Integral Formulation for Rectangular Wires in a Magneto-quasi-static Field

Luca Di Rienzo
Dipartimento di Elettronica, Informazione e Bioingegneria
Politecnico di Milano
Milano, Italy
luca.dirienzo@polimi.it

Pasquale Cambareri
Dipartimento di Elettronica, Informazione e Bioingegneria
Politecnico di Milano
Milano, Italy
pasquale.cambareri@polimi.it

Massimo Bechis
Prysmian S.p.A.
Milano, Italy
Bechis Massimo
massimo.bechis@prysmiangroup.com

Carlo de Falco
MOX—Modeling and Scientific Computing, Dipartimento di Matematica
Politecnico di Milano
carlo.defalco@polimi.it

***Abstract*— The formulation presented in [1] valid for magnetic and conductive wires of circular cross-sections is extended to the case of rectangular cross-sections. The new formulation is validated with canonical examples and then applied to the computation of eddy current losses in armors of tripolar submarine cables.**

Keywords*— *Integral equation formulation, magnetostatics, magneto-quasi-statics, eddy current losses.

I. Introduction

Rectangular wires are extensively used in many technological components that play a fundamental role in the infrastructures of electrical systems. For example, they are used in transformers or in armors of submarine cables. Depending on the application, such wires can be purely conductive or both conductive and ferromagnetic. An accurate numerical simulation of the electromagnetic behavior of these wires is required to calculate the power losses due to induced eddy currents and hysteresis in the case of ferromagnetic materials.

Standard formulations based on the Finite Element Method (FEM) typically lead to a large computational burden. On the other hand, integral formulations are a valid alternative. As a matter of fact, by reducing the computational geometry solely to the wires themselves, without the need of discretizing the surrounding volume as in FEM, they can significantly reduce the computational burden by requiring a lower number of degrees of freedom. An integral formulation for the calculation of eddy currents and hysteresis losses in rectangular wires is presented here.

II. Formulation

A. Magnetostatic Integral Formulation for a Single Wire

The present formulation is derived starting from the work developed in [1]-[3], which focused on wires of circular cross-section. In the presence of a wire with relative magnetic permeability $\mu_r \in \mathbb{R}$, the three-dimensional space can be partitioned into Ω_M, which is the domain occupied by the wire, and Ω_0, which is the remaining part, namely air. Under the influence of an external, known magnetostatic field $\mathbf{B}_0$ the wire is magnetized with a magnetization $\mathbf{M}$. The field $\mathbf{B}_M$ due to such a magnetization can be expressed with the well-known Biot-Savart integral formula [1].

Being $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_M$ the total flux density in the wire, the relation between magnetization and flux density is given by the integral equation

$$\mathbf{M}(\mathbf{x}) = \frac{\mu_r - 1}{\mu_0 \mu_r}\big(\mathbf{B}_0(\mathbf{x}) + \mathbf{B}_M(\mathbf{x})\big), \ \mathbf{x} \in \Omega_M \qquad (1)$$

B. Discretization

In order to apply the collocation method with constant elements, the wire is discretized into rectangular cuboids with thickness d, width w and height h, as shown by Fig. 1.

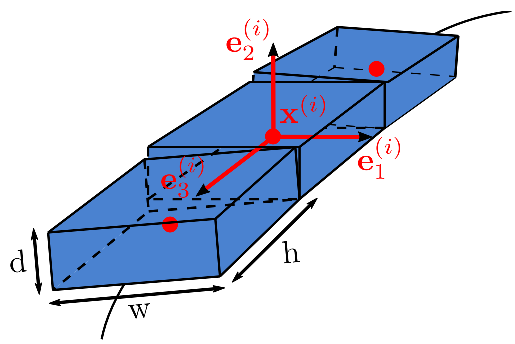

Fig. 1. Rectangular wire discretization. Collocation point $\mathbf{x}^{(i)}$ is the origin for the local coordinate system $\mathbf{e}_1^{(i)}, \mathbf{e}_2^{(i)}, \mathbf{e}_3^{(i)}$.

The geometric center of each cuboid will be indicated as $\mathbf{x}^{(i)} \in \Omega_M$, being $i \in \mathbb{N}$ an index to identify the cuboid. Point $\mathbf{x}^{(i)}$ is the collocation point where the supposed uniform magnetization is considered. In each collocation point a local frame of reference $\mathbf{e}_1^{(i)}, \mathbf{e}_2^{(i)}, \mathbf{e}_3^{(i)}$ is placed. The face of each cuboid perpendicular to the local direction $\mathbf{e}_3^{(i)}$ is referred to as the cross-section, S, and its area is given by $S = w \cdot d$.

It is convenient to formulate the discretized problem in terms of the integral of the magnetization over the cross-section, i.e. $\boldsymbol{\mathcal{M}} = \int_S \mathbf{M}(\mathbf{x}')d^2\mathbf{x}'$. Indicating with $\boldsymbol{m}^{(i)} = \mathbf{M}(\mathbf{x}^{(i)}) \cdot S$, collocating (1) in $\mathbf{x}^{(i)}$ leads to the linear system

$$\left(\boldsymbol{I} - \frac{(\mu_r - 1)S}{\mu_r \mu_0}\boldsymbol{A}\right) \cdot \boldsymbol{m} = \frac{(\mu_r - 1)S}{\mu_r \mu_0}\boldsymbol{b} \qquad (2)$$

where $\boldsymbol{I} \in \mathbb{R}^{3n \times 3n}$ is the identity matrix, with n the number of collocation points, whereas

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}^{(1,1)} & \cdots & \boldsymbol{A}^{(1,n)} \\ \vdots & \ddots & \vdots \\ \boldsymbol{A}^{(n,1)} & \cdots & \boldsymbol{A}^{(n,n)} \end{pmatrix}, \boldsymbol{m} = \begin{bmatrix} \boldsymbol{m}^{(1)} \\ \vdots \\ \boldsymbol{m}^{(n)} \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} \boldsymbol{B}_0^{(1)} \\ \vdots \\ \boldsymbol{B}_0^{(n)} \end{bmatrix}$$

The accurate computation of submatrices $\boldsymbol{A}^{(i,i)}$, defined as "self-elements", is crucial for the overall accuracy of the proposed method. In the full paper a new procedure based on the two-dimensional simulation of an infinite straight wire in a uniform magnetic field will be presented in detail.

C. Extension to the Magneto-quasi-static Case

As shown in [1]-[3], if the external magnetic field is time-harmonic, the effects of the presence of eddy currents in an infinitely long, straight magnetic wire with circular cross section can be mimicked, in an otherwise magnetostatic simulation, by substituting the physical value of the relative permeability of the wires with a complex tensor that can be computed analytically. In the case of rectangular cross section, the equivalent permeability tensor must be computed with 2D FEM simulations.

III. Numerical Results

The formulation is first implemented in magnetostatics for two 2D canonical test cases: two infinitely long straight rectangular conductors with cross-section of width $w = 12$ [mm] and thickness $d = 3$ [mm] (Fig. 2) are immersed in a uniform magnetostatic field oriented as the x-axis (for test case of Fig. 2a) and as y-axis (for test case of Fig. 2b). Both conductors have $\mu_r = 300$. In this case the unknowns are $\boldsymbol{m}^{(1)}$ for fist conductor and $\boldsymbol{m}^{(2)}$ for the second one and for symmetry $\boldsymbol{m}^{(1)} = \boldsymbol{m}^{(2)}$. The reference solution is obtained with FEM commercial software [4].

In Fig. 3 the relative error on the x and y component of $\boldsymbol{m}$ is plotted vs. the normalized distance $\Delta y/d$ for test case of Fig. 2a. Correspondingly in Fig. 4 the relative error on the x and y component of $\boldsymbol{m}$ is plotted vs. the normalized distance $\Delta x/w$ for test case of Fig. 2b.

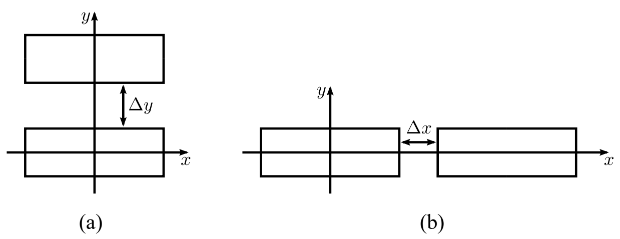

Fig. 2. Geometry of the magnetostatic test cases.

Then, the formulation is implemented to compute eddy current and hystheresis losses of an armor of a three-phase submarine cable made of ferromagnetic and conductive rectangular wires. Results are in good agreement with those of FEM [4] (model in Fig. 5). In the full paper all the details on the geometry and material parameters of the submarine cable will be provided and the proposed method will be compared with FEM for two different geometries.

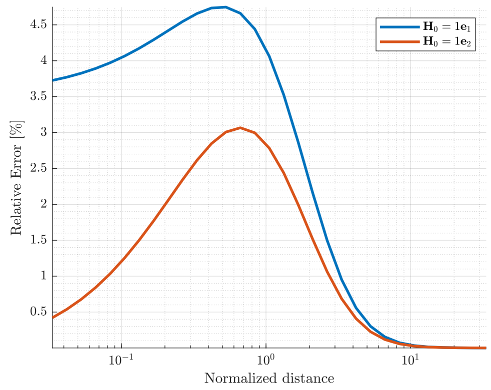

Fig. 3. Relative error on the x and y component of $\boldsymbol{m}$ vs. the normalized distance $\Delta y/d$ for test case of Fig. 2a.

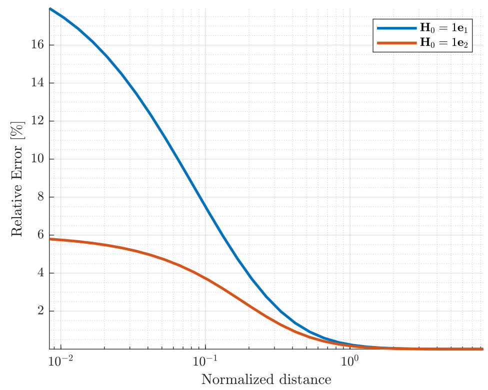

Fig. 4. Relative error on the x and y component of $\boldsymbol{m}$ vs. the normalized distance $\Delta x/w$ for test case of Fig. 2b.

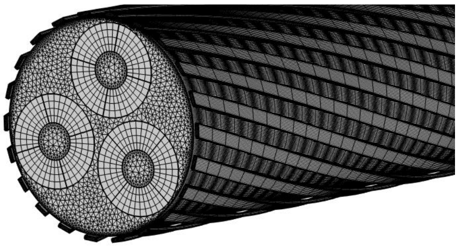

Fig. 5. FEM model of a tripolar submarine cable with armor made of rectangular wires.

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