Demand-Aware Network Function Placement

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***Abstract*—Network function virtualization is an emerging network resource utilization approach which decouples network functions from proprietary hardware and enables adaptive services to end-user requests. To accommodate the network function requests, network function instances are created and deployed at runtime. In this paper, we study a network virtualization scheme to orchestrate and manage networking and network function services. We propose an integrated design for network function instance allocation and end-to-end demand realization sharing the same physical substrate network and demonstrate that the corresponding network design problem is NP complete. A mixed-integer programming formulation is proposed first to find its optimal solution, followed by a two-player pure-strategy game model which captures the competition on physical resources between network function instance allocation and routing. We then design an algorithm based on iterative weakly dominated elimination in Game Theory. Computational results demonstrate the value of the integrated approach and its ability to allocate network function instances supporting end-to-end requests with limited physical resources in optical networks.**

***Index Terms*—Cloud network, game theory, network function virtualization, optical network.**

I. Introduction

CURRENT cloud systems consist of geographically-distributed datacenters, servers hosting content/services, and a wide-area optical network which interconnects them and consumers [1]. Cloud service models, including Software-as-a-Service, Infrastructure-as-a-Service, and Platform-as-a-Service, provide different levels of abstraction on hardware and software [2]. The realization of the cloud services should support global delivery of high-performance network-based applications over high-capacity dynamic optical networks with high data rate [3]. Over a shared physical infrastructure consisting of optical networks and datacenters [4], [5], network virtualization techniques, which support multiple coexisting virtual instances (e.g., virtual machines) and virtual networks over a single physical infrastructure, allow cloud and network service providers to offer a fraction of connection, computation, and storage capacities to one or multiple tenants.

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To support cloud service models, traditional optical network virtualization architectures [3], [6], [7], where optical resources are dynamically allocated through optical switching and transport technologies, require a variety of proprietary interconnecting hardware called middleboxes. These middleboxes deployed in communication networks provide network functions (NFs) such as firewall, network address translation, WAN optimization, and quality-of-service analysis. They are deployed both singularly to provide an isolated function and, more commonly, in conjunction with other NFs [8].

Due to increasing demands to shorten time-to-market for new network services, scale up/down existing services, and reduce capital and operational expenditure, the concept of *network functions virtualization* (NFV) is attracting more attention as it facilitates the cycle of NF induction, modification, upgrade, and removal. In general, NFV replaces proprietary networking hardware with software services running on generalized commercial-off-the-shelf (COTS) equipments such as servers, switches, and storage devices. Hence, virtual network functions (VNFs) can be deployed and removed at runtime on COTS devices at NFV infrastructure's points of presence, including datacenters, network nodes, and end-user premises [9], to accommodate changes in traffic demands and network states [10].

Compared with traditional NF services where traffic can only be routed through different chains of predefined, static, and proprietary middleboxes with fine-grained traffic steering [11], [12], it is desirable to utilize NFV techniques so that different VNFs can be dynamically deployed to an optical network, which enables high-volume, large scale, and agile network services. With the increasing agility of optical network [13], [14], the time to establish paths through wavelength switching is acceptable when considering the time required to instantiate VNFs [12]. Hence, NFV concept can be applied on optical networks through a transport NFV architecture [15].

As mentioned in [16] that "VNF is an abstract entity that allows the software contract to be defined, and a VNF Instance is the runtime instantiation of the (design time) VNF," multiple instances of a single VNF should be considered [16]–[18], where NFV "dynamically redistributes packet processing across multiple instances of an NF" [19]. An example of VNF instances provided in [19] shows that intrusion detection is a VNF where the launch of new VNF instances and rerouting of ingress flow to new VNF instances can be achieved.

All these facts motivate us to explore network virtualization techniques over optical networks for NF orchestration and management targeting agile and flexible cloud services facilitating end users' demands and traffic flows. Fig. 1 illustrates an example of NFV on top of an optical network. Virtual networks (accessed through IP layer) are mapped onto an optical network,

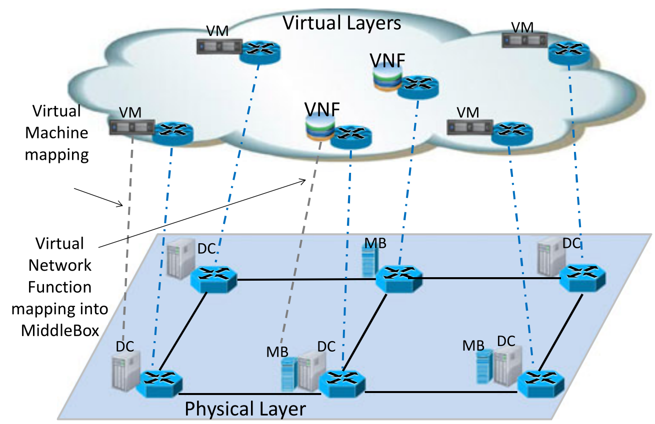

Fig. 1. Network function virtualization.

where virtual nodes accessed through IP routers co-exist with some optical switching nodes. The optical/IP backbone provides connectivity among datacenters and the VNFs are deployed on datacenters and/or routers/switches.

In this paper, we investigate an integrated design for VNF instance deployment and end-to-end demand routing on an optical network based on an established NFV use case in [9], which guarantees the quality-of-service and quality-of-transmission, including high throughput, scalability, and efficiency, and provides VNF services in optical networks. We explore the impacts of NFV realization and routing in a substrate optical network with limited physical resources.

The contributions of this paper lie in the following: (1) we define the problem and propose mathematical models which may serve as the foundation to design a cloud platform supporting NFV and network virtualization; (2) to the best of our knowledge, this is the first paper providing a techno-economic analysis on a consolidated design and provision scheme for VNF instance allocation and traffic routing in optical networks ; and (3) motivated by Game Theory, we model the VNF instance allocation and routing problem over a shared physical substrate network as a two-person, non-zero-sum, pure-strategy game in normal form.

The rest of the paper is organized as follows. In Section II, we categorize related literature on NFV and point out their similarities and differences from our work. In Section III, we provide formal problem statement on NFV realizing end-to-end requests and demonstrate its NP-completeness. We propose a mixed-integer program to compute the optimal solution for the problem in Section IV. We re-model the problem as a two-person pure strategy game and propose solution approaches in Section V. Experiment settings and computation results are presented in Section VI.

This paper is an expanded version of our previous work [20].

II. Literature Review

NFV is still an emerging technology, whose realization through optical networks starts drawing researchers' attention. The optical network architecture considered in this paper consists of fibers connecting optical add-drop multiplexers and optical cross-connect, where incoming requests are routed over the optical (physical) network [21]. Recently, lots of research concentrates on the *routing and spectrum assignment* problems with static and dynamic routing schemes in elastic optical networks [22],[23]. Most of them consider multiple *end-to-end requests* and determine optical flows between the two ends of requests. In this paper, we also consider the realization of end-to-end requests and design single lightpath based routing scheme with NFV.

Major network operators and standard setting organizations are currently leading the development of NFV and presenting it in technical reports and white papers [8], [24]. Compared with cloud infrastructures using proprietary networking hardware, capital expenditures and operating expenses (OPEX) of NFVs are greatly reduced as migrating and scaling-up and down of workloads do not require the deployment of specialized hardware [10]. An important benefit of such flexibility is that network operators can provide more flexible and operationally efficient NFVs to end-users [25], [26], which is beneficial for both operators and end-users [16].

To adapt to the evolving traffic and NF requirements, several new architectures and systems were introduced, for example, the Split/Merge architecture and FreeFlow system in [27], OpenNF framework in [19], StatelessNF architecture in [28], and OpenBox architecture in [29]. Rather than concentrating on the architecture design for NFV control and data planes, our focus is on game-theoretic network virtualization techniques which orchestrate and facilitate the networking and NF services required by customers on the physical substrate network.

Another line of investigation is the routing schemes for end-to-end requests with VNF-enabled nodes. Gushchin *et al.* [30] presented an SDN-based routing approach in networks with middleboxes, which considered the resource constraints on middleboxes and generated routing rules bounded by a polynomial function of the number of nodes, links, and traffic classes for SDN switches. Gupta *et al.* [31] provided the mathematical formulation of the routing scheme for end-to-end demands where VNFs should be deployed in a specific sequence called the service-chain.

The other direction of related research on NFV is the deployment of NFs which selects physical nodes from VNF-enabled nodes for NF deployment. Basta *et al.* [32] discussed the function placement problem on core gateways in LTE mobile networks considering load and delay on the transport networks. Moens and Turck [33] proposed a model where network services are fulfilled by a hybrid physical network composed of dedicated network hardware as well as VNF-enabled physical nodes. Xia *et al.* [34] studied the minimal cost on-demand VNFs deployment which aimed to reduce the number of opto-electro-optical conversions involved in packet/optical data centers. With given traffic flows, Mohammadkhan *et al.* [35] determined the placement of VNF instances. Bari *et al.* [36] studied the VNF orchestration problem which provisioned allocated VNFs on physical nodes to optimize OPEX. Bouet *et al.* [37] considered the placement of a special type of VNF, virtual deep packet inspection, on physical nodes through traffic flow monitoring.

Yoshida *et al.* [38] introduced an algorithm for NFV deployment scheduling. Different from prior research on NFV allocation, the end-to-end demands with VNF requests defined in this paper not only require NFs, but also a specific number of instances of such functions.

Unlike the above works which solely examined either the VNF deployment (selecting physical nodes to deploy NFs) under different settings or the routing for end-to-end requests (with known locations of deployed NFs), in this paper, we consider an integrated approach to deploy multiple VNF instances and realize end-to-end demands simultaneously, whose idea was originated from our prior work [20]. In addition to the framework introduced in [20], to address the competition between VNF instance allocation and end-to-end demand realization over shared physical resources, we show in this paper that the VNF instance allocation and end-to-end demand realization can be modeled as a pure-strategy two-player game, which has not been discussed before. We also investigate the solution approaches based on iterative weakly dominated elimination motivated by Game Theory. In the rest of paper, "VNF instance allocation" and "VNF allocation" will be used interchangeably.

III. Use Cases and Problem Statement

In this section, we first review the use cases for VNFs discussed in [9]. Then, we provide problem definitions and prove their computational complexity.

A. Use Cases: VNF Mapping

Use cases for NFV [9] include: (1) VNF as a Service, which configures the set of VNF instances made available by service providers; (2) Virtual Network Platform as a Service, with which dedicated Access Point Names serve as IP level entry points to private corporate networks of enterprises; and (3) end-to-end services supported by network service providers which involve cross-administrative-boundary operation, interworking, and migration to/from physical NF implementations. Correspondingly, NF virtualization can be categorized into three types: (1) Network-Level VNFs: an entire virtual network is associated with a set of VNFs. For example, all end requests in an enterprise virtual private networks are required to pass through NFs such as authentication and firewall. (2) Node-Level VNFs: to fulfill certain requests, demands are routed through virtual nodes associated with a subset of VNFs. For instance, demands requiring user registration or/and load balancing would be routed through either single or multiple nodes with such functions' instances. (3) Link-Level VNFs: similar to (2), but VNFs are associated with virtual links instead of nodes. Note that the difference between (2) and (3) are that for any demand in (3), the required NFs by a demand in (3) can have their instances reside on any physical node along its physical route (which is a simple path) of the demand; and instances of the required NFs by a demand in (2) are deployed at any physical node residing on a physical simple walk which starts and ends at the physical node mapped by the virtual node.

In this paper, we focus on the third use case and present *network function virtualization realizing end-to-end requests* (NFV-RR) as follows: given a physical substrate network provided by a network operator, end-user's requests may be estimated based on their service contracts. Service providers can then instantiate and deploy required NFs at runtime onto VNF-enabled physical nodes to achieve user's end-to-end requests without the need of proprietary networking hardware. Our goal is to minimize the cost to deploy NF instances and the cost to utilize physical resources on both nodes and links, simultaneously.

TABLE I
Notations for Parameters and Variables

Parameter	Description
$G_P(V_P, E_P)$	Physical substrate network (i.e., optical network connecting data centers). Node set V_P and edge set E_P;
i, j, s, t	Node indices $i, j, s, t \in V_P$;
e	Physical link index, $e \in E_P$;
C_e	Capacity of physical link e, $e \in E_P$;
C_i	Computational capacity of physical node i, $i \in V_P$;
D	End-user request set, i.e., $D = \{(s,t) : s, t \in V_P\}$;
$\mathcal{F}$	Network function set, with network function $f \in \mathcal{F}$;
F_{st}	Network function required to fulfill the request between $(s,t), (s,t) \in D$, i.e., $F_{st} = \{(f, m_{st}^f) : f \in \mathcal{F}, m_{st}^f \in \mathbb{Z}^+\}$, where m_{st}^f is the required instances of function f;
d_{st}^c	Required bandwidth for request between $(s,t), (s,t) \in D$;
d_{st}^ℓ	Required computational resources on node(s) to fulfill the request between $(s,t), (s,t) \in D$;
d_{st}	End-to-end request between (s,t) denoted as a triplet $d_{st} = \{F_{st}, d_{st}^c, d_{st}^\ell\}, (s,t) \in D$;
η_i^f	Required computational resources for an instance of network function f deployed at node i;
p_{st}	Physical path for request d_{st}, $p_{st} \subseteq G_P$, $(s,t) \in D$;
c_i^f	Unit cost to deploy instances of network function f on node i, $f \in \mathcal{F}$ and $i \in V_P$;
c_{ij}	Unit cost to utilize physical resources on edge $(i,j), (i,j) \in E_P$;
c_i	Unit cost for end-to-end request to utilize physical resources on node i, $i \in V_P$
Variable	**Description**
x_i^{st}	Binary variable indicates whether demand d_{st} is routed through node i. If yes, $x_i^{st} = 1$; otherwise, $x_i^{st} = 0$, $(s,t) \in D$ and $i \in V_P$;
y_{ij}^{st}	Binary variable indicates whether demand d_{st} is routed through (i,j). If yes, $y_{ij}^{st} = 1$; otherwise, $y_{ij}^{st} = 0$, $(s,t) \in D$ and $(i,j) \in E_P$;
ρ_{st}^{fi}	Binary variable indicates whether function f is deployed at node i for demand d_{st}
n_i^f	Number of f's instances deployed at physical node i with $f \in F$ and $i \in V_P$;
ς_{st}^{fi}	Number of f's instances deployed at physical node i for demand d_{st}.

B. Network Function Virtualization for End-to-End Requests

Based on the setting of NFV-RR, we introduce the notations for parameters and variables used in this paper in Table I and provide formal definitions for NFV-RR and related problems.

Definition 1: Given G_P and end-to-end requests D. An *end-to-end virtual function request*, denoted as $F_{st} = \{(f, m_{st}^f) : f \in \mathcal{F}, m_{st}^f \in \mathbb{Z}^+\}$, is the NF requirement of the request between nodes s and t, where m_{st}^f denotes the required number of instances of NF f for the request between s and t, $(s,t) \in D$.

An *end-to-end request* d_{st}, a triplet $d_{st} = \{F_{st}, d_{st}^c, d_{st}^\ell\}$, integrates the NF request, its required bandwidth (d_{st}^c), and computational resources (d_{st}^ℓ). In this paper, we consider that the

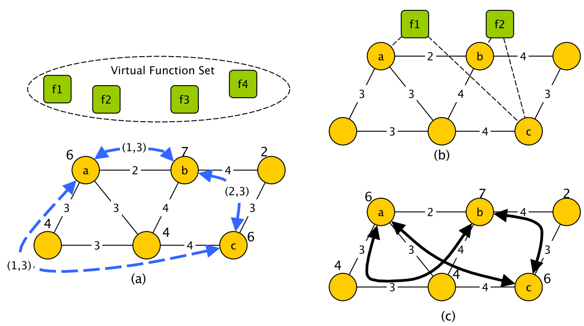

Fig. 2. An instance of NFV for end-to-end requests.

end-to-end request is only routed through a single path connecting the two end nodes of a request.

Let n_i^f be the number of instances of NF f deployed at physical node i, and η_i^f be the computational resources required to fulfill an instance of NF f at node i. Hence, $\eta_i^f n_i^f$ represents the total computational resources required to fulfill all n_i^f instances of function f. We now define NF allocation and the NFV-RR problem.

Definition 2: NF instance allocation is to deploy n_i^f instances of NF $f \in \mathcal{F}$ on physical node $i \in V_P$ and determine $\eta_i^f n_i^f$, the total computational resources required to fulfill all n_i^f instances of function f.

Definition 3: Given a physical substrate network $G_P = (V_P, E_P)$, its link capacity C_e, node capacity C_i, and demand $d_{st} = \{F_{st}, d_{st}^c, d_{st}^\ell\}$, $F_{st} = \{(f, m_{st}^f) : f \in \mathcal{F}, m_{st}^f \in \mathbb{Z}^+\}$, the problem of *NFV-RR* is to determine the placement of NFs' instances satisfying the following conditions.

1) Each demand $d_{st}, (s,t) \in D$, is realized through a physical route $p_{st}, p_{st} \subseteq G_P$;
2) the route of d_{st}, p_{st}, should pass through physical nodes deployed with the required types and number of instances of NFs (specified in F_{st}). That is, $\sum_{i \in p_{st},(s,t)\in D} m_{st}^f \leq n_i^f$, $f \in \mathcal{F}$ and $i \in V_P$;
3) the cumulative bandwidth request on each physical link $e \in E_P$ should not exceed its capacity, i.e., $\sum_{e \in p_{st},(s,t)\in D} d_{st}^c \leq C_e$; and
4) the cumulative computational resources required to process NFs and flows routed through each physical node i should not exceed its capacity, i.e., $\sum_{i \in p_{st},(s,t)\in D} d_{st}^\ell + \sum_{f \in \mathcal{F}} \eta_i^f n_i^f \leq C_i$.

We use Fig. 2 to illustrate an instance of VNF allocation realizing end-to-end requests. Fig. 2(a) shows an NF set $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$, a physical network with node capacity (computation) and link capacity (communication), end-to-end requests (illustrated in dashed blue line), and their computation and communication resources (the two values on the blue line). We assume that all demands require the processing of NFs $\{f_1, f_2\}$. Fig. 2(b) presents feasible NF instance allocation where an instance of function f_1 is deployed to physical nodes a and c, and an instance of f_2 is assigned to nodes b and c. Fig. 2(c) illustrates a virtual link mapping which satisfies VNF requests and does not require extra physical resources.

Theorem 1: The NFV-RR problem is NP-complete.

Based on the problem definition, the two-commodity integer flow problem [39] is a special instance of the NFV problem where NF and node capacity are not considered. Since the two-commodity integer flow problem is NP-complete, our claim holds.

IV. Mathematical Formulation

In this section, we present a mixed-integer program aiming at optimizing NFV-RR problem which satisfies all conditions given in Definition 3 as well as providing the techno-economic analysis for VNF orchestration and management. All variables used in the mathematical formulations are listed in Table I.

A. Formulations for NFV-RR

First, as discussed in [40], each end-to-end request is realized through a physical route generated by flow conservation constraints as follows.

$$\sum_{(i,j)\in E_P} y_{ij}^{st} - \sum_{(j,i)\in E_P} y_{ji}^{st} = \begin{cases} 1, & \text{if } i = s, \\ -1, & \text{if } i = t, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$
$$(s,t) \in D, i \in V_P.$$

Different from [40], we introduce in our formulation node capacity and NF allocation which determines (1) the consumption of physical node/link resources for end-to-end requests, and (2) whether physical routes $p_{st}, (s,t) \in D$, travel through physical node i or not. For (2), we introduce an auxiliary variable x_i^{st} in the following proposition.

Proposition 1: The physical route p_{st} of d_{st} visits physical node i if and only if the following node-based routing constraints lead variable x_i^{st} to 1; otherwise, $x_i^{st} = 0$.

$$x_i^{st} \leq \sum_{(i,j)\in E_P} (y_{ij}^{st} + y_{ji}^{st}), \qquad (s,t) \in D, i \in V_P \quad (2)$$

$$x_i^{st} \geq y_{ij}^{st} + y_{ji}^{st}, \quad (s,t) \in D, i \in V_P, (i,j) \in E_P. \quad (3)$$

If p_{st} is not routed through node i, $\sum_{(i,j)\in E_P} (y_{ij}^{st} + y_{ji}^{st}) = 0$, which forces $x_i^{st} = 0$ in constraint (2). Otherwise, if node i is visited by p_{st}, one of y_{ij}^{st} equals one in constraint (3), which forces $x_i^{st} = 1$. Similarly, if $x_i^{st} = 0$, it forces $y_{ij}^{st} + y_{ji}^{st} = 0$ in constraint (3).

Note here that constraints (2) and (3) build the connection between indicator x_i^{st} and route indicator y_{ij}^{st}.

We propose an assignment-based method and introduce an auxiliary variable ς_{st}^{fi} which represents the number of f's instances deployed at node i required by d_{st}, $(s,t) \in D$. The corresponding constraints are introduced as follows.

Proposition 2: Constraints (4) and (5) determine the allocation of NFs' instances which fulfills the request of NF F_{st}

$$m_{st}^f = \sum_{i\in V_P} \varsigma_{st}^{fi} x_i^{st}, \quad f \in \mathcal{F}, (s,t) \in D \quad (4)$$

$$n_i^f = \sum_{(s,t)\in D} \varsigma_{st}^{fi}, \quad f \in \mathcal{F}, i \in V_P. \quad (5)$$

In constraint (4), $\varsigma_{st}^{fi} x_i^{st} \neq 0$ means that demand d_{st}'s route p_{st} passes through physical node i, and instances of NF f required by d_{st} are deployed at i. Constraint (5) cumulates all instances of NF f placed at i for all requests in D. Note that we can reformulate the nonlinear constraint (4) to linearize it without introducing extra variables.

Proposition 3: Constraints (6) and (7) are equivalent to constraint (4)

$$\varsigma_{st}^{fi} \leq m_{st}^f x_i^{st}, \quad i \in V_P, f \in \mathcal{F}, (s,t) \in D \tag{6}$$

$$m_{st}^f = \sum_{i \in V_P} \varsigma_{st}^{fi}, \quad (s,t) \in D, f \in \mathcal{F}. \tag{7}$$

Proof: We prove this proposition by demonstrating that these two sets of constraints provide the same feasible regions for end-to-end request (s,t)'s NF instance placement. There are two conditions for $\varsigma_{st}^{fi} x_i^{st}$: if $x_i^{st} = 1$, $\varsigma_{st}^{fi} x_i^{st} = \varsigma_{st}^{fi}$; otherwise, $\varsigma_{st}^{fi} x_i^{st} = 0$. With constraint (6), when $x_i^{st} = 0$, $\varsigma_{st}^{fi} = 0$, which forces $\varsigma_{st}^{fi} x_i^{st} = 0$; otherwise, ς_{st}^{fi} is not forced to be 0, and the summation of ς_{st}^{fi} for all $i \in V_P$ equals m_{st}^f. Thus, the proposition holds. ■

Here we assume that NF f required by d_{st} can only be deployed on one physical node (unsplittable/atomic). So, we let $\rho_{st}^{fi} \in \{0, 1\}$ indicate whether function f is deployed at node i for demand F_{st} with $f \in \mathcal{F}, i \in V_P (s,t) \in D$. We add the following constraints

$$\sum_{i \in V_P} \rho_{st}^{fi} = 1, \qquad f \in \mathcal{F}, (s,t) \in D \tag{8}$$

$$\varsigma_{st}^{fi} \leq m_{st}^f \rho_{st}^{fi}, \quad f \in \mathcal{F}, (s,t) \in D, i \in V_P \tag{9}$$

to restrict that only one physical node can be selected to realize NF f required by d_{st}.

Next, we present a mixed integer linear program (MILP) for the NFV-RR problem with single end-to-end physical route for each d_{st}. The objective is to minimize the total cost realizing VNF instance deployment and routing for end-to-end requests

$$\min \sum_{f \in F} \sum_{i \in V_P} c_i^f n_i^f + \sum_{(s,t) \in D} \left(\sum_{(i,j) \in E_P} c_{ij} y_{ij}^{st} d_{st}^c + \sum_{i \in V_P} c_i x_i^{st} d_{st}^\ell \right)$$

s.t. Constraints (1)–(3) and (5)–(7)

$$\sum_{(s,t) \in D} y_{ij}^{st} d_{st}^c \leq C_e, \quad e = (i,j) \in E_P \tag{10}$$

$$\sum_{(s,t) \in D} x_i^{st} d_{st}^\ell + \sum_{f \in \mathcal{F}} \eta_i^f n_i^f \leq C_i, \quad i \in V_P \tag{11}$$

$$y_{ij}^{st}, x_i^{st} \in \{0,1\}, \quad (s,t) \in D, i \in V_P, (i,j) \in E_P$$

$$n_i^f, \varsigma_{st}^{fi} \in \mathbb{Z}^+, \quad (s,t) \in D, i,j \in V_P, f \in \mathcal{F}. \tag{12}$$

Here, the objective is to minimize the cost to deploy NF instances and the cost to utilize physical resources on both nodes and links, simultaneously. Constraints (10) and (11) provide capacity limitation on physical links and nodes introduced as conditions (3) and (4) in Definition 3. Constraint (10) restricts the cumulative flow routed through a physical link to be less than or equal to its capacity. Constraint (11) guarantees the computational resources required to carry out the NFs and flows to be less than or equal to the capacity of physical nodes.

V. Solution Approaches

In this section, we first point out that the potential physical resource competition between VNF instance allocation and end-to-end demand realization can be modeled as a two-player pure-strategy game, where VNF instance allocation and routing are the two players. Then, we propose an algorithm for the NFV-RR problem which iteratively eliminates weakly dominated strategies to reduce the action space of each player, which can eventually help achieve the Nash Equilibrium point (NEP) of the game. In the following discussion, we let $\mathcal{P}_{st}$ be demand (s,t)'s collection of physical routes and Λ be a set of triplets containing physical node, its supported VNFs, and the number of VNF instances deployed at the node.

A. Game Theory Insights of NFV-RR

We consider a two-player non-zero-sum game [41] where two players are an NF placer and end-users with end-to-end requests. We denote them as players 1 and 2.

The action space for player 1, denoted as S_1, is the instances of NFs placed on selected physical nodes by player 1 to provide services to end-to-end requests, i.e., $S_1 = \{s_1\}$ with $s_1 = \{(f, n_i^f) : f \in \mathcal{F}, i \in V_P\}$. The action space for player 2, denoted as $S_2 = \{s_2\}$ with $s_2 = \{p_{st}, p_{st} \in \mathcal{P}_{st}\}$, is the single-path routes selected for each end-to-end request. Note here that both these two actions are bounded by the physical node and link capacity limitations. Here, M is a large constant ($\rightarrow +\infty$).

The utility function of player 1 is

$$u_1(s_1, s_2) = \sum_{f \in \mathcal{F}} \sum_{i \in V_P} c_i n_i^f + M \sum_{i \in V_P} \max \left\{ 0, \sum_{f \in \mathcal{F}} \eta_i^f n_i^f + \sum_{(s,t) \in D} d_{st}^\ell \delta_p^i g_p - C_i \right\} + M \sum_{e \in E_P} \max \left\{ 0, \sum_{(s,t) \in D} d_{st}^c \delta_p^e g_p - C_e \right\}, \tag{13}$$

and the utility function for player 2 is

$$u_2(s_1, s_2) = \sum_{p \in \mathcal{P}_{st}} g_p \left\{ \sum_{e \in E_P} c_e \delta_p^e d_{st}^c + \sum_{i \in V_P} c_i \delta_p^i d_{st}^\ell \right\} + M \sum_{i \in V_P} \max \left\{ 0, \sum_{f \in \mathcal{F}} \eta_i^f n_i^f + \sum_{(s,t) \in D} d_{st}^\ell \delta_p^i g_p - C_i \right\} + M \sum_{e \in E_P} \max \left\{ 0, \sum_{(s,t) \in D} d_{st}^c \delta_p^e g_p - C_e \right\}, \tag{14}$$

where δ_p^i and δ_p^e are new variables indicating whether physical node i and link e are on a demand route p or not. If yes, $\delta_p^i = 1$ and $\delta_p^e = 1$; otherwise, $\delta_p^i = 0$ and $\delta_p^e = 0$. g_p indicates whether a path p between (s,t) is selected or not. If yes, $g_p = 1$; otherwise, $g_p = 0$. Both utility functions are composed of two parts. The first part calibrates the cost to realize players' actions. For player 1, $\sum_{f\in\mathcal{F}}\sum_{i\in V_P} c_i n_i^f$ is the total cost to allocate instances of NFs; for player 2, $\sum_{p\in\mathcal{P}_{st}} g_p\{\sum_{e\in E_P} c_e\delta_p^e d^c st + \sum_{i\in V_P} c_i\delta_p^i d_{st}^\ell\}$ is the total cost to realize all end-to-end demands. And the second part provides penalties for actions which violate physical node or link capacities. If any node or link capacity is violated, then, a penalty M will be imposed on it.

We name this game the *NFV placement and routing game* denoted as $(\{1,2\},(S_1,S_2),u)$. The relationship between the NFV placement and routing game and NFV-RR is that (1) given any feasible solution of NFV-RR, we have $\omega(s_1,s_2) = 0$, where $\omega(s_1,s_2) = M\sum_{i\in V_P}\max\{0,\sum_{f\in\mathcal{F}}\eta_i^f n_i^f + \sum_{(s,t)\in D} d_{st}^\ell\delta_p^i g_p - C_i\} + M\sum_{e\in E_P}\max\{0,\sum_{(s,t)\in D} d_{st}^c\delta_p^e g_p - C_e\}$; (2) with a given action for the NF placement, if a routing action leads to $+\infty$ for either $u_1(s_1,s_2)$ or $u_2(s_1,s_2)$, then, the NF placement and the routing action correspond to an infeasible solution in NFV-RR. Next, we further explore the existence of Nash Equilibrium of the NFV placement and routing game and demonstrate its relation with the optimal solution of NFV-RR.

Proposition 4: Both S_1 and S_2 are in finite strategy space.

Proof: The total number of actions in S_1 is bounded by $|V_P|\sum_{f\in\mathcal{F}}\sum_{(s,t)\in D}\{m_{st}^f\}$, and the total number of actions in S_2 is less than $|D|(2^{|V_P|}-2)$, where $(2^{|V_P|}-2)$ is the upper bound for the total number of possible simple paths in G_P. Hence, even though the strategy space is large, it is still finite. ■

Directly following [41], the NFV placement and routing game is with finite number of players and action profiles. Hence, the Nash Equilibrium holds.

Proposition 5: The Nash Equilibrium exists for the NFV placement and routing game with an NEP (s_1^*,s_2^*) satisfying

$$u_i(s_1^*,s_2^*) \le u_i(s_1,s_2^*) \tag{15}$$

$$u_i(s_1^*,s_2^*) \le u_i(s_1^*,s_2) \tag{16}$$

with i = 1 and 2.

Theorem 2: The optimal solution of NFV-RR, denoted as $(\hat{s}_1,\hat{s}_2)$, is an NEP for the NFV placement and routing game.

Proof: We prove this claim by contradiction. Given (s_1^*,s_2^*) as the NEP of the NFV placement and routing game and $(\hat{s}_1,\hat{s}_2)$ as the optimal solution of NFV-RR. We assume that the optimal solution of NFV-RR is not the NEP, then, one of following three conditions holds: (1) $s_1^* = \hat{s}_1$, $s_2^* \neq \hat{s}_2$; (2) $s_2^* = \hat{s}_2$, $s_1^* \neq \hat{s}_1$; and (3) $s_1^* \neq \hat{s}_1$, $s_2^* \neq \hat{s}_2$. With the fact that $(\hat{s}_1,\hat{s}_2)$ is the optimal solution of the NFV-RR, $(\hat{s}_1,\hat{s}_2)$ is feasible and with fixed $\hat{s}_1$ as $u_1(\hat{s}_1,\hat{s}_2) \le u_1(\hat{s}_1,s_2)$; and same claim holds for u_2 with fixed $\hat{s}_2$.

For condition (1), we have $u_1(\hat{s}_1,\hat{s}_2) = u_1(s_1^*,\hat{s}_2) \le u_1(s_1^*,s_2^*)$. For condition (2), we have $u_2(\hat{s}_1,\hat{s}_2) = u_2(\hat{s}_1,s_2^*) \le u_2(s_1^*,s_2^*)$. For condition (3), with the assumption of the optimality, $u_1(\hat{s}_1,\hat{s}_2) \le u_1(\hat{s}_1,s_2)$ and $u_2(\hat{s}_1,\hat{s}_2) \le u_2(s_1,\hat{s}_2)$ hold. Hence, for all three conditions, either both or none of $(s_1^*,s_2^*),(\hat{s}_1,\hat{s}_2)$ are NEPs, which contradicts our assumption. ■

Next, we demonstrate that Nash Equilibrium conditions for the NFV placement and routing game are reached when both (weakly) dominated strategy and feasible condition on network node and link capacity constraints hold.

To provide feasible solutions for NFV placement and routing game, we introduce the *(weakly) dominant feasible strategy* for NFV placement and routing game: for strategy $u_i(s_i'',s_{-i})$, there exists a strategy $u_i(s_i',s_{-1})$, where

$$u_i(s_i',s_{-i}) \le u_i(s_i'',s_{-i}) \text{ for feasible strategies } (s_i',s_{-i}) \text{ and } (s_i'',s_{-i}), s_i', s_i'' \in S_i, s_{-i} \in S_{-i}, i = 1,2. \tag{17}$$

Here S_{-i} represents a vector of actions for all players except i. Hence, the following conclusion holds.

Proposition 6: The (weakly) dominant condition holds in the NFV placement and routing game for all feasible solutions of NFV-RR.

Proof: We prove this claim based on the definition of weakly dominant condition (17). Given an arbitrary $\hat{s}_2 \in S_2$, one of the two conditions holds: (1) no solution exists for all $s_1 \in S_1$ such that $u_i(s_1,\hat{s}_2) \le M$, i.e., no feasible solution exists for NFV-RR; (2) feasible solutions exist, denoted as $S_1(\hat{s}_2)$, for all feasible $s_1 \in S_1$. With Condition (2), we could obtain $s_1' = \{(f,\bar{n}_i^f)\} = \arg_{s_1\in S_1}\min u_1(s_1,\hat{s}_2)$. Thus, we get $u_1(s_1',\hat{s}_2) \le u_1(s_1'',\hat{s}_2)$ for any $s'' \in S_1(\hat{s}_2)$. Similar conclusion holds for $u_2(s_1,\hat{s}_2)$. The proposition holds. ■

The VNF placement and routing game introduced in this study is a two-player pure-strategy game with finite action spaces. Pure strategy Nash Equilibrium, in a normal-form game [42] with constant number of players, can be obtained in time polynomial in the number of strategies/actions and the number of players [42], where a normal-form game has the constant number of players, each with a finite set of pure strategies, and the utility function of each player is non-negative if each player selects an action in their action space. Without loss of generality, we assume that each player is with $\mathcal{N}$ actions, thus the two-player pure-strategy game can be solved in $\mathcal{O}(\mathcal{N}^2)$ time. Hence, the VNF placement and routing game can also obtain its pure Nash Equilibrium in time polynomial in the number of players and the number of their actions. Note here that the computational complexity of this problem is still based on the parameters of a given network (node and link numbers) which determine the action space of each player. Since the action space of player 2 is the $s-t$ path set of all virtual links, the action space may have exponential number of strategies/actions.

Next, we propose an algorithm for NFV-RR through iteratively eliminating dominated strategies as follows.

B. Algorithm for NFV-RR

Iterative weakly dominated elimination is an approach in Game Theory which reduces the feasible action spaces while leaving the NEPs untouched in the updated action spaces [43], [44]. We introduced the weakly dominated feasible strategy for

Algorithm 1: Integrated network function allocation and end-to-end request realization for NFV-RR

Input: Given $G_P = (V_P, E_P)$, its link and node capacities C_e, C_i, $e \in E_P$, $i \in V_P$; demand $D = \{(s,t)\}$ with its NF request $\{F_{st}, (s,t) \in D\}$, NF set $\mathcal{F}$, and initial path sets $\mathcal{P}_{st}$ for $(s,t) \in D$

Output: Network function placement and end-to-end request routes

1 Let $P_D^1 = \varnothing$, $\Lambda^1 = \varnothing$, q=1

2 Generate initial path set $\mathcal{P}^q = \{\mathcal{P}_{st}, (s,t) \in D\}$

3 Pick P_D^q, $P_D^q = \{p_{st}, p_{st} \in \mathcal{P}_{st}^q, (s,t) \in D\}$

4 **if** $\sum_{e \in p_{st}, p_{st} \in P_D^q} d_{st}^c > C_e$ *or* $\sum_{i \in p_{st}, p_{st} \in P_D^q} d_{st}^\ell > C_i$ **then**

5     $\mathcal{P}^{q+1} = \mathcal{P}^q \backslash P_D^q, q = q+1, \Lambda^q = \Lambda^{q-1}$. Go to Step 3

6 Calculate the residual capacity on physical nodes: $\widetilde{C}_i = C_i - \sum_{i \in p_{st}, p_{st} \in P_D^q} d_{st}^\ell$

7 **if** *no feasible solution exists when solving* $\min_{n_i^f} c_i^f n_i^f$, *s.t.* $\{\sum_{f \in \mathcal{F}} \eta_i^f n_i^f \leqslant \widetilde{C}_i, \sum_{i \in V_P} \varsigma_{st}^{fi} = m_{st}^f, n_i^f = \sum_{(s,t) \in D} \varsigma_{st}^{fi}\}$ **then**

8   Go to Step 5

9 **else**

10     $q = q+1$; $P_D^q = P_D^{q-1}$; $\Lambda^q = \Lambda^{q-1} \bigcup \{(i, f, n_i^f) : i \in V_P, f \in \mathcal{F}\}$

11   Residual capacity: $\widetilde{C}_i = C_i - \sum_{f \in \mathcal{F}} \eta_i^f n_i^f$ for $i \in V_P$

12   Find the solution for $\min_{p_{st}} \sum_{e \in P_{st}} \sum_{P_{st} \in P_D^q \cup P_{st}^q} \sum_{(s,t) \in D} C_e$, s.t. $\{\sum_{e \in p_{st}, p_{st} \in P_D^q} d_{st}^c \leqslant C_e, \sum_{i \in p_{st}, p_{st} \in P_D^q} d_{st}^\ell \leqslant \widetilde{C}_i\}$. If the optimal solution exists, update P_D^q with the solution of P_{st}

13   **if** $P_D^q \neq P_D^{q-1}$ *or* $\Lambda^q \neq \Lambda^{q-1}$ **then**

14   Go to Step 6

the VNF placement and routing game in inequality (17) which evaluates the dominated strategy of each player with its utility function. We demonstrated that the optimal solution of NFV-RR is an NEP. Following iterative weakly dominated elimination, we introduce Algorithm 1 for the NFV-RR game which selects physical routes for each demand, deploys instances of NFs and moves toward smaller total costs for physical route selection and NF instance allocation.

Given a cloud system, its physical resources on links and nodes, and demands and required NFs. For each demand (s,t), we first generate a set of paths, $\mathcal{P}_{st}$, in the physical network using k-shortest path algorithm [45]. In Algorithm 1, we initialize the set of paths ($\mathcal{P}^1$) in Step 2, which provides the potential solution space for routes. Among all available paths for each demand, a single path is selected for each demand and stored in $\mathcal{P}_D^1$. Steps 3–5 are repeated until a feasible solution is found, which satisfies both bandwidth and computational constraints for demands. If the selected set of paths is not feasible, it is removed from the solution space. Based on the solution found, Steps 6–7 determine if there is enough resources on physical nodes which fulfill the required NFs. Again, if no feasible solution exists, the path set is removed from the solution space; otherwise, Step 7 produces a solution $(\mathcal{P}_D^q, \Lambda^q)$ which satisfies both node and link capacity constraints.

After updating the residual capacity in Step 11, we fix the NF instances deployed at physical nodes and now try to find alternative routes for demands which consume less physical link capacity through a second-stage optimization in Step 12. If the optimal solution exists, a path set P_{st} for all demands is selected and used to update $\mathcal{P}_D^q$. In Step 13, if a new solution exists for either $\mathcal{P}_D^q$ or Λ^q, the algorithm would continue with new iterations of path selection and optimization until no better solution exists. In summary, the algorithm is composed of three fundamental blocks: feasible path selection (Steps 3–5), optimization for NF instance deployment (Steps 6–7), and re-optimization to minimize capacity consumption on physical links (Steps 11–12). Note here that Algorithm 1 leads to the optimal solution of NFV-RR if the solution exists and $\mathcal{P}_{st}$ contains all possible paths between (s,t), though enumerating all paths between two end nodes in a network is NP-hard.

Next, we analyze the complexity of the proposed algorithm which includes two subproblems. The first one is the NF instance allocation in Step 7, and the second one is the end-to-end routing selection in Step 12. The first subproblem is a two-layer facility location problem which is NP-hard, but not in the strong sense. For this subproblem, there exists an 1.488-approximation algorithm [46], and a greedy algorithm can solve this problem with reasonably good performance. The second subproblem is a capacitated multicommodity flow problem, which is NP-complete, which also has a fully polynomial approximation algorithm [47]. As considered in the algorithm, given k paths for each $(s,t) \in D$ as the input, $O(|D|^k)$ combinations in total would be evaluated to determine the selected route for each end-to-end demand, where $|D|$ is the total number of end-to-end pairs. The algorithm is executed iteratively between these two subproblems till an optimal solution is found.

We wish to note that instead of decomposing the NFV-RR problem into two subproblems and solving them sequentially, the proposed algorithm is actually taking the information acquired from player 1's strategies (NFV allocation) to improve player 2's strategy (routing) iteratively to cut out weakly dominated strategies, and vice versa.

VI. Experimental Evaluations

In this section, we first introduce our experiment design and then demonstrate the computational results. We select NSF network, denoted as "NSF" and illustrated in Fig. 3, as the physical infrastructure [48] which has 14 nodes and 21 edges. We consider six end-to-end demands whose corresponding physical node pairs are listed in Table II. We list in Table III the means and variances of the required computational resources and bandwidth for end-to-end demands, computational resources consumed by NFs, and the link and node capacities. We consider three types of VNFs indexed as α, β, γ and two VNF sets $\mathcal{F}_1 = \{\alpha\}$ and $\mathcal{F}_3 = \{\alpha, \beta, \gamma\}$. We then introduce two types of

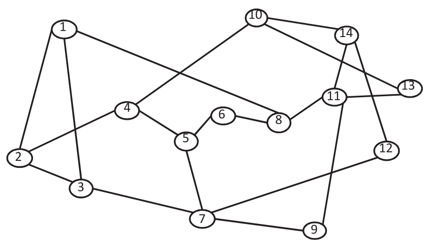

Fig. 3. NSF network.

TABLE II
TESTING SCENARIOS FOR END-TO-END REQUESTS WITH NF DEMANDS

Demand index	1	2	3	4	5	6
Mapped node pair	(1,2)	(1,8)	(2,7)	(7,12)	(8,14)	(13,14)

TABLE III
PARAMETERS FOR TESTING SCENARIOS

Parameters	Mean	Variance
Demand (resource)	10	5
Demand (bandwidth)	15	5
Network function (resource)	10	5
Capacity (node)	40	10
Capacity (link)	25	10

TABLE IV
VNF REQUESTS BY END-TO-END DEMANDS: NON-UNIFORM CASE

Demand index	1	2	3	4	5	6
VNF requests	α	β	γ	β, γ	α, γ	α, β, γ

VNF requests: the non-uniform and uniform. In the non-uniform request, as shown in Table IV, each demand (with different demand indices) requires different VNFs in $\mathcal{F}_3$; for example, demand 2 requires an instance of VNF β and demand 4 requires an instance of VNFs β and γ. In the uniform request, all demands require the same number of instances of VNF α in $\mathcal{F}_1$; for example, demands 1 to 6 all require 3 instances of VNF α.

A. *Impacts of NFV to Existing Network Services*

To illustrate how NFV and its resource consumption impact the realization of end-to-end demands, we try to find the physical routes for each end-to-end demand with and without deploying VNF instances. We aim to show that without the NFV, the problem of finding the routes for end-to-end requests is equivalent to finding the shortest path between each pair of end nodes if there is enough capacity on the physical nodes and links. Also, if the node and link capacities are not enough to support the demands, the problem is equivalent to the multicommodity flow problem.

Since NFV deployment would require additional and dedicated physical resources, we would like to observe that the generated routes for end-to-end demands may vary unless the physical resources are sufficient to support all demands. We report the optimal solutions generated by the MILP formulation for both cases targeting to minimize the total costs for end-to-end demand routing and VNF instance allocation (applicable to the scenario with NFV).

TABLE V
OPTIMAL ROUTING AND NF ALLOCATION IN UNCAPACITATED NETWORK AND WITH UNIT DEMANDS

Index	No NFV Routes	NFV Routes	NFV NF-Node
1	1–2	1–2	1
2	1–8	1–8	1
3	2–3–7	2–3–7	2
4	7–12	7–12	7
5	8–11–14	8–6–5–4–10–14	8
6	13–11–14	13–11–14	13

TABLE VI
NODE CAPACITY AND RESOURCE CONSUMPTION BY END-TO-END REQUESTS AND NETWORK FUNCTION ALLOCATION

Node	1	2	3	4	5	6	7
Capacity	24	22	33	32	22	21	36
NF (S2)	–	–	–	6	–	–	9
Rqt (S2)	22	20	12	20	20	0	27
Node	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Capacity	38	35	27	26	31	38	29
NF (S2)	18	–	–	–	21	27	9
Rqt (S2)	18	0	0	14	7	6	14

The parameters taken in the simulations are given in Tables III and IV with non-uniform VNF requests. Without loss of generality, we assume that all placement and consumption costs are unit cost, i.e., $c_i^f = c_{ij} = c_i = 1$, and each demand requires a single VNF instance specified in Table IV. The results in Table V meet our expectation, where "NF-Node" represents the optimal locations of NFs deployed corresponding to each demand.

We further analyze the capacity consumption of physical nodes while realizing end-to-end demands and VNF allocation simultaneously. Table VI presents the computational results for the NFV-RR MILP formulation with the same parameter settings mentioned above. Note here that "–" in Table VI means no resource consumption for that specific node in the corresponding testing scenario. We also use Fig. 4 to illustrate the ratios of resources consumption on physical nodes as well as VNF instance allocation, in which the x-axis represents the node index and y-axis denotes the node's resource consumption ratio.

B. *Performance of the Proposed MILP Formulation and Algorithm 1*

In this section, we report and compare the performance of the proposed Algorithm 1 and the mathematical formulation introduced in Section IV in solving the NFV-RR problem. We let node and link costs be uniformly distributed in [2, 4] and [5, 10] and randomly generate five testing cases based on parameters in

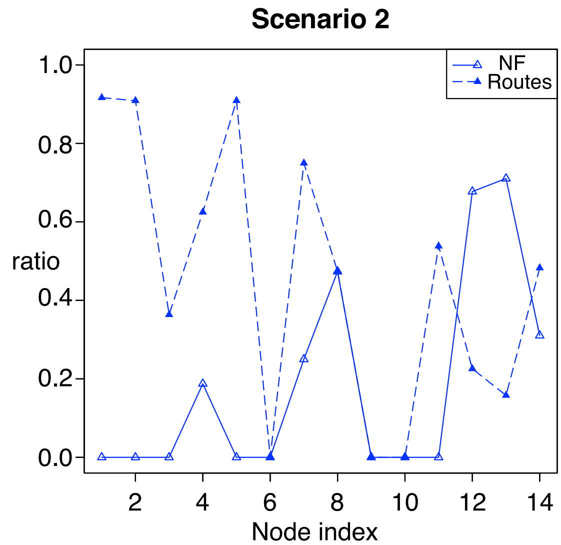

Fig. 4. Node resource consumption ratio comparison between network function and end-to-end-requests.

TABLE VII
PERFORMANCE OF ALGORITHM 1: OPTIMALITY GAP

PNum=2		PNum=4		PNum=6		
Obj	Gap	Obj	Gap	Obj	Gap	Opt
Infb	–	897	18.34%	758	0%	758

TABLE VIII
PERFORMANCE OF ALGORITHM 1: COMPUTATIONAL TIME

PNum=2	PNum =4	PNum =6	Opt
Infb	15.43	16.99	25.03

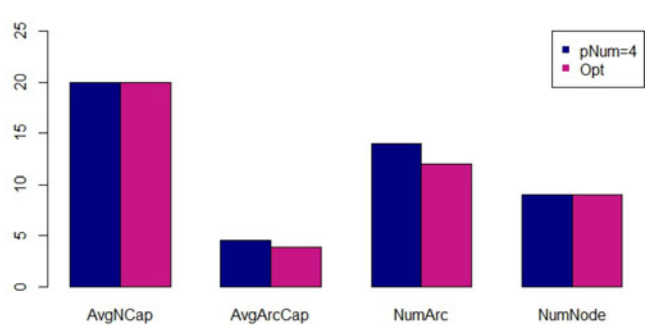

Fig. 5. Algorithm 1 performance comparison.

Tables III and IV. We report the results in Tables VII and VIII and Fig. 5.

We validate Algorithm 1 through the performance assessment as follows. We generate testing scenarios with two, four, and six as the number of initial paths ("PNum") for each demand, which serve as the initial candidate routes for each demands in Algorithm 1. Here, the initial paths are generated using the

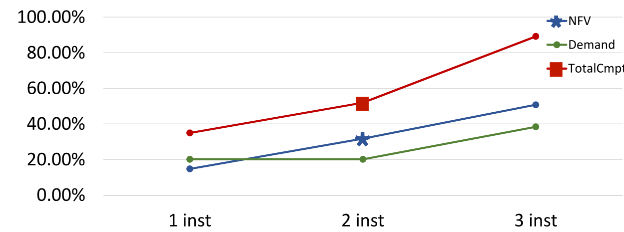

Fig. 6. Physical resource competition.

k-shortest path algorithm [45]. In Table VII, we let "PNum," "Obj," "Opt," "Gap," and "Infb" represent the number of initial paths as the input for Algorithm 1, the objective of Algorithm 1, the optimal objective value of NFV-RR problem calculated by the MILP formulation, the gap between the objective value of Algorithm 1 and the optimal solution of NFV-RR MILP (i.e., Gap $= \frac{\text{Obj}-\text{Opt}}{\text{Opt}}$), and the infeasibility of a testing case, respectively. We observe two facts based on Table VII: 1) when the number of input paths for each demand are sufficient (up to 6 in our testing cases), Algorithm 1 obtains the same objective value as the optimal solution of NFV-RR MILP. Therefore, with all demand routes as inputs, Algorithm 1 can provide the optimal solution for NFV-RR; and 2) when node and link capacities are tight in the physical network, selecting only the shortest path as the demand route may not be feasible as some longer routes are required to fulfill the demands. In Fig. 5, we report the average consumed node capacity on physical nodes deployed with NF instances ("AvgNCap"), the average consumed link capacity on physical links routed through by demands ("AvgArcCap"), and the number of utilized links and nodes ("NumArc" and "NumNode"). In terms of physical resource consumption, the average node capacity consumption is 20 for the nine NF deployed nodes. The average link capacity consumption is 6.262 in the optimal solution of the NFV-RR MILP. The consumption of link capacity in Algorithm 1 is 15.95% higher.

We next compare the computational time (in seconds) for both Algorithm 1 and the NFV-RR MILP formulation. Though Algorithm 1 requires to solve two MILP problems in Steps 7 and 12, its computational time is still shorter than directly solving the NFV-RR MILP formulation.

C. Physical Resource Competition Between NFV Allocation and Realizing End-to-End Demands

To show the competition between VNF instance allocation and end-to-end demand realization, we utilize three testing cases with uniform VNF requests, where each end-to-end demand in the testing case requires one, two, and three instances of VNF α in $\mathcal{F}_1$, respectively. All other parameters are given in Table III. In Fig. 6, physical resource consumptions (in percentage) for end-to-end demand is denoted as "Demand," VNF instance allocation is denoted as "NFV," and the total consumption is denoted as "TotalCmpt." We observe that the physical resources utilized to deploy VNF instances grow linearly with the number of VNF instances required by each demand. However, the resources consumed to fulfill the demands do not increase accordingly. The explanation for such outcome is that realizing end-to-end demands involves deploying VNFs, while deploying VNFs takes away the physical resources that originally can be used to fulfill the demands. The competition between these two parties ends when the physical resources are exhausted and neither can proceed further.

VII. Conclusion

In this paper, we studied NFV-RR based on a use case in [9] and evaluated the performance of the placement of VNFs in terms of its ability to support end-to-end requests with limited physical resources. We proposed a mixed-integer program, solved the NFV-RR problem by MILP formulation, and designed an algorithm based on Game Theory. Computational results demonstrated the value of the integrated approach and its ability to allocate NFs supporting end-to-end requests with limited physical resources in optical networks.

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