

Drive-by frequencies extraction by means of synchrosqueezed wavelet transform

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Abstract

Bridges and viaducts are facing growing traffic, increasing loads as well as ageing and deterioration. Nowadays, vibration-based Structural Health Monitoring (SHM) represents an effective solution to extract quantitative information regarding bridge structural condition. Bridge frequencies are fundamental parameters capable to reflect and summarize bridge dynamics and health condition. Typically, their identification is performed by direct approaches, which rely on the use of sensors installed on the structure under analysis. A cost-efficient, movable, and flexible alternative is offered by drive-by methods, consisting in the use of instrumented vehicles to extract bridge frequencies. This paper presents an experimental campaign carried out on a multiple-span mixed railroad bridge, with the aim to identify its frequencies through onboard measurements. A set of MEMS wireless accelerometers is deployed on the vehicle, firstly allowing for vehicle pitch component mitigation. Vehicle accelerations were then used to extract bridge frequencies, by means of a Most Probable Modal Frequencies approach, relying on synchrosqueezed wavelet transform. At a cruise speed of 30 km/h promising results in terms of indirect frequency estimation were found, with no traffic disruption.

Keywords

Drive-by monitoring, Indirect frequencies extraction, Bridges, Synchrosqueezed wavelet transform.

1 Introduction

Bridges and viaducts are crucial components of the infrastructural network. Nowadays travelling loads [1] and traffic are increasing, while a growing number of bridge structures is approaching service life limits [2]-[3], according to their original design criteria [4]. In this scenario, their regular condition assessment, with the aim to avoid and prevent catastrophic failures, is becoming a task of major concern for infrastructure managers on worldwide scale. Conventionally, bridge maintenance and monitoring operations mainly rely on visual inspections [5], whose outputs are qualitative and subjective [6], since dependent on operators' experience [7]. Moreover, the effectiveness of these practices is strongly related to the nature of the fault and its detectability by human eye [8]. In the attempt to overcome these limitations, thanks to the advancement of sensors and data storage/management technologies, Structural Health Monitoring (SHM) has been widely employed in the last decades. SHM consists of a continuous and automatic process for bridge condition assessment aimed at optimizing and supporting maintenance procedures and decision making [9]-[10]. The objective is the extraction of quantitative information able to reflect the structural status of the bridge structure under analysis.

Among them, bridge frequencies are fundamental parameters, capable to reflect in an immediate way deck conditions as well as bearing supports status [11]. Traditionally, bridge frequencies are identified by means of direct vibration-based approaches which require the installation of sensors network on the bridge itself. Direct monitoring systems are globally adopted and well-established, but they are characterised by high costs, need for traffic disruption (during installation and maintenance operations) and null movability, given their strongly tailored nature [12]. A cost-effective solution is provided by indirect methods, even called drive-by approaches, that employ sensors installed on vehicles rather than on the targeted structure. The idea behind this scanning methodology is to extract bridge dynamic information from vehicle response [13]: in fact, the vehicle works as bridge exciter as well as a moving sensor able to collect bridge vibrating response. After the pioneering work by Yang [14], in which the authors demonstrated the feasibility of indirectly extracting bridge frequency from scanning vehicle response, drive-by approaches have attracted a growing number of researchers in the last two decades [13]-[15]-[16]. Indirect SHM (iSHM) divides into two main categories, i.e., modal and non-modal approaches respectively. The former relies on the identification of modal parameters, i.e., frequencies

[17], mode shapes [18] and damping ratios [19] that are used as diagnostic indexes since directly linked to bridge structural integrity. Instead, non-modal approaches do not require the identification of modal parameters, but rather rely on the computation of other indexes or quantities from vehicle response, such as the apparent profile [20], deck curvature [21] or wavelet coefficients [22]. The present work is inserted in the context of modal-based drive-by approaches. In fact, the paper deals with the indirect identification of the first four vertical frequencies of a combined road and railway bridge. The investigation was entirely performed through an experimental campaign, with the aim to bridge the gap between theoretical findings and results regarding drive-by approaches and their full-scale implementation. Despite having shown great potential and advantages, there still exist few real-world applications, whose great majority apply to highway cases. In this work, the authors aimed to prove the feasibility of an indirect frequency identification approach, addressing all the factors that behave as a source of disturbance in real-world applications, such as asphalt roughness, the presence of expansion joints and vehicle dynamics undermining effect. The experimental campaign was conducted with a Ford Galaxy, travelling at a constant cruise speed of 30 km/h, without requiring any traffic disruption or limitation. To deal with the expansion joints exciting effect, the authors applied Tukey windowing to the vehicle raw accelerations in correspondence of joints crossing. Subsequently, to compensate for the masking effect played by car pitch motion, which may hide bridge information inside vehicle response, the authors propose a summation of accelerations recorded on leading and rear vehicle sides. The obtained signal is then processed by means of synchrosqueezed wavelet transform (SWT) with the purpose to extract bridge vertical frequencies. The proposed methodology led to promising identification results: in fact, the authors were able to effectively extract the first four vertical bridge frequencies with a maximum error of 4%. The paper is organised as follows. Section 2 is intended to provide an overview of the bridge subject of this study. Section 3 describes the methodological approach adopted consisting in a preliminary test for vehicle dynamic characterization and the drive-by campaign. Section 4 illustrates the algorithm used to process onboard accelerations with the purpose to extract bridge frequencies. The results of the experimental drive-by campaign are shown and discussed in Section 5, while conclusions are drawn in Section 6.

2 The bridge: history and structural features

The Bressana bridge, also called Mezzana Corti bridge, is a combined road and rail bridge, crossing the Po River between the small towns of Mezzana Corti and Bottarone. The bridge was originally built in 1866 and designed by Alfredo Cottrau: the superstructure consisted of lattice girders while the piers were made of brick masonry. During the Second World War, the bridge was bombed causing the collapse of the central girders. As a consequence, the original superstructure was completely substituted with a metal truss structure and the construction work ended in 1951. The actual bridge, shown in Figure 1, is composed of 10 double-decked girders (see Figure 2) with 76.5 m in length each and 9.2 m in height. The bridge is part of a research project between Politecnico di Milano and Re-

gione Lombardia [23], within which a fixed monitoring system has been installed on the structure. The latter measures both the static and the dynamic behaviour of the bridge.



Figure 1 A photograph of the Bressana bridge taken from the railway access

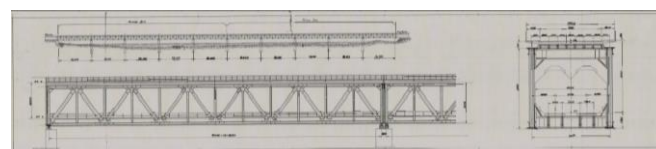


Figure 2 Technical drawing of Bressana bridge

Thus, the modal parameters could be extracted by exploiting the velocimeters placed on the rail deck. Being all the spans featured by the same structural characteristics, their modal parameters resulted to be very similar, as theoretically expected and confirmed by literature [24]. For this reason, the reference to compare the drive-by campaign results with is obtained by statistically aggregating the modal parameters of the spans in terms of a mean value and a related standard deviation. Table 1 reports the first four bending frequencies of the bridge.

Table 1. Natural frequencies of the first four vertical modes obtained through measures from a fixed sensors network installed on the bridge. Frequencies are reported in terms of mean value and standard deviation by aggregating the results of the single spans on a statistical basis

Mode order	Type of mode	Frequency
I	Bending	2.49 ± 0.01 Hz
II	Bending	5.83 ± 0.1 Hz
III	Bending	8.70 ± 0.2 Hz
IV	Bending	11.35 ± 0.1 Hz

3 Methodological approach

The present work is entirely based on an in-field experimental campaign. In fact, a fully data-driven approach has been followed, without performing any numerical simulations and laboratory testing. Two aspects have driven such a methodological choice, one technical, the other referred to the applicability of this technique in the real world. The former is motivated by the strong impact exerted by reality effects on the quality of the results. Drive-by monitoring is highly sensitive to exogenous disturbances, such as vehicle dynamics, road profile, and traffic, to mention

some. Such factors cannot be properly accounted for neither during laboratory tests, which are conducted in a controlled environment, nor through numerical simulations. This partial representation of the complex environment in which drive-by monitoring operates leads to over-optimistic results achieved by reduces-scale experiments and numerical analyses, which seldom find confirmation from the experimental counterpart [17]. From the perspective of a deployment of drive-by monitoring on the real world, a full-scale study is the only way to account for the impact of this technique on the regular traffic on the road. Considering this technique thorough application in a real-case scenario, evaluating its effects on circulation is of paramount importance. However, the decision of not performing prior simulations of the Vehicle-Bridge Interaction phenomenon implied the need for preliminary analyses on the diagnostic vehicle. Such tests served to obtain information on the dynamic behaviour of the vehicle, so as to distinguish between bridge dynamics and vehicle ones, while analysing the data collected within the drive by campaign.

3.1 Preliminary test on the vehicle

Previous studies demonstrated the importance of building knowledge about the dynamics of the diagnostic vehicle before applying drive-by monitoring. Such preliminary information helps with the design of the measurement setup. Moreover, it enables distinguishing between car and bridge contribution when extracting the frequency content from onboard time histories [25]. To this end, a first experimental campaign was conducted to define the dynamic footprint of the vehicle. The vehicle used for the campaign is a Ford Galaxy (Figure 3) and Table 2 presents some of its relevant geometrical features.



Figure 3 Vehicle used to collect drive-by data on the bridge

Table 2 Relevant geometrical characteristics of the diagnostic vehicle

Feature	Dimension
Wheelbase	2850 mm
Length	4853 mm
Width	1916 mm
Height	1811 mm
Curb weight	1978 kg

The experimental test consisted of a one-hour free driving during normal traffic conditions. A wide range of speed and road irregularity allowed to excite the car with an input

featured by a broad frequency spectrum. To get a deep understanding of the vehicle dynamics, a measurement grid of 8 wireless triaxial MEMS accelerometers (G-Link-200) was designed. Figure 4 represents the sensor placement. The rationale behind the setup was guided by the intent of decoupling the most important modes characterizing vehicles: the pitch and the roll motion. Also, a sensor was placed on the carbody in correspondence of each wheel and related suspension system. The sensors are listed below:

- “d” and “e” are on the theoretical neutral axis of the pitch motion;
- “b” and “g” are on the theoretical neutral axis of the roll motion;
- “a”, “c”, “f”, and “h” are attached above each wheel.

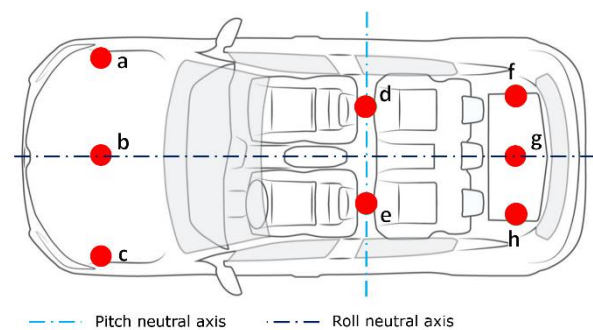


Figure 4 Schematic representation of the sensors network deployed on the diagnostic vehicle. The roll and pitch neutral axes are highlighted in blue and light blue respectively

Bearing in mind the deployment of the technique on a real-world scenario, the design of the system considered also the reachability of the measuring points and the easiness of the positioning. More specifically, “d” and “e” were placed inside the vehicle, grounded in between front and back seats; “a”, “b”, and “c” were attached on top of the car hood; sensors “f”, “g”, and “h” were placed in the car trunk, the closest possible to the rear end. The sensor network was controlled through a base station (LORD Microstrain WSDA-2000) that ensured the synchronization and set the sampling frequency at 128 Hz. This preliminary analysis aimed at identifying the modal characteristics of the car. The interest was in highlighting possible low-frequency dynamics of the carbody frame that could behave as an exogenous disturbance to the methodology. According to this scope, the frequency bandwidth was limited to 0.8 Hz to 10 Hz by applying a bandpass filter with these cut-off frequencies. Figure 5 reports the results of the analyses on the vertical channels of sensors “b”, “d”, and “h”. These transducers were chosen for their location (Figure 4): “d” approximately is on the pitch neutral axis; “b” lies on the roll neutral axis; “h”, instead, does not belong to any motion neutral axis, thus should, in principle, experience bounce, roll and pitch contributions. Such a measurement grid helped in isolating the contribution of the most relevant modes of the carbody. Figure 5 shows the Power Spectral Densities from the vertical accelerations of sensors “b”, “d”, and “h” obtained by through pwelch Matlab function. One may notice that “b” and “h” could detect a strong pitch component at 1.75 Hz. The bouncing contributes to the same frequency, as shown by the PSD computed from sensor “d”. The third accelerometer (“h”) did not unveil any other noticeable peaks other

than the pitch-bounce one. Thus, either the roll motion is not excited enough when the car is running a standard service, or its magnitude is negligible compared to other dynamic contents.

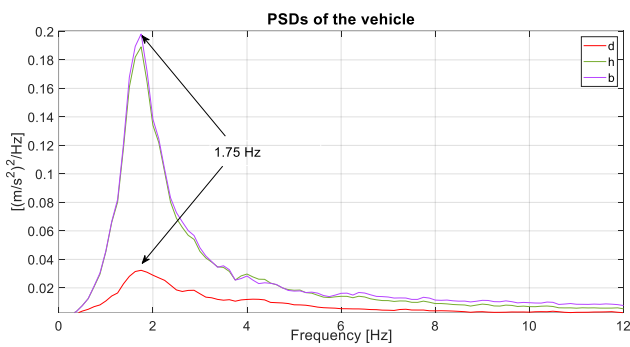


Figure 5 PSD functions computed from sensors “d” (on pitch neutral axis), “b” (on roll neutral axis), and “h” (above wheel). The frequency associated with pitch and bounce motions peaks is highlighted

3.2 Drive-by data collection

The indirect sensing campaign foresaw 15 minutes of onboard acquisition, materialized in 10 crossings of the structure, 5 per direction of travel. The test took place in daytime without imposing any traffic limitation to replicate as much as possible a real deployment condition. Since a study on the impact of the travelling speed was out of the scope of this work, the data collection was realized driving at a constant velocity, namely 30 km/h. This speed value of 30 km/h was chosen following the indication of some successful experimental results from drive-by monitoring presented in literature [25]–[26]. Even though, according to precedent studies, a lower speed would have been beneficial to increase the accuracy of the results, the speed was pushed towards the upper limit to minimize the traffic disruption. For the sake of coherence with the test conducted on the vehicle, the same placement of the sensors and sampling frequency, reported in subsection 3.1, have been used.

4 Algorithms for data analysis

4.1 Pre-processing

Raw data from the drive-by campaign have been low-pass filtered and treated to remove the impulses caused by the expansion joints connecting the spans. Considering the frequencies of interest were lower than 12 Hz, the cut-off frequency of the filter was set equal to 15 Hz. To purge the data from the excitation exerted by the joints, the corresponding portions of the acceleration signals have been removed by applying a Tukey window [27]. Figure 6 shows a time history after pre-processing was applied. The ten spans are visible and well decoupled by the windows. The following step consisted in mitigating car pitch motion effect on vehicle response, which may hide bridge information. According to an ideal model of the car vehicle, the pitch motion acts with the same amplitude but with opposite phase on the front and back of the vehicle. Therefore, the idea was to sum vehicle accelerations (once windowed to remove joints effect) from couples of sensors placed in the front and rear car sides, so that the pitch effect could be reduced. Following this rationale, three new time histories were computed, one per couple of sensors (see Figure

4).

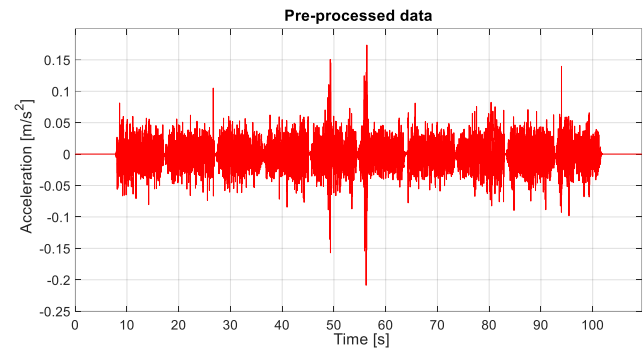


Figure 6 An example of data collected on the bridge after the pre-processing. The ten spans are decoupled by the Tukey windows that have been used to eliminate the portions of the signal in correspondence of the expansion joints

4.2 Most Probable Modal Frequencies

The frequency identification analysis has been performed by applying a wavelet-based method to the signals obtained with the pre-processing procedure presented in the previous section. Among the available sensors, the two on the pitch neutral axis (namely “b” and “g”) were summed and fed to the synchrosqueezing algorithm. Compared to more traditional techniques, e.g., the Welch method, such algorithms are able to capture non-stationary phenomena, possibly providing better results. The algorithm herein presented aims at identifying the most probable modal frequencies (MPMFs) by applying the synchrosqueezed wavelet transform [28], with a Morlet basis [29]. The synchrosqueezed wavelet transform is particularly suitable for this application because it reduces the effects of smearing affecting signal decomposition in the case of closely spaced frequencies. Synchrosqueezing produces a time-frequency representation of the dynamic signal. The transformation from time-frequency to space-frequency is obtained by mapping time to a coordinate system referred to the bridge (by travelling speed). The algorithm operates as follows:

1. The synchrosqueezed wavelet transform is calculated for each of the bridge crossings individually;
2. The time variable is then converted into a linear location on the bridge by exploiting the known value of the speed. The result is a space-frequency representation of the signal. An example of space-frequency map is reported in Figure 7;
3. Ridges (i.e. sequences of instantaneous frequencies) are then identified as peaks at each location on the map;
4. Locations near each other are clustered into aggregated spatial groups for each space-frequency diagram;
5. Maps from all bridge crossings are now aggregated, producing an average space-frequency diagram;
6. The most prominent vibration frequencies from each spatial group are selected and a histogram of these is created;
7. The most commonly occurring values of this histogram are identified using a kernel density estimation (KDE) fit. These values are considered the MPMFs. For a more extensive description of the algorithm, including the mathematical details, one may refer to [30].

5 Results and discussion

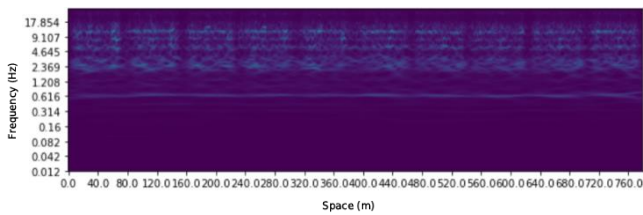


Figure 7 Frequency-space map produced by applying the synchrosqueezing transform to the time histories. Five frequency components are visible. The continuous one represents the so-called driving frequency, while the others contain the first five modal frequencies

Figure 7 represents a space-frequency map produced by the synchrosqueezing transform (steps 1-to-3 of the algorithm). The map illustrates the intensity of signal components at different frequencies and spatial locations on the bridge. Five groups of ridges emerge, among which only one – at 0.65 Hz – is continuous. This contribution comes from the so-called driving frequency [25]. The latter depends on the vehicle speed and span length according to the formula $f = \pi * v/L$ and its effect is visible throughout the entire bridge at a frequency equal to twice the driving frequency. Each span of the Bressana bridge is 78 m long, and the speed was kept constant and equal to 30 km/h (i.e., 8.33 m/s), which results in a driving frequency of 0.33 Hz. The other four groups depend on the bridge dynamics. Indeed, the corresponding ridges are interrupted nearby the joints, where the signal has been filtered. Figure 8 reports the identified frequencies (steps 4-to-7). The algorithm successfully estimated the bridge first four vertical modal frequencies. The obtained results were very promising, with a maximum error lower than 4 % in correspondence with the second vertical mode (i.e., 5.60 Hz). Another frequency has been identified, too, equal to 3.52 Hz. According to the results from the fixed monitoring systems, this frequency relates to the first torsional mode. The proximity of the latter to the first vertical natural frequency contributes to the blur of the ridge around 2.5 Hz on the map. Such a frequency in fact feels the influence of both the two modes mentioned above and the vertical dynamics of the vehicle.

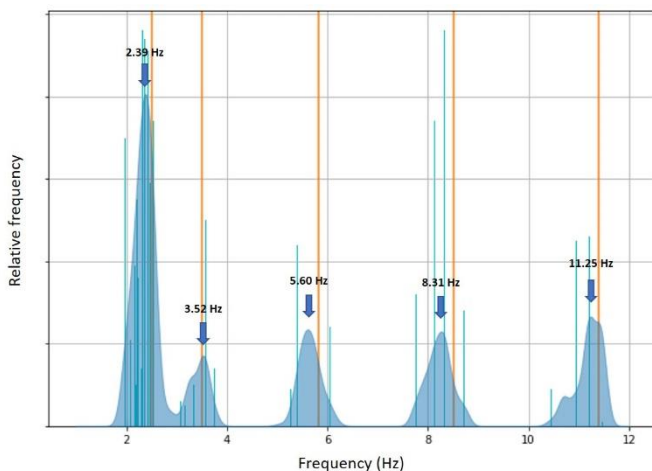


Figure 8 Histograms showing the statistically most prominent frequencies after stacking the maps of the ten trips produced by applying the synchrosqueezed wavelet transform. The solid orange lines indicate the reference bridge frequencies extracted from the fixed sensors installed on the bridge. The arrows highlight the frequencies estimated by the algorithm

6 Conclusions and future developments

The present work illustrates and discusses the outcomes of a drive-by experimental campaign performed on an Italian mixed rail and road bridge. The objective of the work was to assess the feasibility of indirectly extracting bridge vertical frequencies from onboard measurements. The drive-by campaign was conducted on a commercial vehicle, without any modifications applied to the car frame, travelling at a constant speed of 30 km/h, with no traffic disruption. Given the experimental nature of this work, in this study the authors addressed real-world applications problems, such as joints excitation and vehicle dynamics detrimental effects. The former was mitigated by means of Tukey windowing operation, while the latter was reduced by a summation of lead and rear vehicle accelerations. The processed signal was then studied in the time-frequency domain by means of synchrosqueezed wavelet transform (SWT), by which most probable modal frequencies were extracted. The presented drive-by methodology showed promising accuracy in determining the first four vertical bridge frequencies, obtaining a maximum error lower than 4 %, in correspondence of the second vertical mode. Conclusively, it is worth remarking some limitations of this work, already under consideration as further developments. First, the experimental campaign was performed exploiting one single vehicle: to increase the level of generality of the proposed methodology it would be beneficial to repeat the experimental campaign using other cars. To the same extent, testing the technique with different travelling speed is recommended, since the literature highlights the relevance of this parameter on the successfulness of indirect frequency identification. Lastly, albeit the pitch compensation method provided very good results, it might represent a limit to an extensive employment of this technique in a real-world scenario, since it requires an ad hoc configuration of sensors on the diagnostic vehicle.

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