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IFAC PapersOnLine 56-2 (2023) 7090–7095

A steady-state optimal coordination strategy for DERs systems with guaranteed probabilistic performance ⋆

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Abstract: We consider the problem of coordinating multiple Distributed Energy Resources (DERs) so as to supply energy to the grid while minimizing its variability around a reference profile that must also be optimized. We focus on the case when each DER is equipped with solar panels and a battery storage device, and jointly design the disturbance compensation strategies for charging and discharging the batteries on a one-day time horizon. To this purpose, we linearly parameterize the strategies and search for a solution minimizing the fluctuations of the energy exchange with the grid in steady-state, with a bound on their extent that holds in probability given the stochastic nature of the solar energy. Interestingly, the probability measure of the resulting chance-constrained optimization problem depends on the parameters of the disturbance compensation strategies, which makes the application of the scenario approach not standard. The proposed scenario-based solution is feasible for the original steady-state chance-constrained optimization problem and proves effective in numerical simulations.

Keywords: DERs, energy systems, optimal control, stochastic systems

1. INTRODUCTION

Despite the efforts undertaken by countries all around the globe, the world energy demand keeps rising. Consequently, in the last few years the interest in renewable source of energies has grown and it is predicted to increase in the next years. The main factors responsible for this fast development are identified in The secretariat and Van de Graaf (2019). First, the reduction of the cost of renewables is to be considered: today renewable technologies have competitive costs. For example, since 2010, the average cost of electricity from photovoltaics and wind has decreased by 73% and 22%, respectively. Then, the effect of climate change and pollution must be taken into account. Indeed, pollution contributes to the global warming, hence decarbonisation has gained more importance for governments, which elaborate their policies also in order to be environmentally friendly. In fact, the use of renewable energy combined with a more efficient energy management could be an effective way to achieve the Paris Agreement goal. As renewable sources of energy spread in the grid, it is necessary to combine them with innovative energy management strategies that make them work efficiently and suitably integrate them into the grid.

In recent years, Distributed Energy Resources (DERs) systems have attracted much interest, becoming a valid alternative to traditional power plants for many economical and social reasons (see Pepermans et al. (2005)). DERs are small energy generation units situated close to energy consumers, rather than connected to the bulk power transmission systems, Kari and Arto (2006). They produce energy directly in the point of usage avoiding the use of large power plants and transmission lines. They can be used for self-consumption purposes thus allowing consumers to become independent of the grid energy supply. An aggregation of DERs can create a so-called Virtual Power Plant, with a great advantage in terms of scalability and flexibility: since each unit is independent from the others, it is easy to add new ones or remove them if needed (see Kari and Arto (2006)).

DERs systems can integrate different energy sources, including renewable ones (e.g., rooftop solar panels or wind microturbines), and can then drive the transition to a greener electric grid (see Akorede et al. (2010) and Pepermans et al. (2005)). The penetration of DERs systems with renewables is currently a growing phenomenon, that challenges the traditional energy supplier, which are forced to question their service, and eventually adapt it to these new technologies, Tolmasquima et al. (2020).

Unfortunately, DERs exploiting renewable energy sources like photovoltaic panels presents the drawback of being not controllable: they generate electricity only for some hours a day and the energy produced is highly uncertain since it depends on the weather conditions. Generally speaking, intermittency is the main disadvantage of most distributed generation resources. However, it can be mitigated thanks to the usage of Electrical Energy Storage (EES) systems.

EES systems are able to convert energy from one form (mainly electrical energy) to a storable form (mainly chemical energy). When needed, the stored energy is then converted back into electrical energy, Luo et al. (2015). EESs

⋆ This paper is supported by PNRR-PE-AI FAIR project funded by the NextGeneration EU program.

10.1016/j.ifacol.2023.10.576

can be very helpful in power network operation as they allow to compensate time variability in energy production and, hence, alleviate the intermittence of renewable source power generation, ensuring a higher quality and reliability of the energy. In particular, one of the most widely used EES technologies are rechargeable batteries, which are appreciable for their long lifetime, reduced charging time, higher energy density, and shorter response time, da Silva Lima et al. (2021).

The adoption of systems including both EESs and DERs allows to exploiting the advantages of both technologies. One should however devise a suitable control strategy to operate the EEs and compensate the intermittency of the energy produced by renewables.

Contribution

In this paper, the problem of coordinating a Virtual Power Plant resulting from the aggregation of many DERs, each one with solar panels and a battery, is considered. We jointly design disturbance compensation strategies to operate the batteries of the virtual power plant on a one-day time horizon in order to supply energy to the grid while minimizing its variability around a reference profile that must also be optimized. Due to the stochastic nature of the solar energy production, the problem is formulated as a chance-constrained optimization program, where the constraints are expressed in probabilistic terms.

The considered framework is similar to the one studied in Tuissi (2020): the aggregation of multiple DERs is considered and a disturbance compensator is used in the batteries charging policy to cope with uncertainty in the solar production affecting each DER. In this way, the compensation acting on every DER contributes to counteract the variability of the overall power exchange with the grid. Differently from Tuissi (2020), the problem is addressed with reference to stationary conditions so that the obtained control law is guaranteed to be optimal, and feasible, in the long run. This means that the optimization problem must be solved only once, and then the obtained control strategy can be applied day after day.

To overcome the complexity related to the chance-constrained nature of the problem, we employ a randomized technique known as scenario approach, Campi et al. (2009). In this way, the problem is reformulated in a deterministic form and it is possible to find an approximated solution to the original problem that still maintains the desired probabilistic guarantees. Given that probabilistic constraints are given in terms of the steady state probability distribution that depends on the applied control strategy, we adopt the non-standard scenario solution introduced in Falsone et al. (2022).

Interestingly, the resulting scenario optimization problem is suitable to be solved both in a centralized and in a distributed way, for example with the algorithm proposed by Falsone et al. (2020). The most appreciable feature of a distributed resolution strategy is that the computational effort of the resolution is not managed by a unique centralized unit but is divided among all the DERs. This can be helpful in case of systems constitutes by a large number of DERs, where the dimension of the centralized problem can be a limit to the resolution of the posed optimization problem.

2. PROBLEM FORMULATION

In this paper we consider the problem of coordinating a system composed of m DERs, each one equipped with solar panels and a battery storage device. Given the daily periodicity of solar power production, it is convenient to consider a one-day time horizon, discretized into k_f time slots. We first introduce the model of the single DER, then we formulate a compensation-based optimal control strategy for a one-day time horizon, and finally show how to design the compensator to optimize steady-state operations.

2.1 Modeling

Consider the i-th device. For all $k \in \mathcal{K} = \{0, \dots, k_f - 1\}$, let $E_{p,i}(k) \geq 0$ denote the solar energy produced during time slot k, $\xi_i(k)$ the State Of Charge (SOC) of the battery at the beginning of time slot k, and $u_i(k)$ the energy entering (if $u_i(k) > 0$) or leaving (if $u_i(k) < 0$) the battery during time slot k. The battery SOC evolves according to the recursive equation

$$\xi_i(k+1) = a_i \xi_i(k) + u_i(k), \quad k \in \mathcal{K}, \tag{1}$$

where coefficient $a_i \in (0, 1)$ models self-discharging losses. Clearly, it must hold

$$-s_i^{max} \leq u_i(k) \leq s_i^{max}, \quad k \in \mathcal{K}, \tag{2}$$

$$p^{min} C_i \leq \xi_i(k) \leq p^{max} C_i, \quad k \in \mathcal{K}, \tag{3}$$

where $s_i^{max} > 0$ is the maximum energy that can be exchanged with the battery in one time slot, $C_i > 0$ is the battery rated capacity, and $0 < p^{min} < p^{max} < 1$ are limits for safe battery operations and for the validity of (1).

If $E_{g,i}(k)$ denotes the energy injected (if $E_{g,i}(k) > 0$) or drawn (if $E_{g,i}(k) < 0$) by DER i to/from the main grid during time slot k, then

$$E_{g,i}(k) = E_{p,i}(k) - u_i(k) - \eta_u |u_i(k)|, \quad k \in \mathcal{K} \tag{4}$$

where the last term accounts for charging/discharging losses through the coefficient η_u. If we further denote with $E_g(k)$ the energy exchanged by the aggregate of m DERs (with the same sign convention), then

$$E_g(k) = \sum_{i=1}^{m} E_{g,i}(k), \quad k \in \mathcal{K}. \tag{5}$$

2.2 One-day optimal control

With reference to the one-day time horizon, for all $i = 1, \dots, m$, let

$$\xi_i = [\xi_i(0) \cdots \xi_i(k_f)]^\top,$$
$$u_i = [u_i(0) \cdots u_i(k_f - 1)]^\top,$$
$$E_{p_i} = [E_{p,i}(0) \cdots E_{p,i}(k_f - 1)]^\top,$$
$$E_g = [E_g(0) \cdots E_g(k_f - 1)]^\top,$$

be the vectors containing the evolution of the corresponding quantity along the horizon, with $\xi_i(k_f)$ denoting the SOC at the end of time slot $k_f - 1$. To ease the notation, let $x_i = \xi_i(0)$ and $\mathbf{1}$ be the vector containing all ones of suitable dimension. From (1), we obtain $\xi_i = Fx_i + Gu_i$ for suitable matrices F and G, and, recalling (2)-(5), we have

$$|u_i| \leq s_i^{max}\mathbf{1} \qquad \forall i, \tag{6}$$

$$p^{min}C_i\mathbf{1} \leq Fx_i + Gu_i \leq p^{max}C_i\mathbf{1} \qquad \forall i, \tag{7}$$

$$E_g = \sum_{i=1}^{m}(E_{p,i} - u_i - \eta_u|u_i|), \tag{8}$$

with $|u_i|$ denoting the component-wise absolute value of vector u_i.

Since the solar energy production is uncertain, $E_{p,i}$ is a random vector. Let us denote with $\mu_i = \mathbb{E}[E_{p,i}]$ its expected value and $d_i = E_{p,i} - \mu_i$ its deviation from μ_i. Given that the grid energy exchange profile E_g depends on the $E_{p,i}$'s (cf. (8)), and since we want to contain the variability of E_g, we select a battery control law of the form

$$u_i = \gamma_i + \vartheta_i d_i, \forall i, \tag{9}$$

which adapts the battery energy exchange u_i based on the actual variability d_i of the solar energy production, with $\gamma_i \in \mathbb{R}^{k_f}$ and $\vartheta_i \in \mathbb{R}^{k_f \times k_f}$ being the control policy parameters to be optimized. Clearly, ϑ_i must have a lower-triangular structure with zeros on the main diagonal, so that $u_i(k)$ depends only on $E_{p,i}(0), \dots, E_{p,i}(k-1)$ and is thus implementable in practice. Under (9), (8) becomes

$$\begin{aligned} E_g &= \sum_{i=1}^{m} \big((\mu_i + d_i) - (\gamma_i + \vartheta_i d_i) - \eta_u|u_i|\big) \\ &= \sum_{i=1}^{m}(\mu_i - \gamma_i) + \sum_{i=1}^{m} \big((I - \vartheta_i)d_i - \eta_u|u_i|\big), \end{aligned}$$

where the first term do not depend on the uncertainty, while the second one does. We can thus take $E_g^\circ = \sum_{i=1}^{m}(\mu_i - \gamma_i)$ as our reference profile and require E_g to belong to a tube centered in E_g° and with half-width $c_0\mathbf{1} + c_1\overline{\mu}$, where $\overline{\mu} = \sum_{i=1}^{m}\mu_i$ and c_0 and c_1 are two positive scalars parameterizing the tube. This is formalized as

$$-(c_0\mathbf{1} + c_1\overline{\mu}) \leq E_g - E_g^\circ \leq (c_0\mathbf{1} + c_1\overline{\mu}).$$

Unfortunately, due to the presence of $|u_i|$ in the expression of E_g, the upper constraint in the previous expression is non-convex in γ_i and ϑ_i. We therefore approximate it by simply neglecting[1] $|u_i|$ in the upper constraint, and requiring

$$\begin{aligned} &-(c_0\mathbf{1} + c_1\overline{\mu}) \leq \sum_{i=1}^{m} \big((I - \vartheta_i)d_i - \eta_u|u_i|\big), \\ &\sum_{i=1}^{m}(I - \vartheta_i)d_i \leq (c_0\mathbf{1} + c_1\overline{\mu}), \end{aligned} \tag{10}$$

which are both convex. Minimizing the variability in the grid energy exchange profile E_g thus simply amounts to minimize the area of the tube. The optimal controller parameters γ_i and ϑ_i of each DER, along with the minimum-tube parameters c_0 and c_1 can thus be obtained as the solution of the following optimization problem

[1] Note that by neglecting $|u_i|$ we are being more conservative as the resulting constraint is tighter than the original one.

$$\begin{aligned} \min_{\substack{c_0 \geq 0,\ c_1 \geq 0, \\ \{\gamma_i, \vartheta_i\}_{i=1}^m}} \quad & \mathbf{1}^\top(c_0\mathbf{1} + c_1\overline{\mu}) \\ \text{subject to:} \quad & \mathbb{P}\Big\{(6) \wedge (7) \wedge (9) \wedge (10)\Big\} \geq 1 - \epsilon \end{aligned} \tag{11}$$

in which the constraints are imposed in probability as $E_{p,i}$, and thus d_i, are uncertain quantities.

Note that constraint (7) depends on the value of the battery initial SOC $x_i = \xi_i(0)$, $i = 1, \dots, m$. Thus, problem (11) should be solved at the beginning of each day after measuring the SOC of all batteries, which, however, may be impractical.

2.3 Steady-state one-day optimal control

Suppose instead that we would like to compute the controller parameters γ_i and ϑ_i only once, and then use those parameters every day. Given the daily periodicity of the solar irradiation, we can think of the solar energy production as a cyclostationary process with a one-day period. If we denote with $E_{p,i}^t \in \mathbb{R}^{k_f}$ the solar energy production profile of day t, then the cyclostationary assumption implies that $\mathbb{E}[E_{p,i}^t] = \mu_i$ for all t. Let $x_i^t = \xi_i^t(0)$ be the i-th battery SOC at the beginning of day t, $d_i^t = E_{p,i}^t - \mu_i$ the energy production deviation during day t (assumed to be independent across days), and $u_i^t = \gamma_i + \vartheta_i d_i^t$, according to (9), the control law for day t. By iterating (1) within day t and noticing that $\xi_i^t(k_f) = \xi_i^{t+1}(0) = x_i^{t+1}$, we can write the following recursive equation

$$\begin{aligned} x_i^{t+1} &= a_i^{k_f} x_i^t + b_i^\top u_i^t \\ &= a_i^{k_f} x_i^t + b_i^\top \gamma_i + b_i^\top \vartheta_i d_i^t, \end{aligned} \tag{12}$$

with $b_i^\top = [a_i^{k_f-1} \cdots a_i\ 1]$, relating the i-th battery SOC at the beginning of consecutive days. System (12) is an asymptotically stable ($a_i \in (0,1)$) linear system fed by the stationary process $\{d_i^t\}_t$ and its state x_i^t admits a stationary distribution $X_i(\gamma_i, \vartheta_i)$, which depends on the controller parameters γ_i and ϑ_i.

If we want to compute a single value for γ_i and ϑ_i to be used for all days, then it is intuitive to design them for a generic day, with a battery initial SOC distribution equal to the stationary distribution $X_i(\gamma_i, \vartheta_i)$. We formulate this problem as the following mathematical program

$$\begin{aligned} \min_{\substack{c_0 \geq 0,\ c_1 \geq 0, \\ \{\gamma_i, \vartheta_i\}_{i=1}^m}} \quad & \mathbf{1}^\top(c_0\mathbf{1} + c_1\overline{\mu}) \\ \text{subject to:} \quad & \mathbb{P}\Big\{(6) \wedge (7) \wedge (9) \wedge (10)\Big\} \geq 1 - \epsilon \\ & x_i \sim X_i(\gamma_i, \vartheta_i) \quad \forall i, \end{aligned} \tag{13}$$

which is equal to (11) apart from the fact that the initial SOC x_i is now uncertain and is distributed according to the steady-state distribution $X_i(\gamma_i, \vartheta_i)$ instead of being deterministic like in (11).

The optimal solution to (13) results, by construction, into a control law which, at steady-state, satisfies the constraints (up to a probability $1 - \epsilon$) and minimizes the variability in the grid energy exchange profile. Unfortunately, (13) has three complexity aspects that need to be accounted for:

(1) it contains probabilistic constraints;

(2) the distribution of the battery initial SOC is parametric in the optimization variables;
(3) it is potentially large-scale if we consider many DERs.

3. SCENARIO-BASED SOLUTION

We now tackle each complexity issue separately.

3.1 Dealing with chance-constraints

Let us focus on how to handle the the probabilistic constraints first. Suppose N realizations $d_i^{(1)}, \dots, d_i^{(N)}$ and $x_i^{(1)}, \dots, x_i^{(N)}$ of the uncertain quantities d_i and x_i are available, for all $i = 1, \dots, m$. According to the so-called Scenario Theory (cf. Campi et al. (2009)), problem (13) can be approximated substituting the probabilistic constraint with N copies of the constraints inside the probability, where, in each copy, the uncertain quantities are replaced with one realization, i.e.,

$$\begin{aligned} \min_{\substack{c_0\geq 0,\ c_1\geq 0,\\ \{\gamma_i,\vartheta_i\}_{i=1}^m}} \quad & \mathbf{1}^\top(c_0\mathbf{1} + c_1\overline{\mu}) \\ \text{subject to:} \quad & \Big\{(6)\wedge(7)\wedge(9)\wedge(10) \text{ with} \\ & \quad x_i = x_i^{(j)} \wedge d_i = d_i^{(j)} \quad \forall i\Big\} \\ & \quad j = 1,\dots,N. \end{aligned} \tag{14}$$

The main result of the scenario theory states that, if N is selected to satisfy

$$\sum_{j=0}^{n-1} \binom{N}{j} \epsilon^j (1-\epsilon)^{N-j} \leq \beta, \tag{15}$$

where n is the total number of decision variables, then, with confidence $1-\beta$, the optimal solution of (14) is feasible for the original chance-constrained problem (13). Unfortunately, this approach requires to extract samples $x_i^{(1)}, \dots, x_i^{(N)}$ from a probability distribution $X_i(\gamma_i, \vartheta_i)$ which depends on the decision variables and is thus unknown before solving the problem.

3.2 Dealing with steady-state distribution

Luckily, if we assume that d_i has a (zero-mean) multivariate Gaussian distribution, then we know that also the steady-state distribution $X_i(\gamma_i, \vartheta_i)$ will be Gaussian, and, according to Falsone et al. (2022), its mean and variance can be expressed analytically as a function of γ_i and ϑ_i. Since the system in (12) is scalar, the expressions in Falsone et al. (2022) greatly simplifies, leading to

$$\mathbb{E}[X_i(\gamma_i, \vartheta_i)] = \frac{b_i^\top \gamma_i}{1 - a_i^{k_f}}, \tag{16}$$

$$\text{var}(X_i(\gamma_i, \vartheta_i)) = \frac{b_i^\top \vartheta_i S_i S_i^\top \vartheta_i^\top b_i}{1 - a_i^{2k_f}}, \tag{17}$$

where $S_i S_i^\top = \mathbb{E}[d_i^t {d_i^t}^\top]$ is independent from t since $E_{p,i}^t$ was assumed to be cyclostationary. Relations (16) and (17) can be obtained computing the steady-state mean and variance of (12). Given that $X_i(\gamma_i, \vartheta_i)$ is Gaussian, together with (16) and (17), x_i distributed according to $X_i(\gamma_i, \vartheta_i)$ can be expressed as

$$x_i = \frac{b_i^\top \gamma_i}{1 - a_i^{k_f}} + \frac{b_i^\top \vartheta_i S_i}{\sqrt{1 - a_i^{2k_f}}} e_i, \tag{18}$$

where e_i is a k_f-dimensional Gaussian variable with zero mean and a covariance equal to the identity matrix. As can be noticed from (18), a random sample $x_i^{(j)}$ from the steady-state distribution of the initial SOC can be drawn in practice by extracting a sample $e_i^{(j)}$ from the standard k_f-dimensional Gaussian distribution, and is linearly parameterized by the control parameters γ_i and ϑ_i. Problem (14) can thus be solved *exactly* via the equivalent formulation

$$\begin{aligned} \min_{\substack{c_0\geq 0,\ c_1\geq 0,\\ \{\gamma_i,\vartheta_i\}_{i=1}^m}} \quad & \mathbf{1}^\top(c_0\mathbf{1} + c_1\overline{\mu}) \\ \text{subject to:} \quad & \Big\{(6)\wedge(7)\wedge(9)\wedge(10)\wedge(18) \\ & \quad \text{with } e_i = e_i^{(j)} \wedge d_i = d_i^{(j)} \quad \forall i\Big\} \\ & \quad j = 1,\dots,N. \end{aligned} \tag{19}$$

Problem (19) is a standard convex program and can be solved resorting to off-the-shelf algorithms. Unfortunately, when the number m of DERs considered is very high, finding the optimal solution to (19) can become a difficult task for a single processing unit.

3.3 Dealing with large-scale DERs systems

In recent years, distributed optimization has emerged as the paradigm to deal with large-scale optimization problems. For a mathematical program to be solvable in a distributed way, it must have a specific structure. Among the different structures that have been considered in the literature, we are interested in the so-called constraint-coupled optimization problems:

$$\begin{aligned} \min_{\{y_i\}_{i=1}^M} \quad & \sum_{i=1}^M f_i(y_i) \\ \text{subject to:} \quad & \sum_{i=1}^M H_i y_i \leq L \\ & y_i \in Y_i \quad i = 1,\dots,M, \end{aligned} \tag{20}$$

where the decision variables are divided into M small decision vectors $y_i \in \mathbb{R}^{n_i}$, each y_i has its own constraint set $Y_i \subseteq \mathbb{R}^{n_i}$ and gives its own contribution $f_i(y_i)$ to the cost function. Clearly, the challenge in the solution of (20) is the presence of the constraints $\sum_{i=1}^M H_i y_i \leq L \in \mathbb{R}^q$, which relates the different y_i's to each other, thus coupling their optimization (hence the adjective constraint-coupled).

Next, we show that (19) can be slightly reformulated to fit the structure of (20). First, let us set $M = m+1$: one decision vector for each DER control parameters γ_i and ϑ_i (m in total) and one for the tube parameters c_0 and c_1. Consequently, $y_i = (\gamma_i, \vartheta_i)$ for $i = 1, \dots, m$ and $y_M = (c_0, c_1)$. Since the cost function of (19) depends on c_0 and c_1 only, then we set $f_i(y_i) = 0$ for $i = 1, \dots, m$ and $f_M(y_M) = \mathbf{1}^\top(c_0\mathbf{1} + c_1\overline{\mu})$. Then, in Y_i we gather all constraints depending on γ_i and ϑ_i only:

$$Y_i = \bigcap_{j=1}^{N} \Big\{ (\gamma_i, \vartheta_i) : \; (6) \wedge (7) \wedge (9) \wedge (18)$$
$$\text{with } e_i = e_i^{(j)} \wedge d_i = d_i^{(j)} \Big\}.$$

Finally, as for the coupling constraint, it is easy to see that (10) contains a summation over the γ_i and ϑ_i parameters of all DERs plus the c_0 and c_1 parameters of the tube. Clearly, $H_i y_i$ must contain all constraints (10) associated to all the N realizations $d_i^{(j)}$ of d_i. Since the second constraint in (10) is linear, it is easy to see how it can be expressed as $H_i y_i$, whereas for the first constraints, which is nonlinear, we first need to convert it to a linear form via an epigraphic reformulation of $|u_i^{(j)}| = |\gamma_i + \vartheta_i d_i^{(j)}|$.

Since, by the discussion above, (19) fits the structure of (20), we can apply any distributed algorithm that is able to handle problems in the form of (20).

4. NUMERICAL RESULTS

We next validate the proposed strategy and its advantages via numerical simulations. The case of $m = 10$ users is presented with the one-day time horizon discretized into $k_f = 144$ time slots of 10 minutes each.

All batteries are equal, with $C = 18.62$ MJ, $s^{max} = 1.12$ MJ, $\eta_u = 0.02$, $a = 0.9993$, $p^{min} = 5\%$ and $p^{max} = 95\%$.

For each agent i, the policy parameters γ_i and ϑ_i are further parameterized to reduce the number of decision variables. Specifically we set $\gamma_i = c_{\gamma_i}\mu_i$ with $c_{\gamma_i} \in \mathbb{R}$, while ϑ_i is constrained to have only the first $p = 3$ sub-diagonals different from zero and with the elements on the same sub-diagonal being equal. This way the solar production in the latest p time slots is weighted with time invariant parameters and is used for compensation. This results [2] in a control policy of the form

$$u_i(k) = c_{\gamma_i}\mu_i(k) + \sum_{s=1}^{p} c_{\vartheta_i,s} d_i(k-s),$$

with $c_{\vartheta_i,1}$, $c_{\vartheta_i,2}$, and $c_{\vartheta_i,3}$ being the coefficients on the first, second, and third sub-diagonal of ϑ_i respectively.

The scenario approach is implemented setting $\beta = 10^{-4}$, $\epsilon = 0.1$ and the number of scenarios is set equal to $N = 690$ to satisfy (15). For all the agents, the randomly selected realizations $E_{p,i}^{(j)}$ are extracted from a Gaussian model estimated from a dataset of daily photovoltaic energy production profile.

We then compute a solution to the resulting problem (19) using the *IBM ILOG CPLEX* solver. To validate the obtained results, the found optimal policy is applied on 1000 new realizations of d_i (validating scenarios), different from the ones used for solving (19). In Figure 1 we report the obtained tube and the steady-state grid energy exchange profiles corresponding to the validating scenarios. As can be seen from the picture only a few validating scenarios exit the tube. Indeed, an estimate $\hat{\epsilon} = 0.04$ for the violation probability was computed as the percentage of validating scenarios that violates the constraints and observed to be less than the chosen threshold $\epsilon = 0.1$, thus confirming the guarantees provided by the scenario theory.

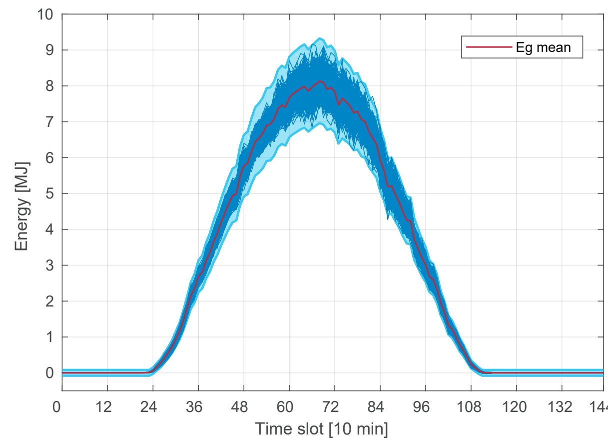

Fig. 1. Grid energy exchange profiles (blue), corresponding mean (red) and minimum tube (light blue area). Computed with validating scenarios.

Then, the evolution of the SOC over $T = 120$ days is tested. At day 1 all the m batteries are initialized at 50% of the capacity and the recursive equation (12) with the optimal γ_i and ϑ_i obtained is applied until convergence. To check whether convergence is reached or not the recursive expressions of the mean and of the covariance of the initial state of charge are found starting from equation (12). At day $t+1$, the probability density distribution of the initial SOC is analytically computed starting from its mean and covariance at day t, then it is checked against the steady-state one. In Figure 2 (top) it is visible that, as days go by, the two distributions get closer, until they are exactly equal, meaning that the initial SOC distribution tends to the steady-state one.

As can be seen from the picture the left and right tails of the distributions crosses the battery capacity limits. Clearly, the limits in (3) cannot be violated in practice, since they represent a physical limitation. Therefore, we recomputed the evolution of the SOC distribution by applying a saturated control strategy, which keeps the battery SOC inside $p^{min}C$ and $p^{max}C$ for all validating scenarios. The resulting histograms are reported in Figure 2 (bottom). By a visual comparison we can see that the evolution of the SOC distribution is basically unaffected and the probabilistic guarantees on constraint satisfaction are preserved.

4.1 Advantages of the aggregation

The advantages of using a network made up of many agents are identified via simulation.

In particular, some simulations are run, changing the value of C_i for some batteries. It is observed that the variation of the single capacities does not influence the total energy exchange with the grid (and the dimension of the tube) as long as the total capacity of the aggregate is unchanged. By modifying the total capacity, it is instead concluded that from the aggregation of the agents derives the ability

[2] The expression is valid throughout the day except for border effects at the beginning of the day.

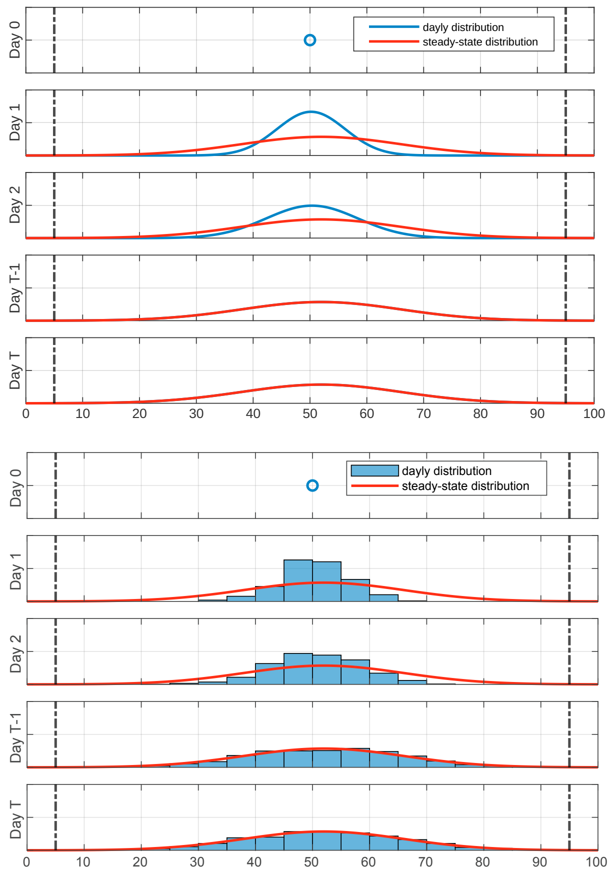

Fig. 2. Initial state of charge distribution (blue) compared to the steady-state distribution (red) in some significant days. Black dashed lines denotes the SOC bounds. Actuation limits are allowed to be violated (top) or not (bottom).

of the system to face the variability of the energy exchange with the grid: the bigger is the total capacity of the aggregate, the smaller are the fluctuations of the energy exchange with the grid.

Besides, a decoupled implementation of problem (19) was also tested, to evaluate the effect of the coupling constraint on the optimization. In the decoupling implementation each agent aims at maintaining its own energy variations below a $\frac{1}{m}$ portion of the tube. This results in an overall exchange E_g with the grid that is the same as in the previous case, but in a tube that is 5 times bigger, proving that the aggregation of the agents helps in the reduction of the dimension of the tube.

5. CONCLUSION

In this work, we presented a solution to the problem of limiting the fluctuations of the energy exchange with the grid of an aggregation of many DERs, giving guarantees in the long run. The most appealing feature of the proposed control policy is the fact that the optimal control action must be computed only once and then it can be applied day after day. This has a benefit in terms of energy management strategy since, when the DERs are deployed, the control strategy is set, and then automatically repeated every day, without the need of further intervention.
As for future developments, an alternative implementation of the problem with a state feedback control law deserves to be studied, so that the control action is set according to the state of charge of the batteries instead of just reacting to the measured solar energy production.

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