STOCHASTIC VEHICLE ROUTING WITH RANDOM TIME DEPENDENT TRAVEL TIMES SUBJECT TO PERTURBATIONS

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Abstract

Assigning and scheduling vehicle routes in a stochastic time dependent environment is a crucial management problem. The assumption that in a real-life environment everything goes according to an a priori determined static schedule is unrealistic, resulting in a planning gap (i.e. difference in performance between planned route and actual route). Our methodology introduces the traffic congestion component based on queueing theory, thereby introducing an analytical expression for the expected travel. In real life travel times are subject to uncertainty, we solve a time dependent vehicle routing problem to find robust solutions, that can potentially absorb such uncertainties. We model uncertainty as perturbations that are randomly inserted on the routes, we optimize the perturbed solutions via Tabu Search. We conduct experiments on a set of 32 cities, and found that the perturbed solutions generally cope better with the uncertainty than the non-perturbed

solutions, with a small increase in expected travel times.

Keywords: vehicle routing problems, time-dependent travel times, uncertainty, robustness.

Introduction

Routing problems have been largely studied due to the interest in different applications in logistics and supply chain management. Not surprisingly, transportation is an important component of supply chain competitiveness since it plays a major role in the inbound, inter-facility, and outbound logistics. Transportation costs represent approximately 4 to 5 percent of total logistics costs and 4 to 1 percent of the product selling price for many companies Coyle et al. [7]. As such, transportation decisions directly affect the total logistic costs. The passage of the transportation deregulation acts in the 198's in the USA and in the 199's in the EU drastically changed the business climate within which the transportation managers operate. Within the EU, the competition is becoming intense between transporters since they often operate at transnational levels and must provide higher levels of service with lower costs to meet the various needs of customers. In this context, assigning, scheduling and routing the fleet of a transportation company is a crucial management problem. In many real-life circumstances, traffic conditions and the related uncertainty can not be ignored in order to carry out a realistic routing optimization. Uncertainty about traffic and thus the travel times is a pervasive aspect of routing and scheduling. As the cost impact due to this uncertainty can be substantial, planners may wish to know to what extent the routes and schedules are risky in terms of travel times.

Here, the traffic congestion component is introduced through a queueing approach, that also guarantees adherence to the FIFO principle, that was emphasized by Ichoua et al. [16] The distribution of the speeds is then calculated based on traffic flows. More specifically, the stochastic

nature of travel times is captured using queueing theory applied to traffic flows (see Vandaele et al. [24] 2; Van Woensel [26], 23). By making use of this analytical approach the necessary data (i.e. traffic flow and some queueing parameters to capture road conditions) to model congestion is limited which opens the door for real-life applications. This analytical approach to congestion based on queueing models allows for the calculation of average speeds. In this paper, Tabu Search is used as the prime tool to generate solutions.

Over the past years optimizing under uncertainty has attracted much research, this is driven from the realization that in a modeling process many parameters are assessed and are inserted in the given constraints, however in many cases a great deal of uncertainty lies in assessments. In principle this means that while sufficiently good solutions are found their applicability is limited, since they do not take into account uncertainty. Robust optimization as presented by Bertimas and Thiele [6], is essentially defined over a convex space, and hence applies in essence to continuous linear programming with uncertainty. Bental and Nemirovski [5] expand the approach to include conic quadratic problems. While this area in combinatorial optimization has dramatically evolved throughout the past years, the techniques developed depend on the symmetric composition of the problem as well as independencies between the arguments of the uncertainty. However more importantly when dealing with an NP-Hard problems (Yu and Yan [29]), such as vehicle routing problem (VRP), techniques are insufficient and solutions are found via heuristics.

In a time dependent vehicle routing problem (TDVRP) setting an actual realization of a route might lead to a far greater travel time than the truly optimal solution. Montemanni and Gambardella [21] address the issue of travel time uncertainty in the TSP by defining a robustness measure, that is incorporated into the target function, which they try to minimize under various scenarios.

Sorensen [23] defines robustness as the insensitivity of a solution with respect to changes in the environment in which this solution is implemented. He further notes that while there are a number of powerful tools to obtain robust solutions, they are very hard to implement using local search techniques such as tabu search. He proposes an approach that incorporates perturbations on given solutions which are then evaluated through the objective function. This approach was presented in the context of robust continuous optimization using local search. We choose to adapt this approach to our TDVRP setting (due to its simplicity, relevance and adaptability to Tabu Search). We show that the travel times and reliability of the travel times can be improved significantly when explicitly taking into account random fluctuations during optimization. The rest of the paper is organized as follows: in Section 2 we present the queuing model used to depict travel times, we then present our robust modeling approach, in Section 3, we present one preliminary experiment and its results, and finally we conclude with Section 4 that highlights main conclusions and discusses potential further research.

Model

Formally, the vehicle routing problem can be represented by a complete weighted graph $G=(V,A,c)$ where $V=\{,1...n\}$ is a set of vertices and $A=\{(i,j): i <> j\}$ is a set of arcs. The vertex 0 denotes the depot; the other vertices of V represent cities or customers. The non-negative weights c_{ji}, which are associated with each arc (i,j), represent the cost (distance, travel time or travel cost) between i and j. For each customer, a non-negative demand qd_i and a non-negative service time d_i is given ($d_0 = 0$ and $qd_0 =0$). The objective is then to find the minimum cost vehicle routes where the following conditions hold: every customer is visited exactly once by exactly one vehicle; all vehicle routes start and end at the single depot; every vehicle route has a total demand not exceeding the vehicle capacity Q; every vehicle route has a total route length not exceeding the maximum length L (Laporte [20]). If it seems reasonable to assume that the service time at each vertex (customer) is known in advance, it is definitely not the case for the travel time between two vertices. In fact, the travel times are the result of a stochastic process related to traffic congestion. Clearly, travel times depend greatly on the different number of vehicles occupying the road and on their speeds. In this paper, the *VRP* problem considered deals with dynamic travel times. In this case, the non-negative weights c_{ij}^p associated with each arc (i,j), represent the travel time between i and j starting in time zone p. In the rest of this section we start off by presenting the queueing model developed by Van Woensel [26]. Later on we present the perturbation model.

Queuing approach

Vandaele et al. [24] and Heidemann [14], showed that queueing models can also be used to model traffic flows and thus offering a more analytical approach, useful for sensitivity analysis, forecasts, etc. Jain and Smith [17] describe in their paper a state-dependent *M/G/C/C* queueing model for traffic flows. Also a lot of research is done on a travel time-flow model originating from Davidson [9]. The model is based on some concepts of queueing theory but a direct derivation has not been clearly demonstrated (Akcelik [2] and [3]).

Vandaele et al. [24] developed different queueing models that can be used to model traffic flows. The *M/M/1* queueing model (exponential arrival and service rates) is considered as a base case, but due to its specific assumptions regarding the arrival and service processes, it is not useful to describe real-life situations. Relaxing the specifications for the service process of the *M/M/1* queueing model, leads to the M/G/1 queueing model (generally distributed service rates). Relaxing both assumptions for the arrival and service processes results in the *GI/G/m* queueing model. Moreover, following Jain and Smith [17], a special case of the *GI/G/m* queueing model is derived: a state dependent *GI/G/m* queueing model. This model assumes that the service rate is a (linear, exponential,etc.) function of the traffic flow. In this case vehicles are served at a certain rate, which depends upon the number of vehicles already on the road.

In our queueing approach to traffic flow analysis, roads are subdivided into segments, with length equal to the minimal space needed by one vehicle on that road. We define k_j as the maximum traffic density (i.e. maximum number of carson a road segment). This segment length is then equal to $1/k_j$ and matches the minimal space needed by one vehicle on that road. Each road segment is then considered as a service station, in which vehicles arrive at a certain rate λ and get served atanother rate μ (Vandaele et al. [24]}; Van Woensel et al. [27]; Heidemann [14]).

Following Heidemann [14], the arrival rate λ is defined as the product of the traffic density k and the free flow speed v_f or $\lambda = k$ x v_f. Similarly, the service rate μ is defined as the product of free flow speed v_f with the maximum traffic density k_j, or $\mu = k_j$ x v_f. Vandaele et al. [24] and Heidemann [14] showed that the speed v can be calculated by dividing the length of the road segment ($1/k_j$) by the total time in the system (W).

$$v = \frac{1/k_j}{W} \qquad (1)$$

The total time in the system W in formula 1 is different depending upon the specific queueing model used. The total time spent in the system W equals the sum of the waiting time W_q and the service time W_q. Table 1 shows the specific form of W_q for the general queueing models. For the *GI/G/m* queueing models, no exact solutions are available and one must rely on approximations. Here, three approximations are considered: the Kramer-Lagenbach-Belz (KLB) approximation[19] is widely used but is limited to single servers only. To cope with multiple lanes, the heavy traffic or Kingman approximation (K) [18] and the Whitt (W) approximations [25] with multiple servers are used. In this paper, the *GI/G/m* queueing models with the Whitt approximations are used.

Table 1 : The specific form of W_q for each queueing model

$GI/G/m$	W_q
KLB	$\frac{r}{(1-r)} \frac{(c_a^2+c_s^2)}{2} \frac{1}{k_j v_f} \exp\left[\frac{-2(1-r)(1-c_a^2)^2}{3r(c_a^2+c_s^2)}\right]$
K	$\left(\frac{c_a^2+c_s^2}{2}\right)\left(\frac{r^{\left(\sqrt{2(m+1)}-1\right)}}{m(1-r)}\right)\left(\frac{1}{k_j v_f}\right)$
W	$\phi\left(\frac{c_a^2+c_s^2}{2}\right)\left(\frac{1}{k_j v_f}\right) W_{qM/M/k}$

With ϕ a correction factor defined in Whitt [25] and $W_{qM/M/m}$ the formula for the waiting time in an $M/M/m$ queue.

Results show that the developed queueing models can be adequately used to model traffic flows (Van Woensel and Vandaele [28]). Moreover due to the analytical character of these models, they are very suitable to be incorporated in other models, e.g., the *VRP*. In general, formula 1 can be rewritten in the following basic form (see Van Woensel [26] for the details):

$$v = \frac{v_f}{1+\Omega} \tag{2}$$

Formula 2 shows that the speed is only equal to the free flow speed v_f if the factor Ω is zero. For positive values of Ω, v_f is divided by a number strictly larger than 1 and speed is reduced. The factor Ω is thus the influence of congestion on speed. High congestion (reflected in a high Ω) leads to lower speeds than the maximum. The factor Ω is a function of a number of parameters depending upon the queueing model chosen: the traffic intensity r, the coefficient of variation of service times c_s and coefficient of variation of inter-arrival times c_a, the jam density k_j and the free flow speed v_f. High coefficients of variation or a high traffic intensity will lead to a value of Ω strictly larger than zero. Actions to increase speed (or decrease travel time) should then be focused on decreasing the variability or on influencing the traffic intensity, for example by manipulating the arrivals (arrival management and ramp metering).

The major strength of using the queueing models is that, given the physical characteristics of the road network, it can immediately be linked with the parameters of the queueing model. In practice, the jam density and the free flow speed is fixed for a given arc (i,j), leaving only the coefficients of variation to represent the traffic conditions (e.g., bad weather, etc.). The flow q is a parameter that is determined empirically over time, allowing the determination of realistic velocity profiles as a function of time. Analytical queueing models based on traffic counts model the behavior of traffic flow as a function of the most relevant determinants (e.g. free flow speeds, jam density, variability due to weather, etc.). An empirical validation of the queueing approach is provided in Van Woensel and Vandaele [28]. Consequently, the travel times can be modeled much more realistically using these speeds (i.e. expressed in kilometer per hour) and are directly related to the physical characteristics and the geographical location on the arc.

For a more detailed discussion of the queueing models and their results, the interested reader is referred to Vandaele et al. [24] and Van Woensel et al. [27].

Towards Travel Times

To compute the travel times, one should note that in the time-dependent case, the travel speeds are no longer constant over the entire length of the arc. More specifically, one has to take into account the change of the travel speed when the vehicle crosses the boundary between two consecutive time periods. For example, the speed changes when going from time period p to time period *(p+1)* from v_{ij}^p to v_{ij}^{p+1}. The time horizon is discretized into P time periods of equal length *Δp* with a different travel speed associated to each time period p ($1 \leq p \leq P$). The travel speeds are obtained using the

above discussed queueing models for traffic flows. Formally, the travel time T_{ij}^{p0} going from customer i to customer j, starting at some time p_0, must satisfy the following condition:

$$\int_{p_0}^{p_0+T_{ij}^{p_0}} [v^p] dp = d_{ij}$$

With v^p denoting the speed in time period p and d_{ij} the distance traveled. Solving this integral for T_{ij}^{p0} and making use of the discrete time horizon, results in:

$$d_{ij} = \Delta_p (\varphi v^{p_0} + v^{p_0+1} + .. + v^{p_0+(k-2)} + \phi v^{last})$$

Rewriting as a function of the time slices, gives:

$$T_{ij}^{p_0} = \varphi v^{p_0} + (k-2)\Delta_p + \phi \Delta_p)$$

The travel time is thus the sum of the following components:

1.The fraction of travel time still available in the first time zone, given by $(\varphi \Delta_p)$ with φ the fraction parameter $(0 \leq \varphi \leq 1)$.

2. The duration of the (k-2) intermediate time zones passed: (k-2) Δ_p.

3. The fraction of the travel time in the last time zone, given by $(\Phi \Delta_p)$, with Φ the fraction parameter $(0 \leq \Phi \leq 1)$.

Concluding, in total k time buckets are crossed. The number k is totally defined by the equations in the paper and is a function of the distance i to j and the speeds in the different time buckets. In practice, the other values are computed incrementally: Starting at time p_0 (part of the first time bucket), one knows the fraction of time left in the first time bucket and consequently $\varphi = 1 - p_0 / \Delta_p$. Then a number of time buckets Δ_p is added as necessary to reach the destination city j. Of course, one will spend in the last time bucket only part of the time or the fraction Φ. This fraction is totally depending upon the residual distance that needs to be traveled in the last time bucket. Using this incremental procedure, the travel time T_{ij}^{p0} from customer i to customer j, starting at time p_0 can be determined easily based on the distance d_{ij} and the speed v^p for the different time periods p.

Perturbation insertion

Using the above described queueing model that depicts travel timeswe look into finding robust solutions (Sorensen [23]) For a given solution, we generate perturbations with a size δ_i from a given distribution, then we randomly chose a link, and we assess the solution given by the perturbation in the target function. This is done n times, at the end of which the average target function value is associated with the solution. The following equation describes the procedure, where $f_i(x)$ is the target function value associated with route x.

$$f_r(x) = \frac{1}{n}\sum_{i=1}^{n} f(x + \delta_i) \qquad (3)$$

The power of such an approach lies in the fact that the structure of the Tabu search remains the same, however when evaluating each solution a single, yet different, random time perturbation is inserted at the beginning of a random link on the route, for n scenarios. The main idea behind such an approach, is locating a solution that is in a stable environment, meaning that if fluctuations were to occur randomly this solution would on average perform well.

Experiments

First, we calibrate the input used in the optimization step based onreal-life data. limited dataset collected by the ministry of transportation of the Flemish Government (Belgium) is used tocalibrate and validate the model. The original dataset contains minute-per-minute observations the number of trucks, the number of passenger cars and their speeds for four counting points in Belgium for two weeks (which is the longest period for which this type of date is stored on this level of detail). One counting point of this set (Ternat) is selected. All trucks and passenger cars are converted to a vehicle equivalent, i.e. cars are 1 vehicle equivalent and trucks are 2 vehicle equivalents (see e.g. [8]). Based on these data we select best parameter settings for the queueing model. Similar to Ichoua et al. [16], we take into account multiple road types e.g. to represent highways versus rural roads. Due to the lack of real-life data on both speeds and flows for different road types, we used the same observed speeds by applying a re-scaling to represent different road types. If there is only one road type, the original series is used; if there are two road types, both the original as 60% of the original series is used; if there are three road types, the original, 60% of the original and 30% of the original series was used. In the two road types setting, all even to even node arcs are of the faster type. In the two road type setting, all even to even node arcs are of the fast type, all odd to odd node arcs are of the middle type and everything else is slow. The parameter settings for the queueing approach are obtained via the same procedures as described in van Woensel et al. [27].

We experiment with a data set from Augerat [4]. This set contains 32 customers with two road types, including the depot. The duration of a time zone for this experiment is set equal to 1 minutes. Note that the choice of the time settings is purely arbitrarily, i.e. in the extreme case, time zones of 1 minute can be considered. All capacities of the trucks are set to 1 (following Augerat [4]). All coordinates are multiplied with a constant factor of 1 to ensure that multiple time zones are covered over longer distances. The starting time is allowed to be different (comparable to the real-life decisions of leaving earlier or later due to congestion) in the dynamic case: trucks are allowed to start their routes anywhere between 6AM and 11AM.

In the second stage we set the perturbation parameters, given the fact that we choose to uniformly assign the perturbation at the beginning of a random link, two other parameters need to be set: n being the number of perturbations for each solution, and the perturbation size δ_i which is taken from a distribution. We choose to experiment with $n=\{10,20,..80\}$. As for the size of the perturbation, we chose to work with a deterministic perturbation of size 36.36, as well as a normally distributed perturbation with ($\mu=36.36$, $\sigma=3.64$) (36.6 is the average travel time per link in the initial non perturbed setting).

The solution strategy based is on local search is proposed. According to Aarts and Lenstra [1], local search is a solution process that tries to improve a given initial solution by making relatively small changes in several steps in the solution space. The quality of the solutions is determined with the cost function of the problem. Local search techniques will result in a good but not necessarily optimal solution within reasonable computing time. In this paper, the tabu search heuristic is used for obtaining solutions for the vehicle routing setting presented. The success of this methods is due to several factors: general applicability of the approach, flexibility for taking into account specific constraints in real cases and ease of implementation (Pirlot [22]).

Tabu Search can be described as a local search technique guided by the use of adaptive or flexible memory structures. Tabu search was first proposed by Glover ([12] and [13]) and involves the examination of all the neighbors of a solution of which the best is selected. To prevent cycling, solutions that were recently examined are forbidden and inserted in a constantly updated tabu list. For our tabu search implementation the following references where used as a basis: Gendreau et al. [10]; Gendreau et al.[11]; Hertz et al. [15]. The only change made to this basic algorithm is to replace distance by dynamic travel time. We programmed our implementation in JAVA.

Unlike in the static VRP where the gain is calculated based on distances, in the time dependent VRP, the gain is calculated in terms of travel time. As the evaluation is done in terms of travel times, the arrival times at a certain node will be affected by an exchange of two arcs. Therefore, the potential gains of the solution has to be re-evaluated. Note that in principle, one does not need to re-evaluate the complete solution as the travel times of the arcs traversed before the exchanged ones, do not change. Neither are the subtours not involved in the arc swap, affected by this operation.

The solution found under the perturbed setting is expected to have higher expected travel times, in comparison with expected travel times in the original setting (otherwise it would have been the same), however the solution under the perturbed setting is expected to perform much better with regards to uncertainty. For the data set used the solution found by the Tabu search, without any perturbations denoted as *TT(initial)*, has the value of 138.5. Table 2 depicts the travel times, for the deterministic perturbation, the travel times of the best found solution in a perturbed setting is denoted by *TT(pert)*, this solution's associated expected travel is denoted by *TT(pert no pert)*. The last column *TT(initial pert)* shows the result of the initial solution (1308.5) when subjected to the set of perturbations that led the Tabu search to its chosen solution.

Table 2 : Deterministic perturbation size

| n	Total iteration number	TT(pert)	TT(Pert no per)	TT(initial)	TT(initial of perturbed)
10	1840	1332.2	1309.3	1308.5	1341.9
20	1232	1335.2	1309.9	1308.5	1340.0
30	1310	1336.7	1309.0	1308.5	1340.7
40	1074	1336.3	1308.9	1308.5	1340.4
50	1744	1338.0	1308.9	1308.5	1341.7
60	1627	1337.1	1309.7	1308.5	1340.7
80	1507	1337.7	1308.9	1308.5	1340.4

We can observe from tables 2 and 3 that there is a significant difference between the initial perturbed solution *TT(initial pert)* and the perturbed solution found by the Tabu search *TT(pert)*, *p*=4.1858E-5 and (p=0.001) for tables 2 and respectively.

Table 3 : Normally distributed perturbation sizes

| n	Total iteration number	TT(pert)	TT(Pert no per)	TT(initial)	TT(initial of perturbed)
10	892	1330.4	1309.7	1308.5	1342.1
20	1529	1336.6	1310.8	1308.5	1339.3
30	2081	1335.6	1309.0	1308.5	1340.1
40	1319	1338.3	1309.3	1308.5	1339.0
50	1726	1337.1	1309.2	1308.5	1340.8
60	1517	1337.5	1309.6	1308.5	1340.4
80	1746	1339.0	1309.3	1308.5	1342.3

In order to illustrate the potential benefits of using the method, we plot the added expect travel time (without perturbations) i.e. the added planning time, along with the difference of both solutions when subjected to perturbations, i.e. the saved travel time when subjected to perturbations by using TT(pert).

Figure 1 : Tradeoff between solutions with and without perturbations (Det)
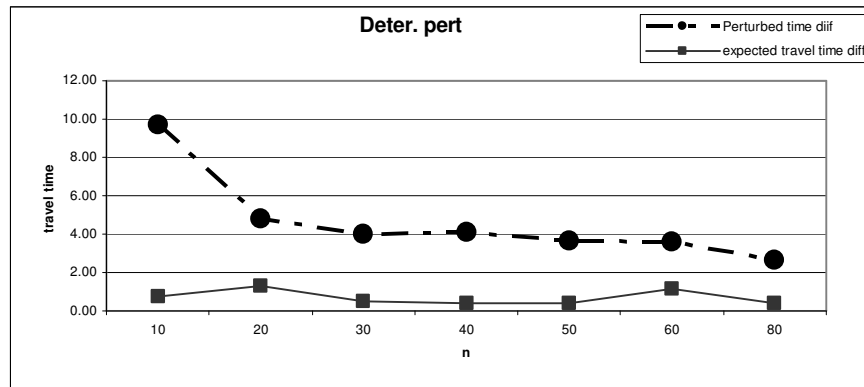
It can be observed that the generated solutions on expectation are longer than the initial one, however their potential saving under perturbations is more than this difference. In other words the addition of time on the expected travel times is far less than the potential savings when subjected to perturbations.

Figure 3 : Tradeoff between solutions with and without perturbations (Norm)
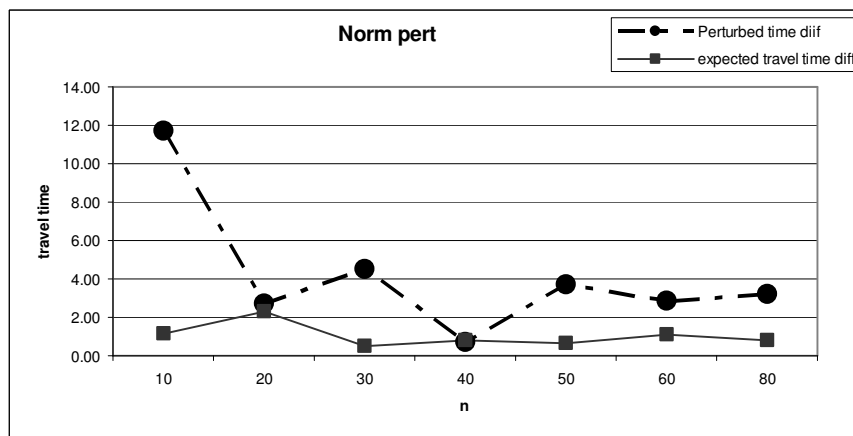
Conclusions and further research

We conducted a preliminary analysis on a time dependent VRP subject to perturbations, we managed to find more robust solutions that are immune to perturbations. We showed that there is a tradeoff between the expected travel and the realization of perturbations under such a solution. Furthermore,

we note that while a perturbation with an average of size 36.36 was added the resulted solutions, for both experiments, have an average of 27 minutes more than the initial solution (138.5). This means that the resulting travel time is smaller than the added perturbation.

There is room for conducting further research first on more data sets, with a number of perturbations and distributions, furthermore there is a need to identify common characteristics for the solutions, more properties should be explored such the length of each sub route.

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