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WAYPOINT-BASED ZEM/ZEV FEEDBACK GUIDANCE: APPLICATIONS TO LOW-THRUST INTERPLANETARY TRANSFER AND ORBIT RAISING

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Low-thrust guided trajectories for space missions are extremely important for fuel-efficient autonomous space travel. The goal of this paper is to design an optimized, waypoint-based, closed-loop solution for low-thrust, long duration orbit transfers. The Zero-Effort-Miss/Zero-Effort-Velocity (ZEM/ZEV) feedback guidance algorithm which has been demonstrated to exhibit great potential for autonomous onboard implementation is applied in a waypoint fashion. Generally, ZEM/ZEV is derived by solving an optimal guidance problem under well-defined assumptions, where the gravitational acceleration is either constant or time-dependent and the thrust/acceleration command is unlimited. If gravity is not constant, the target state is generally achieved in a suboptimal fashion. A way to improve the performances is to divide total trajectory into many segments, and determining with a rigorous optimization method near-optimal waypoints to connect the different segments. Here we consider two possible scenarios, i.e. 1) a low-thrust transfer Earth-Mars and 2) a low-thrust orbit raising from LEO to GEO. For both cases, open-loop energy and fuel-optimal trajectories generated by L.Ferrella and F.Topputo[13] are considered as reference trajectories where a set of arbitrary points are targeted by the ZEM/ZEV guidance in a sequential fashion. An initial parameteric study is conducted to evaluate guidance performances as function of the number of the selected waypoints. Subsequently, a global optimization problem, parametrized with the position of the points on the trajectory is solved using a genetic algorithm to determine the minimum set of waypoints necessary for close-to-fuel-optimal waypoint space guidance. The optimization results are compared with the parametric analysis for both scenarios to show that the proposed approach is feasible in achieving quasi-optimal performances even for challenging cases where 500 revolutions are required for low-thrust orbit raising in the

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Earth gravitational field. Finally, the proposed waypoint-based guidance algorithm is simulated in a more realistic scenarios including perturbing acceleration to verify the robustness of the system via a Monte Carlo analysis.

INTRODUCTION

Over the past few years, low-thrust propulsion has received a lot of attention from the scientific community investigating new technologies designed to improve space flight performances. The high specific impulse afforded by low-thrust engines enables a dramatic reduction of the propellant mass fraction needed to transfer spacecraft to a desired target therefore resulting in a reduced mass at launch or an increased payload mass. The effectiveness of low-thrust propulsion in driving the spacecraft to the desired targeted state has recently been demonstrated by both NASA and ESA in two missions, i.e. the NASA's Deep Space-1 and the ESA's SMART-1. Although low thrust-based propulsion systems results in lower propellant mass, the trajectory design and consequent guidance approach is necessarily more complex and requires sophisticated mathematical approaches with respect to their chemically-propelled counterparts. Whereas chemical propulsion systems are assumed to produce instantaneous velocity changes, low-thrust propulsion systems generate continuous thrust for a long time during the transfer. Consequently, optimal control theory must be employed to find trajectory and piecewise continuous solutions that both minimize a specified performance index (e.g. fuel) and satisfy the desired mission constraints. Finding optimal low-thrust transfers can be tackled using two classes of approaches, i.e. direct and indirect methods. Historically, the optimal trajectory design has been approached first with indirect methods and subsequently with direct methods. The former stem from the Pontryagin's maximum principle that uses the calculus of variations [1,2]. Conversely, direct methods use a transcription approach where the continuous problem is transformed in a standard nonlinear programming procedure and solved via appropriate local optimizers[3]. Direct and indirect methods methods can be shown to be equivalent[4], and both methods have different pros and cons. Nevertheless, they require solving of a complex set of equation, i.e. the Euler-Lagrange equations (indirect methods) and the Karush-Kuhn-Tucker equations (direct methods).

In the context of space guidance, optimal trajectories found via direct and/or indirect methods are open-loop, i.e. the optimal path and thrust profiles are do not respond to perturbations that could potentially alter the state of the spacecraft. Importantly, optimal trajectories and trust are generally function of time and are found by solving an optimization problem constrained by a necessarily simplified dynamical model (e.g. motion of the spacecraft moving on a central field). Perturbations such as spherical harmonics, drag, effect of other bodies of the solar systems, are not generally included in the optimal trajectory calculation. Consequently, corrections are continuously required to keep the spacecraft on the desired path. Indeed, one would be interested in generating possible closed-loop optimal trajectories that autonomously keep the trajectories on the nominal path. Indeed, low-thrust guided (feedback) trajectories for space missions are extremely important for fuel-efficient autonomous space travel. A standard approach to autonomous space guidance has been to design an open-loop optimal trajectory via direct/indirect methods and then use standard linear control methods to continuously keep the spacecraft to the designed path (e.g. via a Linear Quadratic Regulator as in5). However, although the planned trajectory is generally optimal, closing the loop around the trajectory may not be optimal in the general sense. Moreover, losing-the loop require storing on-board the full state trajectory and thrust profile on the on-board computer that shall be readily available to the guidance system for proper spacecraft steering.

In this paper, we explore an alternative waypoint-based closed-loop guidance approach for space travel that is based on the recently developed generalized ZEM/ZEV feedback guidance law.

The generalized Zero-Effort-Miss/Zero-Effort-Velocity (ZEM/ZEV) feedback guidance[6] and its robustified version known as Optimal Sliding Guidance[7] have been developed and applied for both planetary landing and general space guidance. The ZEM/ZEV feedback guidance has been studied extensively and can be found in the literature for intercept, rendezvous, terminal guidance and landing applications. Recently, it has been investigated for relative motion guidance[8]. Such analytical closed-loop guidance has been originally conceived by Battin[9] who devised an energy optimal, feedback acceleration command for powered planetary descent. Ebrahimi et al.[10] introduced the ZEV concept, as a partner for the well-known ZEM and integrated it with a sliding surface for missile guidance with fixed-time propulsive maneuvers. Furfaro et al.[11] extended the idea to the problem of lunar landing guidance and set the basis for the theoretical development of a robust closed-loop algorithm for precision landing. ZEM/ZEM feedback guidance is attractive because of its analytical simplicity as well as potential for quasi-optimal fuel performance for constant gravitational field. Additionally, when robustified by a time-dependent sliding term, the resulting OSG can be proven to be Globally Finite-Time Stable (GFTS) in spite of perturbation with known upper bound[7]. The closed-loop analytical ZEM/ZEV solution has been rigorously derived by solving an energy-optimal problem assuming constant or time-dependent gravity and unconstrained thrust. Guo et al.[6], explored applying ZEM/ZEV to the orbit raising problem where analytical algorithm has been employed to generate feedback trajectories as potential Earth-Mars transfer via guided low-thrust. Comparisons between the open-loop fuel-efficient solution found via direct transcription and the ZEM/ZEV algorithm demonstrated that the latter can be 50%-60% suboptimal. The authors showed that the performance can be generally improved by implementing a waypoint scheme. Indeed, the adverse effects from nonlinear terms coming from the transfer in a central field and coupled dynamics can be reduced by breaking the mission up smaller portions exhibiting shorter flight time such that the quasi-optimal performances of the ZEM/ZEV can be preserved.

Here, we are interested in further exploring these ideas and develop as well as test a waypoint-based ZEM/ZEV targeting scheme that can be implemented in a closed loop fashion. More specifically, we compute open-loop trajectories for selected space transfers and we investigate the ability of ZEM/ZEV guidance to sequentially targeting points on the optimal trajectory. For two low-thrust transfer scenarios (i.e. Earth-Mars transfer and Earth-based LEO to GEO transfer), we study the optimal number and location of points on the open-loop trajectories that need to be sequentially targeted. Fuel performances and final targeting accuracy are evaluated to show that the method is both accurate and quasi-optimal.

GUIDANCE APPROACH: ZERO-EFFORT MISS (ZEM) AND ZERO-EFFORT VELOCITY (ZEV) ERRORS

The physical model employed to generally derive the basic guidance equations for orbital transfer can be synthetically represented as follows:

$$\frac{d}{dt}\boldsymbol{r}_L = \boldsymbol{v}_L \tag{1}$$

$$\frac{d}{dt}\boldsymbol{v}_l = \boldsymbol{a}_L(t) = \boldsymbol{g}(\boldsymbol{r}_L, t) + \boldsymbol{a}_{COMM}(t). \tag{2}$$

Here, the vector $\boldsymbol{g}(\boldsymbol{r}_L, t)$ represents all forces acting on the spacecraft (except for the thrust) whereas $\mathbf{a}_{\mathrm{COMM}}$ is the acceleration command (i.e. thrust-to-mass ratio) that drives the spacecraft to the desired state. Eq. (11)-(12) can be integrated starting from knowledge of position and velocity at time t to formally determine position and velocity of the spacecraft at a specified final time t_f:

$$\boldsymbol{v}_L(t_f) = \boldsymbol{v}_L(t) + \int_t^{t_f} \left(\boldsymbol{g}(\boldsymbol{r}_L, \tau) + \boldsymbol{a}_{COMM}(\tau)\right) d\tau \tag{3}$$

$$\boldsymbol{r}_L(t_f) = \boldsymbol{r}_L(t) + \boldsymbol{v}_L(t) t_{go} + \int_{\mathrm{t}}^{\mathrm{t_f}} \int_{\tau'}^{t_f} \left(\boldsymbol{g}(\boldsymbol{r}_L, \tau') + \boldsymbol{a}_{COMM}(\tau')\right) d\tau' d\tau. \tag{4}$$

Here $t_{go} = t_f - t$ is the time-to-go, i.e. the time required to reach the desired position (target) with the desired velocity. Next, we define the following quantities:

Definition #1: Given the time t, we define the Zero-Effort Miss (**ZEM**) as the distance (vector) the spacecraft will miss the target if no acceleration command (guidance) is generated after t:

$$\boldsymbol{ZEM}(t) = \boldsymbol{r}_{Lf} - \boldsymbol{r}_L(t_f), \qquad \boldsymbol{a}_{COMM}(\tau) = \boldsymbol{0}, \tau \in [t, t_f] \tag{5}$$

Definition #2: Given the time t, we define the Zero-Effort Velocity (ZEV) as the error in velocity at the final time, if no acceleration command (guidance) is generated after t, i.e.

$$\boldsymbol{ZEV}(t) = \boldsymbol{v}_{Lf} - \boldsymbol{v}_L(t_f), \qquad \boldsymbol{a}_{COMM}(\tau) = \boldsymbol{0}, \tau \in [t, t_f] \tag{6}$$

Here, $\boldsymbol{r}_{Lf}$and $\boldsymbol{v}_{Lf}$ are the desired position and velocity at the final time. Both $\boldsymbol{ZEM}(t)$ and $\boldsymbol{ZEV}(t)$can be explicitly expressed as functions of the current position, velocity and time-to-go by substituting Eq. (12, 13) with $\mathbf{a}_{COMM} = \mathbf{0}$ into Eq.(14) and Eq.(15):

$$\boldsymbol{ZEV}(t) = \boldsymbol{v}_{Lf} - \boldsymbol{v}_L(t) - \int_t^{t_f} \boldsymbol{g}(\boldsymbol{r}_L, \tau) d\tau \tag{7}$$

$$\boldsymbol{ZEM}(t) = \boldsymbol{r}_{Lf} - \boldsymbol{r}_L(t) - \boldsymbol{v}_L(t) t_{go} - \int_{\mathrm{t}}^{\mathrm{t_f}} \int_{\tau'}^{t_f} \boldsymbol{g}(\boldsymbol{r}_L, \tau') d\tau' d\tau \tag{8}$$

ZEM/ZEV FEEDBACK GUIDANCE: BASIC EQUATIONS

The basis of our algorithm development is the ability to generate an optimal guidance law as a function of $\boldsymbol{ZEM}$and $\boldsymbol{ZEV}$. Indeed, given the actual spacecraft position and velocity, both quantities can be estimated on-line by the numerical integration of the (unperturbed) equations of motion as functions of the time-to-go and the targeted conditions. One of the key ingredients is the ability to obtain a closed loop guidance law that minimizes the overall guidance effort, i.e. a guidance law that minimizes the overall acceleration command. The problem can be formulated as follows:

Find the $\boldsymbol{a}_{COMM}$ *as a function of* $\boldsymbol{ZEM}(t)$ *and* $\boldsymbol{ZEV}(t)$*that minimizes the following performance index:*

$$J(\boldsymbol{a}_{COMM}) = \int_t^{t_f} \boldsymbol{a}_{COMM}(\tau)^T \boldsymbol{a}_{COMM}(\tau) d\tau \tag{9}$$

Subject to Eq. (9,10) as physical constraints, with initial conditions (at time t) r(t) and v(t) and final conditions (at time t_f*)* r_L *and* v_L*.*

The acceleration command is assumed to be unconstrained, i.e. the thrust generated by the propulsion system is unbounded.   The problem can be solved by either applying the Pontryagin Minimum Principle (PMP) to determine the necessary conditions for the existence of an optimal solution (Two-Point Boundary Value Problem, TPBVP) or by a direct application of calculus of variations. It is found that the acceleration command is linear in time, i.e.:

$$\boldsymbol{a}_{COMM}(t) = \boldsymbol{A}_1 t - \boldsymbol{A}_2 \tag{10}$$

The constants $\mathbf{A}_1$ and $\mathbf{A}_2$ are determined by substituting $\mathbf{a}_{COMM}$ in Eq. (2)-(3). Finally, the optimal acceleration command can be expressed as a function of $\mathbf{ZEM}$(t), $\mathbf{ZEV}$(t) and t_{go} as follows:

$$\boldsymbol{a}_{COMM}(t) = \frac{k_R}{t_{go}^2}\boldsymbol{ZEM}(t) + \frac{k_V}{t_{go}}\boldsymbol{ZEV}(t) \tag{11}$$

Where $k_R = 6$, and $k_V = -2$ are the optimal guidance gains.[6,7]

The methodology employed to determine the optimal guidance law as a function of $\boldsymbol{ZEM}$and $\boldsymbol{ZEV}$ is very similar to the analysis presented by D'Souza[12], who derived the optimal acceleration command for a power landing descent as a function of error in position (actual position minus target position), velocity (actual velocity minus target velocity) and time-to-go. Both formulations do not impose any constraints in term of maximum thrust or minimum altitude. Nevertheless, both algorithms are easy to implement and mechanize which may justify the attractiveness of the guidance approach. Numerical simulations of the closed-loop trajectories may be analyzed a-posteriori to verify that both constraints are never violated or that the guidance algorithm works (i.e. guides the spacecraft to the target) even in the presence of thrust saturation.

WAYPOINT-BASED ZEM/ZEV GUIDANCE FOR LOW-THRUST TRANSFER

The feasibility of applying ZEM/ZEV feedback guidance in a waypoint fashion is analyzed in two low-thrust orbital transfer scenarios, i.e. 1) transfer Earth-Mars and 2) transfer from LEO to GEO via GTO. In both scenarios, a set of open-loop, energy-optimal solutions have been generated and employed as reference trajectories[13]. Waypoints (position and velocity) are selected on the reference trajectories and are employed as the sequential target set by the ZEM/ZEV feedback guidance algorithm. The goal is to show that the waypoint approach can generate quasi-optimal closed-loop trajectories that autonomously guide the spacecraft during the transfer to the desired final state. In both scenarios, a study on the performance of the proposed approach is executed as function of the 1) number of selected waypoints on the reference trajectory and 2) position of the waypoints on the reference trajectory. A Genetic Algorithm is subsequently employed to find the best location and the relative time-of-flight between waypoints.

Scenario 1: Earth-Mars low-Thrust transfer

A reference energy-optimal trajectory for a low thrust Earth-Mars transfer is first considered. The open-loop solution has been computed assuming a spacecraft mass of 1000 kg and a maximum thrust of 0.75N (saturated), using an indirect method as described in Ferrella and Topputo[13]. An initial parametric study is conducted by increasing the number of waypoints selected for sequential targeting. The minimum number of waypoints (WP) to allow ZEM/ZEV targeting is 2WP; increasing the number up to 8WP, the best case with the greatest value of final mass will be reached. Continuing to increase the number up to a maximum of 73 WP the mass consumption almost linearly increases (Fig.1).

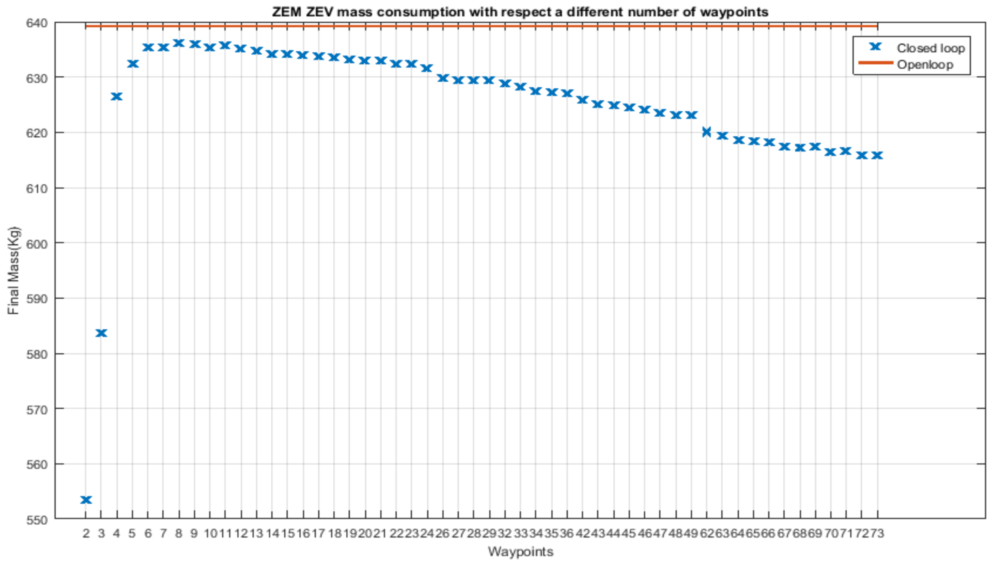

Figure 1: Trend of the final mass of the spacecraft that varies with the number of the waypoints (Energy-optimal case with Tmax=0.75N)

In the following, details about the three more interesting cases (2WP, 8WP, 73WP) are shown i.e. the ZEM/ZEV feedback targeting trajectory and the comparison with the open loop of the acceleration command, spacecraft position among the three coordinates and the thrust. There are two main observations common to all simulations:

1. The plot of the acceleration is discontinuous with respect to the smoother one in the open loop guidance. This is due to the typology of the control action that is continuous in the single "flight segment", but not overall.
2. In the closed-loop case, the saturation of the thrust increases with the increasing of the waypoints, this is probably the reason why from a certain number onwards the mass consumption increases.

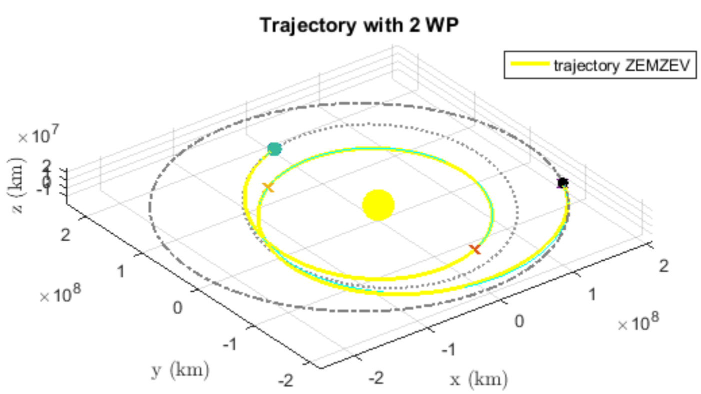

Figure 2: Trajectory with 2WP (Energy-optimal case with Tmax=0.75N)

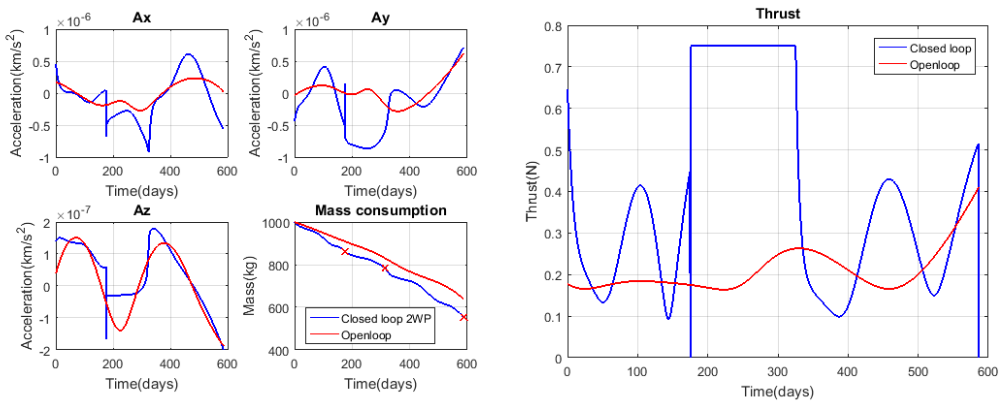

Figure 3: Comparison open loop/closed loop of the command acceleration, final mass and thrust in the case of 2WP (Energy-optimal with Tmax=0.75N)

The ZEM/ZEV trajectory (in yellow) overlaps the open loop one (in cyan) meaning that the control is targeting the waypoints (Fig.2) – this is also evident in the plot of the position along the three coordinates x, y and z (Fig.4). Conversely, the final mass is displaced from that of the open loop (Fig.3). Conversely, the case of 8WP (Fig.5) presents a trend of the mass consumption very similar to the one of the open loop, indeed the difference of final mass is minimal (Fig.6). The greater is the number of waypoints, the greater is the number of peaks, and better the thrust follows the trend of the open loop one. This is most likely the reason why the two extreme cases, i.e. a low number of waypoints (2WP) and high number of waypoints (73WP), exhibit a very high mass consumption (see also fig.7 and Fig.8).

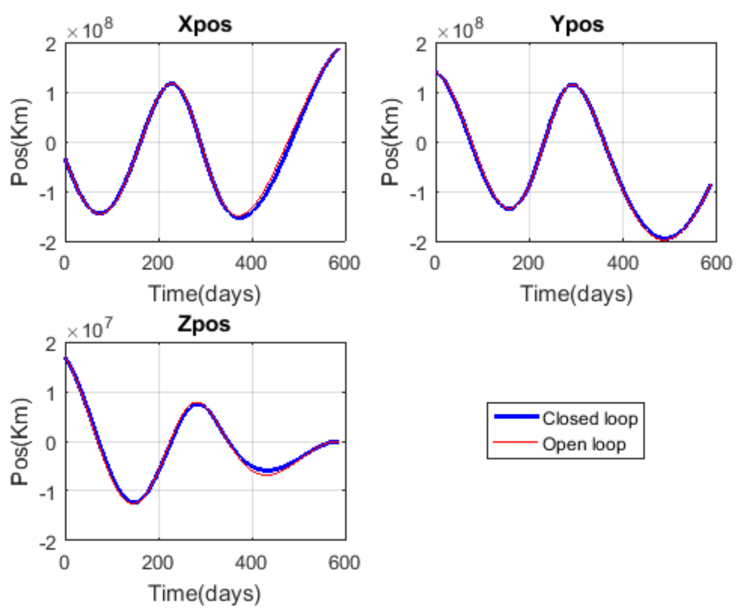

Figure 4: Position of the spacecraft with 2WP (Energy-Optimal case with Tmax=0.75N)

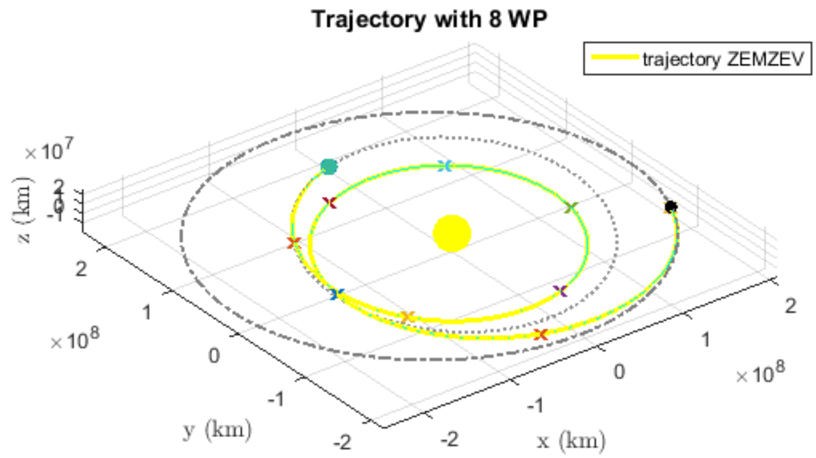

Figure 5: Trajectory with 8 WP (Energy-Optimal with Tmax=0.75N)

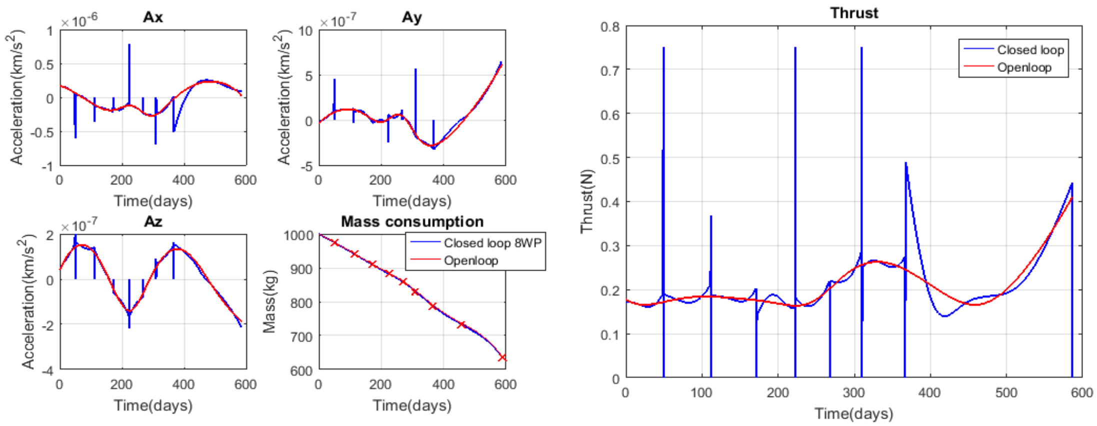

Figure 6: Comparison open loop/closed loop of the command acceleration, final mass and thrust in the case of 8WP (Energy-optimal with Tmax=0.75N)

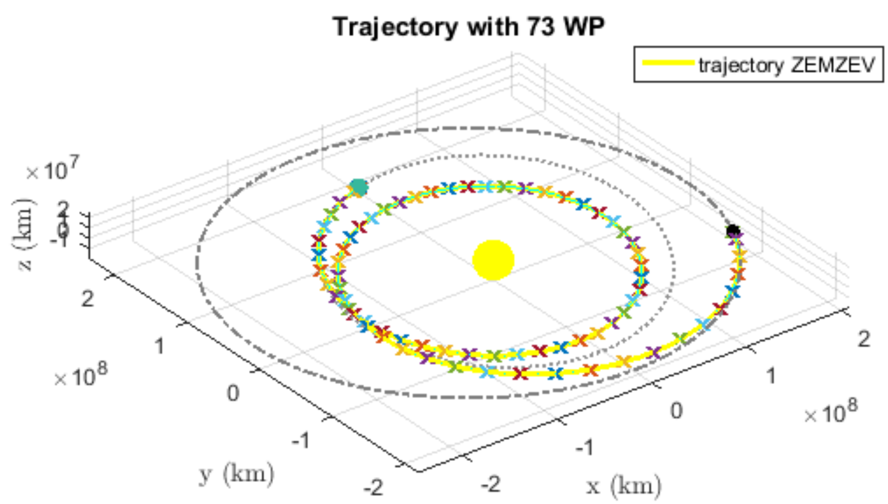

Figure 7: Trajectory with 73 WP (Energy-Optimal with Tmax=0.75N)

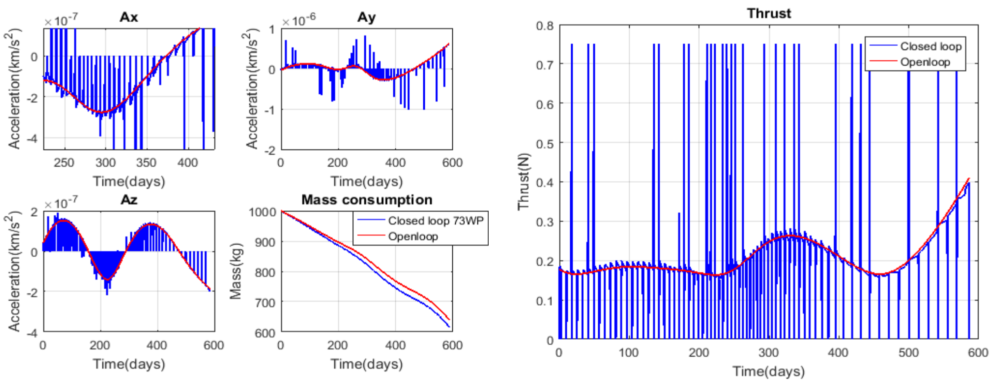

Figure 8: Comparison open loop/closed loop of the command acceleration, final mass and thrust in the case of 73WP (Energy-optimal with Tmax=0.75N)

As a final analysis of the Energy-Optimal case with T_{max}=0.75N, the targeting accuracy in terms of position and velocity errors as function of numbers of waypoints are reported in Fig. 9 and Fig. 10.

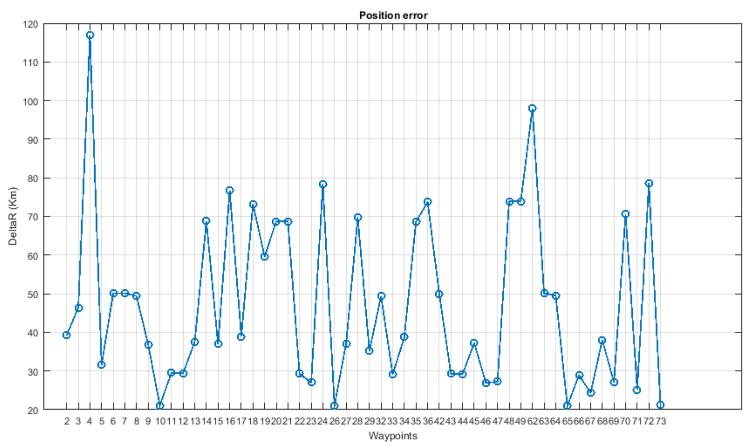

Figure 9: Final position error (Energy-Optimal Tmax=0.75N)

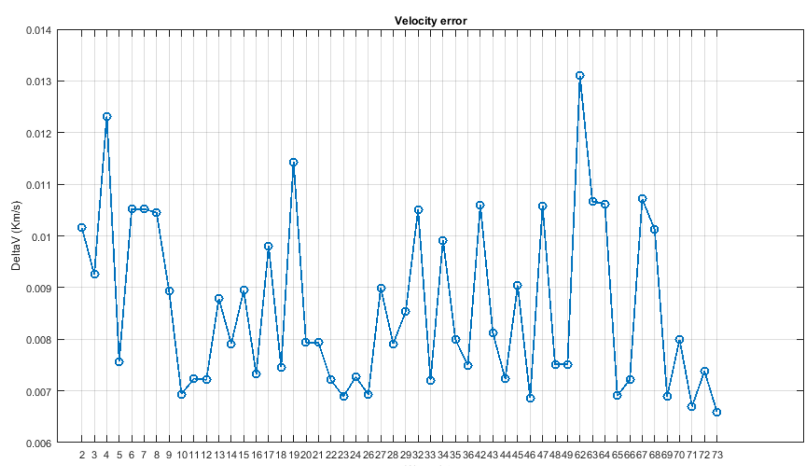

Figure 10: Final velocity error (Energy-Optimal Tmax=0.75N)

The best case is with 8WP, where the difference between open loop and closed loop is about 3kg (Fig. 11).

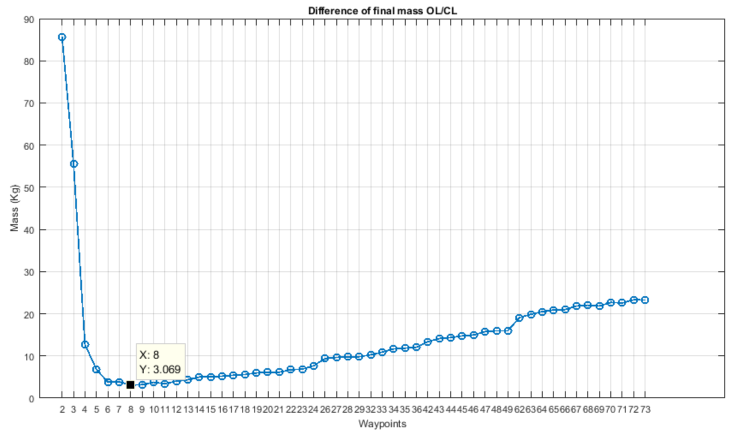

Figure 11: Difference of final mass between open loop and closed loop (Energy-Optimal with Tmax=0.75N)

Scenario 2: LEO to GEO Low-Thrust Orbit Raising

In this case, we present an extreme case where a 1000 kg spacecraft is required to transfer from LEO to GEO using a low-thrust engine generating a maximum thrust of 0.25N.

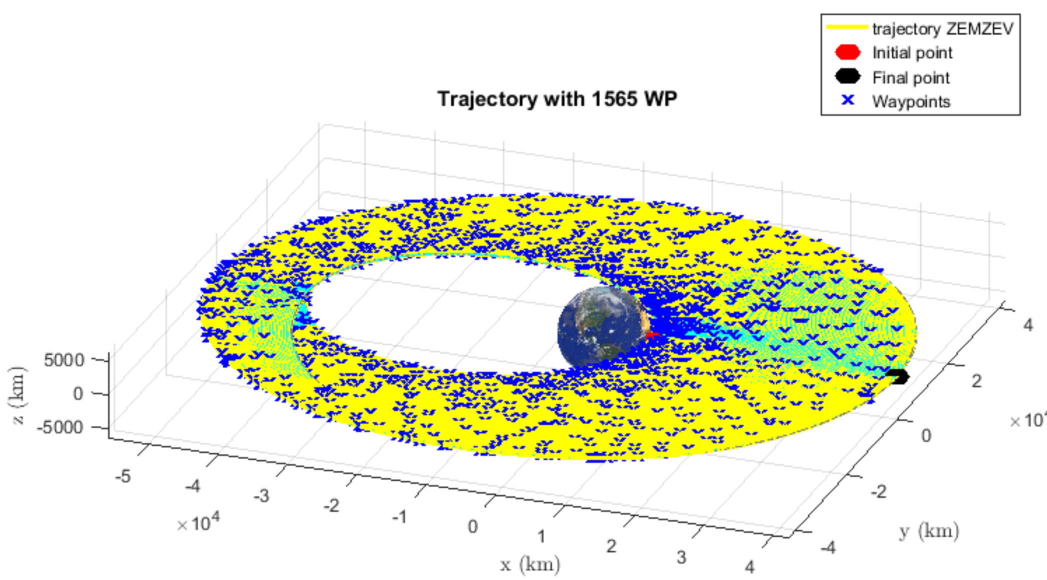

Figure 12: Trajectory with 1565 WP (GTO to GEO with Tmax=0.8N)

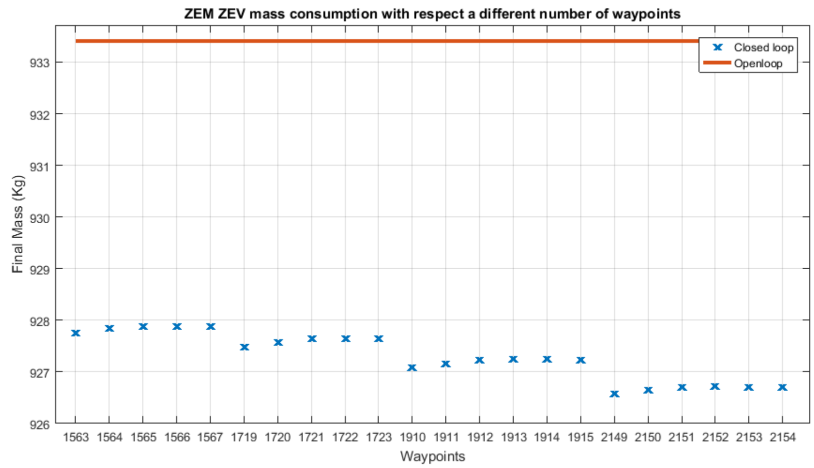

Figure 13: Trend of the final mass of the spacecraft that varies with the number of the waypoints (GTO to GEO case with Tmax=0.25N)

In this critical case, the minimum number of waypoints needed is generally very high. Several tests are executed by running ZEM/ZEV targeting up to 3000 WP. The fuel performance and comparison with the open-loop energy-optimal orbit raising trajectory as function of the number of WPs is reported in Fig. 13. Importantly, the most satisfactory mass consumption and accuracy corresponds to the case with 1565WP (Fig. 12, Fig.14, Fig.15, Fig.16, Fig. 17), which reaches a final mass of 927.9 kg, a position accuracy of 50 m and a velocity accuracy of 7 cm/s.

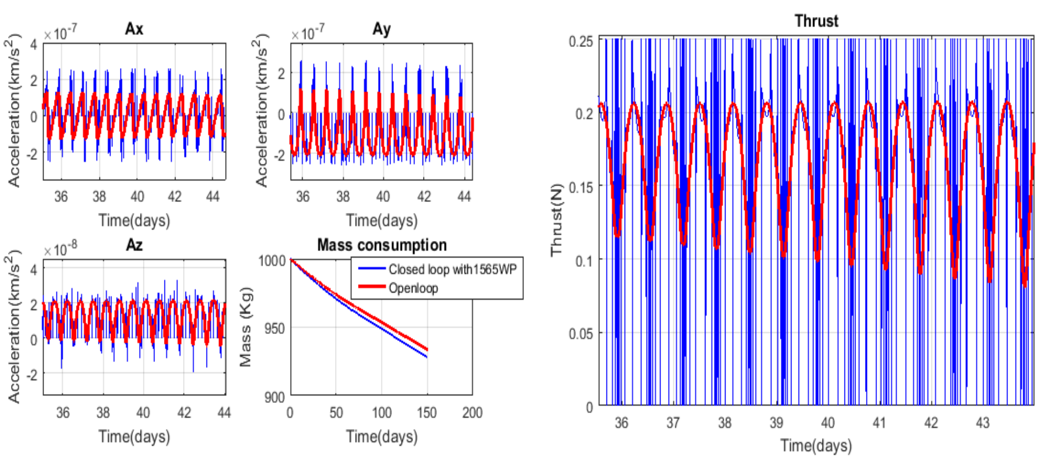

Figure 14: Comparison open loop/closed loop of the command acceleration, final mass and thrust in the case of 1565WP (GTO to GEO with Tmax=0.25N)

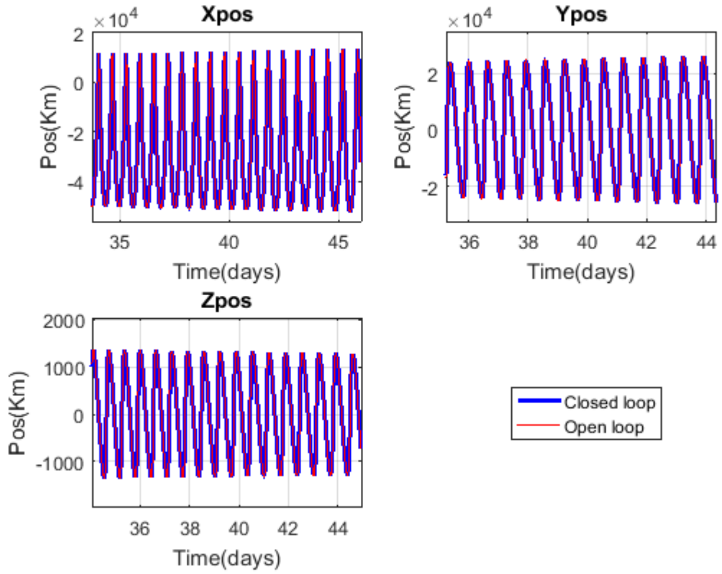

Figure 15: Position of the spacecraft (GTO to GEO with Tmax=0.25N)

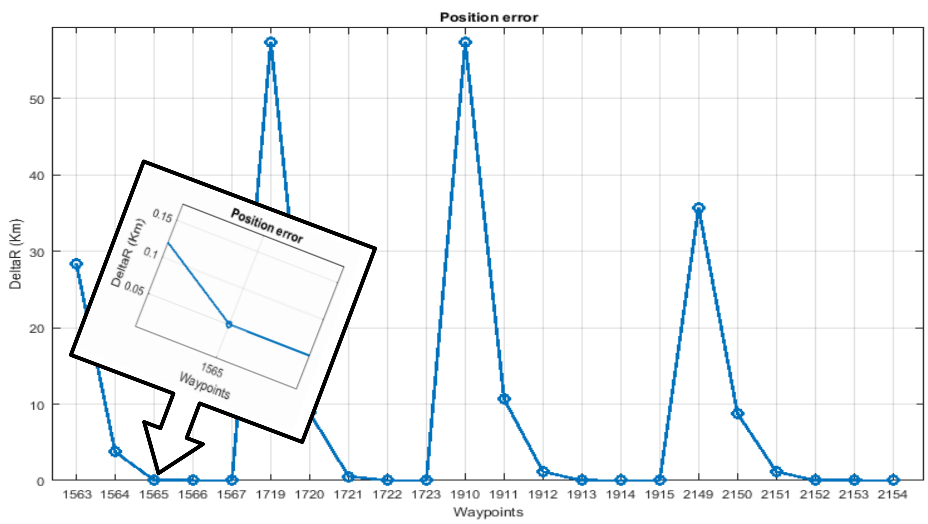

Figure 16: Final position error (GTO to GEO with Tmax=0.25N)

Since for this scenario the level of saturation is lowered to 0.25N, an increasing of saturation frequency is observed. The histories of accelerations, thrust and position (Fig. 14 and Fig. 15) maintain the same behavior of previous cases, i.e the waypoint-based, closed-loop trajectory tracks the open loop one except for the acceleration peaks achieved by the guidance as $t_{go} \rightarrow 0$.

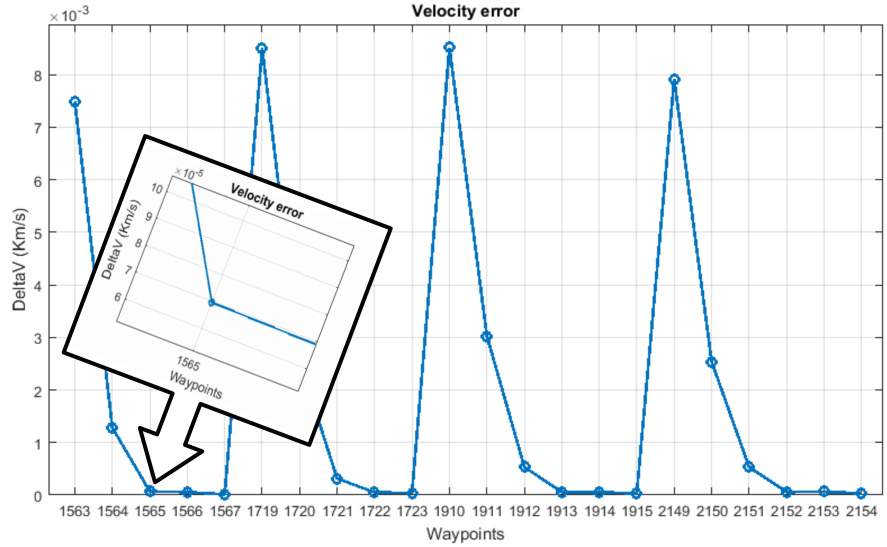

Figure 17: Final position error (GTO to GEO with Tmax=0.25N)

WAYPOINTS OPTIMIZATION

The waypoint-based ZEMZEV feedback guidance scheme is able to achieve the final target and keep close-to-optimal performances in terms of mass consumption and velocity accuracy, but it still suffers from relatively low position accuracy. This problem is solved by a waypoint-placement optimization through which it is possible to reduce the position error peaks. Such errors are due to the position of waypoints which changes from case to case and in some circumstances they may negatively affect the ZEM/ZEV performances. Furthermore the criteria of evenly spaced disposition is performed by the position vector point of view so that an approximation of the division of the length of the position vector by the number of waypoints was necessary, which results in a false evenly spaced positioning.

The optimization problem is defined by selecting a cost function and a proper set of decision variables to be found by the optimization procedure. The cost function is defined as follows:

$$J(x) = -m_f(x) + w_1 \cdot \|r_f - r(x)\| + w_2 \cdot \|v_f - v(x)\| \tag{12}$$

Here, we maximize the final spacecraft mass (i.e. minimize the overall fuel) as function of the flight times between the waypoints along the open-loop trajectory. A weighted penalty function is introduced to minimize the final target position error (weight w_1) and final target velocity error (weight w_2). The decision variables are defined as $x = [t_{wp1}, t_{wp2}, \dots\dots, t_{wpN}]^T$, where $t_{wpi}, i = 1, \dots\dots, N_{wp}$ is the time corresponding to the i-th waypoint on the open-loop optimal trajectory. Indeed, for a given optimal trajectory, there is a one-to-one correspondence between the time instant and a point on the trajectory. Consequently, it is possible to fix the position of the waypoint by fixing a time instant which represents the minimization variable x of the cost function. Additionally, the decision variables (waypoint time on the trajectory) are subjected to the following (linear) constraints:

$$t_{wp(i)} < t_{wp(i+1)} \qquad i = 1 \dots \dots \dots N_{wp} \tag{13}$$

$$t_{init} < t_{wp(i)} < t_{final} \quad i = 1 \dots \dots \dots N_{wp} \tag{14}$$

The optimization problem is solved by a genetic algorithm to compute the decision variable, where the cost function is computed by running the waypoint-based ZEM/ZEV algorithm at each iteration for any element of the population.

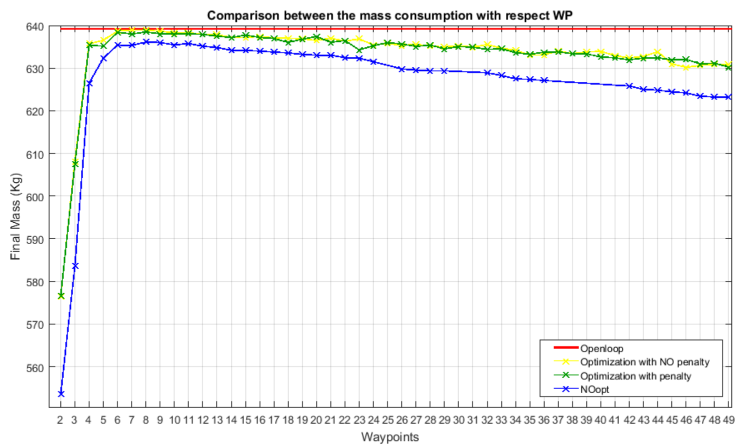

Figure 18: Optimization effect on the final mass with basic cost function

For the Earth-Mars low-thrust transfer, the optimization routine has been executed for a minimum number of 2WP up to 49WP with penalty (weights selected to be $w_1 = 100, w_2 = 50$) and without penalty ($w_1 = w_2 = 0$). Fig. 18 shows the final spacecraft mass a s function of the number of way-points for all case. For both cases, the maximum is achieved for 8 waypoints. Increasing the number of waypoints decreases the spacecraft final mass. For the case with no penalty function, the is a considerable improvement of the final mass; indeed it is possible to save up to 2.85Kg with the optimize waypoints placement. However, the final targeted position and velocity error is not reduced. Conversely, by using the weighted penalty function, it is possible to achieve significant improvement in the final position and velocity accuracy (Fig. 19 and 20).

Figure 19: Final position error of the spacecraft with optimization (Energy-Optimal with Tmax=0.75N)

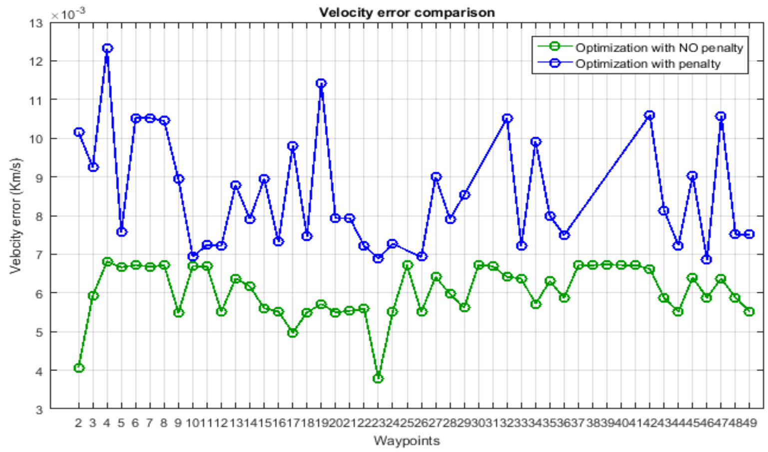

Figure 20: Final velocity error of the spacecraft with optimization (Energy-Optimal with Tmax=0.75N)

The average targeting position error is reported to be 20 km with a targeted velocity error less than $7\ m/sec$. Fig. 20 shows the closed-loop trajectory with 8 waypoints for the evenly spaced and waypoints optimized cases.

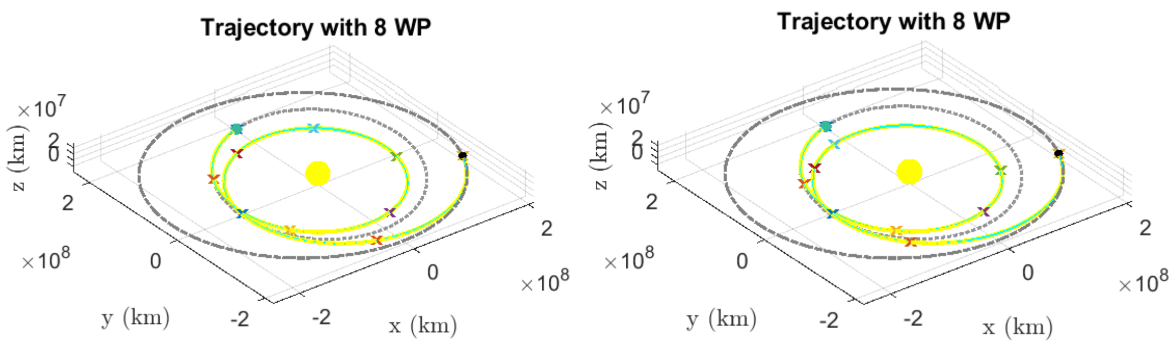

Figure 21: waypoint-based ZEM/ZEV guidance algorithm for Earth-Mars low-thrust transfer. The transfer uses 8 waypoints (Energy-Optimal with Tmax=0.75N). Left: evenly-spaced (time) waypoints. Right: Optimal way-point placement via GA

CONCLUSIONS

In this paper, we developed and tested a waypoint-based generalized zero-effort-miss/zero-effort-velocity (ZEM/ZEV) feedback guidance algorithm for space guidance applications. The approach has been investigated on two low-thrust scenarios of interest, i.e. 1) a Earth-Mars transfer and 2) a LEO to GEO transfer. In both cases, the gravitational acceleration depends on the state of the spacecraft and the thrust has been subjected to a saturation to make the simulation practical on the real environment. In particular a low-thrust limit has been set considering the use of electric propulsion that allows us to lower the propellant consumption by means of its intrinsic high specific impulse. Although the assumptions behind the derivation of the basic ZEM/ZEV algorithm are violated, we have shown that using a waypoints-based approach one, one can achieve quasi-optimal performances in terms of fuel efficiency with relatively accurate targeting capabilities.

By considering several open-loop optimal trajectories, which differ from each other in the limit thrust and in the way in which they have been generated (energy-optimal or fuel-optimal), we have investigated the overall performance as function of number and position of the waypoints. Firstly, a minimum number of waypoints is always necessary to achieve sufficient final target accuracy. Such number depends on the number of open-loop revolutions. As a general trend, the final spacecraft mass increases with number of waypoints until it reaches a maximum value (minimum fuel) and subsequently decreases, i.e. a maximum exists (i.e.minimum fuel). Consequently, the fuel optimal number of points can be determined parametrically. Overall, the waypoint ZEM/ZEV guidance seems to be very sensitive to the position of waypoints due to the fact that the range of controllability is restricted, especially considering that the assumptions of the ZEMZEV optimal control are not verified in the case studies. By solving a global optimization problem, the optimal placement of waypoints along the open-loop trajectory was determined, generally resulting in improved performances. Finally, Numerical simulations have confirmed the effectiveness of the ZEM/ZEV algorithm also in a higher fidelity environment. It can approach the performances of the open-loop optimal solution, while maintaining the robustness of a closed-loop feedback algorithm.

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