Efficient computation of losses in metallic sheaths and armor of AC submarine cables

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ABSTRACT

The problem of computing efficiently and accurately the armor losses in three-core AC submarine cables has been of great interest to the offshore cable community for some years. Indeed, the efficiency of the computation method is crucial to allow comprehensive parametric analysis as might be needed in the design study.

An algorithm based on integral equations for performing such calculations with an accuracy comparable (within acceptable tolerances) to state-of-the-art Finite Element Method (FEM) approaches is presented.

In particular, we discuss how exploiting the cable structure periodicity and symmetry leads to huge computational cost reductions.

KEYWORDS

Submarine cables, Eddy currents, Numerical simulation.

INTRODUCTION

The installed capacity of offshore wind farms has steadily increased in recent years. This has led to a growing interest in the accurate rating of submarine power cables that connect offshore generators between themselves and the mainland.
In particular, armored three-core AC submarine cables are quite difficult to model accurately. Both core conductors and armor wires are twisted, with different pitches, and armor wires are stranded either with opposite (contralay) or same (equilay) orientation as the phase cables. This intrinsically 3D geometry affects in a non-trivial way the magnetic behavior of the cable, rendering 2D models inaccurate.
The first studies using 3D FEM revealed that 2D FEM simulations are not reliable for their limited accuracy.
Recently there have been efforts toward the improvement of 3D computational models using FEM [1-3]. These models have the potential of being highly accurate so that they can be considered as a benchmark for validating alternative methods but are computationally demanding both in terms of computer memory (hundreds of GB of memory) and simulation time (hours of computation).
To lower the computational requirements of these 3D FEM models, several recent works have proposed to exploit the symmetries which characterize a submarine cable. By means of progressively more sophisticated use of these symmetries, these approaches have allowed shortening the length of the cable model [1], [4].
In this article, we present an alternative 3D computational method, based on integral equations, which can be used to compute losses in armored three-core cables with good accuracy.
The method considers the presence of both wire armor and conductor sheaths. For the armor, we use the formulation proposed in [5], while for the sheaths we employ a formulation for thin conducting surfaces [6]. These two formulations can be easily coupled to take into account the mutual interaction between armor and sheaths. Both the formulations are based on integral equations, thus, compared to FEM models, they have the advantage of requiring the discretization of the magnetically active regions only.
The exploitation of cable symmetries is taken one step further with respect to [4]: symmetries are used not only for reducing the size of the computational domain but also to confer a block-circulant structure to the matrix of the linear system. This structure can be exploited for reducing computational time and memory requirements during the assembly and solution of the fully coupled linear system of equations arising from the discretization of the proposed integral method.
The algorithm described in this paper has been implemented as part of a suite of simulation tools that are currently in use by industrial system designers in their real industrial activity. The required training for the use of the developed code is very low.

THE SYMMETRY OF CABLE GEOMETRY

The geometry of a submarine three-phase cable features a regularly repeating structure, arising from the helical twisting with pitch P_S of the core conductors and sheaths and the helical twisting with a pitch P_A of the external armor.

This regularly repeating structure allows building the cable geometry by replicating, with appropriate geometrical transformations, a single elementary unit cell. This fact can be exploited to reduce the computational requirements of numerical simulations.

To find the unit cell and the set of transformations that allow describing the entire geometry of the cable, we analyze the geometrical transformations which leave the cable armor Ω^A and the metallic sheaths Ω^S unchanged, at first separately and then jointly.

We consider a xyz cartesian frame whose z-axis coincides with the axis of the cable. Let $R\,(\Theta)$ denote the matrix associated to a rotation by an angle Θ around the z-axis

$$R(\Theta) = \begin{bmatrix} cos(\Theta) & -sin(\Theta) & 0 \\ sin(\Theta) & cos(\Theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [1]$$

and $\boldsymbol{t}(l)$ denote the vector associated to a translation by a distance l along the z-axis

$$\boldsymbol{t}(l) = \begin{bmatrix} 0 \\ 0 \\ l \end{bmatrix} \quad [2]$$

If the number of armor wires is N, then the armor geometry is invariant by the transformations

$$\tau_n^{A_1}(\boldsymbol{x}) = R_n^{A_1}\boldsymbol{x}, \quad n = 0, 1, \dots, N-1 \quad [3]$$

$$\tau_n^{A_2}(\boldsymbol{x}) = R_n^{A_2}\boldsymbol{x} + \boldsymbol{t}_S^{A_2}, \;\; s \in \mathbb{R} \qquad [4]$$

where $\boldsymbol{x} \in \mathbb{R}^3$ and

$$R_n^{A_1} = R\left(2\,\pi\frac{n}{N}\right) \qquad [5]$$

$$R_S^{A_2} = R\left(2\,\pi\frac{s}{P_A}\right) \qquad [6]$$

$$\boldsymbol{t}_S^{A_2} = \boldsymbol{t}(s) \qquad [7]$$

Transformations [3] form a finite cyclic group of order N, generated by the rotation along the cable axis which brings an armor wire to coincide with the next one. Transformations [4] form a continuous screw group. Geometrically they can be visualized as the armor wires sliding along themselves with a screw motion. These symmetries are shown in Fig. 1-2.

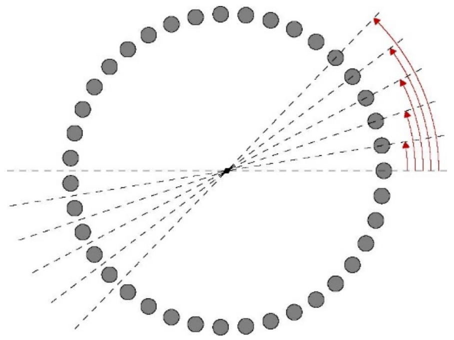


Fig. 1: Discrete rotational symmetries

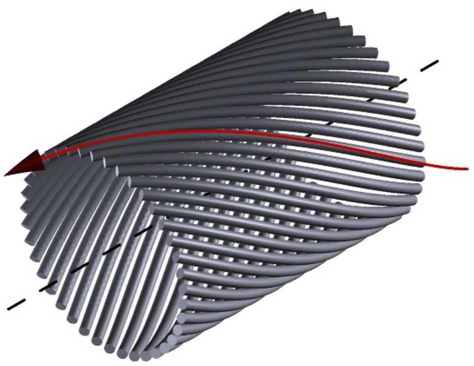


Fig. 2: Continuous screw symmetry

The composition of [3] and [4] is commutative and results in

$$\tau_n^A(\boldsymbol{x}) = R_{n,s}^A\boldsymbol{x} + \boldsymbol{t}_{n,S}^A, \;\; n = 0,1,\dots,N-1, \;\; s \in \mathbb{R} \qquad [8]$$

where

$$R_{n,s}^A = R\left(2\,\pi\left(\frac{s}{P_A} + \frac{n}{N}\right)\right) \qquad [9]$$

$$\boldsymbol{t}_{n,S}^A = \boldsymbol{t}(s) \qquad [10]$$

The three phase conductors and sheaths are characterized by symmetries which are analogous to the symmetries of the armor

$$\tau_m^{S_1}(\boldsymbol{x}) = R_m^{S_1}\boldsymbol{x}, \quad m = 0,1,2 \qquad [11]$$

$$\tau_q^{S_2}(\boldsymbol{x}) = R_q^{S_2}\boldsymbol{x} + \boldsymbol{t}_q^{S_2}, \;\; q \in \mathbb{R} \qquad [12]$$

where

$$R_m^{S_1} = R\left(2\,\pi\frac{m}{3}\right) \qquad [13]$$

$$R_q^{S_2} = R\left(2\,\pi\frac{q}{P_S}\right) \qquad [14]$$

$$\boldsymbol{t}_q^{S_2} = \boldsymbol{t}(q) \qquad [15]$$

Sheath symmetry [11] is shown for completeness, but for simplicity only symmetry [12] is considered in the rest of the article. Considering also [11] would decrease the size of the elementary unit cell by a factor three, but at the cost of a greater complexity in the implementation and exposition.

For certain combinations of the parameters n, s, and q, [8] and [12] describe the same geometrical transformation. To systematically find such combinations of parameters it is necessary to verify when

$$R_{n,s}^A\boldsymbol{x} + \boldsymbol{t}_{n,S}^A = R_q^{S_2}\boldsymbol{x} + \boldsymbol{t}_q^{S_2} \qquad [16]$$

for each $\boldsymbol{x} \in \mathbb{R}^3$. From the definitions of $R_{n,S}^A$, $\boldsymbol{t}_{n,S}^A$, $R_q^{S_2}$ and $\boldsymbol{t}_q^{S_2}$, [16] is equivalent to

$$\begin{cases} 2\pi\left(\frac{q}{P_S}\right) = 2\pi\left(\frac{s}{p_A} + \frac{n}{N} + k\right) \\ q = s \end{cases} \qquad [17]$$

The set of solutions of [17] can be parametrized with $n \in \mathbb{Z}$ as

$$s = q = \frac{n}{N}P_{cp} \qquad [18]$$

where

$$P_{cp} = \left(\frac{1}{P_S} - \frac{1}{P_A}\right)^{-1} \qquad [19]$$

is the cross-pitch length, i.e. the length of cable after which an armor wire returns to the same relative position with respect to the phase conductors.

From the previous discussion, we can conclude that the entire geometry of the cable is invariant by the following set of transformations

$$\tau_n(\boldsymbol{x}) = R_n\boldsymbol{x} + \boldsymbol{t}_n, \;\; n = 0,1,\dots,N-1 \qquad [20]$$

where

$$R_n = R\left(2\,\pi\frac{n}{N}\frac{P_{cp}}{P_S}\right) \qquad [21]$$

$$\boldsymbol{t}_n = \boldsymbol{t}\left(\frac{n}{N}P_{cp}\right) \qquad [22]$$

As a consequence, the entire geometry of the cable can be partitioned in the following way

$$\Omega = \cup_{n\in\mathbb{Z}}\tau_n(\Omega_0) \qquad [23]$$

where Ω_0 is an elementary unit cell consisting of a slice of cable of length P_{cp}/N, as shown in Fig. 3.

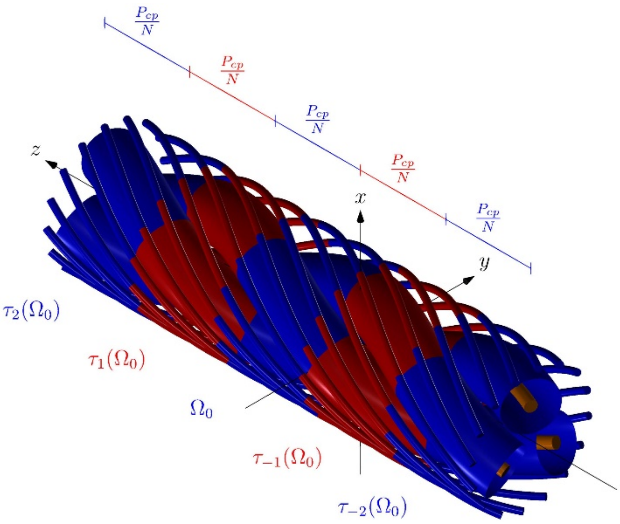


Fig. 3: Partition of the cable geometry into unit cells

FORMULATION

Conductor sheaths in submarine cables are usually metallic, non-magnetic, and thin with respect to the skin depth at mains frequency. Therefore with good approximation the current density can be considered uniform across their thickness. Moreover their thickness is small with respect to other geometrical dimensions, and enables to represent the current as a surface distribution. With these hypotheses, it is possible to employ surface integral equation formulation [6] to compute the current density in the sheaths.

Armor wires are usually made of ferromagnetic material, hence, when they are exposed to the magnetic field produced by the core conductors, they become magnetized. Moreover, since the magnetic field is time-varying, eddy currents develop inside the wires.

In the proposed formulation eddy currents are not directly and explicitly considered, but they are modeled as an additional contribution to the wire magnetization [5]. This can be done by modifying the actual value of complex relative magnetic permeability μ_r of the armor wires with a tensor-valued permeability coefficient which takes two different values $\mu_r^{\parallel}$ and $\mu_r^{\perp}$ in the axial and transversal direction of the wire respectively. The formulation is described in detail in [5].

It is well known that there is a mutual electromagnetic interaction between the armor and the sheaths. On the one hand the presence of the armor contributes to concentrating the magnetic field inside the cable, which leads to greater losses in the sheaths. On the other hand, the presence of the sheaths shields the magnetic field generated by the core conductors, leading to lower losses in the armor. To evaluate this effect it is necessary to introduce a bidirectional coupling between the sheaths formulation and the armor formulation, as explained in [7].

NUMERICAL RESULTS

The cross-section of the geometry of the cable under analysis is shown in Fig. 4.

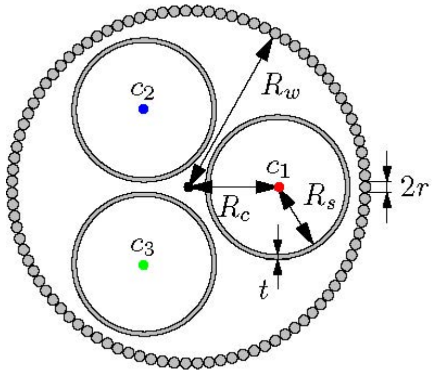


Fig. 4: Geometry of cable section

The geometrical dimensions reflect those of a typical high-voltage submarine cable and physical parameters are consistent with internal measurements performed by Prysmian S.p.A. The magnetic behavior of the armor is modeled using a complex-valued permeability $\mu_r = |\mu_r|e^{-j\,\phi}$. The physical and geometrical parameters used for the simulations of this section, unless otherwise stated, are those summarized in Table I.

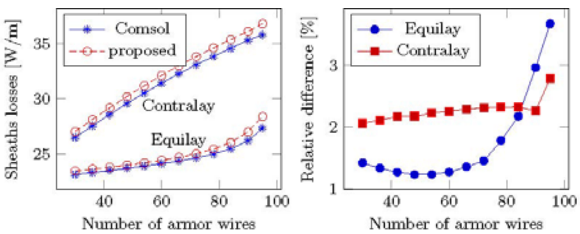
Quantity	*Value*
R_c	57 mm
R_w	110 mm
R_s	43.3 mm
r	3.5 mm
t	2.9 mm
P_A	2000 mm
P_S	3000 mm
$\|\mu_r\|$	300
ϕ	60°
ρ_{arm}	$2.8 \times 10^{-7}\ \Omega m$
ρ_{sh}	$2.14 \times 10^{-7}\ \Omega m$

Table I: Cable geometrical and physical parameters

The inherent simplicity of the algorithm described in the paper allowed to implement most of the code in the interpreted language of Octave (which is mostly compatible with that of Matlab), thus maximizing its maintainability and extensibility. Input and Output are given in the form of MS Excel spreadsheets to allow for simple integration in existing design workflows.

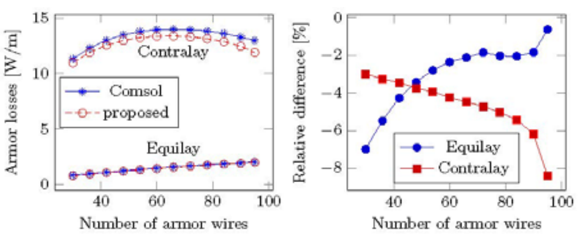
Since the proposed method cannot be validated experimentally because it does not provide conductor losses as they are assumed like filaments, its results are compared to those obtained with FEM models developed using the software COMSOL Multiphysics [8]. Following [1], the employed FEM model exploits a rotated periodicity which ensures that field quantities have a periodic repetition of a cross-pitch. Actually, the model length is three times the cross-pitch, i.e. 3600 mm for the contralay configuration and 6000 mm for the equilay configuration. Losses are then computed from the fields evaluated in the one-third in the middle, to drastically reduce the effects of the finite size of the computational domain. Magnetic insulation conditions are imposed at its boundaries, and sheaths are discretized using two layers of quadratic FEM elements. Second-order finite elements are used for a total number of 115.266.416 degrees-of-fredom. An extruded mesh is built inside the phase cables, the armor, and the surrounding domain. The length of the extruded finite elements is equal to 20 mm. Free tetrahedral elements are used in the remaining part of the model. Two finite elements are placed through the thickness of metallic sheaths and 16 elements discretize the cross-section of each armor wire.

Fig. 5 and Fig. 6 show the comparison between the losses computed with the proposed method and with the FEM models as a function of the number of armor wires, both in the sheaths and in the armor and for both the contralay and equilay configurations.



(a) (b)

Fig. 5: Sheaths losses for different numbers of armor wires in both contralay and equilay configurations. (a) Sheaths losses. (b) The relative difference between the proposed model and FEM.



(a) (b)

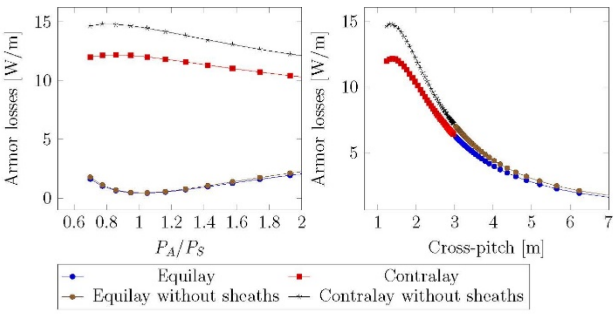
Fig. 6: Armor losses for different numbers of armor wires in both contralay and equilay configurations. (a) Armor losses. (b) The relative difference between the proposed model and FEM.

The agreement between the two methods is generally good, with relative differences always below 4 % for sheath losses and 10 % for armor losses. It can be observed that the discrepancy gets larger when the number of armor wires increases, hence when wires are very close to each other. This has already been observed in [5].

The contralay configuration leads to higher losses both in the armor and in the conductor sheaths. Losses exhibit a nonlinear behavior with respect to the number of wires. This is particularly noticeable in contralay armor losses, which, exhibit a maximum when the number of wires is equal to 66 and then decrease slightly but non negligibly.

In every configuration, the presence of the armor leads to higher losses in the sheaths. This effect is particularly marked for cables in equilay configuration with closely spaced armor wires: adding more wires leads to a steep increase in sheath losses.

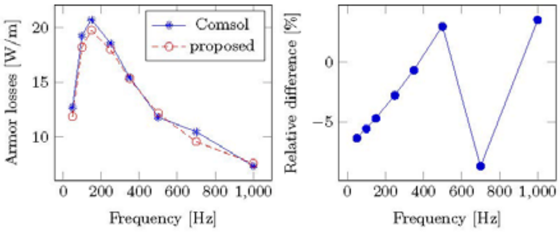
In Fig. 7 the behavior of armor and sheaths losses is investigated as the value of armor laying pitch P_A is varied between 2.1 m and 6 m (parameter P_A/P_S is of interest in the mechanical design phase), while the sheaths laying pitch is held fixed at P_S=3m. The values of losses (both for contralay and equilay cable configurations) are plotted both against the ratio P_A/P_S and against the cross-pitch length P_{cp} defined in [7]. From this figure it can be noted that the equilay configuration always guarantees lower losses both in the armour and in the sheaths. In case a contralay configuration is to be adopted, higher values for P_A/P_S provide lower losses.



(a) (b)

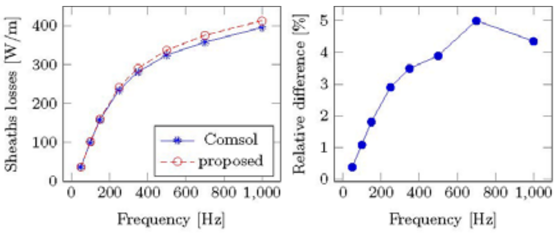
Fig. 7: Losses per unit length of cable for different values of armor pitch: on the left as a function of the ratio P_A/P_S, on the right as a function of the cross-pitch.

Fig. 8 and Fig. 9 represent a validation of our results in a frequency range between 50 Hz and 1 KHz, which is the typical range of interest in the computation of armor and sheath losses to consider harmonics in the current waveforms. When frequency increases, conductive sheaths are more effective in shielding the magnetic field of the phase conductors so that armor losses decrease. This happens at the expenses of losses in the sheaths. As can be noted, the difference with FEM in the computation of armor losses does not show any monotonic behavior vs. frequency, showing that the assumptions at the basis of the proposed formulation for the armor are valid in a wide frequency range. On the other hand, the difference with FEM in the estimation of sheath losses increases with frequency, which can be attributed to the simplifying hypothesis of uniform current density in the sheaths, which is less valid at a higher frequency. Comparison with FEM at frequencies higher than 1 KHz would require expensive refining of the mesh.



(a) (b)

Fig. 8: Armor losses in contralay configuration at various frequencies. (a) Armor losses. (b) The relative difference between the proposed model and FEM.



(a) (b)

Fig. 9: Sheaths losses in contralay configuration at various frequencies. (a) Sheaths losses. (b) The relative difference between the proposed model and FEM.

CONCLUSION

The new fully coupled formulation can be applied to estimate armor and sheath losses in actual submarine cables with a low computational burden. With the help of the new fast approach, extensive parametric numerical simulations needed in a design stage become affordable. They show how the equilay configuration of the armor wires is always characterized by lower armor and sheath losses. Furthermore, they also indicate that both armor and sheath losses increase with the increase of the absolute value of the complex relative permeability, for all the considered typical values of electrical resistivity.

The efficiency of the proposed method is based on two main properties. The first one is its fully integral approach when modeling the armor and the sheaths, avoiding the need to mesh the air as required by FEM. The second one is exploiting the geometrical symmetries of the cable structure and their proposed theoretical analysis can be also applied to other numerical methods.

It is true that since FEM-shortened models became available, one of the advantages of the proposed integral formulation, namely the reduction of the computational burden, became less apparent. However, the proposed formulation is relevant for two reasons. Firstly, based on integral equations, it has some advantages compared to FEM simulation: simpler construction of the computational mesh, no need to discretize non-magnetic and non-conductive domains, no need for a fine mesh to properly discretize skin effect in armor wires or thin airgaps between armor wires. Secondly, the integral formulation, in a setting dominated by FEM simulations, provides an additional, independent method that can be used to validate and increase confidence in the accuracy of results obtained using FEM simulations.

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