



Confirmation by analogy

Francesco Nappo¹ 

Received: 2 May 2021 / Accepted: 15 January 2022 / Published online: 23 February 2022
© The Author(s) 2022

Abstract

This paper proposes a framework for representing in Bayesian terms the idea that analogical arguments of various degrees of strength may provide inductive support to yet untested scientific hypotheses. On this account, contextual information plays a crucial role in determining whether, and to what extent, a given similarity or dissimilarity between source and target may confirm an empirical hypothesis over a rival one. In addition to showing confirmation by analogy compatible with the adoption of a Bayesian standpoint, the proposal outlined in this paper reveals a close agreement between the fulfillment of Hesse's (Models and analogies in science, University of Notre Dame Press, 1963) criteria for analogical arguments capable of inductive support and the attribution of confirmatory power by the lights of Bayesian confirmation theory. In this sense, the Bayesian representation not only enriches a framework, Hesse's, of enduring relevance for understanding scientific activity, but may offer something akin to a proof of concept of it.

Keywords Analogical reasoning · Bayesian epistemology · Mary Hesse

1 Introduction

Thinking by analogy involves drawing comparisons between the features of two or more domains and is a form of cognition often used in scientific investigation (Bartha, 2009). An especially notable trait of many analogies in the history of science is that they were first invoked, not merely for the function of illustration, nor purely as heuristic devices, but to provide support to yet untested empirical hypotheses: some rational, though defeasible, ground in their favor. For instance, Darwin's argument for natural selection in *Origin of Species* revolved around an analogy with artificial selection, the selective process exercised by breeders on desirable traits of plants and animals; Maxwell (1890) supported his formulation of the electromagnetic laws

✉ Francesco Nappo
fr.nappo@gmail.com; francesco.nappo@polimi.it

¹ Department of Mathematics, Politecnico di Milano (Polytechnic University of Milan), Piazza Leonardo da Vinci, 32, 20131 Milano, MI, Italy

through a comparison with fluid dynamics; even today, many physicists place their bets on yet another analogy, first noted by Bekenstein (1972), between black holes and thermodynamic systems. In these and many other examples, we find that the scientific community is engaged in an attentive examination of an analogy's virtues and limits, in a comparison of its assumptions with known facts, and in the evaluation of the arguments drawn from it as more or less strong. These facts indicate an implicit commitment to the view (henceforth: the 'inductive view') that, in at least some cases, an analogy with a more familiar domain can be a source of non-negligible inductive support for hypotheses about the yet unknown (cf. Shelley, 2002; Wylie, 1985).¹

Is it possible to capture in a precise way the role of scientific arguments from analogy in supporting empirical hypotheses within Bayesian confirmation theory (henceforth: BC)? This question has received considerably less attention in the philosophy of science literature than the parallel issues about representing abductive reasoning in Bayesian terms (cf. Lange, 2020 and references therein). To find a suitable precedent, we must go back to the account that Mary Hesse put forward in her ground-breaking work on analogy in science (Hesse, 1963, pp. 107–112).² In brief, Hesse showed that analogical arguments can be confirmatory in BC's sense (of raising the probability of a hypothesis to more than the background knowledge alone) when some formal conditions are satisfied. However, as Hesse herself recognized, the conditions singled out by her model showed no clear relation to any of the informal criteria for when analogical arguments in science are considered capable of inductive support. That is why she noted: "this method adds nothing [...] for what it gains in precision it loses in the number and obscurity of assumptions that have to be made" (113). More recent attempts either depart from the aim of formalizing the contribution of an analogical argument in incremental terms, i.e., in terms of additional degrees of confirmation (Bartha, 2009; Pearl & Baremboim, 2013) or provide Bayesian analyses limited to specific modes of analogical inference in science (e.g., Dardashti et al., 2019; Sober, 2015).

Our aim in this paper is to show that there is reason to be more optimistic: there is, in fact, a general Bayesian pattern that promises to capture in a perspicuous way the central aspects of the logic of analogical inference in science. As we will present it, the probabilistic model only applies to circumstances in which some evidence contained in an analogical argument can be treated as new; we will not try to solve the problem of old

¹ Some philosophers (e.g., Agassi 1964; Crowther, 2019) are skeptical about the notion of inductive support by analogy—Thébaud 2019 goes as far as calling it the current "consensus view in philosophy" (188). We will not evaluate their arguments here (but see Bartha 2009, pp. 53–57). As discussed in Sect. 4.2, the arguments of the next few sections put indirect pressure on the skeptical view. A more moderate position is defended by Nyrup (2019), who claims that sometimes analogical arguments are not used to provide inductive support but merely to establish the scientific 'pursuit-worthiness' of a hypothesis. This position is consistent with the inductive view. See sub-Sect. 4.1 below; cf. also Nyrup (2019): "nothing I have said is meant to rule out that analogies can sometimes form the basis of a plausibility argument" (20).

² There exist several attempts at representing the confirmatory work of analogical reasoning within the framework of inductive logic by Carnap (1950): e.g., Niiniluoto (1981), Skyrms (1993), Festa (1997), Hutteger (2019). A source of difficulties for attempts in this tradition is the role of disanalogy in confirmation. Hesse noted this issue already in her (Hesse, 1963), arguing that Carnap's preference for "perfect analogies" obscured the more typical case of "imperfect" analogies, where both similarities and differences can confirm. Cf. also Sects. 3.2 and 4.1 below. A full evaluation of Carnapian accounts falls outside of the scope of this paper.

evidence for BC.³ Under the assumption that the analogical argument introduces some new evidence, precise conditions will be given regarding when a newly introduced similarity or dissimilarity between a source and a target may confirm (in BC's sense) a given hypothesis. Even though we will deny that the resulting probabilistic analysis offers a self-standing justification of the use of analogy in empirical investigation, it is an important feature of the proposal below that it reveals a close agreement between the fulfillment of Hesse's criteria for inductively strong analogical arguments in science and the attribution of confirmatory power from BC's standpoint. In this sense, the proposed representation may offer something akin to a proof of concept of Hesse's account: a verification of its consistency as a template for evaluating analogical arguments in science.

The discussion will proceed as follows. In section two, we will provide a first outline of the probabilistic model (which exploits a transitivity theorem by Roche & Schogenji, 2013) by showing how it captures some paradigmatic instances of good and bad reasoning from analogy. The examples will help illustrate exactly how contextual information (background knowledge) plays a role in licensing or disallowing confirmation by analogy. In section three, we will illustrate how the same framework extends to various other phenomena related to inference from analogy, including the role of dissimilarities and the relation between degree and weight of similarities. In section four, we will explain why the probabilistic representation advanced in this paper is not excessively demanding and why it can be useful despite not yielding a self-standing justification of the use of analogy in empirical investigations. In section five, we will provide a summary of what has been achieved and indicate directions for future research in this area.

2 The basic framework

It will be useful to start by clarifying exactly what we want from an account of confirmation by analogy (Sect. 2.1). We will then proceed to illustrating our proposed framework in a relatively clear case where such confirmation occurs: Galileo's argument for the Moon's irregular morphology (Sect. 2.2). Finally, we will consider how the same framework treats analogical arguments having the same structure as Galileo's, but where confirmation is either absent or weaker due to features of the background knowledge (Sect. 2.3). This will conclude our preliminary presentation of the account.

³ Cf. Bartha (2009, 2019) for the observation that analogical arguments often mobilize 'old evidence', i.e., information that is already part of the background knowledge when the argument is considered (cf. Glymour, 1980). We believe that such cases are easier to treat once we have a clear understanding of the way in which an analogical argument that introduces new evidence (as in the examples below) can provide confirmation. Solutions to the problem of old evidence for BC are readily available in the literature (e.g., Garber, 1983; Lange, 1999).

2.1 Desiderata

A ‘Bayesian model’ of confirmation from analogical argument must be understood in a specific way. Consider, as an illustration, a relatively clear case where such confirmation occurs. As Galileo (1610) observes in his *Sidereus Nuncius*, about three or four days after new Moon:

The boundary that divides the [Moon’s] dark part from the bright does not extend smoothly in an ellipse, as would happen in the case of a perfectly spherical body, but it is marked out in an irregular, uneven, and very wavy line. Now we have an appearance quite similar on the Earth at sunrise, when we behold the valleys, not yet flooded with light, but the mountains surrounding them on the opposite side to the Sun already ablaze with his beams. (Galileo, 1610, p. 53)

Galileo’s reasoning to the Moon’s irregular morphology consists in inferring a new similarity (irregular morphology) between model and target (the Earth and the Moon, respectively) from some observed similarities between them (the line patterns on the Moon’s surface and the similar shadow line patterns familiar on Earth). In this sense, it is analogical. Galileo’s is also a plausible example of confirmation by analogy since the argument’s conclusion is supported to some non-negligible degree by the similarities (in shadow patterns) figuring in the argument’s premises.

What we would like from a Bayesian account of confirmation by analogy is not some generic assurance that the telescopic observation of the wavy lines on the Moon confirms Galileo’s hypothesis in BC’s sense. For that, we may simply note that, if ‘E’ names Galileo’s observation and ‘H’ his hypothesis of the Moon’s irregular morphology, then plausibly $P(H|E) > P(H)$.⁴ The aim is, instead, to show that confirmation occurs solely in virtue of the fact that a relatively strong analogical argument is at work. For a comparison, consider the fact that, from knowing that both Wilhelm and George became rich and that Wilhelm has brown hair, one does not plausibly conclude that George likely has brown hair, too. A Bayesian model should be able to capture *precisely* what makes Galileo’s a relatively strong analogical argument and the one about George a weak one. Moreover, its diagnoses should be *perspicuous*. If the model’s verdicts are uninterpretable in informal terms, then, though we may have replaced the question about whether and how a given analogy confirms with a set of precise probabilistic conditions, we are arguably no more prepared to address the original question since we do not understand those conditions.

To illustrate the kind of diagnostic capacity that we require of a probabilistic account, it will be useful to consider how an informal evaluative template discerns good and bad reasoning from analogy. According to the influential framework developed by Hesse (1963), an analogical argument in science can provide non-negligible inductive support to the hypothesis that stands as its conclusion when (and only when) the following conditions are met⁵:

⁴ More precisely, $P(H|E \& K) > P(H|K)$. I will keep omitting K (viz. the background knowledge) below.

⁵ Note that for Hesse analogical arguments in science can be genuinely confirmatory, as opposed to merely establishing the prima facie ‘acceptability’ of a conclusion. See also Dardashti et al., (2019, p. 2) on this point.

- (1) *Materiality*: the similarities figuring in the analogical argument must be “material” or “pre-theoretic” (Hesse, 1963, p. 32), i.e., they must be cases of sharing of features that can be recognized as genuine respects of similarity, as opposed to artificial or gerrymandered respects (e.g., those of the ‘grue’ variety made famous by Goodman, 1955)⁶;
- (2) *Relevance*: the similarities and dissimilarities mentioned in the premises of the analogical argument must be relevant to the occurrence of the similarities mentioned in the argument’s conclusion, i.e., it must be a serious epistemic possibility, which may be further supported by the known similarities, that roughly the same non-accidental connections (stronger than correlation) that exist between the properties of the source also obtain between the known and the predicted properties of the target (Hesse, 1963, p. 77)⁷;
- (3) *No-Critical-Difference*: there must be no known differences between the analogy’s source and target which directly and in the first instance undermine the tenability of the analogical argument’s conclusion (Hesse, 1963, p. 70).

As widely acknowledged (e.g., Bartha, 2009, 2019; Chen, 2020), Hesse’s is a framework of enduring significance for understanding the logic of analogical inference in science.⁸ It plausibly explains, for instance, the difference between an argument such as Galileo’s and one such as George’s hair color. In the former, the known similarities (in wavy lines) were clearly relevant to the hypothesis of an irregular morphology on the Moon, since it was a live possibility that roughly the same causal connection that exists between mountains and shadows on Earth is replicated on the Moon; this fact, together with the expectation that lunar physics would not differ radically from terrestrial in ‘critical’ respects (e.g., in the way light behaves), made Galileo’s argument to the Moon’s irregular morphology a relatively strong one. Conversely, failure to satisfy Hesse’s relevance condition plausibly explains why the analogical inference from Wilhelm’s to George’s having brown hair is a weak one: the known similarities (becoming rich) arguably have no non-accidental connection to the predicted similarity (brown hair).

The question remains if we can do at least as well as the informal template, but using the precise probabilistic language of BC. The next subsection will spell out our new proposal.

⁶ It is a mistake to think that Hesse’s ‘material analogy’ means that source and target are similar in their matter; see, e.g., her (Hesse, 1963, p. 32) example of material analogy between light and sound. Hesse’s ‘materiality’ is tied to language (Hesse, 1963, p. 15) and serves mainly to rule out ‘gruesome’ respects of similarity that an analogical argument may introduce but are not countenanced as part of the accepted vocabulary of the scientific community.

⁷ Hesse (1963, p. 77) cashes out the relevance requirement in terms of the epistemic possibility that *causal connections* of the same kind as the ones that obtain between the properties of the source also obtain between the known and the merely predicted properties of the target. Bartha (2009) has shown that Hesse’s appeal to causal connections is too restrictive: sometimes entailment relations are at work. In what follows, we will speak generically of ‘relevance’ relations, setting aside the issue of precisely which relations fall into the class.

⁸ Bartha (2009, p. 39) raises objections to Hesse’s materiality requirement based on the frequent use of inferences from formal analogy in physics. We are skeptical that Bartha’s examples are formal analogies in Hesse’s sense but will set this issue aside in the following discussion. See also Nappo (2020) for some relevant distinctions.

2.2 The probabilistic conditions

Our aim is to show that Hesse's account has a natural representation in probabilistic terms and that satisfaction of those probabilistic conditions *entails* confirmation in BC's sense. To illustrate the central idea, let's consider how the proposed framework applies to Galileo's example. We start by defining the hypothesis that stands as the conclusion of Galileo's argument, as follows:

H The Moon has mountains and valleys.

The next step is to define the evidence contained in the analogical argument. This is arguably:

E Both the line patterns observed on the Moon's surface and those separating the dark from the bright parts of a valley at dawn are irregular and wavy.

Finally, we consider a hypothesis that serves as a 'bridge' between E and H, defined as follows:

G Earth and the Moon belong to the same kind of physical bodies (in having roughly similar irregular shape).

Confirmation of H by E can then be shown to depend probabilistically on three conditions:

A1 $0 < P(H), P(E) < 1$.

A2 $0 < P(G) < 1$, where:

(a) $P(H|G) > P(H)$.

(b) $P(H|E \ \& \ G) \geq P(H|G)$.

(c) $P(H|E \ \& \ \neg G) \geq P(H|\neg G)$

A3 $P(E|G) > P(E|\neg G)$

A1–A3 correspond at least roughly to the informal conditions defended by Hesse (1963). Specifically, A1 can be read as offering a minimal probabilistic rendition of the materiality requirement, which states that the similarities figuring in the analogical argument must be in scientifically acceptable respects. The role of the background knowledge is crucial here, since we want to exclude cases in which either the premises or the conclusion of the argument contain respects of similarity not countenanced in scientific language (such as the 'gruesome' predicates of Goodman, 1955). This can be translated into the assumption that both H (the conclusion) and E (the premise) are well-defined in the language \mathcal{L} over which the probability function P is defined, and such that they possess non-zero credence in light of the background K.⁹ To this, A1 adds the obvious further requirement that neither P(H) nor P(E) are equal to unity (which we can take as a tacit 'zeroth' requirement on inductive support by analogy on Hesse's account).

⁹ This approach is in line with standard treatments of the 'grue' problem in a Bayesian setting (cf. Sober, 1994), where hypotheses containing illegitimate language (e.g., 'grue') are assigned no or negligible prior probability. See also Sober (2015:119) on treating similarities and dissimilarities as evidence in a probabilistic framework.

A2 explicates probabilistically the relevance of the similarities figuring as the premises of the analogical argument. This is represented by one's assigning non-zero credence to a hypothesis G , stating roughly that causes of the same kind as the ones obtaining in the source also obtain in the target, and such that it can serve as a bridge for the transfer of confirmation from E to H .¹⁰ Specifically, A2(a) states that the bridge hypothesis is positively relevant to the conclusion H . This is obviously satisfied in Galileo's case-study since H is one (indeed, the only) hypothesis compatible with G and the background knowledge.¹¹ A2(b) and A2(c) add that G 'weakly screens' off E from H , which occurs just in case E does not disconfirm H conditional on G or (alternatively) on $\neg G$. Both A2(b) and A2(c) are plausible. Specifically, A2(c) can be justified on grounds that E is irrelevant to H when $\neg G$ is given (i.e., if it is known that the similarities in E have entirely different causes); hence, $P(H | E \& \neg G) = P(H | \neg G)$.¹² Altogether, the transitivity assumptions A2(a)-(c) ensure that, as one would expect in accordance with Hesse's relevance requirement, the new similarities contained in E bear in a non-negligible way on the predicted similarity contained in H only (or at least mostly) by way of bearing on the bridge hypothesis G , whereby the properties of source and target are linked non-accidentally.¹³

Finally, assumption A3 can be read as encoding probabilistically the judgment that lunar physics will not turn out to differ radically from terrestrial physics, so as to satisfy Hesse's no-critical-difference requirement.¹⁴ If, for instance, light probably behaves in the same way on the Moon as on Earth, and no other contextually important differences between the Moon and the Earth are known, then it is plausible that observed similarities between lunar wavy lines and terrestrial shadows are due to their having similar causes, as opposed to being the accidental result of different causes.¹⁵ In probabilistic terms, this is expressed by the claim that $P(E | G)$ is greater than $P(E | \neg G)$. Intuitively, the striking similarities in the wavy line patterns are to be expected if G is the case; they are much more surprising, instead, if the wavy lines observed

¹⁰ We exclude the case of $P(G) = 1$ since, as Hesse notes, if there is certainty that the same causes in the source also hold in the target, "the inference [to H] is purely causal and need not make any reference to [S]" (1963:73).

¹¹ In this case, G together with background knowledge *entails* H . Hence, $P(H | G)$ is unity and A2(a) holds. See 2.2 and 3.1 for examples in which there is no entailment between the bridge and the target hypothesis.

¹² See also the discussion in Sect. 4.1 on 'simple analogy' and the irrelevance of unsupported similarities.

¹³ Conditions A2(b) and (c) are weaker than if we had adopted a Bayesian network representation as in Bovens and Hartmann (2003). In particular, the ' \geq ' sign in A2(b) and (c) would have to be replaced by ' $=$ '. This is because conditional independence is assumed between any two child nodes on the network approach. In most cases, the difference is negligible since the known similarities confirm the predicted ones *only* by way of bearing on the intermediary G (and hence both A2(b) and (c) will be equalities). However, our weaker assumptions avoid questions about how strong claims of conditional independence are justified in some cases (see below), which instead arise for network approaches such as Feldbacher-Escamilla and Gebharter's (2020).

¹⁴ Strictly speaking, A3 also partly expresses the relevance of the similarities S to G (and hence corresponds partly to Hesse's relevance requirement). The correspondence with Hesse's conditions is not perfectly one-one.

¹⁵ Cf. Sober (2015, pp. 110–117) on the no-coincidence intuition in examples of inferences to common causes.

on the Moon had entirely different causes from the shadows produced by terrestrial mountains.¹⁶

It follows from a transitivity theorem by Roche and Shogenji (2013; see Appendix) that A1-A3 entail $P(H | E) > P(H)$. Importantly, this is not to say that an agent's posterior in H upon introducing E ought to be higher than any of H's rivals—whether this is so or not depends on the priors. What the proof shows is that there is incremental confirmation by analogy from BC's standpoint. This is important not only because it shows that granting confirmatory power to analogical arguments is compatible with adopting a Bayesian standpoint, but also because the conditions singled out by the model correspond roughly to Hesse's (1963) requirements. In particular, the representation by means of the 'bridge' hypothesis helps specify in what sense the similarities mentioned as premises in the analogical argument are confirmatory only if relevant: roughly, the similarities are confirmatory to the extent that they bear on the hypothesis of commonality in non-accidental features. The model therefore makes precise (and can be interpreted via) an independently plausible informal account for evaluating analogical inference in science. To show that the proposal just outlined has exactly the features that we require, let's now turn to illustrate how the same framework treats arguments weaker than Galileo's.

2.3 How the conditions work

Let's start with a spurious case. Let's imagine replacing every instance of H, E and G in A1-A3 with, respectively, H*, E* and G*, where H* is the hypothesis that George has brown hair, E* the evidence that George and Wilhelm became rich, and G* the bridge hypothesis:

G* Wilhelm and George belong to the same class of people (in owing their fortunes to their hair color).

As discussed in informal terms, the argument represents a violation of Hesse's requirement of relevance. Specifically, what seems to be missing in this scenario is a reason for thinking that becoming rich is linked non-accidentally to hair color. In probabilistic terms, we can note that, while H* is a live hypothesis, G* can be safely assumed to possess negligible probability in light of our background knowledge, i.e., a probability of either zero or infinitesimal.¹⁷ Accordingly, we have a violation of A2*. To be clear, this is not to say that there may never be agents whose background knowledge justifies A2*. For such agents, observing that both George and Wilhelm made a fortune may be evidence of George's brown hair (assuming that A1* and A3* are also acceptable for them). Our point here is merely to explain why, with no additional information

¹⁶ Note that A3 is not nearly as plausible if one substitutes the similarity E with a description of Galileo's observations considered in isolation: e.g., 'there are wavy lines on the Moon'. Intuitively, what is most striking about Galileo's discovery is not that wavy lines are observed on the Moon's surface (which may well be due to a defect of the telescope), but that the line patterns look exactly like the shadows cast by terrestrial mountains.

¹⁷ Note that this is a different notion than the relativized notion of 'negligible' probability defended by Bartha (2009, chapter 8). Here and elsewhere, 'negligible' probability means a probability of zero or infinitesimal.

other than what we would ordinarily assume, the reasoning to George's brown hair is flawed.¹⁸

Of course, not every analogical argument is as recognizably strong as Galileo's or as recognizably weak as the hair color case. The examples where the diagnostic capacity of the Bayesian framework is most useful are the intermediate ones. To illustrate, consider Schauman's reasoning (discussed in Bartha, 2009, p. 97) to the narcotic effects of meperidine on mice from the fact that it produced an effect on their tails—an S-shaped curvature—that was previously observed only with another narcotic having a similar chemical structure, morphine. Schauman's inference from the analogy with morphine led to the discovery that meperidine is an effective painkiller.¹⁹ In this case, one might be uncertain as to whether, and to what extent, the similarity in tail curvature *confirmed* the claim that meperidine has narcotic powers, as opposed to merely suggesting (in the psychological sense) a hypothesis that later turned out to be correct.

The conditions that our account proposes for confirmation by analogy can help us make the question more precise (and still perfectly intelligible). Let 'U' be the hypothesis that meperidine has narcotic powers on mice. Let 'L' be the evidence that both morphine and meperidine produce the same S-shaped curvature on mice's tails. Finally, let 'M' be the bridge hypothesis:

M Meperidine and morphine belong to the same family of chemicals (in having roughly similar pharmacological powers).

Confirmation of U by L then occurs just in case the following conditions are met:

B1 $0 < P(U), P(L) < 1$.

B2 $0 < P(M) < 1$, where:

- (a) $P(U | M) > P(U)$.
- (b) $P(U | L \ \& \ M) \geq P(U | M)$.
- (c) $P(U | L \ \& \ \neg M) \geq P(U | \neg M)$

B3 $P(L | M) > P(L | \neg M)$

The above conditions help us identify potential sources of uncertainty about the confirmatory status of the argument. In this case, unlike the one about hair color, it seems plausible to treat M as a live hypothesis since it is not ruled out by experience. Yet, it may still be questioned whether the new similarities are sufficiently relevant to confirm the predicted similarity. This issue is reflected neatly in the probabilistic representation. One problem is that, whereas A2(a) in Galileo's example was ensured by the fact that the bridge hypothesis G entailed H, we have no such assurance in the case of B2(a); indeed, someone may question the justification for the inequality, on

¹⁸ Cf. Dardashti et al (2019), who note that negligible confirmation by analogue simulation occurs between any two systems since "strictly speaking, background assumptions regarding different systems are never probabilistically independent" (6). As the authors note, these weak forms of confirmation can be neglected.

¹⁹ A second analogical inference, from the similarities between mice and humans, confirmed that meperidine is efficacious as a pain-killer in human beings. In this sub-section, we will focus solely on the confirmation of the narcotic powers of meperidine on tested mice from the similarities in tail curvature; see also Sub-sect. 3.1.

grounds that a narcotic capacity is a very specific power that may not transfer from morphine to meperidine (even if the two have other similar powers). Moreover, even though B3 sounds plausible, it may be argued that $P(L | M)$ is only ever slightly greater than $P(L | \neg M)$ on grounds that the similarity in S-shaped tail curvature is a fairly superficial resemblance (see also fn. 14); in this case, the confirmation would only be negligible. Accordingly, the analogical argument may be thought of as fulfilling a heuristic, rather than an inductive, function.

However, the confirmatory potential of Schauman's reasoning may be rescued if we were able to identify contextual information that would justify B2(a) and B3. For instance, it may be thought that in 1939, when Schauman's discovery was made, significant evidence had been gathered with regards to the existence of families of substances with similar chemical structures targeting the same biological processes. For instance, in 1893 the structurally similar substances amphetamine and methamphetamine were isolated, both of which turned out to be powerful stimulants of the central nervous system.²⁰ If, based on background knowledge, Schauman had reason to believe that meperidine and morphine were related in the same way, then some ground for accepting B2(a) would have been present. Moreover, if Schauman was justified in regarding the S-shaped tail curvature as a relatively rare effect, then the fact that meperidine causes the same effect as morphine would have been ground for taking $P(L | M)$ to be greater than $P(L | \neg M)$. As in the example of Galileo's wavy lines, the difference in likelihoods would have reflected an expectation that a similarity in such a rare respect as the S-shaped curvature would be relatively unlikely to observe if morphine and meperidine were chemicals of entirely different kinds, but would be more likely if morphine and meperidine belonged to the same chemical family. Complemented with the right kind of supporting information, then, the analogical inference to meperidine's narcotic powers could be understood as genuinely confirmatory.

Our preliminary presentation of our Bayesian model for confirmation by analogical argument is now complete. As the previous examples aim to show, the proposal spells out in a precise probabilistic language exactly which parts of the background knowledge make a difference to whether, and to what extent, confirmation by analogy occurs. The account on offer can not only distinguish clear cases of good and bad analogical arguments, but also help us diagnose the key sources of uncertainty in less clear cases. In this way, the proposal serves to identify the pieces of extra contextual information that would be required for non-negligible confirmation by analogy. The next section will present extensions of this basic framework.

3 Extensions

It is useful to start by showing how the same Bayesian model outlined in section two treats analogical inferences of a different kind from the ones discussed in the previous section: 'predictive' analogical arguments (Sect. 3.1). The case of disconfirmation in analogical argument will then be discussed (Sect. 3.2). Finally, some comments will

²⁰ See also Brown (2004) for an extensive discussion of other examples of analogical inferences in chemistry.

be made about the notion of weight of similarities and dissimilarities and its relation to graded similarity (Sect. 3.3). These extensions are evidence of the high generalizability and non-adhocness of the proposed Bayesian analysis.

3.1 Predictive Analogies

The inferences reviewed in the previous section are all instances of analogical arguments ‘from effects to causes’– ‘abductive’ analogies in Bartha’s (2009) taxonomy: from the similarities between some *effects* (e.g., the wavy line patterns), one reasons that the *causes* may well be similar (e.g., that the Moon has an irregular morphology like the Earth’s). Examples of this kind have been discussed in detail by Sober (2015, p. 118), who has put forward a more complex representation of them in probabilistic terms. The simpler recipe indicated above, which consists in identifying a ‘bridge’ hypothesis (such as G) positively correlated to both the evidence contained in the analogical argument (E) and the hypothesis that stands as its conclusion (H), can be easily adapted to the examples of abductive analogical arguments that Sober discusses. Since the application to the analogical arguments considered by Sober is a fairly uninteresting extension of the discussion of the previous section, we will simply omit it in what follows.

A more interesting result for the purposes of this paper is that the Bayesian representation offered in section two may also fit analogical arguments of a different kind than those discussed by Sober (2015) and exemplified by Galileo’s case. We have in mind analogical arguments ‘from causes to effects’—‘predictive’ analogical arguments in Bartha’s (2009) classification. An example would be cases of reasoning from a simulation: from the known similarities in ‘causes’ between the simulation and the target system being simulated, one reasons that roughly the same effect observed in the simulation may well obtain in the unfamiliar target system.²¹ As a way of illustrating how this category of analogical inferences may be represented by the same Bayesian model of section two, let’s consider a somewhat simplified version of an actual episode from the history of medicine: specifically, of how evidence about the efficacy of inactivated polioviruses (the cause of poliomyelitis) in a population of rhesus monkeys confirmed Salk’s (1955) yet untested hypothesis of the efficacy and safety of the same vaccine in human populations.²²

²¹ Dardashti et al. (2019) have argued for the existence of a distinctive category of inductive inferences that they call from ‘analogue simulation’ and construe as based on syntactic isomorphisms (as opposed to material similarities). In this paper, we will set aside the question whether Dardashti et al. are right to construe analogue simulations as *sui generis* and non-material analogical inferences. But notice that, even if they are right, their framework for analogue simulations does not extend to ordinary cases of confirmation by analogy (*pace* Feldbacher-Escamilla and Gebharder 2020): e.g., Galileo’s argument, which is clearly based material similarities and not on precise mathematical isomorphisms between the models for the Moon and the Earth.

²² Analogical inferences of this type are the object of analysis by Pearl and Baremboim (2013), who adopt formal tools in the attempt to establish precise conditions under which results obtained in the source are ‘transportable’ onto a target. Because judgments of transportability are subject to pragmatic considerations, we are skeptical that an algorithmic procedure can be identified. At any rate, our focus in this paper is not absolute judgments of transportability but on the incremental notion of confirmation by similarity or

As a preliminary point, we note that Hesse's (1963) requirements for analogical arguments capable of inductive support (1–3) plausibly hold for the new case-study. (Hesse's conditions are stated precisely so as to apply to both abductive and predictive analogies.) First, the similarities between rhesus monkeys and humans figuring in the analogical argument were all resemblances in scientifically respectable properties, such as sharing similar immune systems. Second, the similarities figuring as the premises of the analogical argument were plausibly relevant to the prediction that the vaccine is efficacious in human beings, since the biological features in which humans and rhesus monkeys are similar may be causally connected to the vaccine response. Finally, no critical differences between humans and rhesus monkeys were known with regards to the reaction to the poliovirus: indeed, rhesus monkeys were known to suffer from the very same symptoms as humans (paralysis) when the poliovirus was transmitted to them. These features make it plausible that confirmation by analogy occurs, meaning that the observed efficacy of the vaccine in rhesus monkeys is at least some evidence for its efficacy in human beings.

An important difference with abductive analogical arguments remains, which is crucial for the purpose of building an appropriate Bayesian model. This has to do with precisely how inferences from predictive analogical arguments work. As Galileo's case-study illustrates, the similarities and dissimilarities which figure in an abductive analogical argument are relevant to the extent that they bear on the hypothesis (e.g., G) that roughly the *same kinds of causes* that obtain in the source also obtain in the target; this, in turn, makes it plausible to expect further similarities between source and target. For instance, the similarities between lunar wavy lines and terrestrial shadows make plausible the hypothesis that the same irregular morphology that we observe on Earth is realized on the Moon, from which we can in turn derive that the Moon has an irregular morphology.²³ In predictive analogies, instead, the similarities and dissimilarities in, e.g., the immune system between rhesus monkeys and humans, are relevant to the extent that they bear on the hypothesis that a given *effect is relatively robust* to differences (within a certain range), from which we can defeasibly infer that the effect observed in the source will have an analogue in the target: e.g., humans will respond to the vaccine in the same way as monkeys. Hence, the focus in predictive analogical arguments is mostly on the *robustness* of the effect.²⁴

With this in mind, let's consider how an argument from the efficacy of a vaccine in rhesus monkeys to a similar outcome in human beings can be represented in the Bayesian framework. For this purpose, let's stipulate that 'O' is new evidence that

Footnote 22 continued

dissimilarity. See also Van Eersel et al. (2019) for an informal discussion of predictive analogical arguments in psychology.

²³ As model A1–A3 clarifies, confirmation is fueled partly by the judgment that it would be something of a striking coincidence to find such similarities between the shadow patterns unless they were produced by the same kinds of causes. See also fn. 16 and Sober (2015, p. 116) for discussion on the no-coincidence intuition.

²⁴ This is consistent with the fact that abductive and predictive analogical argument are subject to the same requirement of relevance: in both cases, the similarities mentioned as premises must be relevant to those mentioned in the conclusion. Our point here is simply that *the way in which* the similarities are relevant differs.

inactivated polioviruses are safe and efficacious on rhesus monkeys.²⁵ Let ‘T’ be the prediction of a similar outcome in human populations. We now introduce the bridge hypothesis ‘X’, defined as follows:

X Within the range of variation that contains rhesus monkeys and humans, the vaccine’s effects are relatively robust to differences at the level of proprietary biological features.

In other words, X is the bridge hypothesis that the vaccine’s effects will tend, under a variety of changes in background conditions, to remain the same (in this sense, they are ‘relatively robust’).

Given these stipulations, the following inequalities plausibly hold in this case-study:

$$C1 \quad 0 < P(T), P(O) < 1,$$

$$C2 \quad 0 < P(X) < 1, \text{ where:}$$

$$(a) \quad P(T | X) > P(T)$$

$$(b) \quad P(T | O \& X) \geq P(T | X)$$

$$(c) \quad P(T | O \& \neg X) \geq P(T | \neg X)$$

$$C3 \quad P(O | X) > P(O | \neg X).$$

C1–C3 correspond respectively to the three informal requirements by Hesse (1–3 above) on analogical arguments capable of inductive support. C1 is a probabilistic rendition of the requirement of materiality, stating that the similarities figuring in the analogical argument must be in scientifically respectable features. This gets translated into the claim that both T and O possess well-defined, non-extremal probabilities given the background knowledge.

C2 encodes probabilistically Hesse’s requirement of relevance: non-negligible credence to the bridge hypothesis X is justified, which ties the evidence O together with the prediction T from the standpoint of confirmation. In this case, as with B2(a), we do not have an entailment between X and T, which would make the C2(a) trivially satisfied; however, conditioning on the claim that the vaccine’s effects tend to remain the same under a large variety of antecedent conditions, as required for satisfying the ‘robustness’ claim, makes the hypothesis that the vaccine is efficacious and safe in humans likely. Hence, C2(a) is plausible. The remaining assumptions, C2(b) and (c) are also plausible in the example. As for C2(b), new evidence that inactivated polioviruses are safe and efficacious on monkeys is arguably *irrelevant* to T if we already know that the vaccine’s efficacy is robust to differences between monkeys and humans; hence $P(T | O \& X) = P(T | X)$. As for C2(c), the hypothesis that the vaccine is safe and efficacious in humans is arguably not made *less* probable by new evidence that inactivated polioviruses are safe and efficacious on monkeys, even if the vaccine’s effects are not robust to differences between monkeys and humans; plausibly, then, $P(T | O \& \neg X) \geq P(T | \neg X)$.

²⁵ Note that, in this case, the new evidence O does not express a fact of similarity or dissimilarity (as E does in Galileo’s example). This is how it should be: in predictive analogical arguments, the relevant similarities are often best understood as part of the background knowledge (embodied in the prior probability assignments) rather than as constituting the new evidence that the argument introduces (cf. also Dardashti et al., 2019).

Finally, C3 encodes probabilistically one's acceptance of the no-critical-difference condition. Specifically, our background knowledge about the features of monkeys and humans and the fact that no critical differences were known regarding the expected reaction of rhesus monkeys to poliovirus compared to humans made it plausible to set $P(O|X)$ greater than $P(O|\neg X)$.²⁶

Altogether, C1-C3 entail that $P(T|O) > P(T)$, which is exactly what we would expect from a predictive analogical argument that meets Hesse's requirements for inductive support.

As the reader may appreciate by comparing C1-C3 to A1-A3, the conditions for confirmation by analogy are formally the same. The main difference with the Bayesian model of section two lies with the content of the 'bridge' hypothesis. However, we contend that this is not an ad hoc fix. It is supported by the observation that abductive and predictive analogies have different inferential profiles (cf. Bartha, 2009, p. 167): in abductive analogies, the inference from the observed to the merely predicted similarities is mediated by the claim that some effect in the target has *causes of the same kind* as those in the source (e.g., the same kind of irregular morphology); in predictive analogies, instead, the inference from observed to predicted similarities proceeds through the intermediate claim that some salient *effect* (e.g., vaccine efficacy) is *robust* to low-level differences in causes between source and target (e.g., differences in the specific details of the immune system between monkeys and humans). The Bayesian model relies on this plausible observation to describe the mechanism of confirmation for each kind of analogical argument.

To sum up, what we have sketched in C1-C3 is a plausible account of how a predictive analogical argument in science can be confirmatory from BC's standpoint. Although the extension to other realistic examples will have to be left for a separate occasion, we hope that the illustration above helps clarify the general recipe that we are proposing. In each case, the idea is to identify a plausible bridge hypothesis stating that a salient effect (the one figuring in the argument's conclusion) is robust to differences in some low-level detail between source and target and such that confirmation of the bridge hypothesis can in turn bear on the argument's conclusion.²⁷ Based on its correspondence with independently plausible ideas about when and how inductive support by predictive analogical argument occurs, we have reasonable confidence that this recipe may be widely applicable. In order to complete our case that the Bayesian analysis proposed in this paper deserves serious consideration, in what follows we will highlight a few more aspects of the logic of analogy that the current proposal may be able to capture.

²⁶ C3 is equivalent to the claim that $P(X|O) > P(X)$, which is highly plausible in this case-study due to the similarities in biological features between monkeys and humans that are part of the background. Again, we must also stress that C3 partly encodes a judgment of relevance in addition to one of no-critical-difference.

²⁷ Robustness can in turn be analyzed in terms of some counterfactual property. We take no stance here as to how to formalize this notion. Cf. also hypothesis 'X' in the Bayesian model of Dardashti et al. (2019).

3.2 Dissimilarity

An adequate treatment of dissimilarity is one of the *desiderata* of an account of confirmation by analogy. In what follows, we will offer hints towards accommodating the role of dissimilarities within the same Bayesian framework proposed in the previous sections. The most common case, though by no means the only one, is when a dissimilarity weakens the conclusion of an analogical argument. Consider, for instance, Darwin's argument for natural selection in the *Origin of Species*: from the fact that domesticated animals possess a wide variety of traits due to human breeding, and the similarly wide trait variety found in wild animals and plants, Darwin argued that a selection process analogous to the one imposed by human breeders on domesticated animals may be responsible for the wide variety in heritable traits found in nature (cf. Kitcher, 2003:61). One fact of dissimilarity that Darwin considered in spelling out his argument was the fact that artificial selection had not been observed to produce events of speciation (Darwin, 1859:389). This dissimilarity plausibly disconfirms the hypothesis of natural selection to some degree (but see 3.3 on the weight of the dissimilarity). Let's consider how this disconfirmation by dissimilarity can be represented within the framework outlined in the previous sections.

We will make use of the following stipulations. Let 'W' be Darwin's hypothesis of natural selection. Let 'D' be the observation (which we assume to be new) that artificial selection has not given rise to new species of animals or plants. As Darwin's argument has the form of an abductive analogical argument, the relevant bridge hypothesis will be defined as follows:

- J Trait variation in nature and in domesticated animals belong to the same class of processes (in undergoing selection).

The formal conditions for disconfirmation by dissimilarity can be derived immediately by symmetry with the conditions of confirmation by similarity (1–3) and are as follows:

$$D1 \quad 0 < P(W), P(D) < 1.$$

$$D2 \quad D2. \quad 0 < P(J) < 1, \text{ where:}$$

- (a) $P(\neg W \mid \neg J) > P(\neg W)$
- (b) $P(\neg W \mid D \ \& \ \neg J) \geq P(\neg W \mid \neg J)$
- (c) $P(\neg W \mid D \ \& \ J) \geq P(\neg W \mid J)$

$$D3 \quad P(D \mid \neg J) > P(D \mid J)$$

Conditions D1-D3 state three plausible conditions for disconfirmation by dissimilarity that have natural explications in informal terms: namely, the dissimilarities in the argument are in scientifically respectable properties (D1), they are negatively relevant to the conclusion of the analogical argument (D2), and no critical difference is known between source and target that would undermine the conclusion of the analogical argument even before the new dissimilarity is introduced (D3). It is a trivial corollary of Roche and Shogenji's transitivity theorem that D1-D3 together entail that $P(\neg W \mid D) > P(\neg W)$, which means that D disconfirms Darwin's hypothesis of natural selection. This is exactly what we would expect in the case in which we learn some new fact of dissimilarity that disconfirms the conclusion of an analogical argument.

Even though this covers the most typical case, the treatment of dissimilarity just outlined is not complete, since not all dissimilarities weaken an analogical argument (even though they weaken the analogy): some of them may even strengthen the argument.²⁸ Consider, for instance, Darwin's point that natural selection is a much more pervasive force than artificial selection, acting at various stages of the life of an organism (e.g. at the embryonal stage as well as at adult stage), with respect to so many parts of an organism, whereas artificial selection depends on the time, commitment, and physical limitations of the breeder. This dissimilarity figures prominently in Darwin's argument for the natural selection hypothesis in the *Origin*. As Darwin writes:

Why, if man can by patience select variations most useful to himself, should nature fail in selecting variations useful... to her living products? What limit can be put to this power, acting during long ages and rigidly scrutinizing [the traits of] each creature? (1859, p. 379)

In this case, it is plausible that the dissimilarity confirms, rather than disconfirms, Darwin's hypothesis that trait variation in nature is due to an underlying process of natural selection.

The Bayesian representation of this paper gives us precise conditions for confirmation by dissimilarity. Letting 'Z' be the fact that natural selection is a more pervasive force than artificial selection, Z's capacity to confirm W can be cashed out in the (by now) familiar way:

E1 $0 < P(W), P(Z) < 1$.

E2 $2. 0 < P(J) < 1$, where:

(a) $P(W | J) > P(W)$

(b) $P(W | Z \& J) \geq P(W | J)$

(c) $P(W | Z \& \neg J) \geq P(W | \neg J)$

E3 $P(Z | J) > P(Z | \neg J)$

The relevant conditions here are E2(b), E2(c), and E3. Stated informally, the formal model tells us that Z can confirm W only if Z bears confirmation-wise on J (as E3 requires) and Z does not disconfirm W conditional on J or on $\neg J$ (as E2 requires). These conditions are independently credible requirements for a given dissimilarity to provide confirmation; moreover, they are all plausibly satisfied in the specific case of Darwin's argument for the theory of natural selection.

3.3 Of weight and degree

The proposal on offer makes yet no room for a notion that is often associated with the idea of confirmation from analogy: *degree* of similarity (Carnap, 1980). In itself, this need not be a defect of the account. Hesse (1963:115) already questioned the relevance of the notion of graded similarity for understanding confirmation by analogy,

²⁸ Similarly, similarities may sometimes disconfirm a hypothesis. Typically, the requirement of relevance fails in those cases. From a probabilistic perspective, the transitivity assumptions 2(a) or (b) then fail to hold.

on grounds that dissimilarities may sometimes weigh just as much as similarities in favor of the conclusion of an analogical argument. In this sub-section, our aim is to use the formal framework developed in the previous sections to explain why there is at least a defeasible association of confirmation by analogy with greater degree of similarity, even though it is not always the case that more similarity means more confirmation. In other words, we will offer the sketch of a *reductive* account of graded similarity by means of a theory of weight of similarity. This account is further evidence of the capacity of the proposal on offer to correctly represent the logic of analogical inference.

To do so, we propose to introduce the notion of *weight* of a given similarity or dissimilarity. Paralleling Good's (1984) standard account of the weight of evidence, we note that, for any (set of) similarities or dissimilarities S , bridge hypothesis B bearing on the conclusion of an analogical argument L , S 's weight w is plausibly given by the following ratio:

$$w: P(S | B) / P(S | \neg B)$$

The more this ratio diverges from unity, the more we say that S weighs either favorably or unfavorably in confirmation. This proposal is in line with the informal account by Hesse (1963) that we take as our starting point: a given similarity or dissimilarity has more weight the more it bears on the claim that observations in the target have the same kinds of causes as those in the source (in abductive analogical arguments) or the claim that the effects observed in the source are robust to the known differences with the target system (in predictive analogical arguments).

With the notion of weight of similarity, we can recover a role for graded similarity indirectly (as it were) in confirmation. To adopt an intuitive illustration, consider three potential telescopic observations of shadow patterns on the Moon's surface: let 'M' be a potential observation of extremely wavy lines on the surface of the Moon (much more irregularly shaped than terrestrial shadows); 'E' a potential observation of moderately wavy lines on the Moon, similar to the shadows cast by terrestrial mountains; finally, let 'C' be a potential observation of mostly elliptical wavy lines. The following inequalities plausibly hold in this scenario:

$$m: P(M | G) / P(M | \neg G) \approx 1 \text{ (read: it is almost equal to 1)}$$

$$e: P(E | G) / P(E | \neg G) > 1$$

$$c: P(C | G) / P(C | \neg G) < 1$$

What this comparison is meant to show is that, under a plausible assignment of conditional probabilities, it follows that the more similar the lunar wavy lines are to terrestrial shadows, the more the evidence will favorably weigh on the bridge G .²⁹ Of course, this is by no means true in all cases: as discussed in Sect. 3.2, sometimes dissimilarities confirm more than similarities. The point of this illustration is merely to explain why, by considering some realistic examples, there exists at least a defeasible association between more similarity and more confirmation. This is because, as illustrated by Galileo's example, in many cases a larger match in observable features (as in e) bears comparatively more in confirmation. We contend that this 'reductive'

²⁹ For questions about the justification of the relevant conditional probability assignments, see sub-Sect. 4.2. Cf. also Sober (2015, p. 121) for a discussion of similarity in confirmation in the spirit of the current proposal.

account is all what we could ever want with regards to graded similarity and its relation to confirmation.

Before leaving the discussion of the formal details, one final feature of the Bayesian representation may be noted. In some cases, an argument by analogy may be reinforced, not by pointing out some further similarities between the domains being compared, but by arguing that some of the known dissimilarities are *unimportant*. For instance, consider again the objection that artificial selection has not been observed to produce events of speciation. A defender of natural selection might point out that: (a) the practice of breeding is relatively recent, in geological times, when compared to the timeframe of natural selection; (b) breeders have not attempted to produce events of speciation (or may have even acted against it). We may represent this formally by noting that, although plausibly $P(J | D) < P(J)$, it is also true that:

$$d: P(D | J) / P(D | \neg J) \approx 1$$

This gives us a way of representing how, under a reasonable choice of prior probabilities, dissimilarity D would not weigh significantly against Darwin's bridge hypothesis J , and therefore be also relatively unimportant in disconfirming the hypothesis of natural selection.

To sum up the discussion of the previous sub-sections, the proposal of section two turns out to naturally extend to a surprising number of phenomena related to the logic of analogical inference. In particular, we have offered sketches of how the Bayesian analysis may extend to predictive analogical arguments, of how it could offer a perspicuous and compelling account of the role of dissimilarities in analogical reasoning, as well as provide a neat probabilistic rendering of the notions of weight and grade of similarity and dissimilarity (which avoids various logical pitfalls). In the next section, we will respond to two important objections to our proposal.

4 Objections and replies

The previous sections advanced a proposal for a Bayesian analysis of the confirmatory work of analogical arguments in science. Even though we have focused on the case in which an analogical argument introduces a single piece of similarity or dissimilarity, the generalization to the case of a plurality of similarities and dissimilarities is straightforward. For a bridge hypothesis B which bears on the confirmation of a conclusion T , and a finitely long list of similarities and dissimilarities $S1, D1, S2, D2, \dots$, the odds formulation of Bayes' theorem holds:

$$P(B | S1, D1, S2, D2, \dots) / P(\neg B | S1, D1, S2, D2, \dots) = [P(B) / P(\neg B)] \times [P(S1 | B) / P(S1 | \neg B)] \times [P(D1 | B) / P(D1 | \neg B)] \times [P(S2 | B) / P(S2 | \neg B)] \times [P(D2 | B) / P(D2 | \neg B)] \dots$$

This formulation captures the (often diverging) contributions of newly introduced similarities and dissimilarities in an analogical argument to the confirmation of the argument's 'bridge' hypothesis, which we assume to be positively relevant to the argument's main conclusion.

An important feature of the Bayesian analysis proposed in this paper is that it is unitary: *modulo* changes in the salient ‘bridge’ hypothesis, the same formal conditions (which in turn correspond to Hesse’s general criteria) promise to represent many different phenomena related to confirmation by analogy. But an equally important feature of our proposal, which did not emerge in the previous discussion, is its lack of artificiality. Specifically, we refer to the fact that the conditions for confirmation from a material analogical argument turn out to be instances of the more general phenomenon of transitivity of confirmation. We believe that this is far from being a coincidence. That there exists a connection between analogy and transitivity of confirmation was already noted by Hesse, who speculated (Hesse, 1970, p. 57–58) that the judgment of material analogy between a source and a target is a possible enabling condition for evidence collected in the source to bear confirmation-wise on hypotheses about a target.³⁰ In addition, one must stress that Roche and Shogenji’s (2013) theorem states the *weakest* conditions for transitivity of confirmation (cf. their discussion on 2013, p. 7). The proposal of this paper can therefore be understood as vindicating Hesse’s conjecture: material analogies allow for the transfer of confirmation precisely by satisfying the most general conditions for transitive confirmation.

Let us now consider and respond to two important lines of objection to our proposal. They concern, respectively, the issue of the demandingness of the conditions for confirmation by analogy and the issue of whether the account solves the fundamental problems of justification.

4.1 Too demanding?

Our proposal embraces Hesse’s idea that, as stated in the relevance requirement, an argument from similarities A, B, C to conclusion D “rests on the presumption that if AB is connected with D in the [source], then there is some possibility that B is connected with D, and that this connection will tend to make D occur with BC in the [target]” (1963:85). The presumption of ‘possibility’ that Hesse refers to is represented probabilistically by the requirement of a non-zero credence to a salient bridge hypothesis, typically stating that a commonality in causal features exist between the analogy’s source and target; the relevance of the latter to the argument’s conclusion is encoded in the transitivity assumptions 2(a)–(c). Although the idea that strong reasoning by analogy requires assumptions about relevance is popular (Weitzenfeld, 1984; Wylie, 1989), someone may object to it. It seems at least conceivable that an analogical argument may possess non-negligible confirmatory power even when we cannot (yet) articulate a plausible ‘bridge’ whereby the known similarities are connected non-accidentally to the predicted ones.

Our response to this objection is to bite the bullet: there are no such arguments. Consider any example where the objection seems most compelling. For instance, it may

³⁰ Cf. Hesse (1970, p. 57) on: “the cases of... predicting the properties of a full-sized object from experiments on a replica in, say, a wind tunnel [...] This implies... that ‘theoretical terms’ are used in something like the same sense of unobservable systems as they are of observable systems [so as to satisfy materiality]. At least the sense in which these are respectively ‘the same property’ is a sense which permits analogical argument. For if this is not so, it is difficult to see *how* confirmation can be transmitted through a theory to its consequences”.

be argued that archaeologists may sometimes reason from the observed social function of an artefact in one culture to the probable social function of a similar artefact in a long-extinct culture, without being able to formulate any plausible ‘bridge’ hypothesis that underwrites the confirmation.³¹ We contend that any such argument is either (a) not confirmatory or (b) not really a violation of the relevance requirement. Either way, the account’s insistence on relevance is justified.

To illustrate, let’s consider the example in more detail. We are supposed to imagine an agent who reasons to an artefact’s social function in a long-extinct culture from its social function in a more familiar culture, but where no plausible bridge hypothesis whatsoever can be identified. If this is indeed a fair description of the agent’s epistemic situation, then our view is that her reasoning is not confirmatory. It would be equivalent to inferring to George’s brown hair from his becoming rich. Inferences from ‘mere’ similarity might have a heuristic function, as well as perhaps establish a hypothesis’ ‘pursuit-worthiness’ (Nyrup, 2019); however, we deny that it can be an instance of confirmatory reasoning in science (setting aside any negligible enumerative evidence that they might afford). Among other things, an argument by Weitzenfeld (1984) suggests that licensing non-negligible confirmation from mere similarity leads to triviality: unless restrictions are placed on the similarities that can yield confirmation, any number of analogues will confirm that a different property is present in any given target. Accordingly, we agree with Weitzenfeld (1984) that practically “nothing...follows from mere similarity” (138).³²

More typically, what happens in scientific contexts is different: although it might *seem* that no plausible bridge hypothesis can be identified, upon examination such bridges turn out to be contextually available. Wylie (1989) argues that this is the case with many archaeological examples. Thus, an archaeologist who inferred an artifact’s social function from knowledge of its function in an entirely unrelated culture may be implicitly assuming that, because of the relative uniformity of the needs of human life, similar environmental and social pressures may have led people living in different times and cultures to devise the same solutions. Confirmation by analogy can then be seen to occur if there is at least some plausibility to the (tacit) ‘bridge’ hypothesis that roughly the same kinds of pressures as the ones obtaining in the familiar culture also obtained in an unrelated and old-extinct target culture. As Wylie (1989) writes: “archaeological uses of analogy... illustrate how central a role ‘vague general premises’ concerning [relevance relations] play in the development of analogical inferences” (138).³³

To sum up, we reject the allegation that our framework imposes too demanding conditions on confirmation by analogy. Whenever it appears that an analogical argument confirms without any bridge hypotheses, we insist that either the argument is not confirmatory, or bridge hypotheses were available and only tacitly considered. This reflects our view that Hesse’s relevance requirement holds; or, as Wylie (1989) put it, that: “the ground for analogical inference is not necessarily just that similarities

³¹ Many thanks to an anonymous referee for suggesting archaeology as a seemingly compelling example.

³² Among other things, this conviction explains our claim that, as encoded in assumption A2(c) in Galileo’s example, the mere similarities in line patterns are irrelevant to H’s confirmation if one assumes that $\neg G$ holds.

³³ Cf. also Wylie’s (1989) claim that defenses of confirmation by ‘simple analogy’ are based on a non-sequitur.

hold between source and [target] but that the association of observed (compared) and inferred properties is in some sense *non-accidental*” (136).

4.2 The problem of justification

It may still be asked: what does the Bayesian representation above accomplish besides offering a unitary and non-ad-hoc formulation of analogical inferences of various degrees of strength in probabilistic terms? After all,—one might think—a probabilistic model would seem to be useful if it offered a *justification* for the use of analogical arguments in supporting hypotheses. But it is questionable whether the model achieves this much. At the same time, if the model does not actually justify the use of analogical arguments in confirming yet untested hypotheses, it appears to add nothing to what we already knew—nothing but the same arguments rephrased in a probabilistic language. In this section, our aim is to show that this reasoning is mistaken.

In order for the probabilistic representation to yield a justification of the use of analogical arguments in science, it would have to analyze confirmation by analogy into a set of simple assumptions each of which is more acceptable than the analogical argument itself. Admittedly, it is hard to see how the proposed analysis does such a thing. Consider Galileo’s argument for the irregular morphology of the Moon. According to the representation provided in section two, confirmation depends on our acceptance of (among other things) the inequality $P(E | G) > P(E | \neg G)$. In section two, we have claimed that this inequality was plausible at the time of Galileo: the striking similarities between lunar wavy lines and terrestrial shadows are likely if both the Earth and the Moon have similar irregular morphologies. But this assumes that, at the very least, lunar physics will not differ radically from terrestrial physics in basic respects, such as in the way that light behaves. This is not an innocent assumption: lacking access to the Moon, Galileo had no direct empirical evidence for backing this up. In such a case, it is plausible that an appeal to the idea of the uniformity of nature was required to support his assumption.³⁴

The point of bringing up this example is not to retract our previous assertions, but to show that there is a limit to what the Bayesian representation can do: even though we can confidently say that confirmation by analogy depends on assumptions such as A1-A3, and that these assumptions are plausible in specific case-studies, not all questions about justification are solved by the Bayesian analysis. In many cases, the hard questions about the ultimate justification for the relevant assumptions remain open—for instance, the question about what makes it plausible to assume uniformity between lunar and terrestrial physics in the absence of observational evidence. These questions are directly relevant to the problem of justification for analogical arguments in science

³⁴ Discussing an abductive analogical argument similar to Galileo’s, Sober (2015) notes that confirmation requires an assumption of uniformity between source and target of the analogy: “this specific assumption”, he writes, “can be seen as the broader and vaguer idea of the uniformity of nature” (117). In Galileo’s argument, this is the assumption that lunar physics will not turn out to differ radically from terrestrial. Cf. Spranzi (2004) for an excellent discussion of this subtle logical point about Galileo’s reasoning. Cf. Lange (2017) and Evans and Thébault (2020) on the inherent ‘circularity’ in inductive reasoning and its philosophical significance.

(and, more generally, to the well-known problems about induction); unfortunately, they are not resolved but merely restated by adopting a probabilistic model.

Even though the Bayesian representation does not yield the sort of analysis that would fully justify the inductive use of analogy in science, it does not follow that it is useless. At the very least, the model puts pressure on philosophical claims to the effect that there may be an in-principle tension between Bayesianism and the recognition of confirmatory potential to analogical arguments: e.g., Bartha's (2019) claim that "the definition of confirmation in terms of Bayesian conditionalization seems inapplicable" (Sect. 2.3) to confirmation by analogy (cf. also Bartha, 2009, p. 32).³⁵ These claims have arguably contributed to making skepticism about analogical reasoning a highly popular view in the recent literature.³⁶ With the Bayesian account on offer, instead, the burden falls on skeptics about analogical reasoning to show that the probabilistic conditions above are, for systematic reasons, not satisfiable in any given example.

In addition, the Bayesian model has a useful diagnostic function, in that it specifies exactly which probability assignments must be in place in order to have confirmation by analogy (or for us to have any given amount of it). This can prove useful in any situation of scientific inquiry in which the foundational questions about the justification for inductive reasoning are not salient. A prime example is when two scientific parties disagree over whether confirmation of empirical hypotheses about a target occurs from the study of a given scientific model, but where neither party embraces a form of inductive skepticism.³⁷ In these cases, a probabilistic representation adds precision to the assessment of the relevant arguments. Once the formal conditions have been precisely laid out, one can leave the matter to the relevant scientific parties to dispute whether (or to what extent) those conditions are satisfied, given the available evidence.³⁸

Still another reason why the account is not useless, despite failing to solve the problem of justification, is that it reveals a significant agreement between the conditions for confirmation by analogy from BC's standpoint and a prominent philosophical account of analogical reasoning in science, namely Hesse's (1963). The coherence between our intuitive criteria for strong analogical arguments, the philosophical account that systematizes those criteria, and the Bayesian model that formalizes them in probabilistic language makes for an enviable bundle for addressing *descriptive* problems about

³⁵ In addition to the old evidence problem (see fn.3), Bartha (2009) justifies his claim partly on grounds that often "analogical arguments are applied to novel hypotheses H for which the prior probability $P(H|K)$ is not even defined" (32). However, this does not exclude confirmation by analogy in cases in which H's prior is defined. Moreover, even when H's prior is not defined, it seems that a Bayesian agent could always extend her credence function to include H and then update on any new evidence contained in the analogical argument.

³⁶ See Crowther et al. (2019): "The consensus in the philosophical literature (see Bartha [2019] for a review) is that analogue reasoning only establishes the *plausibility* of a hypothesis" (3705); cf. Thébault (2019): "the philosophical consensus is that...the role of evidence produced by analogical arguments is null" (188).

³⁷ The scientific debate over the evidential status of analogue simulations reviewed in Dardashti et al. (2019) is an example of such disagreement – one that an account of confirmation by analogy can help resolve; cf. fn. 21.

³⁸ Cf. Halpern and Hitchcock (2014) for a similar approach to modelling judgments of actual causation, based on the idea that a formal model can be useful to clarify sources of agreement and disagreement among experts.

analogical reasoning in science: i.e., questions about which analogical arguments, and under what conditions, are regarded as strong or weak in various contexts of scientific inquiry (quite independently of the question about justification).

Finally, even though the Bayesian analysis does not solve the problem of justification, it arguably points towards new and potentially fruitful directions. In this regard, we may note that the literature on the problem of justification for analogical reasoning has been dominated by two opposing tendencies. On the one hand, there have been attempts at justifying the use of analogy in supporting empirical hypotheses by appeal to some *global* principle of inductive inference – such as the principle of limited variability defended, in different ways, by Keynes (1921) and Hesse (1974).³⁹ The problem often raised with these principles is that, being a priori and separate from the contextual information that often accompanies analogical arguments in science, they prove too much, mistaking bad analogical arguments for good ones (cf. Bartha, 2019). On the other hand, some philosophers (e.g., Norton, 2020) have argued for highly *local* solutions. On these views, there is no general ‘schema’ that all analogical arguments capable of inductive support satisfy; the justification for the use of analogy in scientific inquiry must rather be sought on the ground: in specific aspects of the contextual information available in a given example.

The probabilistic representation that we have proposed points towards an interesting middle ground. On this alternative, there is a general confirmatory pattern that different analogical arguments in science obey quite regardless of the context – in this sense, there is a ‘logic’ of analogical inference. The existence of such a pattern helps explain something that defenders of highly local solutions have trouble accounting for, namely, how is it that we are relatively reliable in our assessments of analogical arguments across distinct contexts (Bartha, 2020). At the same time, the use of analogical arguments still depends crucially upon contextual information – in this sense, the justification for the use of analogy in science is not a priori. In what way these brief remarks can be elaborated into a solution to the problem of justification is a topic for a different work (cf. also Bartha, 2020 for a related discussion). For the purposes of this paper, it is sufficient to note that the passage to a Bayesian representation helps us identify a third way between highly global and highly local solutions, one that has at least the potential to avoid the most dissatisfying features of the traditional approaches to the problem of justification.

5 Conclusion

In this paper, we have proposed a compatibilist account of the confirmatory work of analogical arguments of various degrees of strength within a Bayesian framework. By showing that the probabilistic representation naturally accounts for a variety of phenomena related to confirmation by analogy, we hope to have made clear its generalizability and non-adhocness. While not all questions about the justification for the inductive use of analogy are solved by the Bayesian representation, the latter is arguably far from being useless: first, it adds precision to our reconstruction and

³⁹ This takes the form of an a priori stipulation that the amount of variety in the universe domain is limited.

assessment of analogical arguments in science; it is also coherent with an independently plausible informal account of analogical reasoning in science (namely Hesse's, 1963); finally, it points towards fruitful avenues for solving the problem of justification for analogical reasoning. Altogether, these features give considerable strength to our proposal.

Since analogical reasoning is involved in a wide variety of circumstances in which we may infer to features of real-world targets from the study of a scientific model, the probabilistic analysis advanced in this paper is immediately serviceable to methodologists interested in the epistemology of scientific modelling. It is a question deserving future research how the same Bayesian analysis can be extended to contexts of multiple analogies in scientific investigation (cf. Bartha, 2009:158) as well as to confirmation from analogical arguments in mathematics (cf. Polya, 1954) and related issues regarding the epistemic role of diagrams at the early stages of geometrical research (cf. Manders, 2008). Because the complexities associated with these further applications are considerable, especially when we bring probabilistic notions into the realm of mathematics, a careful treatment of those extensions will require a separate discussion.

Acknowledgements I am very grateful to Marc Lange and Matthew Kotzen for their incredible support and encouragement at the early stages of the writing of this paper. Special thanks also go to Gustavo Cevolani, Luc Bovens, Alan Nelson, Ram Neta, Jerry Postema, Giovanni Valente, Samantha Wakil, and two anonymous referees for their extremely helpful comments. Finally, I am very grateful to Raphael Ginsberg and Pavel Nitchovski for their precious support and solidarity at a crucial stage when this research was underway.

Funding The author acknowledges financial support from the Italian Ministry of Education, University and Research (MIUR) through the grant n. 201743F9YE (PRIN 2017 "From Models to Decisions").

Declarations

Conflict of interest The author declares that they have no Conflict of interest.

Ethical approval Not applicable.

Informed consent Not applicable.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

Appendix

The *transitivity theorem* by Roche and Shogenji (2013) employed in this paper is the following:

TT. Given any three propositions with non-extremal credences X , Y and Z , if:

- (i) $P(X | Y) > P(X | \neg Y)$;
- (ii) $P(Z | Y) > P(Z)$;
- (iii) $P(Z | X \ \& \ \neg Y) \geq P(Z | \neg Y)$;
- (iv) $P(Z | X \ \& \ Y) \geq P(Z | Y)$, then:
- (v) $P(Z | X) > P(Z)$.

Cor. In the limiting case in which Y entails Z , assumptions (i) and (iii) guarantee (v).

See Roche and Shogenji (2013) for a proof of this theorem – which is a generalization of a transitivity theorem by Hesse (1970). Note that Roche and Shogenji's theorem arguably states the *weakest* conditions for transitivity of confirmation – see their (2013) and references therein.

References

- Agassi, J. (1964). Analogies as generalizations. *Philosophy of Science*, 31, 351–356.
- Bartha, P. (2019). Analogy and analogical reasoning, in Zalta, E. (ed.), *Stanford Encyclopedia of Philosophy*, Fall 2020 Edition.
- Bartha, P. (2009). *By parallel reasoning*. Oxford University Press.
- Bartha, P. (2020). Norton's material theory of analogy. *Studies in History and Philosophy of Science Part A*, 82, 104–113.
- Bekenstein, J. (1972). Black holes and the second law. *Lettere Al Nuovo Cimento*, 4, 737–740.
- Bovens, L., & Hartmann, S. (2003). *Bayesian epistemology*. Oxford University Press.
- Brown, T. (2004). *Making truth: Metaphor in science*. University of Illinois Press.
- Carnap, R. (1950). *Logical foundations of probability*. Chicago University Press.
- Carnap, R. (1980). A basic system of inductive logic, part II. In R. Jeffrey (Ed.), *Studies in Inductive Logic and Probability*. Berkeley: University of California Press.
- Chen, R. (2020). Natural analogy: A Hessean approach to analogical reasoning in theorizing. *Erkenntnis*, published online.
- Crowther, K., Linnemann, N., & Wütrich, C. (2019). What we cannot learn from analogue experiments. *Synthese*, 198, 3701–3726.
- Dardashti, R., Hartmann, S., Thébault, K., & Winsberg, E. (2019). Hawking radiation and analogue experiments: A Bayesian analysis. *Studies in the History and Philosophy of Science Part C*, 67, 1–11.
- Darwin, C. (1859) [1999], *The origin of species*. Oxford University Press.
- Evans, P., & Thébault, K. (2020). On the limits of experimental knowledge. *Philosophical Transactions of the Royal Society A*, 378(2177), 20190235.
- Feldbacher-Escamilla, C., & Gebharter, A. (2020). Confirmation based on analogical inference: Bayes Meets Jeffrey. *Canadian Journal of Philosophy*, 50(2), 174–194.
- Festa, R. (1997). Analogy and exchangeability in predictive inference. *Erkenntnis*, 45, 89–112.
- Galilei, G., 1610 [2008], *The sidereal messenger*, in Finocchiaro, M. (ed.), *The Essential Galileo*. Hackett Publishing Company.
- Garber, D. (1983). Old evidence and logical omniscience in bayesian confirmation theory. *Minnesota Studies in the Philosophy of Science*, 10, 99–131.
- Glymour, C. (1980). *Theory and evidence*. Princeton University Press.
- Good, I. (1984). The best explicatum for weight of evidence. *Journal of Statistical Computation and Simulation*, 19, 294–299.

- Goodman, N. (1955). *Fact*. Harvard University Press.
- Halpern, J., & Hitchcock, C. (2014). Graded causation and defaults. *The British Journal for the Philosophy of Science*, 66(2), 413–457.
- Hesse, M. (1963). *Models and analogies in science*. University of Notre Dame Press.
- Hesse, M. (1970). Theories and the transitivity of confirmation. *Philosophy of Science*, 37(1), 50–63.
- Hesse, M. (1974). *The structure of scientific inference*. MacMillan.
- Hutteger, S. (2019). Analogical predictive probabilities. *Mind*, 128, 1–37.
- Keynes, J. M. (1921). *A treatise on probability*. MacMillan.
- Kitcher, P. (2003). *In Mendel's mirror: Philosophical reflections on biology*. Oxford University Press.
- Lange, M. (2020) Putting 'explanation' back into 'inference to the best explanation'. *Nous*, available online.
- Lange, M. (1999). Calibration and the epistemological role of Bayesian conditionalization. *The Journal of Philosophy*, 96(6), 294–324.
- Lange, M. (2017). Idealism and incommensurability, in T. Goldschmidt & K. Pearce (Eds.), *Idealism: New Essays in Metaphysics*. Oxford University Press.
- Manders, K. (2008). Diagram-based practice. In P. Mancosu (Ed.), *The Philosophy of Mathematical Practice*. Oxford University Press.
- Maxwell, J. C. (1890) *The scientific papers of James Clerk Maxwell*, vol. I and II, W.D. Niven (ed.), Cambridge University Press.
- Nappo, F. (2020) Learning from non-causal models. *Erkenntnis*, published online.
- Niiniluoto, I. (1981). Analogy and inductive logic. *Erkenntnis*, 4(1), 1–34.
- Norton, J. (2020). Analogy, in his *The Material Theory of Induction*, available online.
- Nyrup, R. (2019). Of water drops and atomic nuclei: analogies and pursuit-worthiness in science. *The British Journal for the Philosophy of Science*, 71(3), 881–903.
- Pearl, J., & Baremboim, E. (2013). A general algorithm for deciding transportability of experimental results. *Journal of Causal Inference*, 1, 107–134.
- Polya, G. (1954). *Mathematics and plausible reasoning*. Princeton University Press.
- Roche, W., & Shogenji, T. (2013). Confirmation, transitivity and moore: The screening-off approach. *Philosophical Studies*, 3, 1–21.
- Shelley, C. (2002). Analogy counterarguments and the acceptability of analogical hypotheses. *The British Journal for the Philosophy of Science*, 53, 477–492.
- Skyrms, B., et al. (1993). Analogy by similarity in Hyper-Carnapian inductive logic, in J. Earman (Ed.), *Philosophical Problems of the Internal and External Worlds*. University of Pittsburgh Press.
- Sober, E. (2015). *Ockham's razors*. Cambridge University Press.
- Spranzi, M. (2004). Galileo and the mountains on the moon: Analogical reasoning, models and metaphors in scientific discovery. *Journal of Cognition and Culture*, 4, 451–483.
- Thébault, K. (2019). What can we learn from analogue experiments? in R. Dardashti, R. Dawid, & K. Thébault (Eds.), *Why Trust a Theory? Epistemology of Fundamental Physics*. Oxford University Press.
- Van Eersel, G., Reiss, J., & Koppenol-Gonzales, G. (2019). Extrapolation of experimental results through analogical reasoning from latent classes. *Philosophy of Science*, 86(2), 219–235.
- Weitzenfeld, J. (1984). Valid reasoning by analogy. *Philosophy of Science*, 51(1), 137–149.
- Wylie, A. (1985). The reaction against analogy. *Advances in Archaeological Method and Theory*, 8, 63–111.
- Wylie, A. (1989). 'Simple' analogy and the role of relevance assumptions: Implications of archaeological practice. *International Studies in the Philosophy of Science*, 2(2), 134–150.