MEAN IMPULSE RESPONSE IN A TURBULENT CHANNEL FLOW

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Summary The mean linear response of a turbulent channel flow to a small enough, impulsive (in space and time) body force is defined and measured through direct numerical simulations, by considering the continuous range of wall distances from the wall to the centerline. A zero- mean, white-noise body force is used to probe the turbulent flow, and the response function is obtained efficiently by accumulating the space-time correlation between the white forcing input and the velocity field obtained as output. Three different responses are measured, at the same Reynolds number, for a laminar channel flow, a channel flow where the mean velocity profile has the turbulent shape but no turbulence is present, and a true turbulent channel flow.

INTRODUCTION

The linear impulse response function (LIRF) is a classic tool for the description of linear, time-invariant dynamical systems. Its use in relation to physical phenomena that involve moving fluids is not particularly widespread, as it is well known that the Navier–Stokes equations which govern the fluid motion are highly non-linear. The analysis of the LIRF of the flow to small perturbations is a natural approach in the (linear) hydrodynamic stability theory. In this context, the Navier–Stokes equations are linearized about an equilibrium solution, the base flow. The non-normal nature of the linearized Navier–Stokes (LNS) operator in wall-bounded shear flows implies the possibility for a transient growth of the perturbation energy, which explains how e.g. the Hagen–Poiseuille flow or the plane Couette flow undergo laminar-to-turbulent transition even though the linear modal stability theory predicts that the critical Reynolds number for transition is infinite. The importance of transient growth for the transition to turbulence prompted researchers to consider whether such linear mechanisms play an important role also in the dynamics of fully-fledged turbulent flows. Here, the obvious difficulty arises that turbulence is highly non-linear, implying that the stochastic "background'' turbulence can affect the amplified disturbances. The present works aims at measuring the time-mean response of a turbulent channel flow to a impulsive body force in space and time, while accounting for the full non-linearity of the system. Beside assessing the importance of non-linear turbulent transport, the mean impulse response is of greatest interest, thanks to the relevance of linear mechanisms in turbulent flows and the potential for assisting in the design of control laws for turbulence.

METHOD

The linear impulse response function $\mathcal{H}$ (in the frequency or the time domain) is defined for a non-linear system, under the condition that the input is small enough. The linear response $\mathcal{H}$ links the input and the output of a system in the time domain through a convolution. The extension to the channel flow involves a body force as input, one for each Cartesian direction and the three components of the velocity field as output. As explained by Quadrio & Luchini (2002), in the present context there are diverse and equivalent strategies to measure the LIRF. However, most of the approaches available for the laminar regime result computational unaffordable in the turbulent case. The approach used in the present work, originally introduced by Luchini *et al.* (2006) overcomes the issues of the classic strategies combining a decent S/N ratio with the ability to carry out a complete measurement in one shot. It relies the well-known result from signal theory that, when a white noise (i.e., a delta- correlated signal) is passed through a linear system, the correlation between input and output is proportional to the impulse response of the system. Three sets of direct numerical simulations are carried out at a bulk Reynolds number based on the half-channel height of 2280: a laminar flow, a fully turbulent flow, and a pseudo-turbulent flow where turbulence is absent but the base flow is the mean flow of the turbulent case. In each set of simulations, independent cases are run where the impulsive forcing is placed at different distances from the wall. We validate our approach against the work of Jovanović & Bamieh (2005).

RESULTS

The LIRF $\mathcal{H}_{i\to j}$ is a four-dimensional tensor with four independent variables (posiiton in streamwise, wall-normal and spanwise direction and time), plus one parameter made by the wall-normal position y_f where the forcing is applied, and with nine components given by the combination of three inputs i and three outputs j. The main outcome of the analysis is that the impulse response is largely anisotropic.The response $\mathcal{H}_{i\to j}$ changes depending on the couple

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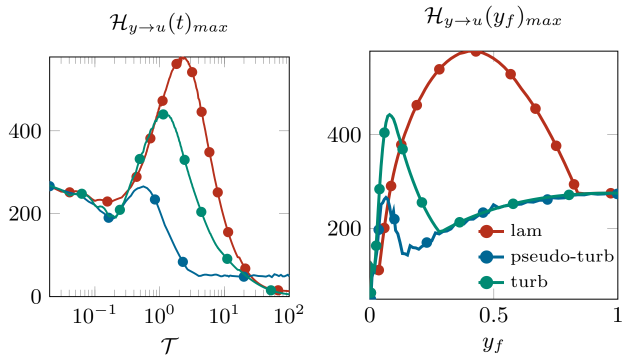

Figure 1. Absolute maxima as a function of the non-dimensional time $\mathcal{T}$ (left) and as a function of the forcing location y_f (right) for the laminar, pseudo-turbulent and turbulent cases. Positions $y_f = 0$ and $y_f = 1$ refer to the wall and the centerline, respectively.

input/output, on the regime i.e. laminar, pseudo-turbulent and turbulent and on the forcing location y_f. Overall the diagonal components of the tensor ($i = j$) are the largest, followed by the components involving the response in streamwise direction $\mathcal{H}_{y\to u}$ and $\mathcal{H}_{z\to u}$. They are of particular interest in the context of flow control for drag reduction via opposition control or spanwise forcing, respectively.

Observing the temporal evolution (see figure 1 left) and the dependence on the forcing location in wall-normal direction (see figure 1 right) of the component-wise maxima of the LIRF in physical space is an effective means to appreciate its anisotropy. Maxima refers to maxima over streamwise, wall-normal and spanwise locations for both, and over the forcing location and the time, respectively. Here only the response for $\mathcal{H}_{y\to u}$ is shown as an example. It shows a transient growth that can be explained by recalling the non-normal property of the eigenvectors of the Orr–Sommerfeld's linearized system. Laminar, pseudo-turbulent and turbulent show different behaviours with the turbulent case having the smallest peak in the response and the the fastest decay of the disturbance. This is due to the turbulent diffusion overcoming the amplification phenomena, particularly close to the wall. About how the maxima of $\mathcal{H}_{y\to u}$ depend upon the forcing distance, the general picture is that the position nearest to the wall yields the smallest response, and the centerline yields the largest. The right panel of 1 shows how the maxima of $\mathcal{H}_{y\to u}$ do not follow this paradigm. The peaks for the three regimes again differ with the turbulent one having the smaller amplification and placed closest to the wall. The position of the maximum response for the pseudo-turbulent and turbulent cases is compatible with the amplification of the streaks of the near-wall cycle. It means that the evolution of the near-wall structures have a linear component, but that it is not sufficient to describe their behaviour.

CONCLUSIONS

The present work has introduced the first DNS based measurement of the mean linear impulse response function (LIRF) for a channel flow, considering the response to an impulsive body force locate at various wall-normal positions, thus extending the work of Luchini *et al.* (2006), where an impulsive wall-normal velocity forcing at the wall was considered. The full time-space structure of mean linear response computed and measured in the present work by DNS is the best linear estimator of the linear dynamics of the turbulent channel flow when a body forcing is impulsively applied. This is of particular interest for the design of a feedback control for which the model of the plant (the channel flow in this case) is needed. This study highlights that relying on the impulse response of a laminar or a pseudo-turbulent (laminar with a superimposed base mean flow of the turbulent flow) case to model the dynamics of a turbulent channel flow can be useful to approximate the behaviour of the response, but fails to catch the temporal behaviour and the dependence on the the wall-normal forcing location of the response. Thus the approach presented here paves the way to more reliable estimation of the linear response of the turbulent flow and consequently a more effective control design.

References

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