Event-Triggered Sliding Mode Control Algorithms for a class of Uncertain Nonlinear Systems: Experimental Assessment

Michele Cucuzzella, Gian Paolo Incremona and Antonella Ferrara

Abstract—An experimental assessment of the recently introduced event-triggered sliding mode control approach is presented in this paper. The major design requirement, in this approach, is to reduce the number of transmissions over the network, while guaranteeing that the sliding mode control is stabilizing with appropriate robustness in front of matched uncertainties. In the present paper a novel Event-Triggered Sliding Mode Control algorithm is first introduced and discussed and then it is compared with two different Model-Based Event-Triggered Sliding Mode Control algorithms. Finally, their experimental assessment is reported, obtaining satisfactory performance consistent with the theoretical treatment and fulfilling all the design requirements.

I. INTRODUCTION

Sliding Mode Control (SMC) is a well-known control methodology able to guarantee satisfactory performance of the controlled system in spite of the presence of matched uncertainties [1], [2]. By virtue of its low complexity implementation and robustness, it can be regarded as an effective solution also in case of Networked Control Systems (NCSs), i.e., feedback systems including communication networks. Indeed, the presence of the network in the control loop can cause the occurrence of packet loss, jitter, and delayed transmissions, which can deteriorate the performance of the control system as long as this is designed in the conventional way, that is neglecting the network presence [3].

Instead, one of the methodologies which is very appreciated to design NCSs is the so-called event-triggered control [4], [5]. Event-triggered control, in contrast to time-triggered control, which features periodic transmissions of the measurements, enables measurements transmissions only when a pre-specified triggering condition is satisfied (or violated, depending on the adopted logic). So, while SMC can be efficacious in NCSs making the effects of the network induced nonidealities negligible by virtue of its robustness, event-triggered control can reduce such effects by significantly limiting the network overload.

By using event-triggered control, in spite of the aperiodic transmission of the measurements, satisfactory stability properties can be enforced. Specifically, in [4], it was proved that in case of nonlinear systems, relying on a triggering condition depending on the system state and based on the measurement error between the current state and the last state transmitted over the network, the Input-to-State Stability (ISS) of the controlled system can be guaranteed.

In the literature, the basic event-triggered control approach have been further elaborated so as to take into account the possible knowledge of a nominal model of the plant. This has produced the so-called model-based event-triggered control discussed in [6], [7]. This approach has been recently effectively exploited in conjunction with SMC [8]–[10], and model predictive control (MPC), even in case of mixed logical dynamical (MLD) systems [11]–[14]. Moreover, a novel genuine Event-Triggered Second Order Sliding Mode (ET-SOSM) control strategy has been recently proposed for affine nonlinear systems [15].

In the present paper, making reference to the class of nonlinear systems affected by matched uncertainty, three event-triggered sliding mode control strategies are introduced and discussed. In particular, a novel genuine Event-Triggered SMC (ET-SMC) strategy is proposed. Then, it is compared with two different Model-Based Event-Triggered SMC (MBET-SMC) strategies, already published in the literature [8], [9]. The aim of the paper is to provide an assessment of the three strategies on the basis of experiments performed on a real setup. This is a water tank with a communication network between the plant and the control unit. Satisfactory results have been obtained for all the strategies. The results confirm the theoretical properties of the considered event-triggered sliding mode control approaches and their suitability.
to be used in a networked control context.

II. Problem Formulation

Consider the single input system given by

\[
\begin{align*}
\dot{x}_i(t) &= x_{i+1}(t) \quad i = 1, \ldots, n-1 \\
\dot{x}_n(t) &= f(x(t),t) + b(x(t),t)(u(t) + w(x(t),t))
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) is the state vector and \(u(t) \in \mathbb{R}\) is the control variable, while \(f(x(t),t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}, b(x(t),t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}\) and \(w(x(t),t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}\) are bounded uncertain functions, with, in particular

\[
|f(x(t),t)| \leq F
\]

(2)

\[
0 < B_{\min} \leq b(x(t),t) \leq B_{\max}
\]

(3)

\[
|w(x(t),t)| \leq W
\]

(4)

where \(F, B_{\min}, B_{\max}\) and \(W\) are positive known constants. Select a variable \(\sigma(x(t)) : \mathbb{R}^n \to \mathbb{R}\), as

\[
\sigma(x(t)) = \sum_{i=1}^{n-1} m_i x_i(t) + x_n(t)
\]

(5)

with \(m_i, i = 1, \ldots, n-1\), real positive constants such that the characteristic equation \(\sum_{i=1}^{n-1} m_i z^{i-1} + z^{n-1} = 0\) has all roots with negative real part. The variable \(\sigma(x(t))\) will be called sliding variable in the following. Consider a nominal model of system (1) defined as

\[
\begin{align*}
\dot{\hat{x}}_i(t) &= \hat{x}_{i+1}(t) \quad i = 1, \ldots, n-1 \\
\dot{\hat{x}}_n(t) &= \hat{f}(\hat{x}(t),t) + \hat{b}(\hat{x}(t),t)u(t)
\end{align*}
\]

(6)

where \(\hat{x} \in \mathbb{R}^n\) is the state of the model, \(\hat{f}, \hat{b}\) are the nominal parts of \(f, b\) (see (1)-(3)), and \(u \in \mathbb{R}\) is the same input fed into the plant. Let \(\hat{\sigma}(t)\) be the nominal sliding variable defined as in (5) but computed relying on the state of (6).

Then, the problem to solve can be stated as follows: with reference to system (1), the nominal model (6), and the control scheme in Fig. 1, find a bounded control law such that the sliding variable \(\sigma\) results in being ultimately bounded, allowing the amplitude of the convergence boundary layer to be arbitrarily set. Moreover, the overall control strategy has to be of networked type, guaranteeing a significant reduction of the state transmissions over the network with respect to a conventional (i.e., non event-triggered) implementation.

III. The Considered Event-Triggered Sliding Mode Control Strategies

As a preliminary step, it is worth recalling that in practical implementation the state is sampled at certain time instants \(t_k, k \in \mathbb{N}\), and the control law, computed as \(u(t_k) = u(t_k), \forall t \in [t_k, t_{k+1}[\), is held constant between two successive samplings. In conventional implementation, the sequence \(\{t_k\}_{k \in \mathbb{N}}\) is typically periodic and the control approach is classified as time-triggered. Instead of relying on time-triggered executions, the idea is to introduce a triggering condition which depends on \(\sigma(x(t))\), so that the state of the controlled plant is transmitted over the network only when such a condition is verified. This implies that the control law is updated and sent to the plant only at the triggering time instants, and the overall control strategy is of event-triggered type. In the following subsections the proposed ET-SMC is first described, then two MBET-SMC strategies [8], [9] are recalled.

A. Strategy 1: ET-SMC

Consider the control scheme reported in Fig. 1. It contains two key blocks: the smart sensor and the sliding mode controller. We assume that the considered sensor is smart in the sense that it has some computation capability, i.e., it is able to compute \(\sigma(x(t))\) and verify a triggering condition. The triggering condition adopted in this case is the following

\[
|\sigma(x(t))| \geq \lambda_1
\]

(7)

Only when the triggering condition (7) holds, is the actual state \(x\) transmitted by the sensor over the network, so that the control law is updated and sent to the plant (switches \(S_1\) and \(S_2\) closed).

By regarding (5) as the controlled variable, associated with system (1), it turns out that the relative degree of the input-output map is 1. Indeed, one has

\[
\sigma(x(t)) = \sum_{i=1}^{n-1} m_i x_i(t) + f(x(t),t)
\]

(8)

where \(f, m_i\) are bounded uncertain functions, \(x_i(t)\) is assumed to be bounded (see [16]), i.e.,

\[
|\sigma(x(t),t)| < \Phi_{\text{sup}}
\]

(9)

with \(\Phi_{\text{sup}}\) positive constant. The proposed SMC law at the triggering time instants, namely \(u(t_k), k \in \mathbb{N}\), can be expressed as

\[
u(t_k) = -U_{\text{max}} \text{sgn}(\sigma(x(t_k)))
\]

(10)

with

\[
U_{\text{max}} > \frac{\Phi_{\text{sup}}}{B_{\min}}
\]

(11)

The control law (10), and the triggering condition (7) give rise to the Event-Triggered SMC (ET-SMC) strategy (Strategy 1). Its aim is to steer the sliding variable \(\sigma(x(t))\) to a boundary layer \(B_{\lambda_1}\), defined as a vicinity of the sliding manifold, i.e.,

\[
B_{\lambda_1} \triangleq \{ \sigma(x(t)) \in \mathbb{R} : |\sigma(x(t))| \leq \lambda_1 \}
\]

(12)

with \(\lambda_1\) positive constant.

Remark 1: Note that, to reduce the number of triggering events when the sliding variable \(\sigma(x(t))\) enters the boundary layer, the amplitude of the control law can be reduced, i.e.,

\[
u(t_k) = -U_{\text{max}} \begin{cases} 
\text{sgn}(\sigma(x(t_k))) & \text{if } |\sigma(x(t_k))| > \lambda_1 \\
K \text{sgn}(\sigma(x(t_k))) & \text{if } |\sigma(x(t_k))| \leq \lambda_1
\end{cases}
\]

(13)

\(K\) being a positive constant less than 1.
B. Strategy 2: MBET-SMC

The second strategy is characterized by the use of the nominal model (6) within the controller indicated in Fig. 1, with switch $S_2$ always closed. The controller includes also the triggering condition, which is written in terms of the nominal sliding variable, i.e., $\dot{\sigma}(t) = \sigma(\dot{x}(t))$, as

$$|\dot{\sigma}(t)| \leq \lambda_2$$

(14)

where $\lambda_2$ is a positive constant, such that the corresponding boundary layer of the sliding manifold is defined as

$$B_{\lambda_2} := \{ \sigma(t) : |\sigma(t)| \leq \lambda_2 \}$$

(15)

Assume, at the initial time instant $t_0$, that $\dot{\sigma}(t_0) = \sigma(x(t_0))$ and that $\dot{\sigma}(t_0) \notin B_{\lambda_2}$. The strategy switches between two operative modes, Mode 1 and Mode 2 (see [8] for more details).

Mode 1 (Condition (14) is violated). The state $\dot{x}$ of the nominal model is provided to the triggering condition block which computes the sliding variable relying on the nominal model state $\dot{x}$ (switch $S_1$ is open), which is used by the sliding mode controller, and the control law

$$u(t) = -U_{\text{max}} \text{sgn}(\sigma(x(t)))$$

(16)

is sent as input to the plant and fed into the nominal model.

Mode 2 (Condition (14) holds). The switch $S_1$ is closed and the measured state $x$ is sent over the network and the following control variable

$$u(t) = -U_{\text{max}} \text{sgn}(\sigma(x(t)))$$

(17)

is used as input for both the plant and the nominal model. When the event occurs, the nominal model is reinitialized with the plant state.

Remark 2: A peculiar aspect of the present ETMB-SMC strategy with respect to a standard SMC law is the fact of using the state of the nominal model outside the boundary layer, so that, in practice, during the portion of the reaching phase in which the sliding variable approaches $B_{\lambda_2}$, no state transmission is needed. When $\dot{\sigma}$ enters $B_{\lambda_2}$, robustness issues becomes priority, and this motivates the fact of using the actual plant state for the feedback. Indeed, the nominal model based control variable cannot guarantee disturbance rejection.

C. Strategy 3: MBET-SMC with Chattering Alleviation

Now, consider a second MBET-SMC strategy. In such a scheme, the plant is modelled by equation (1), the “smart sensor” is a sensor (fastened to the plant) equipped with some computation capability, and switch $S_2$ in Fig. 1 is always closed. In particular, the smart sensor measures the plant state, it determines the sliding variable $\sigma(x(t))$ as in (5), then it checks if the triggering condition

$$|\sigma(x(t))| < \lambda_3$$

(18)

with $\lambda_3$ being a small positive constant, is verified. If this is the case, no state transmission is performed, so that the control law needs to be generated relying on the nominal model of the plant indicated in (6). Moreover, a boundary layer, namely

$$B_{\lambda_3} := \{ \sigma(x(t)) : |\sigma(x(t))| \leq \lambda_3 \}$$

(19)

can be defined, which will have the role of convergence boundary layer. Also this strategy switches between two different operative modes, Mode 1 and Mode 2 (see [9] for more details).

Mode 1 (Condition (18) is violated). The actual state $x$ is sent over the network (switch $S_1$ is closed) and, when the event occurs, the nominal model of the plant is reinitialized. In this case, the controller computes the control law (17), which is sent to the plant and also fed into the nominal model.

Mode 2 (Condition (18) holds). The smart sensor does not send the actual state $x$ over the network (switch $S_1$ is open). In this case, the state $\dot{x}$ of the nominal model is used to compute the sliding variable $\dot{\sigma}$. Note that, since the nominal model in (6) is perfectly known, a nominal pseudo-equivalent control, can be computed by posing $\dot{\sigma} = 0$, i.e.,

$$\dot{\sigma}(t) = \dot{x}_n(t) + \sum_{i=1}^{n} m_i \dot{x}_i(t) = 0$$

(20)

$$\dot{\sigma}(t) = \ddot{f}(\dot{x},t) + \dot{b}(\dot{x},t)u(t) + \sum_{i=1}^{n} m_i \dot{x}_i(t) = 0$$

(21)

which differs from the theoretical equivalent control [1] since it does not take into account the action of the matched unknown term and it is computed using the nominal model state.

IV. Stability Analysis

With reference to the proposed event-triggered sliding mode control strategies, the following results can be proved, but (because of space limitation) the corresponding proofs are omitted. For the readers’ convenience, we introduce the following definition:

Definition 1: The solution $\sigma(x(t))$ to the uncertain system (8) is said to be ultimately bounded with respect to the set $B_{\lambda_1}$ if in a finite time it enters the bounded set $B_{\lambda_1}$ and there remains for all subsequent time instants.

Theorem 1: Given system (1)-(5), controlled via (10), (11), with the triggering condition (7), then, the solution $\sigma(x(t))$ to (8) is ultimately bounded with respect to $B_{\lambda_1}$, $\lambda_1$ being a positive constant arbitrarily set.

Theorem 2: Given system (1)-(6), controlled via (16) or (17), depending on the triggering condition (14), then, a sliding mode on $\sigma(x(t)) = 0$ is enforced in a finite time.

Theorem 3: Given system (1)-(6), controlled via (17) or (21), depending on the triggering condition (18), then, the solution $\sigma(x(t))$ to (8) is ultimately bounded with respect to $B_{\lambda_3}$, $\lambda_3$ being a positive constant arbitrarily set.
V. EXPERIMENTAL ASSESSMENT

In this section, the results of the experimental verification and validation of the proposed algorithms are discussed. The considered setup is the water tank illustrated in Fig. 2, with a communication network between the plant and the control unit. The plant is composed of a polycarbonate tank with a capacity of 25L. Two pumps are the plant actuators. They are driven by brushless motors connected to an electronic driver which uses the Pulse-Width-Modulation (PWM) technique. The measure of the water level is provided by an ultrasonic sensor. The tank has a tap (used as a disturbance generator) on the bottom, and a collection basin with a capacity of 75L is placed under the tank.

Let \( h \) be the level of water in the tank, and \( q_e \) be the water flow delivered by the left pump or aspirated by the right pump. Let \( A \) be the area of the tank, while let \( A_u \) and \( h_u \) be the area and the height of the tap, respectively. The dynamic of the system can be described by

\[
\dot{h}(t) = \frac{1}{A} \left( q_e(t) - A_u \sqrt{2g(h(t) - h_u)} \right)
\]

where, making reference to system (1), \( x = h, u = q_e, w = -A_u \sqrt{2g(h(t) - h_u)} \), while \( f = 0 \) and \( b = 1/A \), respectively. The tank’s parameters are reported in the caption of Fig. 2.

With reference to system (22) the sliding variable \( \sigma(t) \) is chosen as the tracking error between the controlled variable \( h(t) \) and its reference, i.e.,

\[
\sigma(t) = h_{ref}(t) - h(t)
\]

\( h_{ref}(t) \) being the desired level of water in the tank. Let \( \tilde{u} \) be the control variable (dimensionally a voltage) driving the brushless motors of the pumps. Since the operative voltage range for the motors is \([0, 5]\) V, the control amplitude is selected as \( \tilde{U}_{\text{max}} = 5\) V, the thresholds as \( \lambda_1 = \lambda_2 = \lambda_3 = 0.01 \) m and the modulating factor as \( \lambda = 0.25 \). Note that, if the control input \( \tilde{u} > 0 \), then the control input is fed into the motor of the left pump, which delivers water to the tank; if \( \tilde{u} < 0 \), then the control input \( -\tilde{u} \) is fed into the motor of the right pump, which aspirates water from the tank. In the experiments \( h_{ref} = 0.1 \) m and the initial condition was \( h(0) = 0 \) m. The sampling time and the control horizon are \( T_s = 0.1 \) s and \( T_h = 360 \) s, respectively.

To be able to evaluate the frequency of state transmissions, a flag function \( f_{\text{up}} \), representing the number of the state updates, is introduced. More specifically, \( f_{\text{up}} \) is equal to 1 when (7) or (14) are verified or (18) is violated. Three scenarios are considered: with the tap closed, open and open only during the time interval \( t \in [180, 220] \) s, respectively. Figures 3, 4 and 5 show the behavior of the actual sliding variable and of the flag function \( f_{\text{up}} \) by applying Strategy 1. The behaviour of both the actual and nominal sliding variable, and of the flag function \( f_{\text{up}} \) are shown in Figures 6 and 7 when Strategy 2 is applied, and in Figures 8, 9 and 10 when Strategy 3 is applied.

In order to evaluate the closed-loop performance of the proposed control strategies, four indices are considered. They are reported in Table I, where \( n_s \) is the number of integration steps, i.e., \( n_s = T_h/T_s \). The values of these performance indices are reported in Table II. The histogram in Fig. 11 compares the three control strategies in terms of relative values, normalized with respect to the worst results. Finally, one can conclude that Strategy 2 allows to obtain the smallest error of the sliding variable, Strategy 1 allows to obtain the smallest number of state transmissions over the network and Strategy 3 results the best in terms of control effort. All the strategies confirm their capability of steering the system state towards the desired sliding manifold and of generating an ultimately bounded evolution of the sliding manifold.

### Table I

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\text{RMS}} )</td>
<td>( \frac{1}{n_s} \sum_{i=1}^{n_s} \sigma^2(t_i) )</td>
<td>RMS value of the sliding variable</td>
</tr>
<tr>
<td>( E_c )</td>
<td>( \frac{1}{n_s} \sum_{i=1}^{n_s} \tilde{u}^2(t_i) )</td>
<td>average control effort</td>
</tr>
<tr>
<td>( n_{\text{up}} )</td>
<td>( \sum_{i=1}^{n_{\text{up}}(t_i)} )</td>
<td>number of updates</td>
</tr>
<tr>
<td>( \Delta_{\text{up}} )</td>
<td>( \frac{n_{\text{up}}(t_i) - n_{\text{up}}(t_i-1)}{n_{\text{up}}(t_i)} )</td>
<td>updates reduction</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Tap</th>
<th>( \sigma_{\text{RMS}} )</th>
<th>( E_c )</th>
<th>( n_{\text{up}} )</th>
<th>( \Delta_{\text{up}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>closed</td>
<td>0.0233</td>
<td>2.3809</td>
<td>175</td>
<td>95.1%</td>
</tr>
<tr>
<td></td>
<td>open</td>
<td>0.0277</td>
<td>3.4145</td>
<td>2265</td>
<td>31.5%</td>
</tr>
<tr>
<td></td>
<td>closed/open</td>
<td>0.0228</td>
<td>2.4755</td>
<td>447</td>
<td>87.6%</td>
</tr>
<tr>
<td>2</td>
<td>closed</td>
<td>0.0207</td>
<td>4.9785</td>
<td>3084</td>
<td>14.3%</td>
</tr>
<tr>
<td></td>
<td>open</td>
<td>0.0270</td>
<td>4.9882</td>
<td>2688</td>
<td>25.3%</td>
</tr>
<tr>
<td>3</td>
<td>closed</td>
<td>0.0233</td>
<td>1.9071</td>
<td>546</td>
<td>84.8%</td>
</tr>
<tr>
<td></td>
<td>open</td>
<td>0.0298</td>
<td>3.8345</td>
<td>2201</td>
<td>38.9%</td>
</tr>
<tr>
<td></td>
<td>closed/open</td>
<td>0.0254</td>
<td>2.2788</td>
<td>775</td>
<td>78.5%</td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS

In this paper the experimental assessment of sliding mode control algorithms based on the event-triggered control approach has been presented. Specifically, three control strategies have been introduced and discussed in the paper. The three algorithms are a novel event-triggered sliding mode control strategy and two model-based event-triggered sliding mode control strategies. The experimental tests have been performed on an electro-hydraulic testbed. All the three strategies demonstrate to be able to enforce an ultimately bounded evolution of the sliding manifold even in presence of the uncertainties.

VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the contribution and the full help provided by the senior lab technician Gianluca.
with the convergence boundary layer and the flag function $f_{up}$ when the tap is always closed.

Fig. 8. Strategy 3. Time evolution of the actual and nominal sliding variable with the convergence boundary layer and the flag function $f_{up}$ when the tap is always closed.

Fig. 9. Strategy 3. Time evolution of the actual and nominal sliding variable with the convergence boundary layer and the flag function $f_{up}$ when the tap is always open.

Fig. 10. Strategy 3. Time evolution of the actual and nominal sliding variable with the convergence boundary layer and the flag function $f_{up}$ when the tap is open only during a limited time interval.

Fig. 11. Comparison between the proposed control strategies.

De Felici and by the bachelor student Angelo Rendiniello during the experimental tests.

REFERENCES


