Observer based finite time sliding mode control strategies for single-stage grid-connected PV system with LCL filter

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\textbf{A B S T R A C T}

The escalating global demand for clean and sustainable energy sources has trusted solar energy into the forefront of renewable energy (RE) solutions. Solar power generation, however, is profoundly influenced by ever-changing meteorological conditions, notably solar irradiation and temperature. These fluctuations have a direct impact on the voltage and current characteristics of photovoltaic arrays. To achieve optimal power transfer, a harmonious interface between the utility grid or load and solar panels is essential. In this research, the authors propose the integration of LCL filter to ameliorate harmonics emanating from inverter outputs. Additionally, a nonlinear control methodology is employed to maximize power extraction from Photovoltaic (PV) arrays at the Maximum Power Point (MPP) and to ensure a stable and less fluctuating injection of power into the grid. The primary goal of this work is to examine the effectiveness of nonlinear control techniques that assist in reducing harmonics and smoothing the output of solar power systems utilizing LCL filter. The controllers used here are nonlinear Super Twisting Reachability law-based Finite Time Sliding Mode Control (STR-FTSMC) and Arbitrary Order Finite Time Sliding Mode Control (AO-FTSMC) for the three-phase single-stage on-grid PV system in addition to sliding mode observer. The efficacy of the employed control techniques is highlighted via a comparative study in a MATLAB/Simulink environment.

\section{Introduction}

The widespread adoption of solar energy systems has fueled the latest developments in control systems and power electronics technologies. In this context, voltage source inverters (VSIs) find common usage in a wide spectrum of power conversion applications, encompassing distributed generation (DG) systems rooted in PV technology, energy storage systems, wind turbines, and so on (Blaabjerg et al., 2006). The landscape of PV electricity generation is typically categorized into two primary domains: stand-alone and grid-connected systems. However, grid-connected systems, owing to their inherent advantages, emerged as the preferred choice for large-scale power generation endeavors. Notably, one significant advantage of grid-connected systems is their ability to operate without the necessity of high-capacity energy storage batteries (Menaga and Sankaranarayanan, 2020). This paper delves into the technical intricacies of these developments, presenting a comprehensive analysis of the advancements in control systems and power electronics, particularly within the context of grid-connected PV systems.

The preference for single-stage grid-connected PV systems over their two-stage counterparts can be attributed to several compelling factors. Notably, single-stage systems tend to offer a higher degree of cost-effectiveness by eliminating the need for an additional DC-DC converter stage, resulting in an overall reduction in system cost (Wu et al., 2011; Zeb et al., 2019). This cost-saving attribute is particularly advantageous for smaller-scale residential or commercial installations where budget constraints are crucial in decision-making. Furthermore, single-stage PV systems boast a simplified design and installation process. This streamlining of the installation procedure renders it more straightforward and less time-consuming. Although, two-stage systems may provide certain advantages in terms of optimizing power generation under varying solar conditions, a well-designed single-stage system can still deliver with sufficient efficiency for a wide array of applications. This paper provides an in-depth exploration of the merits of single-stage grid-connected PV systems, shedding light on their cost-effectiveness, simplicity of design, and suitability for various practical applications.

In the realm of grid-connected PV systems, two pivotal control objectives take central stage: maximizing PV power generation and...
minimizing harmonics in the inverter output voltage (Erasam and Chapman, 2007). Achieving these goals is essential for ensuring the efficiency and reliability of PV systems. To attain maximum power point (MPP) operation, various control techniques have been devised. These include constant voltage control, the perturb and observe (P & O) algorithm, the incremental conductance algorithm, and their modified versions (Erasam and Chapman, 2007). However, worth noting that implementing a multi-stage power conversion system, although it may offer certain advantages, introduces cost implications and can potentially compromise the dependability and overall energy efficiency of the PV installation (Rivera et al., 2017). For the reduction of harmonics, two types of filters are commonly employed: the L filter and the LCL filter. In this context, the LCL filter takes precedence due to its superior harmonic reduction capabilities compared to the L filter (Rivera et al., 2017).

Augmenting these control strategies, nonlinear control techniques are leveraged to optimize power extraction from the PV system and seamlessly inject it into the grid (Wu et al., 2012; Beres et al., 2015; Bao et al., 2013). This paper provides a comprehensive exploration of the strategies and methodologies employed to achieve the dual objectives of maximizing PV power output and reducing harmonics in grid-connected PV systems.

In Mahmud et al. (2013), a three-phase single-stage grid-connected photovoltaic inverter system with nonlinear control law is developed through an inclusive structure for the synchronous dq reference frame to track the maximum power regardless of climate conditions and to control active and reactive power without the use of extra power controller. Distributed generation and the incorporation of RES into the grid have both benefited from the advancement of power electronics technology. Nonlinear controllers deal directly with nonlinearities, there are various sophisticated and efficient nonlinear control approaches and schemes applied to grid-connected PV systems in the literature (Mahmud et al., 2013).

Sliding mode controllers (SMC) stand out as one of the most advanced control techniques, ensuring the fulfillment of control objectives even in the presence of nonlinearities, variations in model parameters, and external disturbances. Within the realm of grid-connected inverters, SMC has garnered substantial attention due to its notable attributes, including rapid dynamic response, robustness, and excellent regulation qualities (Hao et al., 2012; Vieira et al., 2017; Komurcuğil et al., 2015). This method finds significant applications in the control of grid-connected photovoltaic (PV) systems, as exemplified in Naddami and Ababssi (2023), Kim (2006, 2007). In the context of single-phase grid-connected systems (Kim, 2006), the focus is on achieving maximum power extraction, with a sliding mode controller devised based on current control. Similarly, in the case of three-phase grid-connected systems (Kim, 2007), integral sliding mode techniques are harnessed to craft the sliding surface, underscoring the versatility and effectiveness of SMC in achieving robust control objectives in grid-connected converter systems.

In Zeb et al. (2020, 2018), an investigation into the modeling and design of a Super Twisting Sliding Mode Controller (ST-SMC) is conducted, focusing on the efficient injection of both active and reactive power in a three-phase grid-connected PV system under various operating conditions, including abnormal scenarios. Furthermore, Huang et al. (2019) introduced an observer-based sliding mode control approach tailored for LCL-filtered grid-connected inverters, enhancing system stability. Notably, this control strategy minimizes the need for multiple state variable feedback, reducing sensor requirements by incorporating a state observer. The combination of a state observer with discrete sliding mode control in digital implementation streamlines the control architecture. Additionally, the utilization of estimated capacitor voltage within this framework contributes to active damping, effectively mitigating LCL resonance in the system. This research underscores the advancements in sliding mode control techniques, offering improved control and stability for grid-connected PV systems.

Yu et al. reviewed Terminal Sliding Mode Control (TSMC), including TSMC basics, TSMC developments, the state of art of TSMC theory, and its applications (Yu et al., 2020). The fundamental difference between the TSMC and conventional SMC has been investigated. The important features and advantages of TSMC have been analyzed, and the challenges in TSMC and their future trend in theory and applications have been outlined. Numerous control techniques have been devised for the efficient control current of LCL filter-based Grid-Connected Voltage Source Inverter (GC-VSI) systems. Among these, proportional-integral (PI) control (Jamil et al., 2020), repetitive control (RC) (Husey et al., 2019), proportional resonant (PR) control (Bo et al., 2009), and deadbeat control (DB) (Benyoucef et al., 2014) are the most common. Due to the inherent nonlinear nature of the LCL filter type GC-VSI system, these control techniques only achieved limited objectives and have their own merits and demerits. PI control is the simplest one; however, it suffers from poor sinusoidal current reference tracking performance and lack of disturbance rejection ability. Alternatively, various nonlinear controllers such as backstepping control, Lyapunov-based control, and SMC are being applied to meet the desired performance requirement. In Kale et al. (2016), continuous SMC is applied in the application of a shunt active filter to prevent distorting harmonics components due to high switching frequency.

Dehkordi et al. (2016) focused on a new harmonic and interharmonic compensation strategy proposed for DG-interfacing converters with LCL filters. The proposed method combines a backstepping control system based on a high order sliding mode differentiator. The controller aims to regulate the grid current, irrespective of the load dynamics, grid impedance, grid frequency, and grid voltage. To achieve the desirable performance and to reject any disturbance signal, a new backstepping control based on a high order sliding mode differentiator is proposed.

For improving the dynamic performance and quality of injected power of single stage single phase grid-connected photovoltaic system, a robust terminal mode control was designed (Chigane and Ouassaid, 2024). The model adapted for achieving improved dynamic performance helps the control of active as well as reactive power. For ensuring the voltage control of inverter under islanding condition, a robust SMC was designed (Barzegar-Kalashani et al., 2023). The proposed design was tested under different loading conditions as well as tested with different design parameters. This design showed fast transient response switching from no-load to full load and vice versa.

Voltage and current based Super twisting algorithm-sliding mode controller (STA-SMC) is interfaced with Distributed energy resources (DERs) designed for single-phase voltage sources inverter (Barzegar-Kalashani et al., 2022). The study was done for both grid connected and islanded operation incorporating the effect of load current and output voltage of filter as external disturbances. STA-SMC used, showed stability and robustness under different loading conditions. A novel SMC method for microgrid inverters, employing a compound reaching law to enhance voltage stability amidst load disturbances presented (Yuan et al., 2023). Adaptive SMC further mitigates disturbances, offering robust performance even with unknown disturbance boundaries. This approach significantly improves inverter system performance, making it promising for microgrid applications requiring precise voltage regulation under dynamic operating conditions.

1.1. Motivation and contributions

The main objective of this paper is to design and simulate the nonlinear control technique with a single-stage three-phase grid-connected photovoltaic system inverter and to extract the maximum power at MPP from PV by implementing the Maximum Power Point Tracking (MPPT) control technique. So, the system performance no longer remains appealing without a self-sufficient controller. For this purpose, Super Twisting Reachability law-based Finite Time Sliding Mode Control (STR-FTSMC) and Arbitrary Order Finite Time Sliding Mode Control
(AO-FTSMC) strategies will be adopted to perform robustly in uncertain scenarios. Sometimes system states are not available all the times so a state estimator will also be designed to provide the estimated states. As a result, the problem will be "observer-based finite-time sliding mode control strategies for grid-connected PV systems with LCL filter". The nonlinear control technique will extract the maximum power from the PV system and inject it into the grid.

1.2. Paper layout

The subsequent sections of the paper are structured as follows: Section 2 provides the mathematical modeling of three-phase grid connection and presents the problem formulation. In Section 3, we delve into the explanation of the proposed controller design. Section 4 is dedicated to present the results and engage in discussions regarding the proposed controller. Finally, the paper concludes with Section 5.

2. Mathematical modeling of three phase grid connected inverter with LCL filter

In this section, mathematical model for the proposed system is presented. Fig. 1 depicts a three phase grid connected model which consists of a PV system and an inverter that is connected to the grid through an LCL filter. An LCL filter is used to feed the grid with the inverter’s output of AC power. By applying Kirchhoff’s voltage and current law, the three-phase dynamic model of the system is obtained in stationary abc reference frame which is given as follows:

\[
\begin{align*}
\frac{d}{dt}(v_{abc}) &= \frac{1}{C}i_{abc} - \frac{1}{C}i_{abc} \\
\frac{d}{dt}(i_{abc}) &= \frac{1}{L_i}v_{abc} - \frac{1}{L_i}i_{abc} \\
\frac{d}{dt}(i_{abc}) &= \frac{1}{L_g}v_{abc} - \frac{1}{L_g}i_{abc}
\end{align*}
\]  

(1)

where \(v_{abc}\) is the output voltage across the filter capacitor, \(C\) is output filter capacitance, \(i_{abc}\) is the input current to the inverter, \(i_{abc}\) is the grid current, \(v_{abc}\) is the inverter input voltage, \(L_i, L_g\) are the output filter inductances and \(i_{abc}\) is output grid voltage.

By using equations given in Appendix A, the system’s dynamics (reported in Eq. (1)) are transformed into the following synchronously rotating dq-frame.

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{L_x}x_2 + \omega x_4 - \frac{1}{L_x}v_{gxd} \\
\dot{x}_2 &= \frac{1}{C}x_1 - \frac{1}{C}x_3 + \omega x_5 \\
\dot{x}_3 &= \frac{1}{L_i}x_2 + \omega x_6 + \frac{1}{L_i}d_v dc \\
\dot{x}_4 &= \frac{1}{L_x}x_5 - \omega x_4 - \frac{1}{L_x}u_{dqc} \\
\dot{x}_5 &= \frac{1}{C}x_4 + \frac{1}{C}x_6 - \omega x_2 \\
\dot{x}_6 &= \frac{1}{L_i}x_5 - \omega x_6 + \frac{1}{L_i}d_v dc
\end{align*}
\]  

\(\Sigma : \) \hspace{1cm} (2)

where \(x_1, x_2\) and \(x_3\) are the \(d\)-component of grid current, capacitor voltage and inverter current, respectively. Similarly, \(x_4, x_5\) and \(x_6\) are the \(q\)-components of grid current, capacitor voltage and inverter current, respectively.

In addition, the control inputs are represented by \(u_{d} = d_v dc\) and \(u_{q} = d_v dc\). The \(dq\) model of Eq. (2) in compact form is given by:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
\text{where } x &\in \mathbb{R}^6, u &\in \mathbb{R}^2 & y = h(x) = [i_{gq}]T \\
\end{align*}
\]  

\(\Sigma : \) \hspace{1cm} (3)

where \(x \in \mathbb{R}^n, u \in \mathbb{R}^n\) and \(y \in \mathbb{R}^n\) are states, inputs and outputs vectors. \(A\) is the system distribution matrix and \(B\) is the input matrix. The fundamental goal of a three-phase grid-connected photovoltaic system is to supply the grid with a maximum amount of power from PV generation while maintaining a power factor of unity. The perturb and observe (P&O) MPPT algorithm gives \(i_{MPPT} = i_{MPPT} = i_{gref}\) which is tracked by the proposed controller to inject maximum power into the grid. The \(i_{gref}\) is set to zero to ensure power is injected at unity power factor.

3. Observer based sliding mode control techniques

In this section, the two control techniques i.e., STR-FTSMC and AO-FTSMC will be implemented. The aim is to significantly increase power extraction from PV arrays under changing solar conditions while comparing with the existing control approaches. These approaches enable the PV system’s operation in response to changing solar irradiation and temperature levels. They also maximize energy extraction by employing cutting-edge tracking algorithms and ensure precise navigation of the PV array to its MPP, in the presence of the environmental variations. Furthermore, the sliding mode control algorithms i.e., STR-FTSMC and AO-FTSMC are strong enough to withstand uncertainties and disruptions like rapid weather changes. Now, the observer and both the controllers will be presented in the proceeding subsection.
3.1. Sliding mode observer

The sliding-mode observer (also known as the Utkin observer) was introduced by Vadim I. Utkin in the late 1970s. This observer offers robustness to uncertainties, disturbances, and nonlinear dynamics by continuously driving the estimation error to the sliding surface. The key feature of the Utkin observer is the introduction of a switching function in the observer to achieve a sliding mode and steering of error dynamics to origin (Drakunov and Utkin, 1992). A general structure is shown as follows:

\[
\dot{x} = A\dot{x} + Bu + Ge + Lu
\]

where \(e = [e_1 \ e_2]^T\), such that \(e_1 = (i_d - i_{gd})\) and \(e_2 = (u_d - i_{gd})\). Where the core objective is to steer \(e_1\) and \(e_2\) to zero in finite time despite the nonlinearities (or uncertainties). Furthermore, \(G\) and \(L\) are in designer gains matrices and \(v = k\text{sign}(e)\) such that \(k \in \mathbb{R}^n\). The presence of \(v\) ensures sliding mode in face of uncertainties i.e., enhance robustness to matched uncertainties.

The observer used is a full state observer which consists of \(G\) (proportional injecting term) and \(L\) (discontinuous term gain). By substituting matrices \(A, B, C, L,\) and \(G\) in Eq. (4), one gets the following expanded form.

\[
\dot{x}_1 = \frac{1}{L_i} \dot{x}_2 + \omega \dot{x}_4 + (L_{11} k_1 \text{sign}(x_1)) + (L_{12} k_2 \text{sign}(x_2)) + (G_{11} e_1 + G_{12} e_2)
\]

(5)

\[
\dot{x}_2 = \frac{1}{C} \dot{x}_1 + \omega x_4 + (L_{21} k_1 \text{sign}(x_1)) + (L_{22} k_2 \text{sign}(x_2)) + (G_{21} e_1 + G_{22} e_2)
\]

(6)

\[
\dot{x}_3 = \frac{1}{C} \dot{x}_4 + \omega x_5 + (L_{31} k_1 \text{sign}(x_1)) + (L_{32} k_2 \text{sign}(x_2)) + (G_{31} e_1 + G_{32} e_2)
\]

(7)

\[
\dot{x}_4 = \frac{1}{L_i} \dot{x}_5 + \omega \dot{x}_3 + (L_{41} k_1 \text{sign}(x_1)) + (L_{42} k_2 \text{sign}(x_2)) + (G_{41} e_1 + G_{42} e_2)
\]

(8)

\[
\dot{x}_5 = \frac{1}{C} \dot{x}_6 + \omega x_5 + (L_{51} k_1 \text{sign}(x_1)) + (L_{52} k_2 \text{sign}(x_2)) + (G_{51} e_1 + G_{52} e_2)
\]

(9)

\[
\dot{x}_6 = \frac{1}{L_i} \dot{x}_7 + \omega \dot{x}_4 + (L_{61} k_1 \text{sign}(x_1)) + (L_{62} k_2 \text{sign}(x_2)) + (G_{61} e_1 + G_{62} e_2)
\]

(10)

where

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} i_{gd} \\ \dot{e}_d \\ \dot{e}_u \\ \dot{e}_x \\ \dot{e}_y \end{bmatrix}^T.
\]

3.2. Super twisting reachability law based finite-time sliding mode control

Since, SMC is a well-known robust nonlinear control strategy that has a fast control action, better transient and steady-state performance and is invariant against matched uncertainties and disturbances when the system dynamics are in the sliding phase (Zhang et al., 2023). Finite-time sliding mode control is a control strategy that is used to design controllers for systems and render finite time convergence to the sliding surface.

### 3.2.1. Control design for d-component via SRP-TSMC

The d-component dynamics of the subsystem, \(\Sigma_i\) are defined as

\[
\Sigma_i : \begin{cases} \dot{x}_1 = \frac{1}{L_i} x_2 + \omega x_4 - \frac{1}{L_g} \omega y_{gd} \\ \dot{x}_2 = -\frac{1}{C} x_1 - \frac{1}{C} x_3 + \omega x_5 \\ \dot{x}_3 = -\frac{1}{L_i} x_2 + \omega x_6 + \frac{1}{L_g} u_d \end{cases}
\]

(11)

where \([x_1 \ x_2 \ x_3] = [i_{gd} \ \nu_{gd} \ i_{gd}]^T\). The tracking error is defined as,

\[
e_1 = x_1 - x_{1\text{ref}}
\]

(12)

Taking time derivative of \(e_1\), and incorporating Eq. (2), one has

\[
\dot{e}_1 = \left(\frac{1}{L_i} x_2 + \omega x_4 - \frac{1}{L_g} \omega y_{gd}\right) - x_{1\text{ref}}
\]

(13)

Now taking again time derivative of \(\dot{e}_1\), and involving Eq. (2), one has

\[
\ddot{e}_1 = \left(-\frac{1}{C} x_1 + \frac{1}{C} x_3 + \omega x_4\right) + \omega x_4 - \frac{1}{L_g} \dot{y}_{gd} - x_{1\text{ref}}
\]

(14)

Taking the time derivative of Eq. (14), and using Eq. (2), one finally gets

\[
\dot{e}_1 = -\frac{1}{C} \left(\frac{1}{L_i} x_2 + \omega x_4 - \frac{1}{L_g} \omega y_{gd}\right) + \frac{1}{C} \left(-\frac{1}{L_i} x_2 + \omega x_6 + \frac{1}{L_i} u_d\right)
\]

(15)

Now taking time derivative of \(\dot{e}_1\), and involving Eq. (2), one finally gets

\[
\dot{e}_1 = \left(-\frac{1}{C} x_1 + \frac{1}{C} x_3 + \omega x_4\right) + \omega x_4 - \frac{1}{L_g} \dot{y}_{gd} - x_{1\text{ref}}
\]

(16)

Remark 1. The generalized canonical form is a standard representation used in control system theory to describe the dynamics of nonlinear systems. It is particularly useful because it simplifies the analysis and design of control systems by expressing them in a control convenient form. In the context of our study, the canonical form allows us to represent the dynamics of the control system in a structured manner, making it easier to analyze and design control strategies such as the STR-FTSMC and AO-FTSMC. Specifically, the canonical form helps in understanding the relationships between the system’s inputs, outputs, and states, facilitating the development of effective control algorithms.

To develop canonical form, the following transformation may be defined. So, eventually, one gets the following

\[
\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = f_1(x_1, x_2, x_3) + g_1 u
\]

where

\[
f_1(x_1, x_2, x_3) = \frac{1}{T_g} \left(-\frac{1}{C} \left(\frac{1}{L_i} x_2 + \omega x_4 - \frac{1}{L_g} \omega y_{gd}\right) + \frac{1}{C} \left(-\frac{1}{L_i} x_2 + \omega x_6 + \frac{1}{L_i} u_d\right)
\]

(17)

Now, the control convenient form is ready. Therefore, the sliding surface, \(s_1\) is defined as:

\[
s_1 = e_{12} + C_1 e_{11}
\]

(18)
and

\[ s_1 = e_{13} + C_1 e_{13} \]  \hspace{2cm} (19)

Now based on surface \( s_1 \), a final sliding surface is defined as

\[ s_d = s_1 + a_1 s_1 \]  \hspace{2cm} (20)

The time derivative of Eq. (20) (i.e., \( s_d = s_1 + a_1 s_1 \)), along Eqs. (17)–(19), is given by

\[ \dot{s}_d = \dot{e}_{13} + e_{13}(C_1 + a_1) + a_1 C_1 e_{12} \]  \hspace{2cm} (21)

Comparing Eqs. (21) along Eqs. (16), one has

\[
\begin{align*}
\dot{s}_d &= \frac{1}{L_g} \left( -\frac{1}{C} \left( \frac{1}{L_g} x_2 + ax_4 - \frac{1}{L_g} \dot{v}_{eq} \right) + \frac{1}{C} \left( -\frac{1}{L_g} \dot{x}_2 + ax_5 + \dot{x}_5 \right) \right) + \dot{x}_5 - \dot{x}_6 - x_6 + e_{13}(C_1 + a_1) + a_1 C_1 e_{12} \\
&= -A_1 \text{sign}(s_d) - A_1 \int_0^t \text{sign}(s_d) dt
\end{align*}
\]

By equating Eqs. (22) and (23) and solving for \( u_d \), one gets Eq. (24)

**Remark 2.** This system under study is linear in input, output, and states. So, it supports the separation principles. Therefore, the stabilities of the observer and control components are proved in the respective section. In the implementation context, the states in the \( d \) and \( q \)-components are replaced by the estimated states which support the theory.

\[
\begin{align*}
u_d &= L_g C_L_\nu \left( -\frac{1}{L_g} \left( \frac{1}{C} \left( \frac{1}{L_g} \dot{x}_1 + ax_4 - \frac{1}{L_g} \dot{v}_{eq} \right) + \frac{1}{C} \left( -\frac{1}{L_g} \dot{x}_1 + ax_5 + \dot{x}_5 \right) \right) + \dot{x}_5 - \dot{x}_6 - x_6 + e_{13}(C_1 + a_1) + a_1 C_1 e_{12} \right) \\
&= -A_1 \text{sign}(s_d) - A_1 \int_0^t \text{sign}(s_d) dt
\end{align*}
\]

This is the required control law for the \( d \)-component. The design for the \( q \)-component will be outlined in the following study.

### 3.2.2. Control design for \( q \)-component via finite time sliding mode controller

The \( q \)-component dynamics of the subsystem, \( \Sigma_2 \) are defined as:

\[
\begin{align*}
x_4 &= \frac{1}{L_g} x_5 - ax_1 - \frac{1}{L_g} \dot{v}_{eq} \\
x_5 &= -\frac{1}{C} x_4 + \frac{1}{C} x_6 - ax_2 \\
x_6 &= -\frac{1}{L_g} x_5 - ax_5 + \frac{1}{L_g} \dot{v}_{eq}
\end{align*}
\]

where \( [x_4 \ x_5 \ x_6] = [v_{eq} \ v_{eq} \ i_{eq}] \). The tracking error is defined as,

\[ e_2 = x_4 - x_{aref} \]  \hspace{2cm} (26)

Taking time derivative of \( e_2 \), and incorporating Eq. (2), one has

\[ \dot{e}_2 = \left( \frac{1}{L_g} x_5 - ax_1 - \frac{1}{L_g} \dot{v}_{eq} \right) - x_{aref} \]  \hspace{2cm} (27)

Now taking again time derivative of \( \dot{e}_2 \), and involving Eq. (2), one has

\[ \ddot{e}_2 = \frac{1}{L_g} \left( -\frac{1}{C} x_4 + \frac{1}{C} x_6 - ax_2 \right) - ax_1 - \frac{1}{L_g} \ddot{v}_{eq} - x_{aref} \]  \hspace{2cm} (28)

Taking the time derivative of Eq. (28), and using Eq. (2), one finally gets

\[
\begin{align*}
\ddot{e}_2 &= \frac{1}{L_g} \left( \frac{1}{L_g} x_5 - ax_1 - \frac{1}{L_g} \dot{v}_{eq} \right) + \frac{1}{C} \left( -\frac{1}{L_g} x_5 - ax_5 + \frac{1}{L_g} \dot{v}_{eq} \right) \\
&- ax_1 - \frac{1}{L_g} \ddot{v}_{eq} - x_{aref}
\end{align*}
\]

To develop canonical form, the following transformation may be defined. So, eventually, one gets the following

\[
\begin{align*}
\ddot{e}_2 &= e_{21} \\
\dot{e}_{22} &= e_{23} \\
e_{23} &= f_2(x_4, x_5, x_6) + g_{23} u
\end{align*}
\]

where

\[
f_2(x_4, x_5, x_6) = \frac{1}{L_g} \left( -\frac{1}{C} (x_5 - ax_1 - \frac{1}{L_g} \dot{v}_{eq}) \right) + \frac{1}{C} \left( -\frac{1}{L_g} x_5 - ax_5 \right)
\]

\[
- ax_1 - \frac{1}{L_g} \ddot{v}_{eq} - x_{aref}
\]

Now, the sliding control form is ready. Therefore, the sliding surface, \( s_2 \) is defined as:

\[ s_2 = e_{23} + C_2 e_{23} \]  \hspace{2cm} (31)

where \( C_2 \) is a positive constant. Now, the first and 2nd time derivative of Eq. (31) along Eq. (30) are given by

\[ \dot{s}_2 = e_{23} + C_2 \dot{e}_{23} \]  \hspace{2cm} (32)

and

\[ \ddot{s}_2 = e_{23} + C_2 \ddot{e}_{23} \]  \hspace{2cm} (33)

Now based on surface \( s_2 \), a final sliding surface is defined as

\[ s_q = \dot{s}_2 + a_2 s_2 \]  \hspace{2cm} (34)

The time derivative of Eq. (34) (i.e., \( \dot{s}_q = \dot{s}_2 + a_2 s_2 \)), along Eqs. (31)–(33), is given by

\[ \dot{s}_q = e_{23} + C_2 \dot{e}_{23} + a_2 C_2 \dot{e}_{22} \]  \hspace{2cm} (35)

Comparing Eqs. (35) along Eqs. (30), one has

\[
\begin{align*}
\dot{s}_q &= \frac{1}{L_g} \left( \frac{1}{C} \left( \frac{1}{L_g} x_5 + ax_1 - \frac{1}{L_g} \dot{v}_{eq} \right) + \frac{1}{C} \left( -\frac{1}{L_g} x_5 - ax_5 + \dot{x}_5 \right) \right) \\
&+ ax_1 + \frac{1}{L_g} \ddot{v}_{eq} - x_{aref} + \frac{u_g}{L_1 C_L_\nu} + e_{23}(C_2 + a_2) + a_2 C_2 \dot{e}_{22}
\end{align*}
\]

Using the general structure of super twisting control law as:

\[ \dot{s}_q = -A_2 \text{sign}(s_q) - A_2 \int_0^t \text{sign}(s_q) dt \]  \hspace{2cm} (37)

By equating Eqs. (36) and (37) and solving for \( u_g \), one gets Eq. (38) according to Remark 2.

\[
\begin{align*}
u_g &= L_g C_L_\nu \left( -\frac{1}{L_g} \left( \frac{1}{C} \left( \frac{1}{L_g} \dot{x}_1 + ax_4 - \frac{1}{L_g} \dot{v}_{eq} \right) + \frac{1}{C} \left( -\frac{1}{L_g} \dot{x}_1 + ax_5 + \dot{x}_5 \right) \right) \right) \\
&+ \dot{x}_1 + \frac{1}{L_g} \ddot{v}_{eq} - x_{aref} - e_{23}(C_2 + a_2) - a_2 C_2 \dot{e}_{22} - A_2 \text{sign}(s_q) \\
&= A_2 \int_0^t \text{sign}(s_q) dt
\end{align*}
\]

This is the required control law for the \( q \)-component. Now in the next section, another control design method is discussed.
3.3. Arbitrary order finite time sliding mode controller

The Arbitrary Order Finite Time Sliding Mode Controller (AO-FTSMC) is a control strategy that aims to achieve robustness and fast convergence of the control system in a finite time. The advantage of Arbitrary Order sliding mode control (SMC) is its adaptability to diverse system dynamics and performance needs. Unlike fixed-order SMC, it offers greater flexibility in handling complex systems with uncertain or time-varying dynamics. The key advantage of AO-FTSMC is the establishment of finite time sliding mode as well as the finite time convergence of the error dynamics to zero. This, in turn, results in high precision which is ever demanded in control designs. Now, for the sake of completion, the design for d-component of the system will be presented and the q-component design can be carried out in similar fashion as that of d-component.

3.3.1. Design of d-component and q-component of AO-FTSMC

The AO-FTSMC is also a sliding mode based strategy (Alam et al., 2020). However, it design differs from conventional SMC in the sliding surface. The sliding surface is designed in such a way that it facilitates finite time sliding mode enforcement as well as, after sliding mode establishment, it results in the finite time convergence of error dynamics. As mentioned earlier the finite time convergence results in high precision while retaining the Key features of SMC.

At this stage, the sliding surface for the d-component for the controller is defined as:

\[ s_d = e_{13} + e_{12} + e_{11} + \int (b_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13})) + (b_{12}[e_{12}]^{\beta_2}\text{sign}(e_{12})) + (b_{11}[e_{11}]^{\beta_1}\text{sign}(e_{11})) + (c_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13}))dt \]  

(39)

The reachability condition for the d-component for the control law is used as follows

\[ \dot{s}_d = k_{13}\text{sign}(s_d) + k_{12}[s_d]^{\beta_2}\text{sign}(s_d) + k_{11}[s_d]^{\beta_1}\text{sign}(s_d) + k_{14}(s_d) \]  

(40)

The time derivative of Eq. (39) becomes

\[ \dot{s}_d = e_{13} + e_{12} + e_{11} + (b_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13})) + (b_{12}[e_{12}]^{\beta_2}\text{sign}(e_{12})) + (b_{11}[e_{11}]^{\beta_1}\text{sign}(e_{11})) + (c_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13})) + (c_{12}[e_{12}]^{\beta_2}\text{sign}(e_{12})) + (c_{11}[e_{11}]^{\beta_1}\text{sign}(e_{11})) \]  

(41)

Now, making use of Eq. (16) in Eq. (41), then comparing and solving for \( u_d \), one gets

\[ u_d = L_e C_L \left( -\frac{1}{L_e} \left( \frac{1}{L_e} \dot{s}_d + \omega s_d - \frac{1}{L_e} \dot{I}_{ref} \right) + \frac{1}{C_L} \left( -\frac{1}{L_e} \dot{s}_d + \omega s_d \right) \right) - \omega \dot{x}_d \left( -\frac{1}{L_e} \dot{x}_d + \dot{I}_{ref} - \dot{e}_{12} - e_{11} - (b_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13})) - (b_{12}[e_{12}]^{\beta_2}\text{sign}(e_{12})) - (b_{11}[e_{11}]^{\beta_1}\text{sign}(e_{11})) - (c_{13}[e_{13}]^{\beta_3}\text{sign}(e_{13})) - (c_{12}[e_{12}]^{\beta_2}\text{sign}(e_{12})) - (c_{11}[e_{11}]^{\beta_1}\text{sign}(e_{11})) + k_{13}[s_d]^{\beta_3}\text{sign}(s_d) + k_{12}[s_d]^{\beta_2}\text{sign}(s_d) + k_{11}[s_d]^{\beta_1}\text{sign}(s_d) + k_{14}(s_d) \right) \]  

(42)

Eq. (42) gives the final structure of the controller for d-component. While for the q-component, the control law is given by Eq. (43)

\[ u_q = L_e C_L \left( -\frac{1}{L_e} \left( \frac{1}{L_e} \dot{s}_q + \omega s_q - \frac{1}{L_e} \dot{I}_{ref} \right) + \frac{1}{C_L} \left( -\frac{1}{L_e} \dot{s}_q + \omega s_q \right) \right) - \omega \dot{x}_q \left( -\frac{1}{L_e} \dot{x}_q + \dot{I}_{ref} - \dot{e}_{13} - e_{12} - e_{11} - (b_{23}[e_{23}]^{\beta_3}\text{sign}(e_{23})) - (b_{22}[e_{22}]^{\beta_2}\text{sign}(e_{22})) - (b_{21}[e_{21}]^{\beta_1}\text{sign}(e_{21})) - (c_{23}[e_{23}]^{\beta_3}\text{sign}(e_{23})) - (c_{22}[e_{22}]^{\beta_2}\text{sign}(e_{22})) - (c_{21}[e_{21}]^{\beta_1}\text{sign}(e_{21})) + k_{23}[s_q]^{\beta_3}\text{sign}(s_q) + k_{22}[s_q]^{\beta_2}\text{sign}(s_q) + k_{21}[s_q]^{\beta_1}\text{sign}(s_q) + k_{24}(s_q) \right) \]  

(43)

Now, both the control algorithms are presented elaborately. The closed loop stability presentation is omitted here. However, the readers are referred to study (Ullah et al., 2020).

4. Simulation and results

The performance of the proposed controllers is evaluated using numerical simulation with MATLAB/Simulink. The block diagram of the simulation is depicted in Fig. 2. The system consists of a PV array and an inverter that is connected to the grid through LCL filter. A controller is fed by \( i_{ref} \) which is given by MPPT algorithm for maximum power tracking and the grid outputs after conversion from abc to direct quadrature zero (dq0) frame. The controller output is then converted back to abc and supplied to the inverter through a PWM module. Phase locked loop (PLL) has been utilized so as to synchronize the voltage of inverter with grid voltage. The internal diagram of PLL is depicted in Fig. 3 and the internal block diagram for STR-FTSMC and AO-FTSMC is shown in Figs. 5–8.

In this section, the proposed control system design is analyzed using output current, voltage, active power, reactive power, and total harmonic distortion are monitored for the two controllers to conduct comparative analysis. A comparison of the proposed control strategies with existing MPPT algorithms such as P&O based PI shown in Fig. 4 will be performed to assess their efficiency and reliability in maximizing power extraction at the MPP. It is observed that the proposed STR-FTSMC and AO-FTSMC control strategies outperform existing P&O based PI MPPT algorithm for varying atmospheric conditions. The simulation results of STR-FTSMC and AO-FTSMC are comprehensively discussed. The system parameters are listed in Table 1.

The incorporation of the super twisting reach-ability rule into the STR-FTSMC method considerably improves the robustness of grid-connected PV system, especially in circumstances of abrupt weather changes. The control technique guarantees speed and precise tracking of the required trajectories, even under dynamic operating conditions, by utilizing the inherent robustness of the super twisting algorithm to uncertainties and disturbances. Specifically, during abrupt weather changes, the super twisting reach-ability rule allows for rapid adjustments in control inputs to preserve stable operation and reduce the impact of disruptions on system performance. The exploration of arbitrary order finite time sliding mode control (AO-FTSMC) prompts investigation into how the sliding surface order affects convergence speed. Higher orders may yield faster convergence but heightened sensitivity to uncertainties, while lower orders offer improved robustness but slower convergence. We acknowledge that the use of advanced control approaches may complicate system design and necessitate careful parameter tweaking. The LCI filter parameters are given in Table 2.

The variable irradiance has been applied to the solar panels to get varying PV output. Solar irradiance changes with time as shown in Fig. 9. Irradiance varies for 24 h, maximum in day light and minimum during night time. Variation in irradiance causes a change in current and power. Irradiance has a positive effect on current as well as power i.e., power changes as the current varies according to change in irradiance. The results of two different controllers are given in the next sub sections i.e., Section 4.1 for the results of STR-FTSMC and Section 4.3 for the results of AO-FTSMC.

4.1. Simulation results of STR-FTSMC

The parameters of the given controller are listed in Table 3.

As irradiance changes, the grid current also changes following the behavior of the irradiance profile. Fig. 10 illustrates the output of the PV power as the controller tracks the irradiance change over time which results in tracking the reference power.

The grid voltage and current are in phase as shown in Fig. 11, which demonstrates that the power factor of the grid power is unity. When
Fig. 2. Block diagram of the overall system.

Fig. 3. Internal diagram of PLL.

Fig. 4. P&O based PI.
Fig. 5. Internal block diagram for $u_d$ STR-FTSMC.

Table 1

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Values</th>
<th>Units</th>
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<tr>
<td>Parallel strings</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Series-connected module per string</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Module</td>
<td>SunPower SPR-230NE-BLK-U-ACPV</td>
<td></td>
</tr>
<tr>
<td>Maximum power</td>
<td>230.04</td>
<td>W</td>
</tr>
<tr>
<td>Cells per module</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>Open circuit voltage</td>
<td>48.2</td>
<td>V</td>
</tr>
<tr>
<td>Short-circuit current</td>
<td>6.05</td>
<td>A</td>
</tr>
<tr>
<td>Voltage at maximum power point</td>
<td>40.5</td>
<td>V</td>
</tr>
<tr>
<td>Current at maximum point point</td>
<td>5.68</td>
<td>A</td>
</tr>
<tr>
<td>Temperature coefficient of $V_o$</td>
<td>−0.252</td>
<td>(%/deg. C)</td>
</tr>
<tr>
<td>Temperature coefficient of $I_s$</td>
<td>0.009008</td>
<td>(%/deg. C)</td>
</tr>
</tbody>
</table>
current and voltage are in phase, their maximum and minimum values occur simultaneously. Fig. 12 displays active power, indicating that the grid has received the maximum amount of real power. The reactive component of power, however, becomes zero as the $i_q$ reference is set to zero.

With STR-FTSMC, fewer harmonics are generated and shows variation when the irradiance changes and the THD with STR-FTSMC has been shown in Fig. 13.

### 4.2. Observer-based STR-FTSMC

$i_{gd}$, the d-component of the grid current, and $i_{gq}$, the q-component of the grid current, are the two states that will be used as the observer’s inputs. Under the suggested controller, $v_{cd}$, $v_{cd}$, $i_{gd}$ and $i_{gq}$ have been estimated. The observer’s states are compared to the system’s actual states while using the suggested control method.

An observer using the proposed technique, STR-FTSMC, accurately estimates all the states. The observed as well as actual states are shown
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Fig. 7. Internal block diagram for \( u_d \) AO-FTSMC.

**Table 4** Parameters of AO-FTSMC.

<table>
<thead>
<tr>
<th>( u_j )</th>
<th>Value</th>
<th>( u_j )</th>
<th>Value</th>
<th>( u_j )</th>
<th>Value</th>
<th>( u_j )</th>
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<td>( k_{21} )</td>
<td>10</td>
<td>( k_{21} )</td>
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<td></td>
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<tr>
<td>( k_{12} )</td>
<td>50</td>
<td>( k_{22} )</td>
<td>500</td>
<td>( k_{22} )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( k_{13} )</td>
<td>100</td>
<td>( k_{23} )</td>
<td>300</td>
<td>( k_{23} )</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{14} )</td>
<td>100</td>
<td>( k_{24} )</td>
<td>100</td>
<td>( k_{24} )</td>
<td>10</td>
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<tr>
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<td>0.7</td>
<td>( a_{21} )</td>
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</tr>
<tr>
<td>( a_{12} )</td>
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<td>( a_{22} )</td>
<td>0.3</td>
<td>( a_{22} )</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_{13} )</td>
<td>0.1</td>
<td>( a_{23} )</td>
<td>0.3</td>
<td>( a_{23} )</td>
<td>20</td>
<td></td>
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</tr>
<tr>
<td>( \beta_{11} )</td>
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<td>( \beta_{21} )</td>
<td>1.5</td>
<td>( \beta_{21} )</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_{12} )</td>
<td>1.5</td>
<td>( \beta_{22} )</td>
<td>0.3</td>
<td>( \beta_{22} )</td>
<td>1.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_{13} )</td>
<td>1.5</td>
<td>( \beta_{23} )</td>
<td>0.3</td>
<td>( \beta_{23} )</td>
<td>0.4</td>
<td></td>
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</tr>
</tbody>
</table>

in Figs. 14–19. The states \( \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5 \) and \( \dot{x}_6 \) are used in the control law.

4.3. Simulation results of AO-FTSMC

The simulation results of AO-FTSMC are illustrated for a three-phase grid-connected PV system in this section. Table 4 shows the parameters for the controller which can be used for tuning the controller.

An arbitrary order finite-time SMC is designed for a three-phase grid-connected PV system by adding some special non-linear terms in the control law. These terms ensure that the sliding mode is enforced in a fixed time, which do not depend on the initial conditions. This improves the robustness and performance of the controller.

Fig. 20 illustrates the output of the PV power as the controller follows the irradiance change over time which resulting in AO-FTSMC accurately tracks the reference power.

Fig. 21 shows a phase of grid voltage and current, which are in phase. The in-phase grid voltage and grid current prove that power factor of grid is unity. Current and voltage are in phase means that they reach their maximum and minimum values at the same time. This is desirable when injecting current into the grid, because, it ensures that the active power delivered by the source is equal to the power injected into the grid.

Fig. 22 shows active and reactive power as the maximum real power has been delivered to the grid. The figure also shows an increase in active power when the irradiance increases and vice versa. However, the reactive power is zero as \( i_q \) reference is set to zero, and the reactive component of power becomes zero.

The controller AO-FTSMC shows better harmonic distortion which is less than 5% according to the IEEE standard 519. Fig. 23 shows fewer harmonics and shows variation when the irradiance changes.

4.4. Observer-based AO-FTSMC

The two states which will be used as input to the observer are \( i_{gd} \) (d-component of grid current) and \( i_{gq} \) (q-component of grid current).
Fig. 8. Internal block diagram for $u_q$ AO-FTSMC.

Fig. 9. Irradiance profile changes with time.
The result of the observer is compared with the actual states of the system under the proposed control technique. The estimated value will be utilized by the proposed controller.

The observer with the proposed technique, AO-FTSMC accurately estimates the state $x_1$ grid current d-component ($i_{gd}$), state $x_2$ the voltage across capacitor d-component ($v_{cd}$) and state $x_3$ inverter output current d-component, which is shown in Fig. 24, Fig. 25, and Fig. 26 respectively. These estimates show that d-component of actual states and estimated states matched well.

Figs. 27–29 show the actual states (grid Current $i_{gq}$, voltage across the capacitor $v_{cq}$, inverter Output Current $i_{iq}$) and estimated states ($\hat{i}_{gq}$, $\hat{v}_{cq}$, $\hat{i}_{iq}$), which are matched as well. As $i_q$ reference equals zero, the estimated state $\hat{i}_{gq}$, accurately tracks the actual state $i_{gq}$.

4.5. Comparative analysis

The proposed control techniques are compared with the conventional P&O based PI MPPT algorithms. The ratio of the PV output power
to the reference power over a specific time period is known as dynamic efficiency.

$$\eta_{\text{100}} = \frac{\int_{t_0}^{t_f} P_{\text{PV}} \, dt}{\int_{t_0}^{t_f} P_{\text{PV,ref}} \, dt} \times 100$$

The $\eta_{\text{100}}$ is the present dynamic efficiency and $t_0 = 0$ and $t_f = 24s$ are the initial and final time, respectively. Fig. 30 displays the PV's dynamic efficiency. That is described in the following scenario.

Fig. 30 shows the efficiency of the proposed control techniques and the conventional MPPT strategy. The STR-FTSMC has an efficiency of 92.31% and AO-FTSMC has an efficiency of 98.19% while P&O-PI has an efficiency of 91.85%. The PV output power of all three algorithms are compared in Fig. 31. It is obvious from the figure that AO-FTSMC outperforms and it tracks the reference maximum power closely, accurately and smoothly with rapid variation in the PV output power. The real power injected into the grid is depicted in Fig. 32.
Fig. 16. Inverter output current d-component.

Fig. 17. Grid current q-component.

Fig. 18. Voltage across filter capacitor q-component.
Fig. 19. Inverter output current q-component.

Fig. 20. PV power.

Fig. 21. Grid phase ‘a’ voltage and current.
Fig. 22. Active and reactive power injected into the grid.

Fig. 23. Total harmonic distortion.

Fig. 24. Grid current d-component.
Fig. 25. Voltage across filter capacitor d-component.

Fig. 26. Inverter output current d-component.

Fig. 27. Grid current q-component.
The AO-FTSMC outperforms both the STR-FTSMC and the conventional P&O-PI. It is observed that the AO-FTSMC injects clean active power into the grid and maximum power is dumped into the grid as compared to STR-FTSMC and P&O-PI. The active power into the grid by the P&O-PI is accompanied by chattering. Fig. 33 gives a percent THD comparison of the proposed and conventional technique. It is obvious from the figure that THD of the proposed control techniques is better than the conventional one. Although the %THD of the conventional P&O-PI is high but in the time interval 7–17 s, it is within the IEEE standard limit ≤ 5%.

Based on the analysis given above, AO-FTSMC has outperformed both STR-FTSMC and P&O-PI in terms of maximum power extraction, grid power injection, dynamic efficiency and THD.

5. Conclusion

In this paper, a single-stage 3-phase grid connected PV system is used to inject maximum real power extracted from PV arrays with low THD. The observer based proposed nonlinear techniques, AO-FTSMC outperforms ST-FTSMC and P&O-PI, in term of achieving maximum power extraction, high efficiency and very low THD. The Uktin observer adequately estimates the unavailable states from the grid currents. The performance of the proposed Uktin observer based MPPT paradigms is validated using MATLAB/Simulink. AO-FTSMC injects maximum real power into the grid at 98.19% efficiency and 0.0252% THD as compared to ST-FTSMC with ($\eta_{100} = 92.31\%$, $\%THD = 0.0252$) and P&O with ($\eta_{100} = 91.58\%$, $\%THD = 4.9$).

CRediT authorship contribution statement

Ahmad Khan: Writing – original draft, Visualization, Validation, Software, Methodology, Conceptualization. Laiq Khan: Writing – review & editing, Visualization, Supervision, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. Qudrat Khan: Writing – original draft, Validation, Supervision, Investigation, Formal analysis, Conceptualization. Zahid Ullah: Writing – review & editing, Writing – original draft, Resources, Project administration, Formal analysis, Conceptualization. Adil Latif: Writing – original draft, Resources, Methodology, Investigation, Formal analysis, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.
Fig. 30. Dynamic efficiency.

Fig. 31. PV power.

Fig. 32. Real power injected into the grid of AO-FTSMC and STR-FTSMC.
Appendix A. Supplementary equations

\[
\begin{align*}
\begin{aligned}
T_{\text{dc}} &= T^{-1}v_{\text{dc}} \\
I_{\text{dc}} &= T^{-1}I_{\text{dc}} \\
v_{\text{dc}} &= T^{-1}v_{\text{dc}} \\
\end{aligned}
\end{align*}
\]

where \( T \) is transformation matrix,

\[
T = \frac{2}{3}
\begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

and,

\[
\begin{align*}
\frac{d}{dt}T^{-1}v_{\text{dc}} &= \frac{1}{C}T^{-1}v_{\text{dc}} - \frac{1}{L_i}T^{-1}v_{\text{dc}} \\
\frac{d}{dt}T^{-1}I_{\text{dc}} &= \frac{1}{L_i}T^{-1}v_{\text{dc}} - \frac{1}{L_g}T^{-1}v_{\text{dc}} \\
\frac{d}{dt}T^{-1}I_{\text{dc}} &= \frac{1}{L_g}T^{-1}v_{\text{dc}} - \frac{1}{L_f}v_{\text{dc}}
\end{align*}
\]

Appendix B. System and observer matrices

\[
A = \begin{bmatrix}
0 & \frac{1}{L_i} & 0 & \omega & 0 & 0 \\
-\omega & 0 & \frac{1}{L_i} & 0 & \omega & 0 \\
0 & -\omega & 0 & \frac{1}{L_i} & 0 & \omega \\
0 & 0 & -\omega & 0 & \frac{1}{L_i} & 0 \\
0 & 0 & 0 & -\omega & 0 & \frac{1}{L_i} \\
0 & 0 & 0 & 0 & -\omega & 0
\end{bmatrix},
\quad
B = \begin{bmatrix}
0 & 0 & \frac{1}{L_f} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\quad
C = \begin{bmatrix}
1 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
L_{11} & \cdots & L_{1n} \\
\vdots & \ddots & \vdots \\
L_{m1} & \cdots & L_{mn}
\end{bmatrix},
\quad
G = \begin{bmatrix}
G_{11} & \cdots & G_{1n} \\
\vdots & \ddots & \vdots \\
G_{m1} & \cdots & G_{mn}
\end{bmatrix}
\]

Fig. 33. Total harmonic distortion.

References


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