Model-Based Event-Triggered Robust MPC/ISM

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Abstract—A model-based event-triggered control scheme based on the combined use of Model Predictive Control (MPC) and Integral Sliding Mode (ISM) control is proposed in this paper. The aim is to reduce to a minimum the number of transmissions of the plant state over the network, in order to alleviate delays and packet loss induced by the network overload, while guaranteeing robust stability and constraints fulfillment. The presented control scheme includes a model-based controller and a smart sensor, both containing a copy of the nominal model of the plant. The sensor intelligence is provided by a triggering condition, which enables to determine when it is necessary to transmit the measured state and to update the nominal model. The controller includes an ISM component, which has the role of compensating the uncertainties, and a MPC term which optimizes the system evolution. The control system performance are assessed in simulation relying on an illustrative mechanical example.

I. INTRODUCTION

The use of multipurpose shared networks for remote process control purposes, giving rise to the so-called networked control systems (NCSs), is nowadays a reality, by virtue of its cost effectiveness and flexibility (see, for an overview, [1], [2], and [3]). Yet, the presence of a network in the control loop makes several technical and theoretical problems arise, these being related to the fact that a communication network is a band-limited channel which can features delays and packet loss. To overcome the drawbacks of NCSs, different approaches have been investigated. Event-Triggered Control (see, among others, [4], [5], [6], [7], [8], [9], and the references therein cited) is surely one of the most effective. In this methodology, the control signal is recomputed every time the violation of a certain triggering condition occurs.

Another valid solution is the so-called Model-Based Networked Control (see, for instance, [10] and [11]). In a Model-Based Networked Control System (MB-NCS), an explicit model of the plant is added to the controller in order to determine the control law, whenever possible, on the basis of the state of the model, rather than on the actual measurement. Since the use of the nominal model, with the role of predictor, is typical in Model Predictive Control (MPC), [12], [13], [14], [15], [16], [17], [18], in this paper we have investigated the possibility of designing a model-based event-triggered control scheme of predictive nature. The considered plant is assumed to be affected by matched uncertainties. For this reason, we propose a scheme based on the combined use of MPC and Integral Sliding Mode (ISM) control [19]. Note that the joint use of MPC and ISM has already been investigated in [20] and [21] in a conventional, i.e. non NCS, framework.

![Diagram](image.png)

Fig. 1. Representation of the Model-Based Event-Triggered MPC/ISM control scheme.

The proposed control scheme, Fig. 1, consists of two key elements: the model-based controller and the smart sensor. The MPC component of the controller optimizes the system evolution, fulfilling the constraints on state and control variables, while the ISM component has the role of compensating matched uncertainties. The sensor intelligence is provided by a suitable triggering condition, function of the plant state, which enables to determine when it is necessary to transmit the measured state and to update the nominal model.

In the paper, the proposed control scheme is theoretically analyzed. More specifically, two results are discussed: the first refers to the case in which the plant state is always transmitted to the controller, i.e. the mechanism based on the triggering condition is deactivated, while the second deals with the situation in which the complete event-triggered proposed scheme is applied. In both cases, the asymptotic stability of the origin of the controlled system state space is proved in spite of the matched uncertainty presence.

Finally, the control system performances are assessed in simulation relying on an illustrative example of mechanical type.

II. NOTATIONS

In the following sections we shall use $\| \cdot \|$ to denote the Euclidean norm, and $|\cdot|$ to denote the absolute value. Let $T$ be the sampling time and $t_k$, $k \geq 0$, be the sampling time instant. The generic time instant can be regarded as $t_0 \triangleq t_k + \tau_i$, $0 \leq \tau_i \leq T$, with $\tau_i$ being the numerical integration step. Let $w_{[t_k,t_{k+1}]}$ be a sequence of vector $w(\cdot)$ from time $t_k$ and $t_{k+1}$. Let $I_n$ denote the $(n \times n)$ identity matrix. Given the sets $\mathcal{W} \subset \mathbb{R}^n$ and $\mathcal{P} \subset \mathbb{R}^n$, the Pontryagin difference is defined as $\mathcal{W} \sim \mathcal{P} \triangleq \{ w \in \mathbb{R}^n : (w+p) \in \mathcal{W}, \forall p \in \mathcal{P} \}$. Moreover, let $\sigma : \mathbb{R}^n \to \mathbb{R}$, $\Sigma : \mathbb{R}^n \to \mathbb{R}$ be smooth functions, respectively.
named sliding variable and auxiliary sliding variable, while \( \varphi : \mathbb{R}^n \rightarrow \mathbb{R} \) is a smooth function, called transient trajectory. The relative degree of the system, i.e., the minimum order \( r \) of the time derivative \( \sigma^{(r)} \) of the sliding variable in which the control \( u \) explicitly appears, is considered well defined, uniform, and time invariant. Moreover, we shall use \( \emptyset \) symbol to denote that no variable is sent through the network.

### III. Problem Statement

We consider the dynamics of the plant given by

\[
x(t) = Ax(t) + Bu(t) + f(x(t)), \quad t \geq t_0
\]

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) is the control variable, and they satisfy the constraints

\[
x \in X, \quad \forall t \geq t_0
\]

\[
|u| \leq u_{\text{max}}, \quad \forall t \geq t_0
\]

with \( X \) being a compact set containing the origin, and \( u_{\text{max}} > 0 \). In (1), \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^n \), while \( f(\cdot) \) is an uncertain smooth function such that

\[
f(x(t)) = Bw_m(x(t))
\]

Note that \( f(\cdot) \) represents the so-called matched uncertainty, [22], which in practical applications is due to unavoidable unmodelled dynamics, parameter uncertainties and disturbances. Moreover, we assume that

\[
|w_m| \leq w_{\text{max}}, \quad \forall t \geq t_0
\]

with \( w_{\text{max}} \) being a positive constant.

The corresponding nominal model of the plant can be described as

\[
\hat{x}(t) = A\hat{x}(t) + Bu(t), \quad t \geq t_0
\]

where \( \hat{x} \in \mathbb{R}^n \) is the state vector of the model.

The control objective consists in designing a model-based event-triggered control scheme relying on an MPC/ISM control strategy, so as to ensure that the origin of the state space becomes an asymptotically stable equilibrium point of the controlled system, and that the plant state and control variables comply with constraints (2) and (3), respectively, while ensuring satisfactory closed-loop performance, in terms of robustness with respect to matched uncertainties.

### IV. Model-Based Event-Triggered MPC/ISM: The Proposed Overall Control Strategy

The control scheme considered in the paper (see Fig. 1) includes two key blocks, the model-based controller and the smart sensor.

The model-based controller, detailed in Fig. 2, contains a copy of the nominal model of the plant, along with the MPC/ISM controller. The nominal model is reinitialized when a new instance of the plant state is transmitted over the network.

The ISM controller has the task of rejecting matched uncertainties, while the MPC is used to fulfill the plant constraints and to guarantee the optimality of the control law. Note that, in Fig. 2, the notation \( \emptyset/\hat{x} \) means that the state of the model is used if the actual state has not been received through the network.

The control variable \( u(t) \) is chosen as

\[
u(t) = u_{\text{MPC}}(t) + u_{\text{ISM}}(t)
\]

where \( u_{\text{MPC}}(t) \) is a piecewise-constant signal generated by the MPC component of the controller and acting as an input for the ISM controller. This latter generates the component \( u_{\text{ISM}}(t) \), which is oriented to compensate the uncertainty terms.

#### A. The Event-Triggered Strategy

Also the smart sensor, as detailed in Fig. 3, contains a copy of the nominal model of the plant, which provides the computed state \( \hat{x} \) to the triggering condition block. This block, relying on the measured state \( x \), computes the state error \( e(t) = \hat{x}(t) - x(t) \), and verifies the so-called triggering condition. In the present paper, following the suggestion in [11], we adopt a triggering condition with relative threshold. The threshold is progressively reduced as a function of the measured state, i.e.

\[
\|e\| \leq \varepsilon|x|
\]

where \( 0 < \varepsilon < 1 \). If condition (8) is violated, the actual state is sent to the controller and the state of the nominal model is updated. Note that the notation \( x/\emptyset \) in Fig. 3 means that \( x \) or no variable is sent over the network, depending on condition (8).
B. The Integral Sliding Mode Component

According to the ISM control theory [19], it is possible to force the system to evolve in sliding mode starting from the initial time instant. This property is hereafter briefly recalled. Note that the dependence of $\sigma$ on $x(t)$, and the dependence of all the variables on $t$ will be omitted, when it is obvious, for the sake of simplicity.

Let $\chi \in \mathbb{R}^n$ be the current state used by the controller that, according to the switching condition (8), can be the actual plant state $x$ or the model state $\hat{x}$. To start with, assume that $u_{MPC}$ is the MPC law computed relying on the nominal model of the plant in (6).

Now, select the auxiliary sliding variable as

$$\Sigma(t) = \sigma(t) + \phi(t)$$

with $\sigma$ being a conventional sliding variable, and $\phi$ being the desired transient trajectory specified, with reference to (6), as

$$\phi = -\frac{\partial \sigma}{\partial \chi} \{A \chi + B u_{MPC}\}$$

$$\phi(t_0) = -\sigma(t_0)$$

(10)

(11)

where $\phi(t_0)$ is chosen so as to enforce a sliding mode on the sliding manifold $S = \{x \in \mathcal{X} : \Sigma(t) = 0\}$ (see [23]) from the initial time instant $t_0$. Then, the discontinuous control $u_{ISM}$ in (7) is designed as

$$u_{ISM}(t) = -U_{\text{max}} \text{sgn}(\Sigma(\chi(t)))$$

(12)

with $U_{\text{max}} > 0$ suitably chosen so as to satisfy the sliding condition [23] with respect to the auxiliary sliding variable.

** Remark 1:** Note that, as shown in [19], in order to reduce the so-called chattering phenomenon, which is the major drawback of traditional sliding mode schemes [24, 25], one can use the equivalent value $u_{ISM,eq}(t)$ of the discontinuous control, i.e. the so-called equivalent control (see [23], for a definition), obtained at the output of a first order linear filter with the real discontinuous control as input. Note that, according to [19], if the equivalent control is used, then, the transient trajectory $\phi$ must be redesigned as follows

$$\phi = -\frac{\partial \sigma}{\partial \chi} \{A \chi + B (u_{MPC} + u_{ISM,eq} - u_{ISM})\}$$

$$\phi(t_0) = -\sigma(t_0)$$

(13)

(14)

to ensure $\Sigma(t) = 0$, $\forall t \geq t_0$.

C. The Model Predictive Control Component

By virtue of the rejection of the matched uncertainties produced by the ISM part of the controller, the MPC component can be developed relying on the nominal model of the plant. However, a robust MPC is required in order to cope with the uncertainty introduced by the triggering condition (8).

The adopted MPC is based on the solution to a Finite-Horizon Optimal Control Problem (FHOC) which consists in minimizing, at any sampling time instant $t_k$, a suitably defined cost function with respect the control sequence $u_{[t_k,t_k+N-1]}$, with $N \geq 1$ being the prediction horizon. The adopted robust MPC component is inspired by [26], and includes tightened constraints that can be described, as in [20], via a Pontryagin difference of sets as

$$x_{tT+\tau} = x \sim B_{tT+\tau}$$

(15)

where

$$B_{tT+\tau} \triangleq \left\{ \tau \in \mathbb{R}^n : \|\tau\| \leq \gamma \left( T \mathcal{L}_T^{-1} - \mathcal{L}_T^{-1} \right) \right\}$$

(16)

with $\gamma$ being a positive constant, and $\mathcal{L}_T \triangleq \mathcal{L}(\tau)$ being a positive continuous function defined in $[0;T]$, such that $\mathcal{L}_0 = 1$. Note that, if the nominal state evolution belongs to $x_{tT+\tau}$, then the perturbed state of the system fulfills (2), as proved in [20]. In our case, given two positive definite matrices $Q$ and $R$, the cost function to be minimized is

$$J(\chi(t_k), u_{[t_k,t_k+N-1]}, N) = \int_{t_k}^{t_k+N} [\chi^T(\tau) Q \chi(\tau) + u^T(\tau) Ru(\tau)] d\tau + \chi^T(t_k+N) \Pi \chi(t_k+N)$$

(17)

The minimization has to be accomplished subjected to the state dynamics (6), the constraints on the state variables

$$\chi(t) \in \mathcal{X}_{-t_k}$$

(18)

and in (15), and the constraint $u_{MPC}$ on the control variable $u(t)$, which can be determined considering that a quantity equal to $U_{\text{max}}$ (see (12)) must be subtracted to the control bound in (3), i.e.,

$$\tilde{u}_{MPC} = u_{\text{max}} - U_{\text{max}}$$

(19)

Finally, a terminal constraint $\chi(t_k+N) \in \Omega_f$, with

$$\Omega_f \triangleq \{ \chi \mid \chi^T \Pi \chi \leq \rho_f \}, \quad \Omega_f \subseteq \mathcal{X}$$

(20)

where $\rho_f$ is a positive constant, is considered. The terminal penalty and the terminal constraint are chosen following the idea reported in [20] in order to guarantee stability. Then, according to the Receding Horizon (RH) strategy, the applied control law is the following piecewise-constant signal

$$u_{MPC}(t) = \kappa_{MPC}(\chi(t_k)), \quad t \in [t_k,t_{k+1})$$

(21)

where

$$\kappa_{MPC}(\chi(t_k)) \triangleq u^o(t_k)$$

(22)

with $u^o(t_k)$ being the first value of the optimal control sequence obtained by solving the FHOCP at $t_k$.

** Remark 2:** Note that, as done in [17] and [20], in the FHOCP continuous time constraints are considered. However, as shown in [17], the effect of a discrete time integration step can be explicitly considered at the cost of a further conservativeness that is negligible if $\tau \ll T$ ($T$ being the sampling time, as introduced in Section II, with $t_{k+1} - t_k = T$).
V. STABILITY ANALYSIS

With reference to the previously discussed control scheme, the following results can be proved. They are here reported without proofs, for the sake of brevity.

Theorem 1: Given the plant (1), with the matched uncertainty bound in (5), assume that the mechanism based on the triggering condition is deactivated, so that the model-based controller is always fed with the actual plant state. Then, by applying the control law (7), the origin of the state space of the closed-loop system results in being an asymptotically stable equilibrium point.

Theorem 2: Given the plant (1), with the matched uncertainty bound in (5). Assume that the mechanism based on the triggering condition (8) is active. Then, by applying the control law (7), the origin of the state space of the closed-loop system results in being an asymptotically stable equilibrium point.

VI. ILLUSTRATIVE EXAMPLE

In this section, the proposed control strategy is applied in simulation to a cart moving on a plane, as illustrated in Fig. 4.

The plant is described by the following equations

\[
\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{1}{M} (-kx_1(t) - hx_2(t) + u(t) + w_m(t))
\end{aligned}
\]

where the control variable \(u\) is the force applied to the cart. Moreover, considering (1),

\[
A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{M} & -\frac{h}{M} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

where \(M=1\) kg is the mass of the cart, which is assumed to be known, \(k = 0.33\) N/m is the stiffness of the spring, \(h = 1.1\) Nsm\(^{-1}\) is the damping factor, while the matched uncertain term is \(f(x(t)) = (W_m/M) \sin(x_2)\). Note that \(w_m = W_m \sin(x_2)\) is considered unknown in simulation with \(|W_m| \leq 0.65\) N (see Fig. 5 for a plot of the disturbances versus time in simulation). Accordingly, the nominal model is

\[
\begin{aligned}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) \\
\dot{\hat{x}}_2(t) &= \frac{1}{M} (-k\hat{x}_1(t) - h\hat{x}_2(t) + u(t))
\end{aligned}
\]

The initial condition is \(x(0) = [-2.2, -1.7]^T\).

To perform the simulation tests, the Euler solver is used with a numerical integration step \(\tau_s\) equal to 0.005 s, while the MPC sampling time is chosen as \(T = 0.1\) s. The prediction horizon of the FHOCP is \(N = 5\). The quantities \(Q\) and \(R\) in (17) are chosen respectively as \(Q = I_2\), and \(R = 0.01\), while the auxiliary control law and the matrix \(\Pi\) are equal to

\[
\kappa_f(x(t_k)) = Kx(t_k), \quad K = [5.8653, 6.1545]
\]

and

\[
\Pi = \begin{bmatrix} 11.5513 & 1.0769 \\ 1.0769 & 1.6385 \end{bmatrix}
\]

The considered control and state constraints are \(|u| \leq 8\) N,

\[|x_1|, |\dot{x}_1| \leq 2.5\) m, \(|x_2|, |\dot{x}_2| \leq 3\) m/s. The relative degree of the system is \(r = 1\), since the sliding uncertain term is selected as \(\sigma = m_1x_1 + x_2\), with \(m_1=1\). Moreover, the transient trajectory \(\varphi\) is chosen as

\[
\varphi(t) = -m_1x_1(0) - x_2(0) + \int_{t_0}^t m_1x_2(\tau) - \frac{1}{M} (kx_1(\tau) + hx_2(\tau)) + \frac{1}{M} u_{\text{MPC}}(\tau) d\tau, \quad t \in [t_k, t_{k+1})
\]

The discontinuous control law in (12) has the amplitude \(U_{\text{max}} = 0.65\). The triggering condition in (8) is specified by choosing \(\varepsilon = 0.3\).
In order to evaluate the closed-loop performance, we consider four indexes: i) the number of updates of the actual state, denoted with \( n_{up} \); ii) the root mean square (RMS) value of the plant state, \( x_{RMS} \); iii) the RMS value of the state error, namely \( e_{RMS} \); iv) the RMS value of the auxiliary sliding variable, i.e. \( \Sigma_{RMS} \). These indexes are determined as

\[
\begin{align*}
    n_{up} &= \sum_{i=0}^{n_s} f_{up}(\tau_i), \quad x_{RMS} = \sqrt{\frac{1}{n_s} \sum_{i=1}^{n_s} \sum_{j=1}^{n} x_j^2} \quad (29)\\
    e_{RMS} &= \sqrt{\frac{1}{n_s} \sum_{i=1}^{n_s} e_{ji}^2}, \quad \Sigma_{RMS} = \sqrt{\frac{1}{n_s} \sum_{i=1}^{n_s} \Sigma_i^2}
\end{align*}
\]

where \( f_{up}(\cdot) \) is a flag equal to 1 when the actual state is transmitted over the network, equal to zero otherwise, and \( n_s \) is the number of integration steps during the simulation; \( x_j \), \( e_{ji} \), and \( \Sigma_i \) are the \( j \)-th component of the state and of the error vectors, and the auxiliary sliding variable at the \( i \)-th integration step, respectively.

For the sake of comparison, we report the time evolution of the plant state (i.e. \( x_1(t), x_2(t) \) in (23)), in the open loop case, subject to the uncertain term action in Fig. 6, where it is apparent that the state constraints, explicitly indicated in the picture, are not always respected.

**Case 1:** In this first case, we assume that the state of the plant, which is affected by the matched term, is sent to the model-based controller at any numerical integration step (i.e. the mechanism based on the triggering condition is deactivated). Note that this is the case considered in Theorem 1. Fig. 7 shows the time evolution of the plant state variables, which are steered to zero, since the matched disturbance is rejected by the ISM component of the controller. Fig. 8 shows the control variable \( u(t) \) (see (7)). Both the states and \( u(t) \) respect the pre-specified constraints.

**Case 2:** In this second case, we assume that the mechanism based on the triggering condition is activated (see Theorem 2). Fig. 9 shows the time evolution of the state variables of the plant and of the nominal model, which are both steered to zero. Fig. 10 illustrates the control variable \( u(t) \). In Fig. 11, the relative threshold defined in (8), and the flags values are reported. Also in this case, both the states and \( u(t) \) respect the pre-specified constraints.

Table I summarizes the results obtained in the previously described cases. One can notice that the RMS value of the state is increased of the 1% when the event-triggered mechanism is used (Case 2), but this slight performance degradation is compensated by a reduction of the 99% of the number of transmissions of the state with respect to Case 1.
When the event-triggering condition is active, a suitable designed triggering condition with relative threshold is used in order to compensate the matched uncertainties between plant and nominal model, remains bounded under the event-triggered control scheme is proposed. The integral sliding mode approach is used in order to compensate the matched uncertainties. Moreover, one can notice that the RMS value of the auxiliary sliding variable is evidently smaller in Case 1, due to the fact that, Case 2, when the nominal model state is used in place of the actual plant state, the ISM component cannot properly reject the matched uncertainty since this does not affect the nominal model. Finally the RMS value of the state error, in Case 2, highlights that \(|e(t)|\), i.e. the displacement between plant and nominal model, remains bounded under the action of the model-based event-triggered MPC/ISM control proposed in the paper.

### VII. Conclusions

In this paper, a model-based event-triggered MPC/ISM control scheme is proposed. The integral sliding mode approach is used in order to compensate the matched uncertainties affecting the system, while the MPC component allows one to optimize a given cost function taking into account state and control constraints. The scheme enables to use the actual state measurement only when it is necessary, on the basis of a suitably designed triggering condition with relative threshold. The proposed approach is theoretically analyzed. Finally, a simulation example of mechanical nature is discussed, showing the satisfactory performance of the proposed control scheme.

### References


