EU AND US DESIGN APPROACHES FOR STEEL STORAGE PALLET RACKS WITH MONO-SYMMETRIC CROSS-SECTION UPRIGHTS

Claudio Bernuzzi, Marco Simoncelli
Department of Architecture, Built Environment and Construction Engineering
Politecnico di Milano, Milano-Italy

ABSTRACT

Design, fabrication and erection of industrial steel storage rack systems can nowadays take place at different locations, potentially separated by thousands of kilometers. Consequently, manufacturing engineers need to understand the main code provisions adopted in the country where the rack will be in-service. Frequently, design is carried out in accordance with the European (EU) or United States (US) rack codes, which are the most commonly adopted standards for industrial storage systems. As appraisal of the key differences on structural performance associated with the different code provisions in term of storage performance, a previous study of the Authors was focused on storage pallet racks comprised of bi-symmetric cross-section uprights (vertical members). Now, attention is paid to racks with mono-symmetric uprights, typically influenced by relevant warping effects, which are traditionally neglected by rack provisions and, as a consequence, by manufacturing engineers. The design approaches already considered and compared for bi-symmetric uprights now appear to be inadequate and have been necessarily improved, as suggested by the Authors, including at least the contribution due to the bimoment acting along mono-symmetric uprights. Research outcomes, which are discussed in the paper, are related to a parametric analysis on several racks differing for configurations, geometry of components and degree of rotational stiffness of joints. The associated results regarding the four EU and two US considered alternatives are presented and compared directly to each other to allow for a concrete appraisal of the most relevant differences between the admitted design approaches. In order to highlight the importance of warping effects, which can be evaluated only by means of refined finite element (FE) analysis software, design has been undertaken using also the more traditional FE beam formulation implemented in the commercial analysis packages most frequently used in manufacturing offices. Finally, Appendix A presents a complete design example to be used as benchmark, where all the discussed design options are applied and compared to each other.

Keywords: semi–continuous unbraced frames, adjustable selective steel storage pallet racks, open thin-walled cross-section, warping effects, safety index.
1. INTRODUCTION

One of the main results of increasing globalization is that nowadays design, fabrication and erection of industrial steel structures for the storage of materials and products can take place at different locations, potentially separated by several thousands of kilometres. As a consequence, rack owners might require the use of widely accepted steel design codes and manufacturing designers might be consequently familiar with specifications that have substantial differences between each other. Due to the great interest in the comparison of different standard codes, a research project has been developed on the design methods according to European (EU) and United States (US) steel provisions. At first, attention has been focused on the more traditional carpentry steel frames made by hot-rolled I-shaped members: similarities and differences between the EN 1993-1-1 [1] and AISC360-10 [2] have been discussed by Bernuzzi et al. [3] summarizing the main results of a comparative parametric study carried out on more than 700 multi-story semi-continuous planar frames. In the second step, the approaches adopted for adjustable pallet racks have been considered and the design alternatives admitted by EU [4] and US [5] codes have been discussed and applied with reference to selective pallet racks realized by bi-symmetric cross-section members [6,7]. The third and last phase of the research, which is dealt with by the present paper, is focused on the influence of the mono-symmetry of the upright cross-sections most commonly used in design (figure 1).

![Figure 1: typical mono-symmetric cross-section uprights used in industrial storage systems.](image)

It is worth mentioning that rules currently adopted in Europe for the verification checks of pallet racks under monotonic loading derive from those proposed for hot-rolled bi-symmetric cross-section members. As a consequence, with the exception of design developed according to the Australian rack design standard [8], key features associated with mono-symmetric cross-sections typically used for
vertical (uprights) members are considered in a very simplified way, or frequently ignored. The direct consequence, from the design point of view, is the erroneous assessment of the set of internal forces, moment and displacements and of the rack performance, which in few cases reflects directly in a structural system characterized by a level of reliability lower than the one required by standard codes.

In more detail, the limited availability of FE analysis packages appropriates to simulate via beam elements the effects associated with warping torsion and the low confidence of manufacturing engineers with the application of the thin-walled beam theory [9,10] produce the result that, in routine design, no differences can be observed in case of bi- or mono-symmetric cross-section members. It is worth noting that, the introduction of the 7th DOF in the structural analysis make the rack more flexible but at the same time the presence of bi-moment could change remarkably the coefficient of utilization, or equivalently the safety index, that is a parameter, ranging from 0 to 1 (resistance/stability limit is achieved), of great interest for practical design purposes. As better explained in the following, its definition is associated with the effects of the design actions over the resistance. Also the values of the bending moments should be different from those obtained by neglecting warping.

The presence of the bimoment is currently neglected, as well as the influence of the coupling between bending and torsion on the rack deformability and on the overall buckling mode. Very general requirements are provided by codes, without practical indications for designers which expect from them uniquely applicable procedures. In recent research papers edited by Cardoso and Rasmussen [11] and Sangle et al. [12], the problem of warping in the FE model have been avoided by modelling the racks with shell elements. This solution, that gives quite accurate results, is not easily replicable by the structural engineer, due to the complexity of the modelling phase.

On the other hand, Teh et al. [13] using a beam finite element have studied the warping influence on the global behavior of the racks giving, indications also about the influence on the buckling analysis results. From the experimental point of view dynamic test was made by Dev and Ralukdar [14] on isolated column, carrying out the warping influence on the modal parameters. Some preliminary studies on the stress distribution across cross-sections and on the overall buckling conditions carried out by Bernuzzi et al. [15-17] indicated that the effects associated with the use of mono-symmetric cross-section members (i.e. warping torsion, Wagner coefficients and the eccentricity between the shear center and the cross-section centroid) play an important role on the performance of medium-rise pallet racks and cannot ever be neglected in a safe and reliable rack design.

No comparative analyses related to rack provisions adopted in different countries are available for framed systems made up of mono-symmetric uprights. In the present paper, owing to the practical interest of this matter, key features of four EU and two US design alternatives are briefly discussed.
In particular, a parametric study has been carried out on 216 medium-rise racks differing for the frame upright cross-section, longitudinal layout and connection performances. Structural analyses have been carried out by means of two different finite element (FE) analysis packages, differing in the FE beam formulation. The results of a traditional commercial FE formulation based on the use of the Timoshenko beam element [18] have been compared with a more refined one including the warping of the cross-section as an additional (7th) degree of freedom (DOF) [19]. In total, more than 3000 design cases have been developed focusing attention on the uprights, for which more relevant differences are expected with reference to pallet beams and lacings owing to the direct influence of warping effects with regard to the permissible design alternatives. Furthermore, Appendix A presents a complete benchmark example, where all the design paths admitted by both codes are applied and compared to one another, focusing attention not only on the performance of the different racks but also on the key features due to the presence of one axis of symmetry.

2. THE ADMITTED DESIGN ALTERNATIVES

In the case of steel storage systems, as well for carpentry frames, routine design is carried out in two phases.

- structural analysis of the frame, aimed at evaluating the set of displacements, internal forces and moments on members and joints;
- verification checks of members for displacements, resistance and instability and of joints for resistance and rotation demand.

It is worth noticing that remarkable differences associated with the use of 6 or 7 DOFs FE analysis packages can always be detected in each phase of the design process, starting from the values of the set of displacements, internal forces and moments used to verify the serviceability and ultimate limit states.

In the following, only the essential contents of the codes are briefly revisited, refering readers to previous papers [3,6] also for similarities and differences regarding the effective cross-section properties or directly to the code itself for a more detailed discussion about the principles of the alternatives offered to designers. Independent of the cross-section type, steel storage pallet racks are very flexible structures under lateral loads, owing to various factors such as the high slenderness of the uprights, the modest degree of rotational stiffness of beam-to-column joints and base-plate connections [20] and the absence of vertical bracing in the down-aisle direction, due to the need to provide the maximum storage capability. As a consequence, a second-order analysis is often required in routine design, which could also be carried out on occasion via approximated approaches, now obsolete for the relevant progresses of the commercial FE analysis packages. Moreover, owing to the extensive use of thin-walled cold-formed members, the traditional design methods for the
structural analysis developed for frames comprised of bi-symmetric hot-rolled members cannot be
directly adopted, despite it being proposed for storage systems. An open problem, which is currently
neglected in pallet rack design, is related to the influence of warping torsion on the load carrying
capacity of the uprights, as well as on the performance of the whole framed system. The equations
proposed for the design verification checks are in fact only efficient for bi-symmetric cross-section
members. When mono-symmetric uprights are employed, the set of displacements and of internal
forces and moments are significantly influenced by the cross-section warping. Owing to the
importance of this contribution, the bimoment must be accounted for in the design procedures, as
herein discussed. The Authors propose an improvement of the equations traditionally adopted for
routine design, requiring the assessment of the additional contribution of the bimoment stresses on the
considered cross-section, which can be addressed only by means of FE beam formulation characterized
by the cross-section warping.

2.1 The EU approaches

Structural analysis of racks according to the European practice, should be carried out via one of the
following methods, already identified by Bernuzzi [6]:

- EU-DAM: Direct Analysis Method, (specified in EN15512 sub-chapter 10.1.3);
- EU-RAM: Rigorous Analysis Method, (specified in EN15512 sub-chapter 9.7.6);
- EU-GEM: General Method, (specified in EN1993-1-1 sub-chapter 6.3.4).
- EU-IRAM: Improved Rigorous Analysis Method (proposed by Authors).

EU-DAM. The direct analysis method (DAM) requires an advanced three-dimensional analysis,
including both overall rack and member imperfections and joint eccentricities, where relevant.
Furthermore, as clearly stated by EN15512 in very general terms, rack design has to be carried out
by means of refined FE analysis packages able to capture accurately the coupling between flexure
and torsion and the influence of warping deformations on torsional, and consequently flexural-
torsional buckling, warping torsion and shear centre eccentricity. Therefore, only resistance checks
is required and in absence of indications by the codes, it appears necessary to include also the
bimoment contribution, as recently recommended by the Australian Code [8]. In the more general
case of beam-columns subjected to axial load ($N_{Ed}$), bending moments along the axis of symmetry, $y$,
and along the other principal axis, $z$ ($M_{y,Ed}$ and $M_{z,Ed}$, respectively) and bimoment ($B_{Ed}$), reference
should be conveniently made to the following definition of the safety index, $SI_{7}^{EU\text{-DAM}}$: 
where $A_{\text{eff}}$ and $W_{\text{eff}}$ indicate the area and the section modulus of the effective cross-section, respectively, $f_y$ is the material yield strength, $\gamma_{M0}$ is the material safety factor and $B_{Rd}$ is the resisting bimoment, which is defined as:

$$B_{Rd} = \frac{I_w \cdot f_y}{\omega_{\text{max}} \cdot \gamma_{M0}}$$

where, with reference to the gross cross-section, $I_w$ is the warping constant and $\omega_{\text{max}}$ is maximum value of the sectorial area.

For a better understanding of the paper’s contents, it should be noted that eq. 1a) is proposed in a form that clearly distinguishes between the contribution due to the traditional design approach neglecting warping (subscript 6) and that due to the bimoment.

**EU-RAM.** The **Rigorous Analysis Method** (RAM) takes into account the lack-of-verticality imperfections neglecting the effects of the out-of-straightness of members. EN15512 declares that the structure shall be considered a no-sway frame and buckling lengths shall be put equal to system (geometrical) lengths. This *a priori* imposition of a constant value of the effective length factor does not allow one to efficiently account for the coupling between flexural and torsional buckling. At the same time it ignores the influence of the joint stiffness as well as the possible benefits associated with lean-on effects [21].

Stability checks of uprights are carried out with reference to the following equation:

$$SI^{EU-\text{RAM}}_7 = \left( \frac{N_{\text{Ed}}}{A_{\text{eff}} \cdot f_y} + \frac{M_{y,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} \right) \left( \frac{B_{\text{Ed}}}{B_{Rd}} \right) = SI^{EU-\text{RAM}}_6 + \left( \frac{B_{\text{Ed}}}{B_{Rd}} \right) \leq 1$$

(1a)

Where, in addition to the terms already introduced, $\chi_{\text{min}}$ and $\chi_{\text{LT}}$ are the reduction factors for axial and lateral buckling, respectively.

The term $\chi_{\text{min}}$ depends directly on relative slenderness $\overline{\chi}$ defined as:

$$\overline{\chi} = \sqrt{\frac{A_{\text{eff}} \cdot f_y}{N_{cr}}}$$

(3a)

where $N_{cr}$ is the elastic critical axial buckling load for compressed members evaluated on the basis of the well-established theoretical approaches applied to the gross cross-section.
It is worth mentioning that, according to this method, the flexural buckling loads must always be evaluated considering the system length and the torsional buckling load as to be based on an effective length usually assumed equal to 0.7 the height of the upright panel. As a consequence, the flexural-torsional buckling load depends solely on the rack geometry, independent of the presence of bracing systems and/or by the degree of rotational stiffness of the joints.

In the same way, $\lambda_{LT}$ is the suitable reduction factor accounting for the lateral stability related to the relative slenderness $\bar{\lambda}_{LT}$ defined as:

$$
\lambda_{LT} = \frac{W_{\text{eff}} \cdot f_y}{M_{cr}}
$$

where $M_{cr}$ is the elastic critical bending moment for lateral buckling.

Furthermore, this approach has been presented focusing on the stability checks but the resistance checks according to eq. 1) have also always to be carried out, which generally lead to smaller less severe values of the SI, especially with reference to the fully loaded conditions, which is the case considered in the parametric study herein presented.

**EU-GEM.** The General Method (GEM) takes into account only the lack-of-verticality imperfections. This approach, which appears as very promising owing to its simplicity and efficiency when applied to racks, allows warping effects to be accounted for in both resistance and buckling checks. A frame, or equivalently a rack, is safe when:

$$
\frac{\chi_{\text{op}} \alpha_{\text{ult,k}}}{\gamma_M} \geq 1
$$

(4a)

With reference to the symbols already presented, the verification criterion can be more conveniently expressed as:

$$
S_{I}^{\text{EU-GEM}} = \frac{\gamma_M}{\chi_{\text{op}} \alpha_{\text{ult,k}}} \leq 1
$$

(4b)

where $\alpha_{\text{ult,k}}$ is the minimum load multiplier based on the cross-section resistance, $\chi_{\text{op}}$ is the buckling reduction factor in reference to the overall structural system and $\gamma_M$ is the material safety factor.

The ultimate load multiplier for resistance has to account for key features associated with the use of mono-symmetric cross-section members and hence is herein indicated as $\alpha_{\text{ult,k},7}$. It can be determined as:

$$
\frac{1}{\alpha_{\text{ult,k},7}} = \left( \frac{N_{\text{Ed}}}{A_{\text{eff}} \cdot f_y} + \frac{M_{y,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} \right) + \left( \frac{B_{\text{Ed}}}{B_{\text{Rd}}} \right) = \frac{1}{\alpha_{\text{ult,k},6}} + \left( \frac{B_{\text{Ed}}}{B_{\text{Rd}}} \right)
$$

(5)
Also in this case, it has been decided to split the term associated with the traditional design approaches from the second term \( \frac{B_{Ed}}{B_{Ed}} \), proposed by Authors, to suitably account for the presence of mono-symmetric cross-section members. It is worth noting, that, despite eqs. 1) and 5) are formally identical, the design values (subscripts Ed) are related to load conditions differing for imperfections: local and global in DAM approach and only global in the GEM approach.

**EU-IRAM.** The main difference from the EU-RAM approach regards the upright stability check: reference has to be made to eq. 2) but the evaluation of the critical axial load \( N_{cr} \) of each column is based on the overall critical load multiplier \((\alpha_{cr})\) directly obtained from a buckling analysis of the rack. For the upright subjected to the design axial load \( N_{Ed} \), the critical axial load can be directly evaluated as:

\[
N_{cr} = \alpha_{cr} \cdot N_{Ed}
\]  

(6)

In the case of 7DOFs beam formulations, the assessment of the critical load multiplier \((\alpha_{cr})\) allows for a direct evaluation of the critical load considering flexural-torsional buckling modes. Otherwise, if the critical load multiplier is obtained from a 6DOFs overall buckling analysis \((\alpha_{cr})\) that considering only the flexural buckling mode, the flexural-torsional buckling load is evaluated in a different way by defining the effective length for torsional buckling on the basis of the geometry of the frame.

### 2.2 The US approaches

As to the US rack design practice, RMI [5] states that all computations for safe loads, stresses and deflections, have to be made in accordance with the conventional methods for structural design. As to the design of medium-rise pallet racks, as clearly stated also by Sarawit and Pekoz [22], two methods can be adopted:

- **US – NOLM:** *Notional Load Method*, (defined in AISC360 sub-chapter C1.1);
- **US – ELM:** *Effective Length Method*, (defined in AISC360 sub-chapter C1.2 and discussed in Appendix 7.2).

For the evaluation of the performance of the uprights, in addition to the resistance checks according to eq. 1), stability verification checks are required in both methods, differing from one another mainly with regard to the value of critical buckling load or, equivalently, for the effective length factor K. A common equation is proposed for both methods but the values of the internal forces and moments are not equal due to the different set of imperfections, as discussed in detail in [7]. In particular, the safety index for the \( k^{th} \) method can be defined in a general form, as:
where \( P_n \) is the nominal compression member capacity and \( M_{nx} \) and \( M_{ny} \) are the nominal flexural capacities.

It should be noted that the \( x \)- and \( y \)- axes identifying principal cross-section axes in US codes correspond to the \( y \)- and \( z \)- axes, respectively, according to the EU symbols. In order to avoid confusion, reference can be made to table B1 proposing the equivalence between EU and US symbols identifying the geometric cross-section parameters.

**US – NOLM.** The Notional Load Method is the main suggested method and it requires a geometric non-linear analysis, considering all second-order effects, together with flexural, shear and axial member deformations and modelling the geometric initial imperfections via horizontal forces (notional loads) applied on each floor. All steel properties contributing to the elastic stiffness have to be multiplied by 0.8 imposing a 20% reduction of the structural stiffness to lateral load \([22,23]\), mainly to account for the simplified approach used to evaluate the effective length of uprights, which is based on the system length, i.e. \( K=1 \) as in the EU-RAM.

**US – ELM.** The Effective Length Method requires a second-order analysis, like in US-NOLM, but without any stiffness reduction and imperfections are taken into account only via notional loads. The evaluation of the member strengths is based on the effective length factor \( K \) for moment resisting (sway) frames obtained from alignment charts or, more conveniently, from the use of FE buckling analysis of the overall rack, as in the EU-IRAM.

### 2.3 Comparison of the approaches

The discussed EU and US design approaches for rack design present similarities and differences and to this end reference can be made to table 1 for the aspects of relevant importance. As previously introduced, these alternatives are proposed by the EU and US rack codes and independent of the selected alternative, a second-order analysis is strongly recommended. A common remark is that key features associated with the presence of mono-symmetric cross-section uprights are practically neglected in both codes with regard to resistance checks. As a consequence, the Authors suggestion is that, independent of the design alternatives and/or the considered code, all resistance checks have to be executed mandatory according to eq. 1), in order to include also the bimoment contribution.

**Table 1: similarities and differences between the EU and US rack design approaches.**

<table>
<thead>
<tr>
<th>Feature</th>
<th>EU DAM</th>
<th>US NOLM</th>
<th>EU RAM</th>
<th>US ELM</th>
<th>EU IRAM</th>
<th>EU GEM</th>
<th>US ELM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lack of-verticityality (sway imperfections)</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Member Out-of straightness imperfections</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness reduction</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stability checks (K=effective length factor)</td>
<td>NO</td>
<td>YES (K=1)</td>
<td>YES (K=1)</td>
<td>YES (K=K(Ncr))</td>
<td>YES (K=K(Ncr))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is worth noticing that the main differences between the EU-IRAM, EU-GEM and US-ELM approaches are in the definition of the member imperfections and in the equations checking stability. As to the key features associated with the use of mono-symmetric cross-section uprights, only few of these methods appear adequate for design purposes: the EU-IRAM, EU-GEM and US-ELM consider the presence of mono-symmetric cross-sections in the evaluation of the buckling load of the whole framed system while EU-RAM and US-NOLM are based on the definition of the effective length for flexural-torsional buckling depending solely on the geometry of the rack. Furthermore, the EU-GEM approach seems more than adequate for pallet racks [24], taking adequately into account warping effects in both resistance and stability checks and allowing for consideration of the presence of the perforations [25], which are necessary to accept the end joints of pallet beams in the case of adjustable storage systems.

3. THE CONSIDERED RACKS

The parametric study for the approach comparison has been focused on medium-rise double-entry racks, unbraced in the down-aisle direction with four equal bays of 2.78 m length that typically allows for the location of 3 pallet units per bay: rack depth is 1.00 m and upright frames present Z-panels guaranteeing stability to cross-aisle loads. Four configurations (fig. 2) have been defined, differing for the number of load levels (LL) and for the inter-storey height (hi): two (_2LL with hi = 2500 mm), three (_3LL with hi = 2200 mm), four (_4LL with hi = 1800mm) and five (_5LL with hi = 1330 mm) storeys.

Three lipped channels (identified as MM-, DD_ and TT_ types) have been considered for uprights, each having the same cross-section thickness. With reference to the gross cross-section, the value of the area (A), second moments of area (Iy and Iz) and section modulii (Wy and Wz) are reported in table 2, together with the uniform (Iy) and warping (Iw) torsion constants and the maximum value of the sectorial area (ωmax). Furthermore, on the basis of data provided by manufactures, in the same table the Q_reduction factors associated with the results of stub-column tests, equal in both EU (Q EU ) and
US \( Q_{LU}^N \) codes, and with bending tests along the two principal axes required by the EU design codes \( Q_{LU}^{ME} \) and \( Q_{LU}^{ME} \) are also reported.

It should be noted that, despite the fact that the uprights cross-section commonly used are those presented in figure 1, lipped channels are however used for pallet as well as cladding racks. In addition, please note that the input data for this cross-section require the definition of the eccentricity between the shear center and the cross-section centroid as well as the Wagner’s constants, like for the other mono-symmetric cross-section. Furthermore, this upright cross-section choice makes the research outcomes however of great interest for the rack practice, being analysis and design checks based on the use of 7DOFs beam formulation. A rather exhaustive overview of the cases most frequently encountered in routine rack design is proposed: in fact, the ratio between the second moments of area ranges from 1.0 to 3.0 and the ratio associated with section moduli from 1.0 to 1.5, approximately.

![Figure 2: the considered pallet racks: view along the cross-aisle (a) and down-aisle (b) direction (dimension in millimetres).](image)

Rectangular hollow sections (160x40x1.3 mm \( RHS \)) and square hollow section (35x35x2 mm \( SHS \)) elements have been used for pallet beams and lacings of upright frames, respectively. All these structural components are in S355 steel grade [26], with a yielding strength \( f_y = 355 N/mm^2 \).

Table 2: key features of the considered upright cross-sections.
The case of fully loaded racks has been considered with pallet units acting as a uniform load on all pallet beams. In the numerical study, attention has been focussed on the following parameters:

- the degree of flexural stiffness associated with beam-to-column joints: the selected values of rotational stiffness \( S_{j,btc} \) of interest for practical applications have been expressed as multiples (by means of term \( \rho_{j,btc} \)) of a reference stiffness value \( S_{EC3-LB}^{EC3-LB} \) via the relationship:

\[
S_{j,btc} = \rho_{j,btc} \cdot S_{EC3-LB}^{EC3-LB}
\]

where \( S_{EC3-LB}^{EC3-LB} \) is the stiffness associated with the lower bound of the semi-rigid domain, which is the value corresponding to the transition between the domains of flexible (pinned) and semi-rigid joints according to the classification criteria of part 1-8 of Eurocode 3 [27]. The parameter \( \rho_{j,btc} \) has been assumed to range from 1 to 10, and in addition, values of \( \rho_{j,btc} \) equal to 2, 4, 5 and 8 have been considered.

- the degree of flexural stiffness associated with base-plate connections: as for beam-to-column joints, the values of the base rotational stiffness \( S_{j,base} \) have been selected as multiple, by means of term \( \rho_{j,base} \), of the transition stiffness \( (S_{j,base}^{EC3-UB}) \) between the region of semi-rigid and rigid joints, defined as:

\[
S_{j,base} = \rho_{j,base} \cdot S_{j,base}^{EC3-UB}
\]

Three values have been considered (\( \rho_{j,base} = 0.15 \), \( \rho_{j,base} = 0.30 \) and \( \rho_{j,base} = 0.45 \)) to characterize the rotational behavior of the base-plate having different steel details to connect the rack with the foundation system or to the floor slab.
• the warping restraint at the upright base-plate: because of the different steel details used in
the connection of the steel rack with the supporting base, two different ideal cases for base-
plate restraint have been considered, indicated in the follow as WF (warping free) or WP
(warping prevented).

As already mentioned, two commercial finite element analysis packages have been used for the
structural analysis, SAP2000 [18], offering the classic Timoshenko beam element formulation with
6DOFs per node, and ConSteel [19] where the more refined Eulero-Bernoulli 7DOFs beam
formulation is implemented.

In order to propose design cases comparable to each other and research outcomes of actual interest
for routine design, prior to the design phase, a buckling analysis has been carried out for each rack.
In particular, in the Authors’ opinion, it appears necessary to propose research outcomes directly
comparable in parametric studies similar to the one described in the present paper. For this reason,
owing to the differences in the component performances, the multiplier of the 6DOFs buckling
analysis ($\alpha^\circ_{cr}$) was used to define the load condition multiplier as $0.56 \cdot \alpha^\circ_{cr}$, common to each
considered design case.

As to the modelling of the geometrical imperfections, it has been decided to use notional loads instead
of modelling racks with an imperfect geometry, due to the presence of sway and/or bow defects. In
particular it has to be noted that:

• the lateral loads applied on each rack level to simulate the uprights out-of-plumbness
correspond to 0.28% and 0.5% of the resulting vertical floor load, according to EU and US
code, respectively;

• the bow imperfections, which are considered only by the EU-DAM approach, have been
simulated by a uniform lateral load distributed along the upright and equilibrated in
correspondence of the floor by lateral concentrated forces, as recommended by EN 1993-1-1.
The layout summarizing the key parameters considered in this study is represented in figure 3: in total more than 3500 design analysis have been carried out on 216 racks, modelled via two different FE software packages and by applying 4 EU approaches and 2 US design procedures.

Owing to the large amount of data and to the need to clearly identify main research outcomes, the numerical study has been carried out focusing the attention on the uprights, and neglecting joint and pallet beam verifications, which are not expected to have a significant impact. Furthermore, a preliminary investigation showed that distortional buckling phenomenon on uprights is not of interest for the considered rack layout and as a consequence, it has been neglected.

### 4. TRADITIONAL DESIGN: INFLUENCE OF THE METHODS OF ANALYSIS

As already mentioned, the design approaches introduced and discussed in section 2 have been quite recently applied to racks with bi-symmetric cross-section uprights [7] and it was concluded that the EU-DAM and EU-RAM approaches lead to SI values very close to one another and significantly lower than those associated with the other approaches. Furthermore, it has been noted that a quite accurate prediction of the rack performance seems possible via the EU-IRAM, EU-GEM and US-ELM, approaches that are largely equivalent to one another, leading to reasonably similar values of the load carrying capacity, despite the fact that they differ significantly in the input data for the overall geometrical imperfections and for the equations used for member verification checks. As to racks with mono-symmetric cross-section uprights, it is worth mentioning that in the case of traditional 6DOFs FE analysis packages, the presence of bimoment cannot be evaluated in the output analysis data. Furthermore, the available design approaches, with the exception of EU-DAM, require consideration of the stability check for beam-columns and for the flexural-torsional failure buckling mode. The additional set of design cases considered in the present paper also allows for the evaluation

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Upright</th>
<th>Number of load level</th>
<th>Beam-to-column joint ( \rho_{j,MC} )</th>
<th>Base-plate joint ( \rho_{j,base} )</th>
<th>Degree of freedom</th>
<th>Warping condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU-DAM</td>
<td>MM</td>
<td></td>
<td>1</td>
<td>0.15</td>
<td>6DOFs</td>
<td>WF</td>
</tr>
<tr>
<td>EU-RAM</td>
<td>DD</td>
<td></td>
<td>2</td>
<td>0.30</td>
<td>7DOFs</td>
<td>WP</td>
</tr>
<tr>
<td>EU-IRAM</td>
<td>2LL</td>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU-GEM</td>
<td>3LL</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-NOLM</td>
<td>4LL</td>
<td></td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-ELM</td>
<td>5LL</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: layout of the parametric study.
of the influence of this buckling mode that is expected to govern the design in several cases for which
the shear centre coincides with the centroid.
Key results of the considered method of analysis can be evaluated and directly compared, in terms of
safety index making reference to the more highly stressed upright, which is, in general, in the internal
position of the rack, in the first bay where lateral loads simulating imperfections are applied. It is
worth mentioning that increasing the degree of stiffness of both beam-to-column joints and base-plate
connections also causes the value of the safety index to increases, independent of the adopted method
of analysis. The results related to the EU approaches are sketched in figures 4 (MM_racks), 5
(DD_racks) and 6 (TT_racks). In particular, each figure, where different symbols are used to make a
distinction on the basis of the number of load levels, is divided in four parts:
- part a), comparing the EU-RAM and EU-IRAM approaches;
- part b), comparing the EU-RAM and EU-DAM approaches;
- part c), comparing the EU-GEM and EU-IRAM approaches;
- part d), comparing the EU-GEM and EU-DAM approaches.

The bi-sector line is related to cases associated with equal SI and hence corresponds to a 0%
difference between the two compared approaches. Two sub-domains are defined by the bi-sector line
and the reference axis, allowing for the direct identification of the method providing the maximum
value of the safety index, which is directly indicated by the reference axis itself. Furthermore, in each
reference system, a dashed boundary line is also reported to specify the maximum value of the
difference between the compared approaches. It can be pointed out also that:
- for the lowest beam-to column joint stiffness \( \rho_{b,c} \leq 5.0 \), the lowest values of the SI are
always associated with the EU-DAM and EU-RAM approaches, and both approaches lead to
large differences from the EU-GEM (or equivalently EU-IRAM) approach, up to 70% in case
of EU-GEM vs EU-DAM for DD_racks;
- when the beam-to column joint stiffness increases, the differences between all the EU
approaches decrease and EU-DAM always gives the lowest value of SI;
- as already observed for bi-symmetric cross-section uprights, also in these cases the EU-IRAM
and EU-GEM approaches lead to very limited differences in the SI values, never greater than
10%, independent of the geometry of the racks and components;
contrary to what happens in cases of bi-symmetric cross-sections, the EU-DAM and EU-RAM approaches lead in a few cases to significantly different results, especially in the case of MM_racks (up to 40%). Moreover, the EU-RAM method, despite being based on an incorrect assessment of the buckling load, includes the presence of the coupling between torsion and bending at critical buckling conditions, reducing the differences with the EU-IRAM and EU-GEM approaches;
Furthermore, it can be concluded that, basing analyses on a 6DOF beam element formulation, two sets of SI values (EU-DAM plus EU-RAM and EU-IRAM plus EU-GEM) can be identified which, lead to rack performance extremely different to each other.

Figure 5: SI related to the EU approaches by using a 6DOFs formulation for DD_racks.
As to the US design approaches, reference can be made to figure 7, where the value of the safety index according to the two permitted alternatives can be directly compared. The figure is divided into four parts, each of them related to one of the four considered rack geometries differing for the number of floor levels and divided in two sub-domains, defined by the bisector. Also in these cases, the dashed line allows for identifying directly the maximum differences associated with the method indicated in the reference axes. It can be noted that generally, the safety index associated with the US-ELM approach is greater than that associated with US-NOLM and the differences are non-negligible, ranging from 30% (_5LL cases) up to 60% (_3LL cases). In a very limited number of cases the US-NOLM approach results in the more conservative SI, up to 5% in the case of the stiffer MM_racks (\(\rho_{j,base} \geq 0.15\)). In particular, the differences between the two approaches are very large when \(\rho_{j,base} = 1.0\) and \(\rho_{j,btc} = 0.15\), but increasing the stiffness of the joints decreases the differences significantly. Furthermore, it also appears that the number of load levels influences the SI values: increasing the interstorey height of the racks the differences between SI decreases.
To better appreciate the differences between the EU and US approaches, table 3 can be considered, where all the data have been grouped and summarized only on the basis of the upright cross-section type, independent of the number of load levels and joint stiffness; in particular, the mean value (mean), standard deviation (st. dev.), maximum (Max) and minimum (min) values are presented for the methods that are compared in the same manner used for table 1. Differences between EU-IRAM, EU-GEM and US-ELM approaches are quite limited as it appears from the ratios \( \frac{SI_{EU-IRAM}}{SI_{EU-GEM}} \) and \( \frac{SI_{US-ELM}}{SI_{EU-GEM}} \). In cases of US-ELM and EU-IRAM approaches, the associated SI values differ in mean of 6% and up to 1.11, with a moderate dispersion of the corresponding ratio. If the US-ELM and EU-GEM approaches are considered, slightly greater differences can be observed, especially with reference to the MM_racks, having a mean value of \( \frac{SI_{US-ELM}}{SI_{EU-GEM}} \) equal to 1.19 with a maximum value of 1.22. As to the direct comparison between the EU-IRAM and EU-GEM approaches, reference can be made to the part c) of figures 4-6 showing as already discussed, that the safety indices associated with the EU-

---

**Figure 7: SI related to the US approaches, by using a 6DOFs formulation.**
IRAM are slightly greater, up to 10% of the EU-GEM ones, with a mean value based on all the data of 6%.

Table 3: comparison between the EU and US approaches for 6DOFs analysis.

<table>
<thead>
<tr>
<th></th>
<th>SI_{EU-IRAM}^{VM}</th>
<th>SI_{EU-GEM}^{VM}</th>
<th>SI_{EU-IRAM}^{VM}</th>
<th>SI_{EU-GEM}^{VM}</th>
<th>SI_{EU-IRAM}^{VM}</th>
<th>SI_{EU-GEM}^{VM}</th>
<th>SI_{EU-IRAM}^{VM}</th>
<th>SI_{EU-GEM}^{VM}</th>
<th>SI_{EU-IRAM}^{VM}</th>
<th>SI_{EU-GEM}^{VM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM</td>
<td>mean</td>
<td>1.06</td>
<td>0.98</td>
<td>1.10</td>
<td>0.93</td>
<td>1.52</td>
<td>1.17</td>
<td>1.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0188</td>
<td>0.0064</td>
<td>0.0085</td>
<td>0.0109</td>
<td>0.2844</td>
<td>0.1596</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.11</td>
<td>1.00</td>
<td>1.11</td>
<td>0.95</td>
<td>2.37</td>
<td>1.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>1.02</td>
<td>0.97</td>
<td>1.09</td>
<td>0.91</td>
<td>1.14</td>
<td>0.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>mean</td>
<td>1.06</td>
<td>0.99</td>
<td>1.01</td>
<td>0.95</td>
<td>1.39</td>
<td>1.25</td>
<td>1.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0242</td>
<td>0.0094</td>
<td>0.0086</td>
<td>0.0135</td>
<td>0.2330</td>
<td>0.1741</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.10</td>
<td>1.02</td>
<td>1.06</td>
<td>0.97</td>
<td>2.09</td>
<td>1.76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>1.02</td>
<td>0.99</td>
<td>1.03</td>
<td>0.93</td>
<td>1.11</td>
<td>1.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT</td>
<td>mean</td>
<td>1.07</td>
<td>1.01</td>
<td>1.04</td>
<td>0.95</td>
<td>1.55</td>
<td>1.26</td>
<td>1.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0251</td>
<td>0.0099</td>
<td>0.0085</td>
<td>0.0140</td>
<td>0.2003</td>
<td>0.1654</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.10</td>
<td>1.03</td>
<td>1.05</td>
<td>0.98</td>
<td>1.95</td>
<td>1.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>1.01</td>
<td>0.99</td>
<td>1.03</td>
<td>0.93</td>
<td>1.11</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>mean</td>
<td>1.06</td>
<td>1.00</td>
<td>1.07</td>
<td>0.94</td>
<td>1.42</td>
<td>1.23</td>
<td>1.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0227</td>
<td>0.0086</td>
<td>0.0086</td>
<td>0.0128</td>
<td>0.2392</td>
<td>0.1664</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.11</td>
<td>1.03</td>
<td>1.11</td>
<td>0.98</td>
<td>2.37</td>
<td>1.76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>1.01</td>
<td>0.97</td>
<td>1.02</td>
<td>0.91</td>
<td>1.11</td>
<td>0.98</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Furthermore, with reference to these three methods for design, the mean value $SI_{EU-IRAM}^{VM}$ has been computed, which can be assumed to define the real rack performances. In the same table, the $\frac{SI_{EU-IRAM}^{VM}}{SI_{EU-GEM}^{VM}}$ and $\frac{SI_{EU-IRAM}^{VM}}{SI_{EU-IRAM}^{VM}}$ ratios are reported, which are comprised between 0.91 and 1.11, having mean values ranging from 0.93 up to 1.10. In general, the ratios associated with the EU-IRAM approach differ moderately from unity. If the EU-GEM approach is considered, the $\frac{SI_{EU-IRAM}^{VM}}{SI_{EU-GEM}^{VM}}$ ratio is slightly greater than unity whilst the US-ELM ratios are less than unity. Maximum differences in terms of mean value of the safety index can be determined with reference to the EU-DAM approach: the ratio $\frac{SI_{EU-DAM}^{VM}}{SI_{EU-IRAM}^{VM}}$ ranges between 1.11 and 2.37 with a mean value of 1.52 related to the MM_racks. If the comparison is made with the EU-RAM approach it can be observed that mean values of the $\frac{SI_{EU-RAM}^{VM}}{SI_{EU-IRAM}^{VM}}$ ratio are significantly lower than those for EU-DAM, ranging from 1.17 (MM_racks) to 1.26 (TT_racks). Furthermore, in several cases the ratio is very elevated, up to 1.76 for DD_racks, which allows for a safe use of this approach in routine design.
For what concerns the \( \frac{SI_{6}^{VM}}{SI_{6}^{EU-DAM}} \) ratio, it can be noted that it appears acceptable from a design point of view, with a mean value from 1.03 to 1.08, but presents very high maximum values, up to 43%. As also observed for bi-symmetric cross-sections, when the racks are flexible, i.e. for the lowest values of \( \rho_{j,k} \), the US-NOLM approach gives value of SI that are too low when compared to SI\(^{VM} \), leading to a quite conservative design. As to the EU-RAM method, the \( \frac{SI_{6}^{VM}}{SI_{6}^{EU-RAM}} \) ratios are always greater than those associated with the three equivalent approaches, despite being significantly lower than the ones corresponding to the EU-DAM approach.

5. ADVANCED DESIGN: INFLUENCE OF THE METHODS OF ANALYSIS

As already highlighted, according to the basic principles of steel structure theory, structural analysis and design of members having cross-sections with one axis of symmetry must be based on the use of an adequate FE beam formulation that takes into account the presence of warping [19]. Owing to the importance of this matter, attention is herein initially focused on warping restraint at the column bases and then the results associated with the different methods of analysis are discussed and compared.

5.1 Influence of the warping base restraints

With both EU-DAM and EU-GEM approaches being based on the assessment of a resistance safety index, it appears of paramount importance in the case of mono-symmetric uprights, to also focus attention on the distribution of the bimoment. As expected, the value of safety index associated with resistance check changes along the uprights section by section. In particular, in the case of EU-DAM, the maximum value of the bimoment is generally located at the middle of the first story and not at the top or bottom of the upright, as it is in the EU-GEM method where only global imperfections are modelled. As an example of the different distributions of bimoment along the central and lateral uprights, which is due to the different types of imperfections, reference can be made to the figures A1 and A3 in Appendix A. It can be stated, in general, that the influence of warping depends mainly on the cross-section types and on the number of load levels; and it is relatively independent of the values of the flexural stiffness of both beam-to-column joints and base-plate connections. For better appreciating the contribution of the bimoment at the end and at the top of the more stressed internal upright, reference can be made to table 4. For the free warping (WF) and prevented warping (WP) cases, the mean value of the contribution of the bimoment \( \Delta SI_{BM}^{EU-DAM} = \left( \frac{B_{Ei}}{B_{RI}} \right) \) is presented over the safety index associated with the use of the DAM approach ( \( SI_{7}^{EU-DAM} \)), already defined in eq. 1). All data have been grouped and summarized on the basis of the number of the load levels and joint
stiffness. It appears immediately that the bimoment can never be neglected for design purposes when this approach, which is based solely on the resistance check, is considered. Further key remarks are:

*Table 4: influence of the warping modelling on the bimoment for the EU-DAM approach.*

<table>
<thead>
<tr>
<th></th>
<th>Top_WF</th>
<th>Top_WP</th>
<th>Bot_WP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S_I^{EU-DAM}/S_I^{EU-DAM}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MM_2LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD_2LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT_2LL</td>
<td>mean</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0314</td>
<td>0.0180</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>MM_3LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD_3LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT_3LL</td>
<td>mean</td>
<td>0.24</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0484</td>
<td>0.0410</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>MM_4LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD_4LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT_4LL</td>
<td>mean</td>
<td>0.26</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0999</td>
<td>0.0607</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.41</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>MM_5LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD_5LL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT_5LL</td>
<td>mean</td>
<td>0.24</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.1276</td>
<td>0.0667</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.45</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>All</td>
<td>mean</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.0881</td>
<td>0.1210</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.45</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

- in the WF cases, the bimoment term contributes from 9% to 50% to the value of the SI, with a mean value of 0.24 and a very low standard deviation (0.0881). The lowest values of the ratio are generally associated with the large interstorey height. Increasing the number of load levels causes the influence of the bimoment to increase. Maximum values of the $\Delta S_I^{EU-DAM}/S_I^{EU-DAM}$ ratio range from 0.24 (DD_2LL racks) to 0.45 (TT_5LL racks).

- in the WP cases, warping influence is always maximum at the base level (bottom section): the mean value of $\Delta S_I^{EU-DAM}/S_I^{EU-DAM}$ is quite similar to the values associated with the WF ones.

It can be concluded that, for the EU-DAM approach the bimoment contribution is really important for design purposes, with the contribution to the overall SI being up to 40% and never lower than 5%, with a mean value related to all the cases equal to 24%.

Furthermore, it is worth mentioning that in the EU-IRAM, US-ELM and EU-GEM approaches, where the stability checks are more severe than the resistance ones, the value of the elastic critical load
multiplier influences the safety index and column bases restraining warping cases a slight increase in buckling resistance, i.e. \( \alpha_{cr}^{WP} \) is always greater than \( \alpha_{cr}^{WF} \). As an example, reference can be made to figure 8 where the distribution of the ratio \( \frac{\alpha_{cr}^{WP}}{\alpha_{cr}^{WF}} \) is presented in terms of relative frequency. The ratio is always greater than unity, increasing with the beam-to-column joint stiffness with a high concentration of occurrences ranging between 1.06 and 1.16. Adequate attention must therefore be paid to the correct modelling of the upright base restraints.

![Figure 8: Distribution of the \( \frac{\alpha_{cr}^{WP}}{\alpha_{cr}^{WF}} \) ratio in terms of relative and cumulative frequency.](image)

The previously discussed influence of the warping base restraints impacts the SI values as can be observed in table 5 where the \( \frac{SI_{WP}}{SI_{WF}} \) ratio in terms of mean value and standard deviation are presented together with the maximum and minimum values for the \( k^{th} \) design approach.

**Table 5: influence of the warping modelling on the values of the safety index.**

<table>
<thead>
<tr>
<th></th>
<th>SI_{EU–DAM}^{WP}</th>
<th>SI_{EU–DAM}^{WF}</th>
<th>SI_{EU–RAM}^{WP}</th>
<th>SI_{EU–RAM}^{WF}</th>
<th>SI_{EU–GEM}^{WP}</th>
<th>SI_{EU–GEM}^{WF}</th>
<th>SI_{US–NOLM}^{WP}</th>
<th>SI_{US–NOLM}^{WF}</th>
<th>SI_{US–ELM}^{WP}</th>
<th>SI_{US–ELM}^{WF}</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM_2LL</td>
<td>mean</td>
<td>1.35</td>
<td>1.04</td>
<td>1.02</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>st. dev</td>
<td>0.0625</td>
<td>0.0377</td>
<td>0.0256</td>
<td>0.0425</td>
<td>0.0460</td>
<td>0.0288</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.55</td>
<td>1.15</td>
<td>1.10</td>
<td>1.15</td>
<td>1.19</td>
<td>1.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.27</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.33</td>
<td>1.03</td>
<td>1.02</td>
<td>1.05</td>
<td>1.04</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>st. dev</td>
<td></td>
<td>0.0588</td>
<td>0.0284</td>
<td>0.0187</td>
<td>0.0332</td>
<td>0.0334</td>
<td>0.0208</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.53</td>
<td>1.13</td>
<td>1.07</td>
<td>1.13</td>
<td>1.14</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.25</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.27</td>
<td>1.03</td>
<td>1.02</td>
<td>1.06</td>
<td>1.04</td>
<td>1.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>st. dev</td>
<td></td>
<td>0.0333</td>
<td>0.0283</td>
<td>0.0198</td>
<td>0.0326</td>
<td>0.0356</td>
<td>0.0229</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.35</td>
<td>1.13</td>
<td>1.09</td>
<td>1.13</td>
<td>1.17</td>
<td>1.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.22</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.25</td>
<td>1.04</td>
<td>1.03</td>
<td>1.08</td>
<td>1.05</td>
<td>1.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>st. dev</td>
<td></td>
<td>0.0408</td>
<td>0.0394</td>
<td>0.0302</td>
<td>0.0481</td>
<td>0.0479</td>
<td>0.0342</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.32</td>
<td>1.17</td>
<td>1.13</td>
<td>1.22</td>
<td>1.20</td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.17</td>
<td>1.00</td>
<td>1.00</td>
<td>1.02</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.30</td>
<td>1.04</td>
<td>1.02</td>
<td>1.05</td>
<td>1.04</td>
<td>1.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>st. dev</td>
<td></td>
<td>0.0489</td>
<td>0.0334</td>
<td>0.0235</td>
<td>0.0361</td>
<td>0.0408</td>
<td>0.0267</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>1.55</td>
<td>1.17</td>
<td>1.13</td>
<td>1.22</td>
<td>1.20</td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td></td>
<td>1.17</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be noted that the influence of warping restraint on the SI is quite modest for methods using stability verification criteria, but, as expected from the already discussed data in Table 4, it becomes significant when EU-DAM, first column of the table, is considered. Considering the other approaches, the influence of the base warping restraint is more limited. It is slightly greater for the EU-GEM approach but not relevant for design purposes. For this reason, and owing to the need of limiting the number of variables affecting the research outcomes, the decision has been taken to continue the comparison between various methods referring to only the WP cases, which are always associated with the higher SI values.

### 5.2 Influence of methods of analysis

In cases of FE beam formulations including warping effects, which have to be used to model monosymmetric uprights, the safety index for the resistance checks must be evaluated accounting for the bimoment contribution, as recommended also by the Australian standard [8]. Figures from 9 to 12 are equivalent to figures 4-7 but related to the results obtained by means of the 7DOFs design approaches and hence are based on sets of displacements, internal forces and moments that differ from the 6DOFs values. Furthermore, for a better understanding of the design results, the values associated with the use of 6DOFs analysis are again proposed between brackets.
Figure 9: SI related to the EU approaches by using a 7DOFs formulation for MM_racks.

Figure 10: SI related to the EU approaches by using a 7DOFs formulation for DD_racks.
In general, it can be stated that the differences between the considered methods observed with reference to the 6DOFs results are confirmed and hence the choice of the effective length plays a non-negligible role in determining the rack performance. Furthermore, owing to the introduction of the 7th DOF, in several cases the SI values vary remarkably from the 6DOFs value, depending on the assumptions on which the method is based. In particular, it can be noted that:

- the EU-DAM approach is most influenced by the *beam* formulation, owing to the presence of the non-negligible warping contribution;
- the EU-GEM and EU-IRAM approaches yield to similar results; however these differ to the results obtained using EU-RAM and EU-DAM approaches that are similar to one another. Both the sets of methods are generally characterized by differences of SI values in the range of ±15% with a very limited number of exceptions having difference up to 20%;
- the minimum value of the safety index is always associated with the EU-RAM approach, with the exception of MM_racks with 2 and 3 load levels, where EU-DAM gives the most advantageous performance;
SI values associated with the EU design methods become closer with increasing lateral stiffness of the frames, especially for TT_racks. For the highest degree of stiffness of the beam-to-column joints the maximum discrepancy between the methods is almost 25%; compared with the 6DOFs analysis results, differences between EU-GEM and EU-DAM approaches decrease significantly in these 7DOFs cases, also for the highest values of $\rho_{j,btc}$; the EU-IRAM and EU-RAM approaches shows relevant differences, especially when the more flexible racks are considered (in the case of the MM_rack having $\rho_{j,btc} = 1.0$ and $\rho_{j,base} = 0.15$ the difference is up to 60%).

As to the US design approaches, reference can be made to figure 12, which is equivalent to the figure 7, where the SI values the according to both the US alternatives can be directly compared. Also in these cases, the values between the brackets are related to the 6DOFs limit lines. It can be noted that, also for design according to US code, the differences in the 7DOFs SI decreases, especially for the higher values of beam-to-column stiffness. Furthermore, it is confirmed that for the more flexible racks, the SI associated with the US-ELM approach is slightly greater than the US-NOLM one and differences are never greater than 40%. However, when the beam-to-column joint stiffness increases, the use of the more refined FE formulation gives, in the US-NOLM approach, values up to 15% more conservative.
Also in the case of 7DOFs design, for appreciating the differences in terms of rack performance between the EU and US approaches, table 6 can be considered, where all data have been grouped and summarized on the basis of the upright cross-section. Also when warping is accounted for it seems reasonable to associate the effective load carrying capacity of the rack to the mean value of the safety index $SI_{V7}$ obtained as the mean value of the EU-IRAM, EU.GEM and US-ELM SI values. It can be noted that, as an effect of the bimoment presence, the ratios now become closer to unity; focussing on the EU-IRAM, EU-GEM and US-ELM approaches, the maximum differences are associated with the EU-GEM approach but are limited to 11%.

**Table 6: comparison between the EU and US approaches for a 7DOFs based design.**

<table>
<thead>
<tr>
<th></th>
<th>SI_{US-ELM}^{7}/SI_{EU-IRAM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-GEM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-IRAM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-GEM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-IRAM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-GEM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-IRAM}^{7}</th>
<th>SI_{US-ELM}^{7}/SI_{EU-GEM}^{7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM_</td>
<td>mean 1.06</td>
<td>1.04</td>
<td>1.02</td>
<td>1.01</td>
<td>0.97</td>
<td>1.09</td>
<td>1.13</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>st. dev. 0.0210</td>
<td>0.0282</td>
<td>0.0166</td>
<td>0.0194</td>
<td>0.0107</td>
<td>0.2047</td>
<td>0.1395</td>
<td>0.1144</td>
</tr>
<tr>
<td>Max</td>
<td>1.09</td>
<td>1.11</td>
<td>1.06</td>
<td>1.05</td>
<td>1.00</td>
<td>1.64</td>
<td>1.53</td>
<td>1.31</td>
</tr>
</tbody>
</table>
As to the EU-RAM method, the associated ratios are always greater than those associated with the three equivalent approaches, despite being generally lower than the values corresponding to the EU-DAM approach. Furthermore, it is worth mentioning that the presence of one axis of symmetry imposes consideration of flexural-torsional buckling modes, hence increasing the differences between the EU-DAM and EU-RAM approaches. As in the case of 6DOF analysis, the degree of reliability of the racks designed according to EU-RAM approach, identified with the ratio, could be significantly overestimated, leading to unsafe racks entering the market (mean errors up to 21%, with a maximum overestimation of the safety index up to 69%).

The addition of the bimoment contribution in the equation governing the EU-DAM method significantly reduces the differences the EU-IRAM, EU-GEM and US-ELM 7DOFs approaches; it is confirmed by the mean value of the ratio, always less than 9% but however very high values can be again observed, up to 64% from the unsafe side, in case of racks with more flexible joints. The US-NOLM method follows the same trend of the EU-DAM approach, in fact if the ratio is considered, it can be noted that mean values are always less than 8% but maximum values are too large, overestimating rack performance up to 43%.

6. CONCLUDING REMARKS

A research project has been developed to compare the design approaches admitted for steel racks according to both EU and US steel design provisions. Initially, attention has been focussed on bi-symmetrical cross-section members and the associated research outcomes already discussed by Bernuzzi et al. [6, 7] showed the limits in term of reliability of two European (EU-DAM and EU-RAM) approaches permitted for both the more traditional steel framed system and medium-rise pallet racks comprised of bi-symmetric cross-section members.
The core of the last phase of the research, which has been discussed and summarized in the present paper, is represented by pallet racks built up with mono-symmetric cross-section members. It is confirmed that a role of paramount importance, as shown also in cases of frames with bi-symmetric cross-section members, is played by the choice of the system length, when stability checks are required. The use of the system length recommended by the EU-RAM approach could lead to an overestimation of the rack performance if not balanced by the definition of suitable local imperfections and/or reduction of the member stiffness (as in the US-NOLM approach).

Furthermore, another aspect of relevance is related to the capabilities of FE analysis software. The use of two different FE analysis packages shows that FE beam formulation neglecting warping effects leads to an estimation of set of displacements, internal forces and moments not correct because of the relevant warping effects are ignored. As shown in table 7, which presents the ratio of \( \frac{SI_{j-k}}{SI_{k-j}} \) for all considered \( j^{th} \) and \( k^{th} \) code approaches, the influence of warping torsion cannot be neglected, especially in the methods based on the resistance checks (totally in the EU-DAM and partially in the EU-GEM). As already mentioned in the paper, maximum differences are related to the EU-DAM approach, despite the fact that in the other cases the use of traditional design in general overestimates the rack performance. Excluding this approach, the differences between the SI associated with the other alternatives always remains greater than unity, but it is however limited, as shown in figure 13 in which the ratio \( \frac{SI_{j-k}}{SI_{k-j}} \) is presented with a large number of occurrences is in the range 1.05 – 1.15.

**Table 7: Influence of the warping (7\(^{th}\) DOFs) on the values of the safety index.**

<table>
<thead>
<tr>
<th></th>
<th>( \frac{SI_{j-k}}{SI_{k-j}} ) EU-DAM</th>
<th>( \frac{SI_{j-k}}{SI_{k-j}} ) RAM</th>
<th>( \frac{SI_{j-k}}{SI_{k-j}} ) US-NOLM</th>
<th>( \frac{SI_{j-k}}{SI_{k-j}} ) US-ELM</th>
</tr>
</thead>
</table>
| MM_    | \begin{align*}
\text{mean} & : 1.43 & 1.06 & 0.98 & 1.12 & 1.07 & 0.98 \\
\text{st. dev} & : 0.0095 & 0.0057 & 0.0052 & 0.0310 & 0.0082 & 0.0134 \\
\text{Max} & : 1.50 & 1.09 & 1.00 & 1.18 & 1.10 & 1.01 \\
\text{min} & : 1.29 & 1.01 & 0.95 & 1.03 & 1.04 & 0.95 \\
\end{align*} |
| DD_    | \begin{align*}
\text{mean} & : 1.40 & 1.07 & 1.05 & 1.13 & 1.07 & 1.05 \\
\text{st. dev} & : 0.0181 & 0.0063 & 0.0162 & 0.0362 & 0.0072 & 0.0198 \\
\text{Max} & : 1.46 & 1.10 & 1.09 & 1.19 & 1.11 & 1.09 \\
\text{min} & : 1.33 & 1.02 & 1.01 & 1.04 & 1.04 & 1.00 \\
\end{align*} |
| TT_    | \begin{align*}
\text{mean} & : 1.36 & 1.08 & 1.06 & 1.12 & 1.08 & 1.07 \\
\text{st. dev} & : 0.0170 & 0.0067 & 0.0174 & 0.0345 & 0.0083 & 0.0196 \\
\text{Max} & : 1.42 & 1.11 & 1.11 & 1.18 & 1.11 & 1.10 \\
\text{min} & : 1.28 & 1.02 & 1.01 & 1.03 & 1.05 & 1.02 \\
\end{align*} |
| All    | \begin{align*}
\text{mean} & : 1.39 & 1.07 & 1.03 & 1.12 & 1.07 & 1.03 \\
\text{st. dev} & : 0.0149 & 0.0062 & 0.0129 & 0.0339 & 0.0079 & 0.0176 \\
\end{align*} |
<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>1.50</th>
<th>1.11</th>
<th>1.11</th>
<th>1.19</th>
<th>1.11</th>
<th>1.10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>1.28</td>
<td>1.01</td>
<td>0.95</td>
<td>1.03</td>
<td>1.04</td>
<td>0.95</td>
</tr>
</tbody>
</table>

As a general conclusion, it can be stated that:

- rack design codes could mandatorily require the use of finite element analysis packages offering suitable FE 7DOF's beam formulations;
- with reference to the European Provisions, it is expected that the EU-RAM approach will be eliminate as soon as possible. Furthermore, a better definition of the base assumptions of the EU-DAM alternative is expected. The EU-GEM and the EU-IRAM seem to be very promising approaches that lead to a safe design of the racks, independent of the type of cross-section uprights used;
- with reference to the US alternatives, the US-ELM gives more or less the same results as the EU-IRAM and EU-GEM approaches and seems to be a good design method. Despite the stiffness reduction, the US-NOLM approach seems to be an efficient method only when racks joints are not very flexible, otherwise this approach leads to non-negligible differences with respect to the other approaches.

Finally, it should be noted that all the proposed data, presented in term of SI and/or ratios between them, allow for a detailed appraisal of the safety and of the competitiveness of the racks on the market.

As an example, a difference of the safety index of 50% corresponds to a 20% of difference in the

---

Figure 13: Distribution of $\frac{SI_j^{i+k}}{SI_6^{i+k}}$ ratio, excluding the EU-DAM approach.
weight of the rack, which corresponds to a non-negligible reduction in cost and hence special attention is required to define design rules that are safe and lead to an optimal use of components.

7. REFERENCES


APPENDIX A. Design example

The scope of the present Appendix is to reproduce the main computations associated with the design approaches discussed and applied in the paper, which should be useful as a benchmark for the design and validation of the FE packages for racks with mono-symmetric cross-section uprights.

The considered case has been taken randomly from the numerical analyses described in section 3 and refers to the more stressed internal upright of the DD_5LL rack (fig. 2) with $\rho_{j,\text{base}} = 1.0$ and $\rho_{j,\text{base}} = 0.3$. As it also appears from figure 2, the system length of the upright is 1330 mm ($L_y$) in the longitudinal (down-aisle) direction and reference is made to Z-panels in the transverse (cross-aisle) direction having height of 1200mm ($L_z$) forming the upright frames. Blocked warping (WP case) has been assumed at base plate connections. The main cross-section data of the DD uprights are reported in table 2. The material is S355 steel grade [27] with a yielding strength ($f_y$) equal to 355 MPa.

As to the key design values for the component checks, reference has to be made to table A1, where the axial force, $N_{Ed}$, the bending moments, $M_{y,Ed}$, $M_{z,Ed}$ and the bimoment $B_{Ed}$, at the bottom (BOT) and the top (TOP) of the more highly stressed upright are reported. Values are obtained from second-order analyses according to the discussed design approaches. Both the set of data associated with 6DOFs and 7DOFs FE beam formulations are reported, confirming, as mentioned in the paper, that differences in terms of set of internal forces and bending moments are non-negligible from the design point of view.

*Table A1 – Summary of the key results of the second-order FE analysis.*

<table>
<thead>
<tr>
<th></th>
<th>DOFs</th>
<th>$N_{Ed}$ [kN]</th>
<th>$M_{y,Ed}$ [kNm]</th>
<th>$M_{z,Ed}$ [kNm]</th>
<th>$B_{Ed}$ [kNm$^2$]</th>
<th>$B_{Ed}$ [kNm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU</td>
<td>DAM</td>
<td>6 129.19</td>
<td>1.201</td>
<td>0.226</td>
<td>-0.095</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>7 131.68</td>
<td>1.330</td>
<td>0.340</td>
<td>-0.260</td>
<td>-0.009</td>
<td>-0.053</td>
</tr>
<tr>
<td>RAM</td>
<td>IRAM</td>
<td>6 129.06</td>
<td>0.942</td>
<td>0.123</td>
<td>-0.036</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>GEM</td>
<td>7 131.68</td>
<td>0.980</td>
<td>0.240</td>
<td>-0.260</td>
<td>-0.009</td>
</tr>
<tr>
<td>US</td>
<td>NOLM</td>
<td>6 122.07</td>
<td>1.685</td>
<td>1.038</td>
<td>-0.119</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>ELM</td>
<td>7 126.28</td>
<td>1.715</td>
<td>0.450</td>
<td>-0.260</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

The reference uniform load is $q_s = 9.28 \text{ N/mm}$ and the elastic buckling load multipliers are $\alpha_{\text{cr}}^6 = 1.87$ and $\alpha_{\text{cr}}^7 = 1.79$, which have been evaluated by means of the 6DOFs and 7DOFs FE beam formulations, respectively.

A1. The European approaches.
In accordance with the requirements of EC3 [1], Young’s modulus (E) has been assumed equal to 210000 MPa. Furthermore, as to the stability checks, reference is made to an imperfection factor \( \alpha = 0.34 \) and material safety factors, \( \gamma_M \), have been assumed equal to unity.

**EU–DAM**: only resistance checks are required by the EU-DAM approach. In figure A1 an example of the distribution of bimoment and 6DOFs and 7DOFs bending moment in the down-aisle direction is presented, with reference to the more stressed external and internal uprights. Design checks are referred to the more stressed element that is an internal upright at the base of the cross-section.

With reference to the two considered FE beam formulations, the values of the safety index \( S_{EU-DAM}^{6} \) and \( S_{EU-DAM}^{7} \) are:

\[
S_{EU-DAM}^{6} = \left[ \frac{129.19 \cdot 10^3}{600.95 \cdot 355} + \frac{1.20 \cdot 10^6}{22350 \cdot 355} \right] + \left[ \frac{-0.1 \cdot 10^6}{16115 \cdot 355} \right] = 0.61 + 0.15 + 0.01 = 0.77
\]

\[
S_{EU-DAM}^{7} = \left[ \frac{131.68 \cdot 10^3}{609.95 \cdot 355} + \frac{1.33 \cdot 10^6}{22350 \cdot 355} \right] + \left[ \frac{-0.3 \cdot 10^6}{16115 \cdot 355} \right] = \left[ \frac{-0.053 \cdot 10^6}{717785.8 \cdot 355} \right] = (0.62 + 0.17 + 0.05) + 0.21 = 0.83 + 0.21 = 1.04
\]

**Figure A1**: Distributions of the bimoment (a) and bending moment (b) in the down-aisle direction associated with the EU-DAM approach.

Figure A2 presents, for the cross-section in figure A1, the normal stresses distribution in the gross cross-section. In particular, for each critical point of the cross-section (corner or edge point) the values of the normal stress associated with 6DOFs (\( \sigma_{tot}^6 \)) and 7DOFs (\( \sigma_{tot}^7 \)) FE formulation are presented. In addition, the value of the constant stress due to the axial load (\( \sigma_N^7 \)), practically independent of the
FE beam formulation, is reported in the figure together with the stress associated with the bimoment \( \sigma_B^7 \). It can be noted that significant differences can be observed in the values of the total normal stresses, depending not only on the presence/absence of the bimoment but also by the different values of the bending moments.

![Figure A2: Distributions of the internal stresses in the gross cross-section at the bottom of an external upright associated with the EU-DAM approach.](image)

**EU–RAM:** main contributions of the safety index, not depending on FE formulation, need to be preliminarily evaluated.

Flexural buckling load \( N_{cr,k} = \frac{\pi^2 E \cdot I_k}{L_{0,k}^2} \)

- Down – aisle direction
  \[
  N_{cr,y} = \frac{\pi^2 \cdot 210000 \cdot 1368668}{1330^2} = 1603.66 \cdot 10^3 N
  \]
- Cross – aisle direction
  \[
  N_{cr,z} = \frac{\pi^2 \cdot 210000 \cdot 666368}{1200^2} = 959.11 \cdot 10^3 N
  \]

Torsional buckling load: \( N_{cr,T} = \frac{1}{i_0^2} \left[ GI_T + \left( \frac{\pi^2 EI_w}{L_{0,T}^2} \right) \right] \)

\[
N_{cr,T} = \frac{1}{98.16^2} \left[ 80769 \cdot 1061 + \left( \frac{\pi^2 210000 \cdot 5.17 \cdot 10^9}{(0.7 \cdot 1200)^2} \right) \right] = 1586.16 \cdot 10^3 N
\]

with \( i_0 = \sqrt{l_x^2 + l_z^2 + y_c^2} = \sqrt{44^2 + 30.7^2 + 82.2^2} = 98.16 \text{ mm} \)
Flexural-torsional buckling load:

\[ N_{cr,FT} = \frac{N_{cr,y}}{2\beta} \left[ 1 + \frac{N_{cr,T}}{N_{cr,y}} \sqrt{1 - \left(1 - 0.99\right)^2 + 4 \cdot 0.99 \left(\frac{82.20}{98.16}\right)^2} \right] \]

\[ N_{cr,FT} = \frac{1603.66 \cdot 10^3}{0.598} \left[ 1 + 0.99 - \sqrt{\left(1 - 0.99\right)^2 + 4 \cdot 0.99 \left(\frac{82.20}{98.16}\right)^2} \right] = 867.99 \cdot 10^3 \, N \]

with \( \beta = 1 - \left(\frac{y_c}{t_0}\right)^2 = 1 - \left(\frac{82.20}{98.16}\right)^2 = 0.298 \)

\[ \chi_k = \sqrt{\frac{A_{ef} \cdot f_y}{N_{cr,k}}} \]

\[ \lambda_y = \sqrt{\frac{600.95 \cdot 355}{1603.66 \cdot 10^3}} = 0.365 \]

\[ \lambda_z = \sqrt{\frac{600.95 \cdot 355}{959.11 \cdot 10^3}} = 0.472 \]

\[ \varphi = 0.5 \cdot [1 + \alpha(\lambda - 0.2) + \lambda^2] \]

\[ \varphi_y = 0.5 \cdot [1 + 0.34 \cdot (0.365 - 0.2) + 0.365^2] = 0.595 \]

\[ \varphi_z = 0.5 \cdot [1 + 0.34 \cdot (0.472 - 0.2) + 0.472^2] = 0.657 \]

\[ \varphi_{FT} = 0.5 \cdot [1 + 0.34 \cdot (0.495 - 0.2) + 0.495^2] = 0.673 \]

\[ \chi_y = \frac{1}{0.595 + \sqrt{0.595^2 - 0.365^2}} = 0.940 \]

\[ \chi_z = \frac{1}{0.657 + \sqrt{0.657^2 - 0.472^2}} = 0.897 \]

\[ \chi_{FT} = \frac{1}{0.673 + \sqrt{0.673^2 - 0.495^2}} = 0.886 \]

\[ \chi_{min} = \min \{ \chi_y; \chi_z; \chi_{FT} \} = 0.886 \]

2 Evaluation of the critical moment \( M_{cr} \) associated with the lateral buckling of the column is required.

\[ M_{cr} = C_1 \pi^2 EI_z \left[ \left( \frac{k_z}{k_w} \right)^2 I_z + \left( \frac{k_z L_0}{I_z} \right)^2 \frac{G I_z}{\pi^2 E I_z} + \left( \frac{C_z z_y - C_z z_f}{\pi^2 E I_z} \right)^2 - \left( \frac{C_z z_y - C_z z_f}{\pi^2 E I_z} \right)^2 \right] \]

3

\[ M_{cr} = 2.49 \pi^2 210000 \cdot 666368 \left[ \left( \frac{1}{0.7} \right)^2 5.17 \cdot 10^9 \frac{80796 \cdot 1061}{6.67 \cdot 10^9} + (1 \cdot 1200)^2 80796 \cdot 1061 \right] = 2.45 \cdot 10^9 \, Nm \]

4

\[ \chi_{LT} \sqrt{\frac{M_{cr}}{W_{cr,x} \cdot f_y}} = \sqrt{\frac{22350.7 \cdot 355}{2.45 \cdot 10^9}} = 0.18 < 0.20 \quad \rightarrow \quad \chi_{LT} = 1.0 \]
As to the bending moment \( k_{LT} \) and \( k_z \) coefficients, it is necessary to assess the equivalent uniform moment factors, \( \beta_{M,y} \) and \( \beta_{M,z} \), with reference to the effective moment distribution along the system length, about the \( y \)- and \( z \)-axis, respectively.

**EU-RAM\(_6\):**

\[
\beta_{M,y} = 1.8 - 0.7\psi y = 1.8 - 0.7 \frac{222623}{940602} = 1.63
\]
\[
\mu_{LT} = 0.15 \cdot \beta_{M,y} = 0.15 = -0.03 < 0.9
\]
\[
k_{LT} = 1 - \frac{\mu_{LT} N_{Ed}}{\chi z A_{eff} \cdot f_y} = 1 - \frac{0.15 \cdot 129065}{0.897 \cdot 600.95 \cdot 355} = 1.0
\]
\[
\beta_{M,z} = 1.8 - 0.7\psi z = 1.8 - 0.7 \frac{-6464}{-35746} = 1.67
\]
\[
\mu_z = \bar{\lambda_z} (2\beta_{M,z} - 4) = 0.472 \cdot (2 \cdot 1.67 - 4) = -0.31 < 0.9
\]
\[
k_z = 1 - \frac{\mu_z N_{Ed}}{\chi z A_{eff} \cdot f_y} = 1 - \frac{-0.31 \cdot 129065}{0.897 \cdot 600.95 \cdot 355} = 1.21 > 1.0
\]
\[
\rightarrow k_z = 1
\]
\[
 SI_{6}^{EU-RAM} = \frac{129.06 \cdot 10^3}{0.886 \cdot (600.95 \cdot 355) + 1.0} \cdot \left[ \frac{0.94 \cdot 10^6}{1.0 \cdot (22350.7 \cdot 355)} \right] + 1.0 \cdot \left[ \frac{-0.04 \cdot 10^6}{16115.04 \cdot 355} \right] = 0.68 + 0.12 + 0.01 = 0.81
\]

**EU-RAM\(_7\):**

\[
\beta_{M,y} = 1.8 - 0.7\psi y = 1.8 - 0.7 \frac{240000}{980000} = 1.63
\]
\[
\mu_{LT} = 0.15 \cdot \beta_{M,y} = 0.15 = 0.04 < 0.9
\]
\[
k_{LT} = 1 - \frac{\mu_{LT} N_{Ed}}{\chi z A_{eff} \cdot f_y} = 1 - \frac{0.04 \cdot 131676}{0.897 \cdot 600.95 \cdot 355} = 0.97 < 1
\]
\[
\beta_{M,z} = 1.8 - 0.7\psi z = 1.8 - 0.7 \frac{-900}{-260000} = 1.56
\]
\[
\mu_z = \bar{\lambda_z} (2\beta_{M,z} - 4) = 0.472 \cdot (2 \cdot 1.56 - 4) = -0.415 < 0.9
\]
\[
k_z = 1 - \frac{\mu_z N_{Ed}}{\chi z A_{eff} \cdot f_y} = 1 - \frac{-0.415 \cdot 131676}{0.897 \cdot 600.95 \cdot 355} = 1.28 > 1
\]
\[
\rightarrow k_z = 1
\]
\[
 SI_{7}^{EU-RAM} = \frac{131.68 \cdot 10^3}{0.886 \cdot (600.95 \cdot 355) + 1.0} \cdot \left[ \frac{0.98 \cdot 10^6}{1.0 \cdot (22350.7 \cdot 355)} \right] + 1.0 \cdot \left[ \frac{-0.26 \cdot 10^6}{16115.04 \cdot 355} \right] = 0.70 + 0.12 + 0.04 = 0.86
\]

**EU-RAM**: stability check is based on the effective length here evaluated by means of the use of the critical load multiplier \( \alpha_{cr} \), obtained from a FE buckling analysis. In case of 6DOF analysis, the critical load results associated with flexural mode \( N_{cr,y} = N_{cr,z} = \alpha_{cr} \cdot N_{Ed} = 241.35 \cdot 10^3 \) \( N \). Based on this value it is possible to obtain the flexural-torsional critical buckling load as:

\[
N_{cr,y} = \frac{N_{cr,y}}{2\beta} \cdot \left[ 1 + \frac{N_{cr,y}}{N_{cr,y}} \cdot \left( 1 - \frac{N_{cr,y}}{N_{cr,y}} \right)^2 + 4 \cdot \frac{N_{cr,y}}{N_{cr,y}} \cdot \frac{y}{I_0} \right] =
\]
\[
= 1603.66 \cdot 10^3 \cdot \frac{0.598}{1 + 6.57 \cdot \left( 1 - 6.57 \right)^2 + 4 \cdot 6.57 \cdot \left( \frac{82.20}{98.16} \right)^2} = 217.19 \cdot 10^3 \ N
\]
It is worth noticing that the critical load based on the buckling length is approximately 23% of the one based on the system length assumed in the EU-RAM approach and results slightly greater (approximately 8%) than the one associated with the 6DOFs approach.

For what concerns the 7DOFs analysis, it is found that \( N_{cr} = \alpha^2 N_{Ed} = 234.38 \cdot 10^3 \) N that directly accounts for the flexural-torsional behavior.

**EU-IRAM\(_6\)**

\[
\lambda_{min}^6 = \sqrt{\frac{600.95 \cdot 355}{217.19 \cdot 10^3}} = 0.991
\]

\[\chi_{min} = 0.603\]

\[
\lambda_{z}^6 = \sqrt{\frac{600.95 \cdot 355}{241.35 \cdot 10^3}} = 0.940
\]

\[\chi_{z} = 0.635\]

\[k_{LT} = 1 - \frac{\mu_{cr,N_{Ed}}}{\chi_{z} \cdot A_{eff} \cdot f_y} = 1 - \frac{0.08 \cdot 129065}{0.603 \cdot 600.95 \cdot 355} = 0.92 < 1\]

\[k_{z} = 1 - \frac{\mu_{r,N_{Ed}}}{\chi_{z} \cdot A_{eff} \cdot f_y} = 1 - \frac{0.61 \cdot 129065}{0.635 \cdot 600.95 \cdot 355} = 1.55 > 1\]

\[\lambda_{LT} = 1\]

\[S_{I}^{EU-IRAM\(_6\)} = \frac{129.06 \cdot 10^3}{0.603 \cdot (600.95 \cdot 355)} + 0.92 \left( 0.94 \cdot 10^6 \right) + 1.0 \left( -0.04 \cdot 10^6 \right) = 1.00 + 0.10 + 0.01 = 1.11\]

**EU-IRAM\(_7\)**

\[
\lambda_{min}^7 = \sqrt{\frac{600.95 \cdot 355}{234.38 \cdot 10^3}} = 0.954
\]

\[\chi_{min} = 0.626\]

\[\chi_{z} = 0.897\]

\[k_{LT} = 1.0\]

\[k_{z} = 1.0\]

\[S_{I}^{EU-IRAM\(_7\)} = \frac{131.68 \cdot 10^3}{0.626 \cdot (600.95 \cdot 355)} + 1.0 \left( 0.98 \cdot 10^6 \right) + 1.0 \left( -0.26 \cdot 10^6 \right) = 0.99 + 0.12 + 0.04 = 1.15\]

It is worth noting that \( \chi_{LT} = 1.0 \) has been assumed, as previously calculated.

**EU–GEM:** only the effects of global (sway) imperfection are accounted for also in this case:

**EU-GEM\(_6\)**

\[
\alpha_{wh,k,6} = \frac{1}{N_{Ed} + M_{y,Ed} + M_{z,Ed}} = \frac{1}{0.728} = 1.371
\]

**EU-GEM\(_7\)**

\[
\alpha_{wh,k,7} = \frac{1}{N_{Ed} + M_{y,Ed} + M_{z,Ed}} + \frac{B_{Ed}}{B_{Ed}} = \frac{1}{0.787 + 0.031} = \frac{1}{0.818} = 1.222
\]
\[
\alpha_{ult,k} = \sqrt{\frac{1.371}{1.87}} = 0.856
\]
\[
\chi_{op} = 0.689
\]
\[
SI_{E}^{EU\text{–}GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k}} = 1.07
\]
\[
\alpha_{ult,k} = \sqrt{\frac{1.222}{1.79}} = 0.826
\]
\[
\chi_{op} = 0.708
\]
\[
SI_{T}^{EU\text{–}GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k}} = 1.15
\]

1. A2. The United States approaches.

2. It has been decided to develop US computation according to the international system units, and as a consequence, Young’s modulus E = 199950 MPa has been assumed.

3. **US–NOLM**: check has to be done assuming an effective length factor with \( K = 1 \).

4. Reduction of the yield strength for compression:

5. critical flexural stress:

\[
F_{ex} = \frac{\pi^{2}E}{(KL/r)^{2}} = \frac{1973920.88}{(1.1330/44)^{2}} = 2160 \text{ MPa}
\]

6. critical torsional stress:

\[
F_{ct} = \left( \frac{\pi^{2}EC}{(KL)^{2}} + GJ \right) \frac{1}{A_{y} \cdot r_{0}^{2}} = \left( \frac{1973920.88 \cdot 5.17 \cdot 10^{9}}{(KL)^{2}} + 80769 \cdot 1061 \right) \frac{1}{707 \cdot 9635.25} = 2137 \text{ MPa}
\]

7. critical flexural-torsional stress:

\[
F_{f} = \left( \frac{F_{ex} + F_{ct}}{2H} \right) + GJ \left( 1 - \frac{4F_{ex}F_{ct}H}{(F_{ex} + F_{ct})^{2}} \right) = 1169 \text{ MPa}
\]

8. with:

\[
\frac{x_{0}^{2} + y_{0}^{2}}{r_{0}^{2}} = 82.2^{2} + \frac{1368668 + 666368}{707} = 9635.25
\]

9. \( H = 1 - \frac{x_{0}^{2} + y_{0}^{2}}{r_{0}^{2}} = 1 - \frac{82.2^{2}}{9635.25} = 0.299 \)

10. \( F_{ct} = \min \left\{ F_{ex} ; F_{ct} ; F_{ex} ; F_{ct} \right\} = 1169 \text{ MPa} \)

11. \( \lambda_{i} = \sqrt{\frac{F_{ex}}{F_{r}}} = \sqrt{\frac{355}{1169}} = 0.551 < 1.5 \)

12. \( F_{n} = \left[ \frac{F_{ex}}{0.658F_{r}} \right] F_{y} = \left[ \frac{355}{0.658^{1169}} \right] 355 = 312.63 \text{ MPa} \)

13. \( A_{y} = \left[ 1 - \left( 1 - Q \right) \left( \frac{F_{ex}}{F_{y}} \right) \right] A_{net} = \left[ 1 - (1 - 0.85) \left( \frac{313}{355} \right) \right] 707 = 612 \text{ mm}^{2} \)
$P_n = A_y \cdot F_n = 312.63 \cdot 612 = 191.33 \text{kN}$

For the bending resistance it is necessary to check if the member should be subjected to lateral-torsional instability, according to the AISI [28] formula:

$$F_{e,m} = \frac{C_b \cdot r_f A_y}{S_f} \sqrt{F_{ey} \cdot F_{et}} = \frac{12.3 \cdot 98.15 \cdot 707}{24033} \cdot \sqrt{1292 \cdot 2137} = 7587 \text{MPa}$$

$F_{e,m} = 7587 \text{MPa} > 987 \text{MPa} = 2.78 \cdot F_y$

with:

$$C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} = 1.23$$

with the relationship being fulfilled, the members are not subject to lateral torsional buckling, as already observed for the EU procedures in which $\chi_{LT} = 1.0$ was computed. The upright resistance is given by:

$$M_{ut} = S_{e,ut} \cdot F_y = 22350.7 \cdot 355 = 7.93 \text{kNm}$$

Safety index $S_{IUS-NOLM}$ is:

$$S_{IUS-NOLM} = \frac{P_r \cdot \phi_x N_{Ed}}{\phi_y M_{nx} + M_y} = \begin{bmatrix} \frac{122.07 \cdot 10^3}{0.9 \cdot 191327} + \frac{1.69}{0.9 \cdot 7.93} + \frac{-0.12}{0.9 \cdot 5.72} = 0.71 + 0.24 + 0.02 = 0.97 \\ \end{bmatrix}$$

$US-ELM$: upright check has to be performed with the value of effective length derived from finite element buckling analysis.

$$F_{ey} = \frac{\sigma_x^h N_{Ed}}{A_y} = \frac{218554}{707} = 309 \text{MPa}$$

the flexural-torsional one is

$$F_e = \left( \frac{F_{ey} + F_{et}}{2H} + GJ \right) \left( 1 - \frac{4F_{ey}F_{et}H}{(F_{ey} + F_{et})^2} \right) = 280 \text{MPa}$$

$$\lambda_e = \frac{F_{ey}}{F_e} = \frac{355}{280} = 1.26 < 1.5$$

$$F_n = \left[ 0.658 \frac{F_y}{F_e} F_y \right] = \left[ 0.658 \frac{355}{280} \right] 355 = 208.8 \text{MPa}$$

$$F_{ey} = \frac{\sigma_y^h N_{Ed}}{A_y} = \frac{22570}{707} = 319.6 \text{MPa}$$

$$\lambda_e = \frac{F_{ey}}{F_e} = \frac{355}{319.6} = 1.11 < 1.5$$

$$F_n = \left[ 0.658 \frac{F_y}{F_e} F_y \right] = \left[ 0.658 \frac{355}{319.6} \right] 355 = 223 \text{MPa}$$
A. Resistance check

As already discussed in the paper, in the case of mono-symmetric cross-sections, there is the important contribution due to the bimoment and it is essential to also check the resistance of the more stressed cross-section, which should govern design, especially in cases of partially loaded racks. As an example, reference is made to the EU-RAM, EU-GEM and EU-IRAM approaches, which have the same values of internal forces and moments. The resistance checks have to be referred to the more stressed upright cross-section, like in EU-DAM, but due to the presence of the sole overall (sway) imperfection, different distributions of the moments are expected, as it appears from the comparison between figures A1 and A3. Furthermore, figure A4 is related to the normal stress distribution on the more stressed gross cross-section.

\[
SI_{6}^{\text{REX}} = \frac{129.06 \cdot 10^3}{600.95 \cdot 355} + \frac{0.941 \cdot 10^6}{22350 \cdot 355} + \frac{-0.04 \cdot 10^6}{16115 \cdot 355} = 0.60 + 0.12 + 0.01 = 0.73
\]

\[
SI_{7}^{\text{REX}} = \left( \frac{131.68 \cdot 10^3}{600.95 \cdot 355} + \frac{0.98 \cdot 10^6}{22350 \cdot 355} + \frac{-0.24 \cdot 10^6}{16115 \cdot 355} \right) + \frac{-0.008 \cdot 10^6}{717785 \cdot 355} = (0.61 + 0.13 + 0.04) + 0.03 = 0.78 + 0.03 = 0.81
\]

It can be observed that, as in the cases considered in the numerical study, the resistance checks are less severe than the stability ones and it is a general remark associated with the case of totally loaded racks.
A4. Result comparison

Figure A3: Distributions of the bimoment (a) and bending moment (b) in the down-aisle direction associated with the EU-RAM, EU-GEM and EU-IRAM approaches.

Figure A4: Distributions of the internal stresses in the gross cross-section at the bottom of an external upright associated with the EU-RAM, EU-GEM and EU-IRAM approaches.

Main steps related to the calculations associated with the six considered design alternatives have been presented in the Appendix and for a general comparison, reference can be made to the safety index values: in some cases, SI values are greater than unity but the scope of the present Appendix is to...
propose a comparison independently from the acceptability or not from a designer’s point of view of the verification checks.

Table A2 summarizes key final results of the design procedures in terms of safety index (SI), reporting also the terms related to the axial load (SI_N), bending moments along the y- and z-axis (SI_{My} and SI_{Mz}, respectively) and bimoment (SI_B). In general, it can be noted that the contribution due to bending moments is more limited with respect to the one associated with the axial load, especially for bending moments (M_z) along the cross-aisle direction. Furthermore, as expected, the influence of the bimoment is non negligible on methods based on the resistance checks, like the EU-DAM and the EU-GEM and of the same order of magnitude of the bending contribution.

<table>
<thead>
<tr>
<th>Method</th>
<th>SI_N</th>
<th>SI_{My}</th>
<th>SI_{Mz}</th>
<th>SI_B</th>
<th>SI</th>
<th>SI/\text{SI}_6</th>
<th>SI^{\text{max}}/\text{SI}_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU DAM</td>
<td>6.066</td>
<td>0.151</td>
<td>0.017</td>
<td>-</td>
<td>0.77</td>
<td>1.351</td>
<td>1.571</td>
</tr>
<tr>
<td>EU RAM</td>
<td>6.617</td>
<td>0.168</td>
<td>0.045</td>
<td>0.210</td>
<td>1.04</td>
<td>1.163</td>
<td></td>
</tr>
<tr>
<td>EU IRAM</td>
<td>6.683</td>
<td>0.119</td>
<td>0.006</td>
<td>-</td>
<td>0.81</td>
<td>1.062</td>
<td>1.494</td>
</tr>
<tr>
<td>EU GEM</td>
<td>6.697</td>
<td>0.124</td>
<td>0.044</td>
<td>-</td>
<td>0.86</td>
<td>1.407</td>
<td></td>
</tr>
<tr>
<td>EU IRAM</td>
<td>6.1003</td>
<td>0.109</td>
<td>0.006</td>
<td>-</td>
<td>1.11</td>
<td>1.037</td>
<td>1.109</td>
</tr>
<tr>
<td>EU GEM</td>
<td>6.095</td>
<td>0.122</td>
<td>0.044</td>
<td>-</td>
<td>1.15</td>
<td>1.051</td>
<td></td>
</tr>
<tr>
<td>US NOLM</td>
<td>6.985</td>
<td>0.122</td>
<td>0.044</td>
<td>-</td>
<td>1.15</td>
<td>1.052</td>
<td></td>
</tr>
<tr>
<td>US ELM</td>
<td>6.107</td>
<td>-</td>
<td>1.07</td>
<td>1.075</td>
<td>1.131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US NOLM</td>
<td>6.714</td>
<td>0.236</td>
<td>0.020</td>
<td>-</td>
<td>0.97</td>
<td>1.052</td>
<td>1.247</td>
</tr>
<tr>
<td>US ELM</td>
<td>6.734</td>
<td>0.237</td>
<td>0.050</td>
<td>-</td>
<td>1.02</td>
<td>1.186</td>
<td></td>
</tr>
<tr>
<td>US ELM</td>
<td>6.1017</td>
<td>0.140</td>
<td>0.023</td>
<td>-</td>
<td>1.18</td>
<td>1.025</td>
<td>1.055</td>
</tr>
<tr>
<td>US ELM</td>
<td>6.990</td>
<td>0.169</td>
<td>0.050</td>
<td>-</td>
<td>1.21</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

A quite wide dispersion of the results can be noted with reference to the EU approaches: in the case of 6DOFs formulation, the maximum difference of 38% is reduced to 19% when warping effects are considered. As to the US approaches, the safety index associated with the US-NOLM method is very close to the SI associated with EU-IRAM and EU-GEM ones, as already remarked in the paper, and the dispersion of the data is more limited: the ratio between the US-ELM and US-NOLM ranges between 1.18 and 1.21. Furthermore, the US-ELM approach provides a more conservative evaluation of the member performance, with the safety index being equal to 1.21 on the safe side.

**APPENDIX B: List of symbols**
Latin upper case letters

A = gross cross-section area.

$A_{eff}$ = effective cross-section area.

AISC = American Institute of Steel Construction.

ANSI = American National Standards Institute.

B = bimoment.

$B_{Rd}$ = bimoment resistance.

DAM = Direct Analysis Method.

E = Modulus of elasticity of steel.

$E_d$ = design value.

EC3 = EN 1993-1-1 Eurocode 3 “Design of Steel Structures”.

ELM = Effective Length Method.

RMI = Rack Manufacturers Institute.

EU = Europe, European.

$F_n$ = critical stress.

$F_{el}$ = elastic buckling stress.

$F_y$ = US yielding strength.

G = shear modulus.

GEM= GEneral Method.

LL = load levels.

L = member length.

$L_{eff}$ = effective buckling length.

$L_u$ = member length for lateral buckling instability.

$I_t$ = Saint-Venant torsion constant.

$I_w$ = warping constant.

$I_y, I_z$ = second moment of area.

$K$ = effective length factor.

IRAM = improved rigorous analysis method.

$M_{cr}$ = critical bending moment.

$M_{Ed}, M_{y,Ed}, M_{z,Ed}$ = design bending moment.

$M_n, M_{y,n}, M_{z,n}$ = nominal bending resistance.

$M_{Rk}$ = characteristic bending resistance.

$N_{cr}$ = critical load for the i-member.

$N, N_{Ed}$ = member axial load.

$N_{Rk}$ = characteristic axial resistance.

$N_{Rd}$ = axial stability resistance.

$P_c$ = design axial strength.

$Q_c$ = nominal resistance strength for compression.

$Q^N, Q_{EU}^N, Q_{US}^N$ = reduction factor for axial load.

$Q_{EU}^{M_e}, Q_{EU}^{M_y}$ = reduction factor bending moment.

RAM= Rigorous Analysis Method.

$R_d$ = resistance.

$S_{j,btc}, S_{j,base}$ = stiffness of connection.

$S_{j,base}^{EC3-LB}$ = lower bound of EC3.
**sI**^{EC3-UB} = upper bound of EC3.

**sI**^{EC3-UB} = upper bound of EC3 for base-plate connections.

**sI**^{j,k} = safety index associated with the **j**-code and the **k**- design approach.

**sI** ,  **sI**^{EU} ,  **sI**^{US} = design safety index.

US = United State of America.

**W**_{eff} ,  **W**_{eff,y} ,  **W**_{eff,z} = effective cross-section modulus.

WP = prevented warping at base plate connection.

WF = free warping at base plate connection.

Latin lower case letters

**e** = eccentricity.

**h** = interstorey height.

**k**_j,  **k**_z,  **k**_y = bending interaction factor.

**Max** = maximum value.

**min** = minimum value.

**f**_y = specified minimum yield stress strength.

Greek case letters

**α** = imperfection coefficient associated with the relevant buckling curve.

**α**_{crit} = buckling overall frame multiplier obtained via a finite element buckling analysis.

**β**_{n,k} = minimum load multiplier evaluated with reference to the cross-section resistance.

**β**_{M,f} = bending moment distribution coefficient.

**ψ** = gradient moment coefficient.

**τ**_{eq} = relative slenderness of the whole structure.

**λ**_c = slenderness factor.

**μ**_j = non-dimensional term for beam-column verification check.

**ρ**_{j,btc} = parameter to define the elastic rotational stiffness of beam-to-column joints.

**ρ**_{j,base} = parameter to define the elastic rotational stiffness of base-plate joints.

**χ** = reduction factor for the relative buckling curve.

**χ**_{LT} = reduction factor due to lateral buckling.

**σ** = normal stress.

**χ**_{op} = buckling reduction factor referred to the overall structural system.

**γ**_{M} = **γ**_{M1} = material safety factor.

### Table B1 – Comparison between EU and US codes terminology.

<table>
<thead>
<tr>
<th>EU</th>
<th>Term</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N</strong>_{Ed}</td>
<td>axial force demand</td>
<td><strong>P</strong>_r</td>
</tr>
<tr>
<td><strong>N</strong>_{b,Rd}</td>
<td>design axial strength</td>
<td><strong>P</strong>_n</td>
</tr>
<tr>
<td><strong>M</strong><em>{y,Ed},  <strong>M</strong></em>{z,Ed}</td>
<td>required flexural strength about centroidal axes.</td>
<td><strong>M</strong><em>{rx},  <strong>M</strong></em>{rz}</td>
</tr>
<tr>
<td><strong>M</strong><em>{y,Rk},  <strong>M</strong></em>{z,Rk}</td>
<td>design flexural strength about centroidal axes.</td>
<td><strong>M</strong><em>{cx},  <strong>M</strong></em>{cy}</td>
</tr>
<tr>
<td><strong>N</strong>_{cr}</td>
<td>elastic critical buckling load</td>
<td><strong>P</strong>_c</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$W_{eff}$</td>
<td>elastic section modulus of effective cross-section</td>
<td></td>
</tr>
<tr>
<td>$I_y, I_z$</td>
<td>second moment of Area about centroidal axes</td>
<td></td>
</tr>
<tr>
<td>$I_l$</td>
<td>Saint-Venant torsion constant</td>
<td></td>
</tr>
<tr>
<td>$I_w$</td>
<td>torsional warping constant of cross-section</td>
<td></td>
</tr>
<tr>
<td>$i_y, i_z$</td>
<td>radius of gyration about symmetry centroidal axes.</td>
<td></td>
</tr>
<tr>
<td>$f_y$</td>
<td>specified minimum yield stress strength</td>
<td></td>
</tr>
<tr>
<td>$y$-$y$</td>
<td>cross-section symmetry axis</td>
<td></td>
</tr>
<tr>
<td>$z$-$z$</td>
<td>cross-section non symmetry axis</td>
<td></td>
</tr>
</tbody>
</table>