, the optimal guidance gains are found to be $k_R = 6$ and $k_V = -2$. Following D'Souza[1] or Guo[2], an alternative formulation of the generalized ZEM/ZEV guidance algorithm can be determined:

$$\boldsymbol{a}_C = -\frac{6}{t_{go}^2}\left(\boldsymbol{r}(t) - \boldsymbol{r}_f\right) - \frac{4}{t_{go}}\left(\boldsymbol{v}(t) - \boldsymbol{v}_f\right) - \boldsymbol{g} \tag{17}$$

The two formulations of the guidance algorithm are perfectly equivalent only in the case of constant gravitational field. The time-to-go is determined by applying the transversality condition $H(t_f) = 0$. The later generally results in a quartic equation that yields only one feasible positive solution[14].

WAYPOINT OPTIMIZATION FORMULATION FOR ZEM/ZEV GUIDANCE IN RELATIVE MOTION

As demonstrated by Furfaro et al.[13], the ZEM/ZEV feedback algorithm can be employed to accurately execute closed-loop guidance in relative motion. However, a comparison between closed loop trajectories and fuel-optimal open-loop solutions in relative motion, show that the algorithm is very sub-optimal. The reason is due to the fact that the natural dynamics (both linear and linearized) comprises terms that critically depend on position and velocity. One way to overcome the problem is to introduce a set of waypoints. The latter may generate arcs of intermediate closed-loop trajectories where the spacecraft move in a dynamical fields with natural accelerations varying only mildly. In such a situation, the assumption of constant gravitational acceleration is only weakly violated. The position of the waypoints can be determined via an optimization approach. Here, we are interested in determining a set of states (position and velocity) that can be sequentially targeted by the ZEM/ZEV algorithm with the deputy spacecraft on the way to the rendezvous point. The goal is to determine both the position of the states and the time of flight between waypoints. The optimization problem formulation can be summarized in the following way:

$$Find\ \boldsymbol{X}^*\ such\ that\ J^* = min\, J(\boldsymbol{X}^*)$$

$$J = -m(t_F, X) + \sum_{i=1}^{NWAY} A * \left\| r(t_{fi}) - r_i \right\|_2 + \sum_{i=1}^{NWAY} B * \left\| \boldsymbol{v}(t_{fi}) - \boldsymbol{v}_i \right\|_2 + \\ + A * \|\boldsymbol{r}(t_{FINAL}) - \boldsymbol{r}_{FINAL}\| + B * \|\boldsymbol{v}(t_{FINAL}) - \boldsymbol{v}_{FINAL}\| \tag{18}$$

$$\boldsymbol{X} = \left[t_1\ t_2 \dots\ t_{NWAY+1}\ \boldsymbol{r}_1\ \boldsymbol{r}_2\ \boldsymbol{r}_{NWAY}\ \boldsymbol{v}_1\ \boldsymbol{v}_2 \dots\ \boldsymbol{v}_{NWAY} \right]$$

$$\boldsymbol{LB} \le \boldsymbol{X} \le \boldsymbol{HB}$$

It is important to stress that the number of waypoints $NWAY$ is a constant (or parameter) of the problem. As a consequence, the optimal waypoints parameterization is correlated to a specific number of waypoints. The number of variables involved in the optimization problem is related to the number of waypoint through the following relations

$$nvar = 7 * NWAY + 1 \tag{19}$$

The number 7 comes from the sum of the three position and the three velocity components of each waypoint plus the time needed to reach the waypoint itself. The "+1" is instead associated with the time needed to reach the very final target, whose position and velocity are not optimization parameters, but are constants. As for any optimization problem, a performance index to be minimized and the design variables vector have been defined in Eq. (18). They are denoted respectively with J and X. Notice that the basic objective function is set to $-\ m(t_F, X)$, i.e. we are minimizing the negative of the final mass (equivalent to maximize the final mass of the spacecraft). Moreover, in order to guarantee the targeting of each waypoint and the very final point some Penalty terms are added to the basic objective function. This allows to assign a higher score (and so to penalize) the waypoints parameterizations in which the position and velocity errors at each waypoint or at the final one are high. It is straightforward that although the final target in terms of velocity and position is constant, the final penalty term depends on the waypoint parameterization.

The optimization problem is solved for a variety of constrained and unconstrained relative motion scenarios, using a hybrid combination of particle swarm optimization (global optimizer) and sequential quadratic programming (Local optimizer).Here, the goal is to conduct a comprehensive parametric study of the fuel performance of the waypoint-based ZEM/ZEV feedback guidance as function of number of selected waypoints.

RESULTS

An initial analysis of the fuel consumption and accuracy performance of the proposed algorithm is evaluated as function of the number of waypoints. Here, we employ the particle swarm optimization technique to search for the number of waypoints that minimize the fuel consumption when targeted sequentially by the ZEM/ZEV guidance algorithm. The following scenario is considered. It is assumed that the chief satellite is in an initial circular orbit at an altitude of $7500\ km$. The deputy satellite is located in an initial relative position $\boldsymbol{r}(0) = [7047m,\ 5136m,\ 5013m]^T$ in the LVLH coordinate frame. The initial relative velocity is $\boldsymbol{v}(0) = \left[-2.4\ \frac{m}{s}, -13.7\frac{m}{s}, 4.08\frac{m}{s}\right]^T$. The deputy satellite is driven by the generalized ZEM/ZEV feedback algorithm to rendezvous with the chief satellite, i.e. the target point is the origin of the LVLH coordinate system to be achieved with zero terminal velocity. The deputy spacecraft is assumed to have a mass of $2000kg$, exhibiting a propulsion system with specific impulse of $I_{sp} = 204\ s$ and maximum thrust of $16\ N$. In this case, the fuel-optimal solution is shown to provide a final mass of $1991.6\ kg$. The goal is to study the fuel consumption as function of the number of best waypoints as determined by the optimizer.

Generally, any heuristic global optimization technique evaluates the objective function selecting the candidate optimal design variables in a larger search space, with a certain criterion. No initial candidate solutions are provided as input to the function; in this way and the initialization is determined randomly with a uniform distribution inside a limited but specified set. Moreover, some lower and upper bounds are applied to the design variables, i.e. the waypoint parameters.

$$
\begin{gathered}
LB_{time} \leq t_j \leq HB_{time} \\
-lim_{pos} \leq \boldsymbol{r}_i \leq +lim_{pos} \\
-lim_{vel} \leq \boldsymbol{v}_i \leq +lim_{vel} \\
i = 1, \ldots NWAY \qquad j = 1, NWAY + 1
\end{gathered} \tag{20}
$$

The values of limits are reported in Table 1.

$lim_{pos}[m]$	$lim_{vel}[m/s]$	$LB_{time}[s]$	$HB_{time}[s]$
8000	8	1000	8000

Table 1: Lower and upper bounds of the design variables for the Global Optimization

The waypoint position search space in this case can be represented with a cube centered in the origin with a side of 16 km .

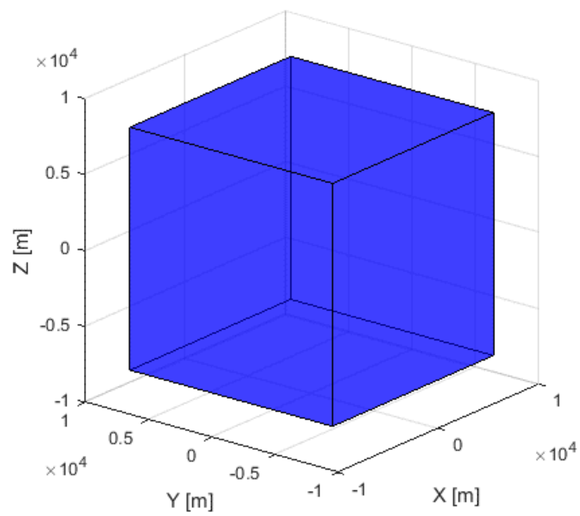


Figure 2: Waypoint position search space of the Global Optimization

The trend of the optimal performances corresponding to the different number of waypoints is shown in Figure 3.

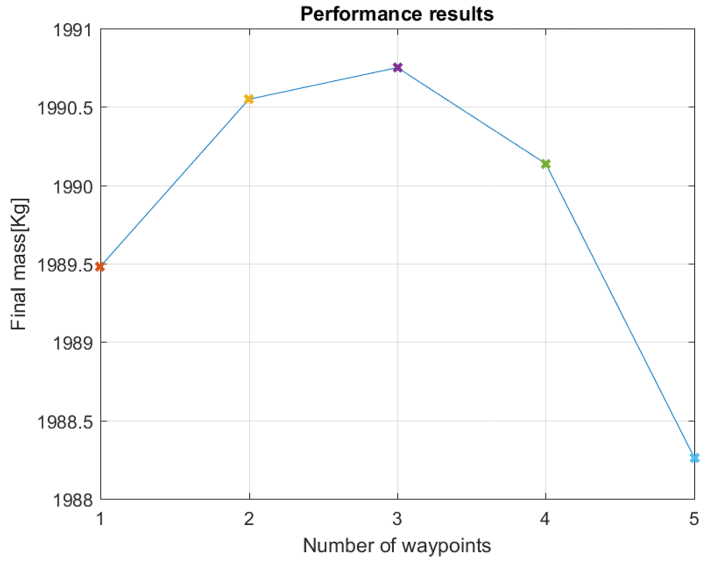


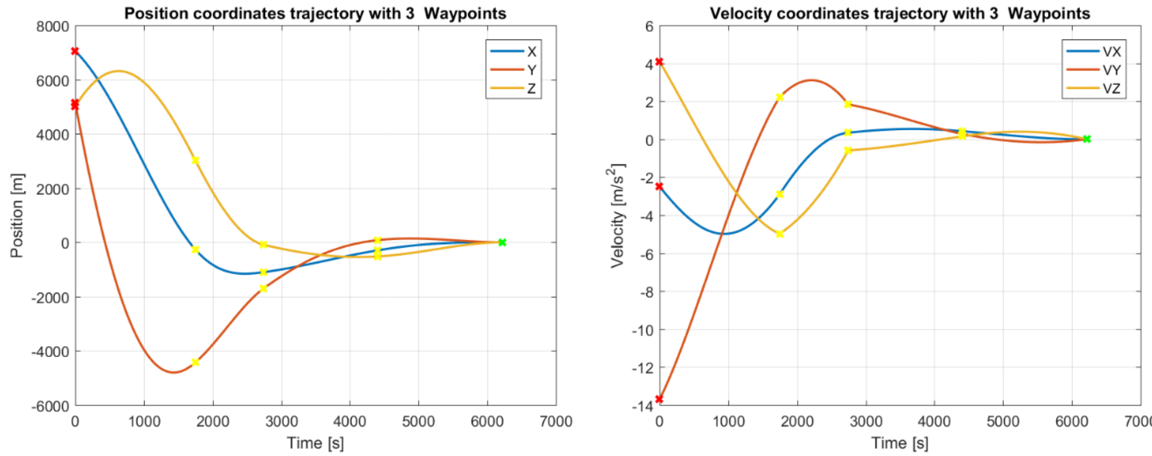
Figure 3: Global optimization performances results with respect to the number of waypoints with CWH

The best (fuel-optimal) solution corresponds to the case with three (3) waypoints. Accuracy and fuel consumption performance are reported in Table 2.

Table 2: Best Result of the Global optimization with CWH

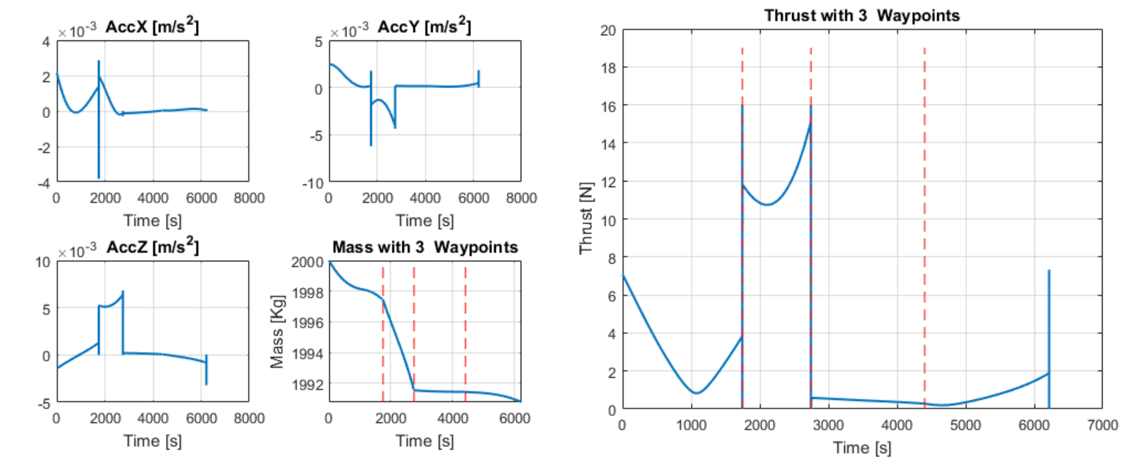
$N°\ wayp$	$FM[kg]$	$ACCP[m]$	$ACCV[m/s]$	$FT[s]$
3	1990,8	2,20e-09	1,02e-05	6217,7

Figure 4 shows the histories of the waypoint-based, guided spacecraft position and velocity. Figure 5 shows the histories of thrust, mass and acceleration command. Figure 6 shows the trajectory in the relative LHLV coordinate frame.



(a) (b)

Figure 4: Position and velocity history along the three coordinates of the best parameterization with obtained with the Global Optimization(CWH)



(a) (b)

Figure 5: Acceleration command components, Mass and Thrust module history of the best parameterization obtained with the Global Optimization(CWH)

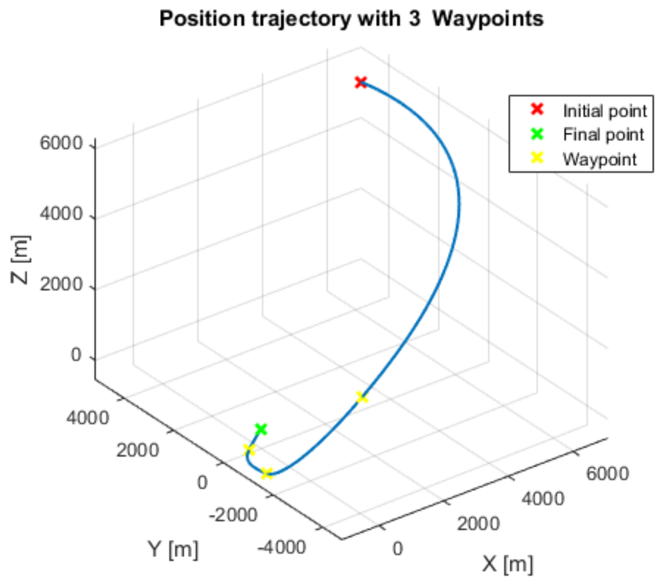


Figure 6: Position trajectory of the best parameterization obtained with the Global Optimization(CWH)

In order to emphasize the improvement obtained with the outcome of the Global Optimization, the results found with the different control strategy and waypoints parameters assignment are reported in Figure 7.

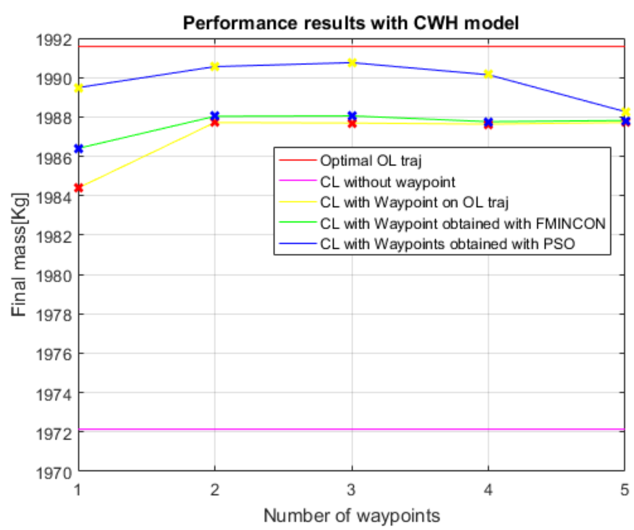


Figure 7: Performance results comparison for unconstrained position trajectory with CWH

The overall fuel performance for the various cases is reported in Figure 8.

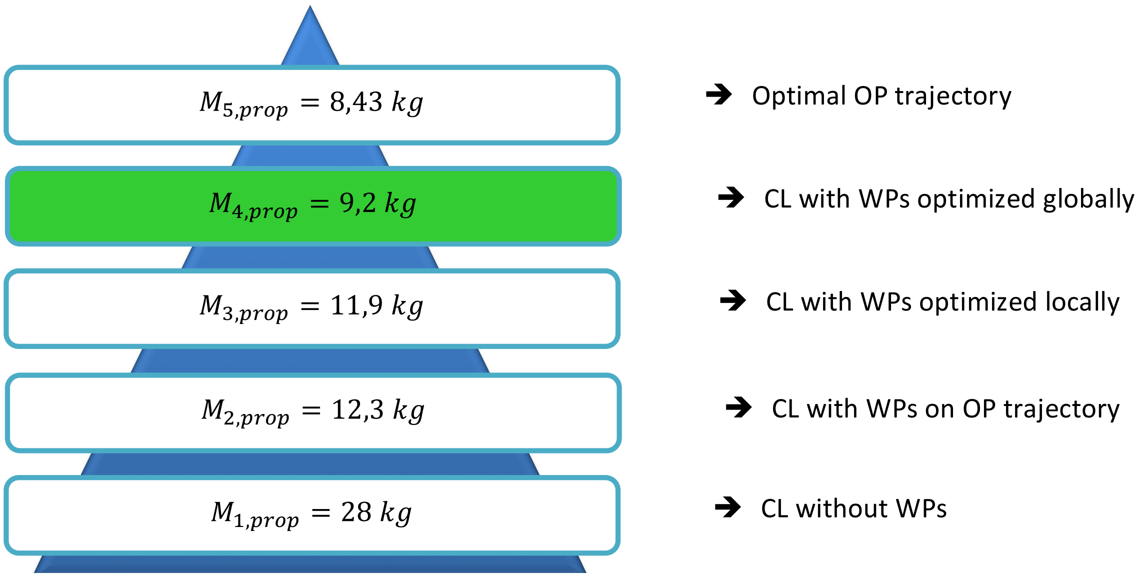


Figure 8: Propellant mass consumed for each guidance strategy. Here, we report details of the closed-loop guidance with WayPoints (WP) optimized via Particle Swarm Optimization

Constrained Way-point Optimization

Most studies assume that the deputy satellite may access any region of the configuration space. However, many scenarios of recent orbital maneuvers involve operations in the proximity of large scale structures, such as a space station. In these case, accidental contact of the flight objects could lead to dangerous situations. Consequently, one of the essential requirements for rendezvous operations is the ability to execute a maneuver which avoid the collision before an eventual docking procedure. Moreover, such scenarios may include additional constraints such as obstructions which are fixed or moving in the neighborhood of the target. Such obstructions may be other spacecraft, station keeping near the target itself or a nonphysical volume, such as an antenna radiation beam.

In this section we report the case where a quasi-optimal waypoint-based ZEM/ZEV closed-loop solution is sought via constrained optimization. The problem consists of forcing the path of the spacecraft inside the same cone but outside a sphere placed between the departure and destination points. Here, it is necessary to introduce in the optimization problem formulation, a constraint which involves the entire position trajectory and not only the waypoints positions. The new non-linear constraints can be expressed considering the Cartesian Equation of the cone and the sphere in the space in the following way:

$$\frac{r_x(t)^2}{a^2}+\frac{r_y(t)^2}{b^2}-\frac{r_z(t)^2}{c^2}<0$$

$$-(r_x(t)-x_c)^2-(r_y(t)-y_c)^2-(r_z(t)-z_c)^2\ +\ R^2<0 \qquad 0<t<t_{FINAL} \qquad (21)$$

The "Constraint function" represents an input of the particle swarm algorithm. In the search of the optimal parameterization, the evaluation of a candidate value of design variables is performed computing the cost function and checking the constraint separately. In order to increase the computational efficiency, the formulation of the optimization problem can be modified transforming the explicit nonlinear constraint in a penalty term in the cost function definition. In this way the integration of the dynamical model differential equations is performed only once in the evaluation of the cost function. The new optimization problem formulation can be summarized as follows:

$$J=-m(t_F,X)+\sum_{i=1}^{NWAY}A*\left\|r\left(t_{fi}\right)-r_i\right\|_2+\sum_{i=1}^{NWAY}B*\left\|\boldsymbol{v}\left(t_{fi}\right)-\boldsymbol{v}_i\right\|_2+$$

$$+A*\|\boldsymbol{r}(t_{FINAL})-\boldsymbol{r}_{FINAL}\|+B*\|\boldsymbol{v}(t_{FINAL})-\boldsymbol{v}_{FINAL}\|+$$

$$+C*\ Traj\_Constr(x,y,z) \qquad (22)$$

$$\frac{{r_{i\,x}}^2}{a^2}+\frac{{r_{i\,y}}^2}{b^2}-\frac{{r_{i\,z}}^2}{c^2}<0\ \ ,\ \ i=1,\dots,NWAY$$

$$-(r_{i\,x}-x_c)^2-(r_{i\,y}-y_c)^2-(r_{i\,z}-z_c)^2\ +\ R^2<0\ \ ,\ \ i=1,\dots,NWAY$$

The new term $C*Traj\_Constr(x,y,z)$ allows to penalize the waypoint parameterization whose position trajectory does not satisfy the constraint. The penalization value is defined through the function $Traj\_Constr(x,y,z)$ which takes as input the position coordinates at every time instants and gives as output the number of points in the space located inside the sphere or outside the cone.Moreover, with the purpose of guaranteeing an appreciable distance between the satellite position trajectory and the spherical surface, a certain tolerance value is added in the Cartesian equation of the sphere.

$$-(r_{i\,x}-x_c)^2-(r_{i\,y}-y_c)^2-(r_{i\,z}-z_c)^2\ +\ (R+tol)^2\ \ ,\ \ i=1,\dots,NWAY \qquad (23)$$

The upper and lower bounds associated to the design variable are kept equal to the previous problem. Also in this case the optimization problem is solved considering a number of waypoint varying from 1 to 5 and with two values of tolerance. The results are summarized in figure 9

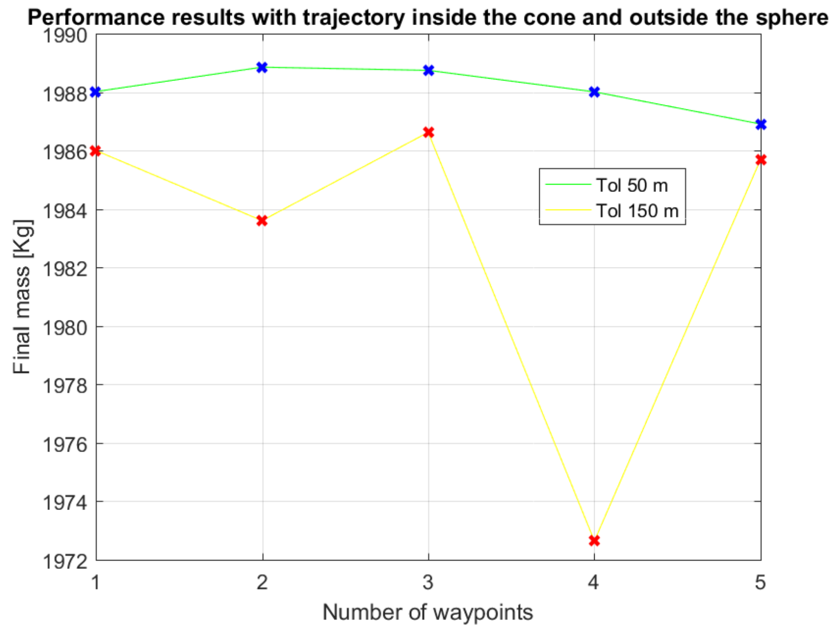


Figure 9: Fuel performance results with the position trajectory inside a cone and outside a sphere as function of the number of waypoints and tolerances.

For each value of tolerance, the best performance is detected in terms of fuel consumption. The resulting data and the corresponding position trajectory are reported in table 3 and 4 for the $50m$ and $150m$, respectively:

Table 3: Best Result Position trajectory inside a cone and outside a sphere case with tolerance of 50 m

$N° \, wayp$	$FM[Kg]$	$ACCP[m]$	$ACCV[m/s]$	$FT[s]$
2	1988,8	6,23e-08	1,61e-05	3004,0

Table 4: Best Result Position trajectory inside a cone and outside a sphere case with tolerance of 150 m

$N° \, wayp$	$FM[Kg]$	$ACCP[m]$	$ACCV[m/s]$	*FT[s]*
3	1986,6	5,76e-08	7,75e-06	4474,7

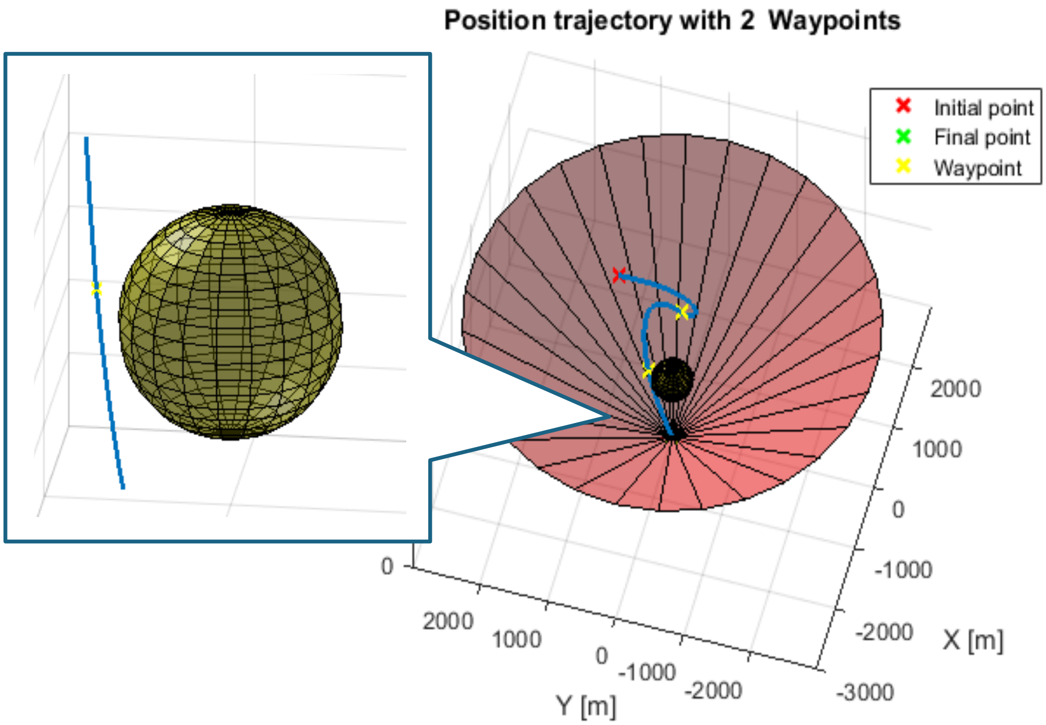


Figure 10: Best state trajectory inside a cone and outside the sphere with tolerance of 50 m.

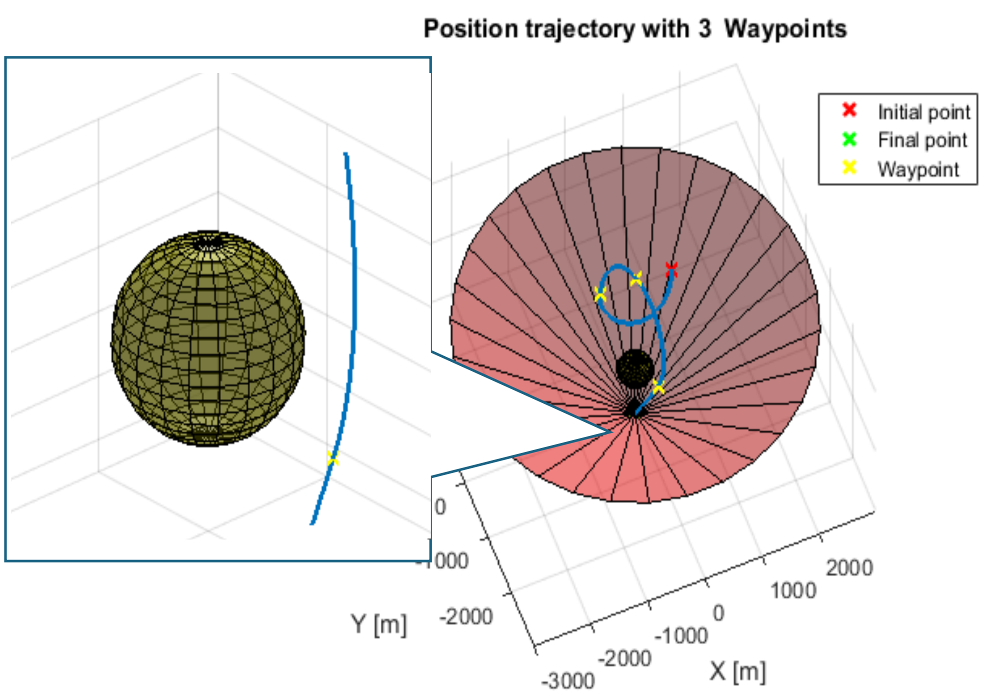


Figure 11: Best state trajectory inside a cone and outside the sphere with tolerance of 150 m.

Figure 10 and 11 show the resulting optimized way-point trajectory. As expected, passing from a tolerance of 50 m to a tolerance of 150 m, the distance between the position trajectory and the spherical surface grows. Furthermore, the results show that by increasing the tolerance, the performances in terms of fuel consumption decrease for all number of waypoint. This is due to the fact that the constraint, in the second case, forces the trajectory in a stronger way with respect to the previous one. The requirement of avoiding the sphere, beside remaining inside the cone, leads to a higher propellant consumption. In conclusion, we demonstrate that the Waypoint based ZEM/ZEV guidance can be adopted also in the collision avoidance field. Even though a comparison with an optimal open loop trajectory is not provided, the performances concerning the fuel expenditure and position and velocity accuracy can be considered acceptable.

CONCLUSIONS

A waypoint-based ZEM/ZEV closed-loop algorithm for relative motion guidance has been presented. The proposed approach was motivated by the fact that previous studies showed that optimal control theory can yield a simple and easy to mechanize feedback guidance algorithm that is energy optimal for the unconstrained motion of the spacecraft in a constant or time-dependent gravitational field. For a more general gravity field encountered in applications such as relative motion guidance, the ZEM/ZEV scheme yields accurate closed-loop trajectories that are generally heavily sub-optimal. An optimized waypoint-based scheme is developed and tested to show that quasi-optimal performances can be achieved by sequentially targeting intermediate points. We presented a study where the performances of the waypoint-based ZEM/ZEV algorithm are analyzed as function of the number of the targeted waypoints, waypoint states and intermediate time-of-flight. It is shown that when compared to numerically generated fuel-efficient optimal trajectories, quasi optimal performances can be achieved. The optimization approach can be modified to include additional state constraints and generate closed-loop trajectories that keep the guided relative motion trajectories in safe zone while avoided prescribed keep-out zones.

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