Efficient PEEC Computation of Losses and Currents in Screens of Round Wires in Submarine Tripolar Cables

L. Giussani [1], L. Di Rienzo [1], *Senior member, IEEE*, M. Bechis [2], P. Cambareri [1], and C. de Falco [3]
[1] Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Milan 20133, Italy, luca.dirienzo@polimi.it
[2]Prysmian S.p.A., Milan 20126, Italy, massimo.bechis@prysmiangroup.com
[3]MOX, Dipartimento di Matematica, Politecnico di Milano, Milan 20133, Italy, carlo.defalco@polimi.it

A PEEC formulation is applied to compute Joule losses and currents in the wires composing the screens of the phase conductors in a submarine tripolar cable. In order to model the infinite extent of the cable and to reduce the computational burden, simmetries of the geometry are exploited. A good agreement is obtained with brute force FEM simulations, with a dramatic reduction of computational times and memory requirement.

***Index Terms*—PEEC, submarine cables, integral equation methods.**

I. Introduction

An IMPORTANT part of the electrical system of an offshore wind farm is the connection between wind turbines and substation(s), where the voltage is elevated before power is transmitted to the land substation. These connections are realized with medium voltage three-phase submarine cables, in which, differently from the case of high voltage tripolar submarine cables [1], [2], each phase has a screen made of conductive wires; the computation of the screen losses is a challenging task and brute force FEM approaches are characterized by a high computational burden.

The Partial Element Equivalent Circuit (PEEC) formulation here presented is applied to a test case where each phase cable has a wire screen of diameter equal to $57.86\,\mathrm{mm}$ made of 55 tiny copper wires (with resistivity equal to $1.7241 \cdot 10^{-8}\Omega \cdot$ m) of diameter equal to $0.9\,\mathrm{mm}$, helically twisted around the cable with pitch equal to $851\,\mathrm{mm}$ (Fig.1). Phase cables are also helically twisted with a pitch of $1740\,\mathrm{mm}$ and a helix diameter equal to $146.56\,\mathrm{mm}$. The tripolar submarine cable carries a balanced three-phase systems of currents ($500\,\mathrm{A}$ rms @ $50\,\mathrm{Hz}$). The wires of the screens can be twisted around the phase cables with opposite (contralay) or same (equilay) orientation.

II. Formulation

Let us consider N distinct screen wires of radius r and conductivity σ, indexed by $n = 1, \ldots, N$. The centerlines of the wires are denoted by Γ_n, and, for the sake of computation, they are represented as piecewise linear curves, each one of them composed by P_n segments $\gamma_n^1, \ldots, \gamma_n^{P_n}$ obtained by joining uniformly distributed nodes along each helix, as shown in Fig.2.

A (possibly zero) voltage ΔV_n is applied to the ends of each wire. Moreover, each wire is subject to a time-harmonic magnetic field represented by magnetic vector potential $\mathbf{A}_0$ and generated by the phase conductors, at angular frequency ω. Let $A_n = \int_{\Gamma_n} \mathbf{A}_0 \cdot d\boldsymbol{\ell}$ be the line integral of $\mathbf{A}_0$ along Γ_n.

Phase conductors are modeled infinitely long, filamentary and helically twisted, each one carrying a known current. Under this assumption, the generated magnetic vector potential at any point of the space can be computed by means of analytical formulas [3].

Even if the formulation is able to consider another external magnetic field in addition to that of the phase conductors, this case is not taken into account because it is not interesting for the application at hand.

The unknowns of the problem are the N wire currents $I_n, n = 1, \ldots, N$ which can be determined by solving the following system of equations [4]

$$R_n I_n + j\omega \sum_{m=1}^{N} L_{n,m} I_m = -j\omega A_n - \Delta V_n \tag{1}$$

where R_n is the resistance of wire n and $L_{n,m}$ is the mutual inductance between wires n and m. In the application at hand $\Delta V_n = 0$. In (1) the proximity effect between wires is neglected, which is possible under the assumption that the current density is uniform on the cross section of each wire (which is satisfied for the application at hand at the frequency of interest).

Under the same hypothesis, the resistance R_n can be simply computed as

$$R_n = \frac{\ell_n}{\pi r^2 \sigma} \tag{2}$$

where ℓ_n is the length of wire n.

Concerning the inductance $L_{n,m}$, since we have discretized the wires into segments, it is useful to express $L_{n,m}$ as

$$L_{n,m} = \sum_{p=1}^{P_n} \sum_{q=1}^{P_m} L_{n,m}^{p,q} \tag{3}$$

where $L_{n,m}^{p,q}$ is the inductance between segments γ_n^p and γ_m^q. When segments γ_n^p and γ_m^q are distinct, their mutual inductance

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Fig. 1. Representation of the cable with screen wires

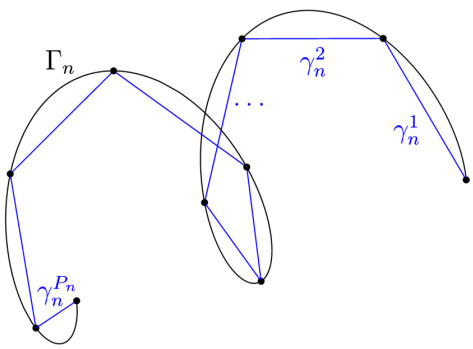

Fig. 2. Geometrical discretization of wire n.

can be computed using the hypothesis of filamentary conductors as

$$L_{n,m}^{p,q} = \frac{\mu_0}{4\pi} \int_{\gamma_n^p} \int_{\gamma_m^q} \frac{d\boldsymbol{\ell}_n \cdot d\boldsymbol{\ell}_m}{r_{n,m}} \tag{4}$$

When segments γ_n^p and γ_m^q coincide (i.e. $n = m$ and $p = q$), the term $r_{n,m}$ tends to 0, making the integral in (4) singular. Therefore $L_{n,n}^{p,p}$ cannot be computed using (4) and we use instead the analytically computed formula for the self inductance of a finite length cylinder of height $h = |\gamma_n^p|$ and radius r given by [5]:

$$L_{n,n}^{p,p} = \frac{\mu_0}{2\pi} \Big[h \log\left(\sqrt{h^2 + r^2} + h\right) - h\left(\log r - \frac{1}{4}\right) + \\ - \sqrt{h^2 + r^2} + 0.905415\, r \Big] \tag{5}$$

III. Exploiting Cable Symmetries

The geometry of the power cable exhibits symmetries [6] which can be exploited for assembling the matrix associated to the linear system of equations (1). In order to describe them, it is necessary to analytically represent the curves of the phase cables and of the screen wires.

The analytical equation of the helix of radius R_1 representing a phase cable is defined by the parametrization $\mathbf{r_1}(s) = (x_1(s), x_2(s), x_3(s))$, with

$$\begin{cases} x_1(s) = R_1 \cos\left(2\pi \frac{s}{K}\right) \\ y_1(s) = R_1 \sin\left(2\pi \frac{s}{K}\right) \\ z_1(s) = p_1 \frac{s}{K} \end{cases} \tag{6}$$

where $K = \sqrt{p_1^2 + (2\pi R_1)^2}$ and p_1 is the laying pitch.

The helix representing a screen wire is twisted along the helix of the corresponding phase cable and its parametrization can be obtained as follows. First the tangent vector of (6) is considered

$$\mathbf{t}(s) = \begin{cases} x_t(s) = -R_1 \frac{2\pi}{K} \sin\left(2\pi \frac{s}{K}\right) \\ y_t(s) = R_1 \frac{2\pi}{K} \cos\left(2\pi \frac{s}{K}\right) \\ z_t(s) = \frac{p_1}{K} \end{cases} \tag{7}$$

together with its derivative

$$\frac{d\mathbf{t}}{ds} = \begin{cases} -R_1 \left(\frac{2\pi}{K}\right)^2 \cos\left(2\pi \frac{s}{K}\right) \\ -R_1 \left(\frac{2\pi}{K}\right)^2 \sin\left(2\pi \frac{s}{K}\right) \\ 0 \end{cases} \tag{8}$$

with $\left|\frac{d\mathbf{t}}{ds}\right| = R_1 \left(\frac{2\pi}{K}\right)^2$.

Knowing that the normal vector is given by

$$\mathbf{n}(s) = \begin{cases} -\cos\left(2\pi \frac{s}{K}\right) \\ -\sin\left(2\pi \frac{s}{K}\right) \\ 0 \end{cases} \tag{9}$$

the binormal vector $\mathbf{b}(s) = \mathbf{t}(s) \times \mathbf{n}(s)$ can be obtained as

$$\mathbf{b}(s) = \begin{cases} \frac{p_1}{K} \sin\left(2\pi \frac{s}{K}\right) \\ -\frac{p_1}{K} \cos\left(2\pi \frac{s}{K}\right) \\ \frac{2\pi R_1}{K} \end{cases} \tag{10}$$

The equation of the curve representing a screen wire is finally given by

$$\mathbf{r_2}(s) = \mathbf{r_1}(s) + \mathbf{n}(s) R_2 \cos\left(\frac{2\pi s}{p_2}\right) + \mathbf{b}(s) R_2 \sin\left(\frac{2\pi s}{p_2}\right) \tag{11}$$

where p_2 and R_2 are the pitch and the radius of the secondary helix twisted along the primary helix (6).

When $s = p_2$ the relative positions between the two helices is the same as in $s = 0$ and the corresponding value for z_1 is defined as the "cross-pitch" (P_{cp}) [2]

$$P_{cp} = z_1(s = p_2) = \frac{p_1 p_2}{\sqrt{p_1^2 + (2\pi R_1)^2}} \tag{12}$$

Similarly as in ([2]), it can be shown that the entire geometry of the sheaths, which we denote by Ω, can be decomposed into a number of cells $\Omega^{(k)}$, $k = 0, \ldots, K$, each one of length equal to $\frac{P_{cp}}{N}$.

Cell $\Omega^{(k+1)}$ is related to cell $\Omega^{(k)}$ by a roto-translation τ composed by a rotation of an angle equal to $2\pi \frac{P_{cp}}{N \cdot p_1}$ and a translation of a length equal to $\frac{P_{cp}}{N}$, so that

$$\Omega = \bigcup_{k=0}^{K} \Omega^{(k)} = \bigcup_{k=0}^{K} \tau^k \Omega^{(0)} \tag{13}$$

which shows that the entire geometry Ω can be obtained by appropriately replicating and transforming the elementary cell $\Omega^{(0)}$, as shown in Fig.3.

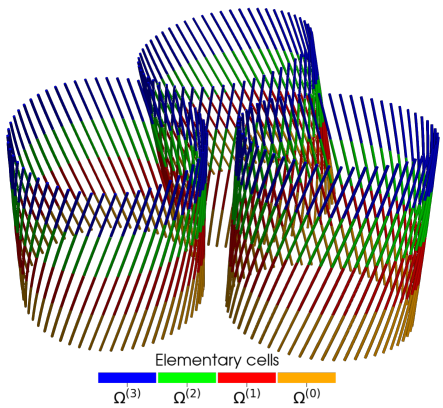

Fig. 3. Partition of geometry into elementary cells

TABLE I
COMPARISON WITH A 2D FEM MODEL IN TERMS OF JOULE LOSSES

Joule Losses [W/m]		Err. [%]
PEEC	FEM	
5.2492	5.2641	-0.28

This regularity in the geometry can be exploited for reducing the computational complexity of the assembly of the inductance matrix L. Let $L^{(n,m)}$ denote the inductance matrix between the wires of cell $\Omega^{(n)}$ and $\Omega^{(m)}$. Then the following property holds:

$$L^{(n,m)} = L^{(n-m,0)} \tag{14}$$

This property can be used to show that L, the inductance matrix of the whole cable Ω can be computed as

$$L = \sum_{k=0}^{K} \sum_{n=0}^{N} c_{k,n} P_{k,n} L^{(k,0)} Q_{k,n} \tag{15}$$

where $c_{k,n}$ are integer constants and $P_{k,n}$ and $Q_{k,n}$ are permutation matrices.

Using (15) for computing the inductance matrix leads to a reduction in computational complexity from $\mathcal{O}(\ell^2 N^2)$ to $\mathcal{O}(\ell N)$, where ℓ is the axial length of the cable, greatly reducing the computational time needed for matrix assembly.

IV. VALIDATION BY COMPARISON WITH A 2D FEM MODEL

The formulation has been firstly validated in the case of straight phase cables and screen wires by comparison with a 2D FEM model built with the help of a commercial software [7] and based on a very fine FEM mesh. For this case self and mutual inductances to be used in the PEEC formulation can be computed analytically. The cross-section of the cable is the same of the submarine cable described in the introduction. A very good agreement can be obtained both in the computation of losses (Tables I) and of currents (Tables II).

TABLE II
COMPARISON WITH A 2D FEM MODEL IN TERMS OF SCREEN CURRENTS

Current Amplitude [A]		Err. [%]	Current Phase [°]		Err. [°]
PEEC	FEM		PEEC	FEM	
55.109	55.172	-0.11	-97.70	-97.73	0.027

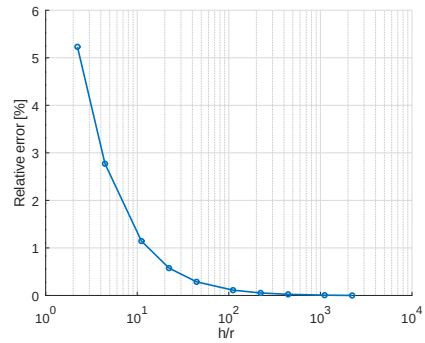

Fig. 4. Relative error of the self-inductance of a straight wire computed with (4) and compared with the analytical solution.

V. NUMERICAL RESULTS

In order to investigate the validity of the filamentary approximation of a wire, the self-inductance of a straight cylinder of length equal to $1\,\mathrm{m}$ is computed according to (4) and compared with its analytical value for different values of parameter h, length of the segments representing the wire, normalized to the wire radius $r = 0.45\,\mathrm{mm}$ [8]. As can be noted, the approximation using filamentary elements shows a low accuracy when $\frac{h}{r} \leq 10$ (Fig. 4).

PEEC results for the real 3D geometries are then compared with 3D FEM results obtained with the same commercial software [7], in the two cases of equilay and contralay configurations.

Table III shows screen losses per-unit-length of cable and Table IV the sum of the currents flowing in the wires which compose the screen of the phase-0°conductor, in the case of contralay configuration.

Corresponding results for the equilay configuration can be found in Tables V - VI.

In order to carefully analyse the role of discretization, three different values of parameter h, length of the segments representing the wires, have been used for the PEEC simulations, ranging from 20 times to 60 times the wire radius r.

TABLE III
COMPUTATION OF JOULE LOSSES AND COMPARISON WITH FEM - CONTRALAY CONFIGURATION

h/r	Joule Losses [W/m]		Err. [%]
	PEEC	FEM	
60	4.5641	4.5663	-0.05
40	4.5765	4.5663	+0.22
20	4.5899	4.5663	+0.52

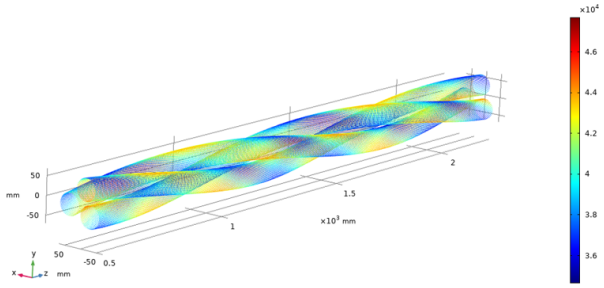

Fig. 6. Density of losses $[W/m^3]$ computed by FEM (reference solution) and represented over the wires.

TABLE IV
COMPUTATION OF SCREEN CURRENT AND COMPARISON WITH FEM - CONTRALAY CONFIGURATION

h/r	Current Amplitude [A]		Err. [%]	Current Phase [°]		Err. [°]
	PEEC	FEM		PEEC	FEM	
60	56.133	54.864	2.31	-96.590	-96.582	-0.008
40	56.181	54.864	2.40	-96.586	-96.582	-0.004
20	56.233	54.864	2.50	-96.597	-96.582	+0.015

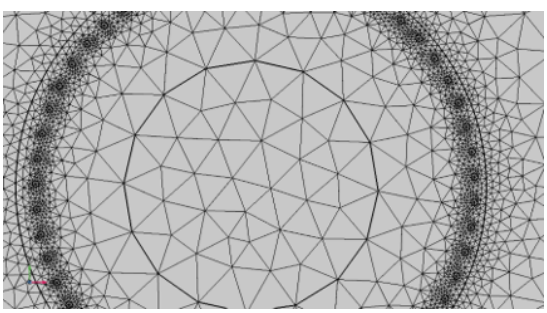

Fig. 5. Representation of the FEM mesh over a cross section of a screen made of wires.

FEM simulations require 74464456 degrees-of-freedom and 426 GByte of RAM and are run on a 4 processor Intel Xeon E7 v2 multi-core with 1.5 TB of RAM. The used mesh is represented in Fig. 5 on a trasversal cross-section of one screen made of wires. The density of losses computed by means of the FEM model is represented in Fig. 6.

The PEEC method (implemented in Octave [9]) only requires 165 degrees-of-freedom (to represent the current in each wire) and a negligible amount of RAM. Computational times reflect the different complexity of the models, with FEM simulations requiring hours of computation and PEEC simulations only a few seconds.

TABLE V
COMPUTATION OF JOULE LOSSES AND COMPARISON WITH FEM - EQUILAY CONFIGURATION

h/r	Joule Losses [W/m]		Err. [%]
	PEEC	FEM	
60	4.2334	4.2645	-0.729
40	4.2122	4.2645	-1.226
20	4.1924	4.2645	-1.6907

TABLE VI
COMPUTATION OF SCREEN CURRENT AND COMPARISON WITH FEM - EQUILAY CONFIGURATION

h/r	Current Amplitude [A]		Err. [%]	Current Phase [°]		Err. [°]
	PEEC	FEM		PEEC	FEM	
60	52.922	51.632	2.4985	-96.376	-96.397	-0.021
40	52.753	51.632	2.1711	-96.346	-96.397	-0.052
20	52.623	51.632	1.9194	-96.333	-96.397	-0.066

VI. CONCLUSION

The proposed PEEC formulation has proven to be accurate in the computation of Joule losses and currents in the wires composing the screens of a medium voltage three-phase submarine cable.

Computational times has been significantly reduced by exploiting the simmetries of the geometry, so that the resulting integral equation approach reveals to be much more efficient than brute force FEM simulations.

The discussed symmetries could applied also to other numerical formulations for the computation of losses in the present and in similar applications.

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