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Buyback and risk-sharing contracts to mitigate the supply and demand disruption risks

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Abstract

The recent widespread of the COVID-19 pandemic has created major disruptions in global supply chains. Events like COVID-19 pose enormous challenges in the movement of people, raw material, and finished products. Companies find it difficult to manage business continuity under supply uncertainties and disruptions. This paper deals with the problem of the optimal ordering policy of the retailer who faces stochastic demand and risks of supply uncertainty and disruption in a single period. The existing literature has suggested that dual sourcing is the best strategy to mitigate the risks of disruptions. In line with this, we consider a two-echelon supply chain consisting of a single retailer and two suppliers (i.e., the main supplier and the backup supplier). The retailer faces the risks of random yield and disruption from the main supplier who provides the product at a cheaper cost. In contrast, the backup supplier is perfectly reliable or has no probability of failure, but is relatively expensive. This paper develops an analytical model using contract-based mechanisms considering the risks of demand uncertainty, random yield, and supply disruption. We consider two typologies of contracts with suppliers, namely risks sharing contract with the main supplier for over-production and under-production due to the random yield —and buyback contract with the backup supplier. A numerical experiment and sensitivity analyses are performed to validate the analytical model and explore the impacts of the major parameters on the decision variables and profits of the supply chain's members. The results provide guidelines for managers regarding how sourcing decisions are influenced by the risks of supply disruption, random yield, and demand uncertainty.

Keywords: Supply disruption; COVID-19; random yield; demand uncertainty; buyback contract; ordering policy; risk-sharing contracts.

1. Introduction

With the intensification of globalization and the rise of India and China as global outsourcing hubs, many firms desire to tap into the cheap resources of these emerging economies. Today's supply chains are therefore longer, complex, and more supplier's dependent than ever, which has exposed the firms to a broader spectrum of supply chain risks. The recent rapid and wide spread of COVID-19 has created significant disruptions in global supply chains. Because of COVID-19, many countries around the world have imposed lockdown, which has disrupted the movement of people, raw material, and finished goods (such as automotive parts, optical-electronics, medical equipment, electronic parts, and consumer goods) from different parts of the world (Financial Times, 2020). This type of disruptive events considerably challenges global supply chains.

Over the years, outsourcing has helped improve various aspects of product and supply chain performances; however, it has also increased several types of risks. Two such important risks are the random yield and supply disruption. Under the risks of random yield, the suppliers cannot provide a certain portion of the demand (Burke *et al.*, 2008). It is common knowledge that the yield of most of the production processes is not deterministic, and it varies widely from one industry to another. For example, in the agriculture industry, it is challenging to predict the exact yield, as it is greatly affected by weather, insects, and parasitic attacks.

Similarly, in many developing countries, the availability of inputs like water for irrigation, power, and credit is quite uncertain. The issue of random yield is also associated with many of the hi-tech manufacturing processes as they are still highly sensitive to ambient conditions and minor fluctuations in process parameters (one such example is microchip fabrication). The random yield can be viewed as uncertainty in the delivery volume, which means that the delivered quantity is not the one that was ordered by the retailer, even though the supplier may be able to provide the goods (Hou *et al.*, 2010).

On the other hand, under the risk of supply disruption, the supplier completely fails due to the occurrence of disruptive events and cannot provide a single unit order quantity (Hou *et al.*, 2010; Meena et *al*., 2011; Meena and Sarmah, 2013). As today's supply chains are expanding globally, natural calamities, pandemics like the COVID-19, currency fluctuation, piracy, and other human-made events are increasing the disruption risks in the supply chain. Supply disruption risks are infrequent or rare but have a high impact on the entire supply chain (Ellis *et al.*, 2010; Meena and Sarmah, 2016; Duffin, 2020). Several examples highlight the shortcomings of the single-sourcing and show the benefits of the dual sourcing for mitigating the risks of supply disruption. This phenomenon is quite common in off-shoring scenarios (Warburton and Stratton, 2005; Sawik, 2018; Venkatesan *et al*., 2018). For example, in 2000, a fire at the plant of the Nokia's and Ericsson's common chip supplier in Albuquerque caused $400 million lost for Ericsson's. Nokia managed the shortage of chips within just three days thanks to the alternative suppliers. Ericsson, however, could not avoid production shutdown as it relied only on a single supplier (*The Economist*, 2006). Recently, the COVID-19 global pandemic has disrupted supply chains and brought a shortage of materials around the world — especially affecting those companies that rely on a dense network of foreign suppliers (e.g., it is estimated that 92% of US companies have a tier-1 supplier in COVID-19 affected regions; Financial Times, 2020). It is expected that, because of the COVID-19, the global gross domestic product (GDP) will decrease by 2.9% in 2020 (Duffin, 2020). Many studies exist in the literature on supply disruption risks, and also one can predict such risks with certain accuracy. On these bases, this paper studies a two-echelon supply chain consisting of a retailer and two suppliers (the main and the backup suppliers) and develops an analytical model for the optimal ordering policy of the retailer under demand uncertainty and risks of random yield and supply disruption. The main supplier is cheap but exposed to both risks of supply disruption and random yield, whereas the backup supplier is reliable but more expensive.

In line with Bose and Anand (2007) and He and Zhang (2007), we consider different risks sharing contracts for random yield between the main supplier and the retailer and evaluate the supply chain partners' performance. This paper defines the risk-sharing contract as a mechanism where the retailer shares the costs of under-production and over-production due to random yield with the supplier. The detailed analytical discussion on how the risk is shared between the retailer and supplier is provided in section 3.1. Similar to Inderfurth and Clemens (2014), we also consider a buyback contract between the retailer and the backup supplier to cope with demand uncertainty and supply disruption. Furthermore, we investigate how the optimal order quantity, buyback quantity, and buyback price of the backup supplier varies under diverse conditions. In particular, the main contribution of this paper is to develop contracts-based mechanisms considering multiple suppliers with different risk profiles in a single model—incorporating three types of supply chain risks, namely, demand uncertainty, random yield, and supply disruption. This paper investigates how different parameters of contracts influence the ordering policy and the profits of the retailer, the main supplier, and the backup supplier. Several numerical studies and sensitivity analyses are performed to test our model and investigate the impact of major parameters on the profits of each supply chain member.

The rest of the paper is organized as follows. The review of the relevant literature is discussed in section 2. The problem description and the mathematical model are presented in section 3. A numerical study and sensitivity analyses are conducted in section 4. Finally, conclusions and scope for future works are provided in section 5.

2. Literature Review

In this section, we review the existing literature about (i) sourcing decisions under demand uncertainty, random yield, and supply disruption risks, and (ii) buyback and risk sharing contracts mechanisms.

2.1 Sourcing decisions under risks

Parlar and Wang (1993) study a random yield problem for two suppliers with different pricing as per the reliability of the supplier. They analyze the diversification in supplies, the economic ordering quantity (EOQ) model, and the newsvendor problem under random yield. Yano and Lee (1995) review the issue of lot-sizing under the risk of random yield and discuss different approaches to model costs and random yield. Swaminathan and Shanthikumar (1999) study demand diversification under supply uncertainty. They find that, under sufficient capacity, it is optimal to purchase a larger share of the supplies from the expensive supplier who is more reliable. Kazaz (2004) analyzes the impact of random yield in olive oil production and demonstrates how to reduce the random yield risks using the spot market purchases. Babich at al. (2007) discuss the trade-off between the single-sourcing and diversification under supplier default risk. Snyder and Shen (2006) find that failing to plan for supply disruptions has more significant implications than that for demand uncertainty. They show how multiple decentralized stocking locations can be beneficial to minimize the risk of supply uncertainty. Chopra *et al.* (2007) focus on the decoupling of recurrent supply risk and disruption risk when risk aversion strategies are planned. They find that a firm should utilize more its reliable suppliers if most of the supply risk growth comes from increasing disruption probabilities. Federgruen and Yand (2008) develop analytical models for solving the problem of supplier selection and demand diversification under random yield risk to minimize the total cost. Yu *et al.* (2009) formulate the sourcing decision problem where the manufacturer chooses between the single and the dual sourcing under disruption risk. Tomlin (2009) provides some crucial insights into disruption risk aversion tactics. He investigates the effect of three strategies, namely supplier diversification, contingent sourcing, and demand switching on the risk exposure and expected profits.

Meena *et al.* (2011) analyze the problem of selecting the optimal number of suppliers considering different probabilities for the disruption risks, different capacity, and the presence of compensation for suppliers who do not fail. Meena and Sarmah (2013) develop a mixed-integer nonlinear programming model to determine the optimal ordering policy for multiple sourcing under different types of disruption risks. Gurnani *et al.* (2014) analyze the dual sourcing problem under disruption risks. They provide the optimal situation to use the single-sourcing strategy and conclude that, in practice, companies prefer to diversify. Meena and Sarmah (2016) study the sourcing decision (i.e., supplier selection and order allocation) problem under disruptions risks and quantity discounts. In recent years, many studies (e.g., Ledari *et al.*, 2018; Namdar *et al.*, 2018; Sabouhi *et al.*, 2018; Sawik, 2019) have been conducted to determine the best sourcing strategies under different types of risks. We refer the interested reader to the study of Hosseini *et al.* (2019), who provides an excellent review on analytical studies conducted for mitigating supply chain risks.

2.2 Risk sharing and buyback contracts

Several studies exist in the literature on the design and implementation of buyback contracts in different supply chain configurations. A buyback contract is defined as "a commitment by the supplier to buy the unsold inventory of the goods back at the end of the selling season to induce the retailer to order more from the supplier" (Mantrala and Raman, 1999). Lau and Lau (1999) consider a scenario where the manufacturer sets the selling price to the retailer and the return price for the returned items. After observing the manufacturer's strategy, the retailer sets the order quantity to the manufacturer. They also investigate the risk attitude of other supply chain members. Yao *et al.* (2005) discuss a manufacturer–retail supply chain consisting of a mix of a traditional retail channel and direct channel and determine how the manufacturer designs its return policy under direct channel option.

Bose and Anand (2007) discuss different types of buyback contracts to cope with supply chain risks under diverse settings. Ding and Chen (2008) analyze a three echelon supply chain with a buyback contract between each of its members. Hou *et al.* (2010) study a buyback contract with the backup supplier under supply disruptions. Zhao *et al.* (2010) investigate a supply chain coordination problem using a contract option approach, where, at the beginning of the period, the retailer reserves a quantity paying the supplier the option price. Based on this, the supplier determines his production quantity Q, and, during the selling season, the retailer purchases a quantity up to Q, paying the supplier the exercise price. They show how to achieve a Pareto optimal situation with a numerical study. Lee *et al.* (2016) analyze and compare a vendor-managed inventory system (where supplier and customer share the stockout cost) with the integrated supplier-customer system. Their results show the equivalence of the two approaches "if and only if the supplier's reservation cost is equal to the minimum supply chain total cost of the integrated system." Ji *et al.* (2017) study a supply chain problem under demand disruption and show that a transshipment-before-buyback contract generates higher benefits (for the entire supply chain) than a traditional buyback contract. Zhao *et al.* (2018) develop a model to capture the interaction between the retailer and the manufacturer, where the retailer designs a risk-sharing contract with the manufacturer under random yield. They find that the retailer cannot benefit from risk-sharing under symmetric information of the random yield. Interestingly, under asymmetric information, the risk-sharing benefits not only the manufacturer but also the retailer under certain conditions.

Peng and Pang (2018) investigate the supply chain coordination problem under yield uncertainty and budget constraints. They propose buyback and risk sharing contracts, where the retailer shares random yield risks with the supplier by purchasing the overproduced quantities at the end of the production period, and the supplier buys back the unsold quantities from the retailer at the end of the period. Their results show that, for the budget-constrained supply chain with random yield, it can be coordinated if the risks of random yield and demand uncertainty are shared between the supplier and the retailer. Chakraborty *et al.* (2018) study a supply chain consisting of a retailer and two suppliers, where suppliers are exposed to disruption risks. They investigate the impact of two coordination strategies, namely emergency supplier and backup supplier options on supply disruption risks and supply chain performance. Their results show that, under supply disruption, even with lower probabilities, the retailer prefers the backup supplier option. They also find the optimal reserve quantity increases with disruption probabilities. Recently, Zare *et al.* (2019) investigate a supply chain that includes a supplier who faces random yield risk and multiple downstream retailers dealing with stochastic demands. They analytically analyze the impacts of risk-sharing mechanisms on the retailers' optimal order decisions and the supplier's production decisions under demand uncertainty and random yield.

This paper differs from the existing literature in the following ways. First, we develop an analytical model for the retailer’s ordering decisions considering three types of risks simultaneously, namely demand uncertainty, random yield, and supply disruptions. To the best of our knowledge, there is a dearth of studies that consider all these risks together. Second, we design two supply chain contracts (risk-sharing and buyback) mechanisms between the retailer and suppliers (i.e., the main and the backup) to maximize the supplier’s and the retailer’s profits. This paper contributes to the ordering policy literature by proposing supply chain contracts in the presence of three supply chain risks together.

3. Problem definition and mathematical formulation

A single-period two-echelon decentralized supply chain problem is considered, with one retailer and two suppliers, namely the main supplier and the backup supplier. The main supplier is cheaper but prone to the risks of disruption (and, in such case, becomes completely unavailable), while the backup supplier is more expensive but perfectly reliable. All the supply chain players are profit maximizers. For ease of exposition, we summarize the notation used in the model in Table 1.

Table 1: Notation

Symbol	**Notation**
q	Order quantity to the main supplier (units), the *decision variable*
I	Order quantity to the backup supplier (units), the *decision variable*
Q	The main supplier's production quantity (units), the *decision variable*
c	The main supplier's unit production cost ($/unit)
c_b	The backup supplier's unit production cost ($/unit)
c_e	Emergency production cost ($/unit)
c_r	Unit return cost of the retailer ($/unit)
c_u	Unit stockout cost of the retailer ($/unit)
i	Node for supply chain player R: the retailer; S: the main supplier; B: the backup supplier
p	Supply disruption probability at the main supplier
s	Unit sales price of the retailer ($/unit)
s_b	Unit salvage value of the backup supplier ($/unit)
w	Unit wholesale price from the main supplier ($/unit)
w_b	Unit wholesale price from the backup supplier ($/unit)
w_r	Unit return price offered by the backup supplier ($/unit)
w_2	Discounted price for over-production ($/unit)
D	Average demand
F(.)	The distribution function of the main supplier's delivery quantity, with f(.) as its density function
G(.)	The distribution function of the demand, with g(.) as its density function
U	Random yield distribution variable
X	Random demand at the retailer
β	Percentage of emergency production cost paid by the retailer
λ	Buyback fraction of the backup supplier's order
$\boldsymbol{\Pi_d^i}$	Expected profit under disruption for player *i*
$\boldsymbol{\Pi_{nd}^i}$	Expected profit under no disruption for player *i*

3.1 The risk sharing and buyback contract mechanism

The retailer places an order of q units from the main supplier at the beginning of the period, and then the main supplier decides his production quantity Q. The main supplier is subject to the risks of disruption (with probability p) and random yield. The main supplier is required to deliver at least q units. However, in the case of under-production because of random yield or disruption, he obtains the shortfall units from other emergency sources. In this paper, we consider a risk sharing contract between the retailer and the main supplier to share the risk of the under-production and over-production units.

In the case of under-production, the retailer's service level will be affected. Therefore, the retailer may choose to share the emergency production cost with the main supplier for the shortfall units. Similar to He and Zhang (2008), we assume the emergency production cost c_e to be higher than the main supplier's production cost ($c_e > c$). As per the risk sharing contract, a fraction β ($0<\beta<1$) of emergency production cost (c_e) is paid by the retailer, and the remaining $1-\beta$ by the main supplier. If random yield results in over-production, the retailer and the main supplier share the extra cost. In this case, the retailer purchases all units from the main supplier, but pays a unit price w for the first q units and a unit price w_2 for the overproduced ones (to avoid triviality, we assume $w_2 < w$, $\mu w_2 < c_e$, and $w_2 < c$).

Let U be the random variable with distribution function F(u), density function f(u), and $E[U] = \mu$, and let UQ be the yield from the production. Following He and Zhang (2008), the production quantity Q is independent of the yield distribution F(u). Let X be the random demand at the retailer, with distribution function G(x), density function g(x), and $E[X] = D$. We assume that f(u) and g(x) are positive except at the boundaries of their domains. In order to mitigate the risk, the retailer purchases I units (which is a portion of the average demand D) from the backup supplier at a unit wholesale price w_b. If, after the delivery from the main supplier, it turns out that the retailer does not need all the I units, she can return part of them (the maximum of which is λI, $0<\lambda<1$) at a unit return cost c_r, and is paid back a unit return price w_r ($w_r < w_b$). The backup supplier resells the unsold items to a secondary market at a unit salvage price s_b ($s_b < c_b$). Following Hou *et al.* (2010), we assume that the main supplier's wholesale price, w, satisfies $w \leq w_r - c_r$, i.e., the main supplier has a strong incentive to be the retailer's first choice. Once the backup supplier determines his prices w_b and w_r, the retailer decides the value of I. In our model, the retailer and the main supplier negotiate three parameters: the wholesale price w, the discounted price for over-production w_2, and the percentage of emergency cost β shared in the case of under-production. The retailer and the backup supplier negotiate two parameters: the buyback price w_r and the maximum portion of buyback items λ.

3.2 Analytical formulation

Consider the backup supplier's expected profit under disruption:

$$\Pi_d^B = w_b I - (w_r - s_b) E_x \, min\{(I-x)^+, \lambda I\} - c_b I. \quad (1)$$

The first term is the revenue of the backup supplier, the second term is the loss when excess quantities are returned by the retailer, and the third term represents the production cost. Similarly, under no disruption, the backup supplier's expected profit is:

$$\Pi_{nd}^B = w_b I - (w_r - s_b) E_{u,x} min\{max((I+uQ-x)^+, (I+q-x)^+), \lambda I\} - c_b I. \quad (2)$$

The terms in equation (2) represent the same quantities as in expression (1), adjusted to the case of no disruption. Since the disruption probability is p, the backup supplier's expected total profit is given by:

$$\Pi^B = p\Pi_d^B + (1-p)\Pi_{nd}^B. \quad (3)$$

The main supplier only generates a profit in case of no disruption (in case of disruption, he becomes completely unavailable). The main supplier's expected profit is given as:

$$\Pi^S = \Pi_{nd}^S = (1-p)[q\,w + w_2 E_u\,[(uQ-q)^+] - c_e(1-\beta)E_u\,[(q-uQ)^+]] - c\,Q. \quad (4)$$

The first term represents the revenue of the main supplier, the second term is the additional revenue in the case of over-production, the third term is the shared emergency cost incurred due to under-production, and the last term is the production cost. The main supplier maximizes his profits, subject to the constraint $\Pi^S>0$. We can now formalize the main supplier's optimal decision, its properties, and its relation to the order quantity q.

Proposition 1: *Under demand uncertainty and yield randomness, the main supplier's expected profit function* $\boldsymbol{\Pi^S}$ *:*

(i) *is concave in Q, and the optimal value Q*(q) satisfies*

$$\int_0^{\frac{q}{Q}} uf(u)du = \frac{c-\mu w_2(1-p)}{(c_e(1-\beta)-w_2)(1-p)}; \quad (5)$$

(ii) *Q*= Kq, where K is a constant determined by c,* w_2*,* c_e*, β, p and f(.);*

(iii) *K increases with* c_e *, and decreases with c, β, and p. Furthermore, if* $\mu < \frac{c}{(c_e(1-p)(1-\beta))}$ *then K decreases with* w_2*; otherwise, it increases with* w_2*.*

Proof of Proposition 1 is provided in Appendix A1.

Proposition 1 reveals that the main supplier's production quantity Q^* is a linear function of the order quantity q and does not depend on the wholesale price w —because the retailer always pays a unit price w for the first q units, independently on the random yield. Furthermore, when the emergency cost is higher, it is more convenient for the main supplier to increase production. We also observe that the production quantity decreases with the production cost and with the percentage of the emergency cost paid by the retailer (if the percentage of emergency cost allocated to the retailer increases, the main supplier is less affected by under-production cost). Similarly, as the disruption probability increases, the main supplier's chances to become unreliable increases, and therefore the optimal production decreases.

In case of disruption, the retailer can rely exclusively on the backup supplier. The retailer's expected profit under disruption is given by:

$$\Pi_d^R = s\, E_x min(I, x) - c_u E_x(x-I)^+ + (w_r - c_r)E_x\, min\{(I-x)^+, \lambda I\} - w_b I. \quad (6)$$

The first term is the sales revenue, the second term is the stockout cost, the third term is the net revenue generated by the units returned to the backup supplier, and the fourth term is the procurement cost from the backup supplier. The retailer's expected profit under no disruption is given by:

$$\begin{aligned}\Pi_{nd}^R = {} & sE_{u,x}\, min\{max(q+I, uQ+I), x\} - wq - w_2 E_u[(uQ-q)^+] \\ & -c_e \beta E_u[(q-uQ)^+] - w_b I - c_u\left[E_{u,x}\, min\{(x-uQ-I)^+, (x-q-I)^+\}\right] \quad (7) \\ & +(w_r - c_r)E_{u,x}\, min\{max[(uQ+I-x)^+, (q+I-x)^+], \lambda I\}.\end{aligned}$$

The first term is the sales revenue, the second term is the procurement cost from the main supplier, the third term is the additional procurement cost from the main supplier in case of over-production, the fourth term is the additional shared procurement cost from the main supplier in case of under-production, the fifth term is the procurement cost from the backup supplier, the sixth term is the stockout cost, and the seventh term is the net revenue generated by the units returned to the backup supplier. The retailer's total expected profit is given by:

$$\Pi^R = p\Pi_d^R + (1-p)\Pi_{nd}^R. \quad (8)$$

Maximizing the retailer's profit function with respect to the two decision variables q and I, we derive the following Proposition for optimality:

Proposition 2: *The optimal ordering decision for the retailer (q*, I*) satisfies the following conditions:*

(i) *If $\Pi^S>0$, the optimal value of q and I can be obtained by simultaneously solving the following two equations:*

$$(1-p)\left[\begin{array}{c}(s+c_u)\left\{\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} f(u)g(x)dxdu+\int_0^{\frac{1}{k}}\int_{q+I}^{\infty} f(u)g(x)dxdu\right\}\\ +(w_r-c_r)\left\{\begin{array}{l}\int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} f(u)g(x)dxdu+\lambda\int_{\frac{1}{k}}^{\infty}\int_0^{ukq+(1-\lambda)I} f(u)g(x)dxdu\\ +\int_0^{\frac{1}{k}}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu+\lambda\int_0^{\frac{1}{k}}\int_0^{q+(1-\lambda)I} f(u)g(x)dxdu\end{array}\right\}\end{array}\right] \qquad (9a)$$

$$+p\left[\begin{array}{c}(s+c_u)\int_I^{\infty} g(x)dx\\ +(w_r-c_r)\left(\int_0^{(1-\lambda)I}\lambda\, g(x)dx+\int_{(1-\lambda)I}^{I} g(x)dx\right)\end{array}\right]=w_b;$$

$$(s+c_u)\left[\begin{array}{l}\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} ukf(u)g(x)dxdu\\ +\int_0^{\frac{1}{k}}\int_{q+I}^{\infty} f(u)g(x)dxdu\end{array}\right]+(w_r-c_r)\left[\begin{array}{l}\int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} ukf(u)g(x)dxdu\\ +\int_0^{\frac{1}{k}}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu\end{array}\right] \qquad (9b)$$

$$=w+w_2\left[\int_{1/k}^{\infty}(uk-1)f(u)du\right]+\mathrm{c}_e\beta\left[\int_0^{1/k}(1-uk)f(u)du\right].$$

(ii) *If $\Pi^S<0$, then we set q*=0 and calculate the value of I* from equation (9b).*

Proof of Proposition 2 is provided in Appendix A2.

We now consider the case where the demand and the random yield are uniformly distributed over [a,b] and [c',d], respectively. We apply the results of Proposition 2 and obtain the following solution:

$$I^* = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}$$
$$q^* = \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1} \tag{10}$$

with

$$a_1 = \frac{k[\lambda(w_r - c_r) - (s + c_u)]}{2(b-a)(d-c')}\left(b^2 - 2\frac{a}{k} + \frac{1}{k^2}\right)$$
$$b_1 = \frac{k^2(s + c_u)}{3(b-a)(d-c')}\left(b^3 - 3\frac{a}{k^2} + \frac{2}{k^3}\right)$$
$$c_1 = \frac{d\,k(s + c_u)}{2(b-a)(d-c')}\left(b^2 - 2\frac{a}{k} + \frac{1}{k^2}\right) - w - \frac{w_2 k\left(b - \frac{1}{k}\right)^2}{2(b-a)} - \frac{c_e \beta k\left(\frac{1}{k} - a\right)^2}{2(b-a)}$$
$$a_2 = \frac{(2-\lambda)\lambda(w_r - c_r) - (s + c_u)}{d - c'}$$
$$b_2 = \frac{k(1-p)[\lambda(w_r - c_r) - (s + c_u)]}{2(b-a)(d-c')}\left(b^2 - 2\frac{a}{k} + \frac{1}{k^2}\right)$$
$$c_2 = \frac{[(s + c_u)d - \lambda(w_r - c_r)c']}{d - c'} - w_b$$

These results will be used in the next session in order to build our numerical study.

4. Numerical study and discussion

In this section, a numerical study is conducted to illustrate our results and investigate the properties of the risk-sharing and buyback contracts. In particular, we analyze the effect of (i) the disruption probability on the retailer's ordering policy and expected profit; (ii) the combined effect of demand uncertainty, supply uncertainty, and disruption probability on the retailer's ordering policy and on the three player's expected profits; (iii) the decision parameters of the risk-sharing contract with the main supplier; and (iv) the decision parameters of the buyback contract with the backup supplier. We set the values of the basic input parameters based on He and Zhang (2008) and Hou *et al.* (2010) studies. The base numerical values considered in the numerical study are shown in Table 2.

Table 2: Values of the different input variable

c	c_e	β	s	w	w_2	c_r	c_u	w_b	c_b	s_b	λ
15	50	0.2	100	55	13	2	10	80	20	10	0.6

Moreover, we let p taking values ranging from 0 to 1. According to the parameters above, we examine the values of w_r within the range of [58, 80] that satisfies the assumptions defined in section 3.1. Demand varies uniformly between 0 and 100, with an average of 50 units.

4.1 The impact of disruption probability on the ordering policies

We start analyzing how different levels of disruption probabilities affect our results, in particular with reference to the ordering policies and the players' profits. In Figure 1, we represent how the optimal order quantities change according to different levels of disruption probability. Initially, at a low disruption probability, most of the order is placed with the main supplier, and a very small or negligible order is placed with the backup supplier. As the probability of disruption increases, the optimal profit zone starts shifting towards the x-axis (i.e., the order to the backup supplier), and, for high values of p, it transcends to a state where onwards almost the entire order is placed with the backup supplier. Therefore, the higher the probability of disruption, the bigger the order to the backup supplier, and the smaller the order to the main supplier.

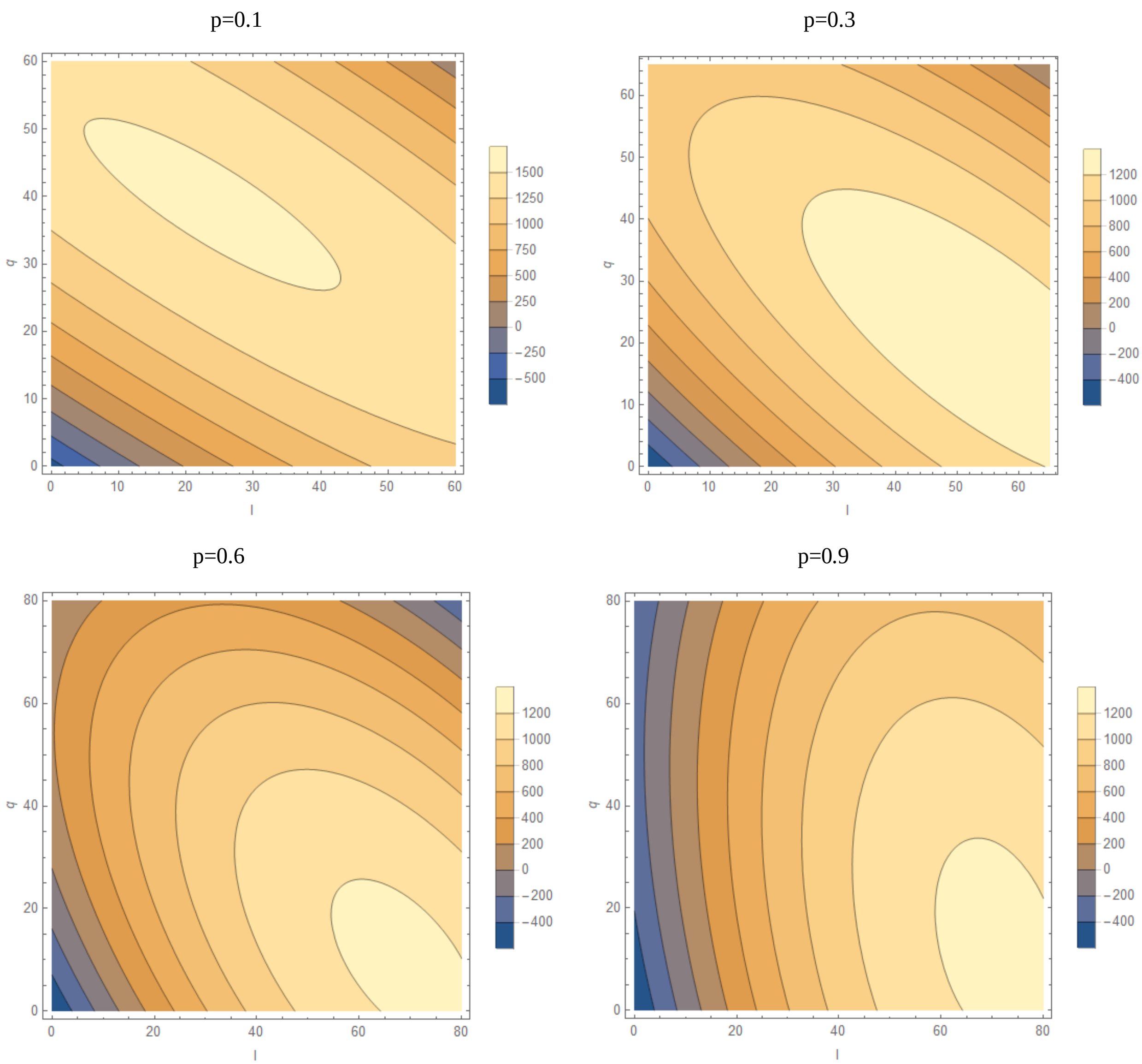

Figure 1: The impact of the disruption probability p on the optimal order quantities I* and q*

4.1.1 The impact of different risks on the ordering policies and profits

We next explore the effect of the three simultaneous risks (i.e., supply uncertainty, demand uncertainty, and supply disruption) on: (i) the order quantity to the main supplier q*, (ii) the order quantity to the backup supplier I*, (iii) the retailer's profit Π^{R}, (iv) the backup supplier's profit Π^{B}, and (v) the main supplier's profit Π^{S}. We let disruption probability (p) vary from 0.1 to 0.9;

demand uncertainty (DU) varies in a way that DU[0,100] represents the largest uncertainty and DU[40,60] the smallest uncertainty; supply uncertainty (SU) varies in a way that SU[0.1, 1.9] represents the largest uncertainty and SU[0.9, 1.1] the smallest uncertainty. We report the results in Table 3.

Table 3: The impact of demand uncertainty (DU), supply uncertainty (SU), and disruption risk (p) on the ordering policy and profits

DU[a,b]		**SU[a,b]**	**q***	**I***	Π^R	Π^B	Π^S
DU [0,100]	*p=0.1*	[0.1,1.9]	35.65	0	1,230.93	0	1,055.64
		[0.5,1.5]	40.51	0	1,236.91	0	1,312.50
		[0.9,1.1]	48.78	0	872.92	0	1,668.00
	p=0.3	[0.1,1.9]	33.11	20.89	627.03	904.15	638.02
		[0.5,1.5]	36.03	18.09	660.97	786.19	762.98
		[0.9,1.1]	35.68	22.56	579.14	974.47	823.43
	p=0.5	[0.1,1.9]	19.91	43.33	419.25	1,865.78	207.76
		[0.5,1.5]	22.17	41.78	435.88	1,801.06	247.60
		[0.9,1.1]	24.14	40.75	447.60	1,757.66	294.41
	p=0.7	[0.1,1.9]	13.82	50.00	371.30	2,213.40	42.08
		[0.5,1.5]	15.00	50.00	377.20	2,206.77	39.63
		[0.9,1.1]	17.50	49.79	389.74	2,187.55	32.41
	p=0.9	[0.1,1.9]	0	50.00	339.00	2,343.00	0
		[0.5,1.5]	0	50.00	339.00	2,343.00	0
		[0.9,1.1]	0	50.00	339.00	2,343.00	0
DU [20,80]	*p=0.1*	[0.1,1.9]	32.65	0.2	1,565.13	8.78	967.11
		[0.5,1.5]	37.57	0	1,624.14	0	1,217.46
		[0.9,1.1]	47.76	0	1327.33	0	1,633.30
	p=0.3	[0.1,1.9]	26.25	31.78	1,020.24	1,322.61	505.93
		[0.5,1.5]	28.28	29.87	1,049.47	1,248.2	599.07
		[0.9,1.1]	29.34	32.09	988.25	1,334.7	677.33
	p=0.5	[0.1,1.9]	17.75	48.26	821.34	2,022.79	185.23
		[0.5,1.5]	19.11	47.33	835.99	1,985.95	213.33
		[0.9,1.1]	20.42	46.70	846.14	1,960.56	249.02
	p=0.7	[0.1,1.9]	15.62	50.00	753.81	2,272.53	47.57
		[0.5,1.5]	16.32	50.00	760.35	2,265.90	43.11
		[0.9,1.1]	17.79	50.00	773.51	2,253.50	32.00
	p=0.9	[0.1,1.9]	0	50.00	685.00	2,505	0
		[0.5,1.5]	0	50.00	685.00	2,505	0
		[0.9,1.1]	0	50.00	685.00	2,505	0
DU [40,60]	*p=0.1*	[0.1,1.9]	20.58	25.86	1,456.88	900.41	609.52
		[0.5,1.5]	26.64	19.19	1,598.52	715.27	862.98
		[0.9,1.1]	37.1	16.59	1500.33	634.34	1,268.65

p=0.3	[0.1,1.9]	19.34	42.69	936.64	1416.67	373.83
	[0.5,1.5]	20.55	41.66	967.02	1,393.72	435.17
	[0.9,1.1]	23.02	41.64	937.65	1393.36	531.25
p=0.5	[0.1,1.9]	17.54	50.00	664.15	1,963.25	183.05
	[0.5,1.5]	17.79	50.00	678.30	1,950.34	198.65
	[0.9,1.1]	18.34	50.00	688.46	1,940.13	223.61
p=0.7	[0.1,1.9]	17.43	50.00	471.87	2,568.24	53.07
	[0.5,1.5]	17.63	50.00	479.07	2,561.57	46.59
	[0.9,1.1]	18.19	50.00	492.70	2,549.54	33.67
p=0.9	[0.1,1.9]	0	50.00	215.00	3,315	0
	[0.5,1.5]	0	50.00	215.00	3,315	0
	[0.9,1.1]	0	50.00	215.00	3,315	0

When DU increases, the retailer increases the order quantity to the main supplier for low values of p. However, when SU increases, the main supplier's order quantity decreases. When considering the simultaneous effect of increasing DU and SU, the order quantity to the main supplier increases, except for very low values of p. When p becomes high, the main supplier's order quantity starts to fall with the increase of DU. This result becomes even more evident for the combined effect of increased DU and SU.

The backup supplier's order quantity decreases as both DU and SU increase (since the retailer places a higher order with the main supplier). However, for high p, it becomes maximum: independently on DU and SU, the main supplier becomes unreliable, and the retailer mainly relies on the backup supplier. For the combination of higher p, DU, and SU, the retailer orders the maximum quantity from the backup supplier (to face the high p)—plus a negligible or small quantity from the main supplier (to face the high DU).

Considering the profits of the supply chain members, we observe that, under higher p, the backup supplier's profit increases; however, the retailer's and the main supplier's profits decrease. Further, the retailer's profit is concave in DU. For low and moderate values of p, the main supplier's profit increases with DU, and for high values of p, it falls with DU. The backup supplier's profit decreases with high DU. For low values of p, the retailer's profit is concave in SU; for moderate and high values of p, it decreases with SU. For low and moderate values of p, the main supplier's profit is decreasing with SU, and then, for high values of p, it increases with SU. The backup supplier's profit is convex in SU for lower values of p; then, for moderate and high values of p, it increases with SU.

We conclude that, for moderate and high p, the backup supplier benefits from higher SU because the retailer places a higher order with him, and, consequently, his profit increases. Furthermore, for very high p, the main supplier cannot produce and the retailer's profit is not affected anymore by the SU since she only relies on the backup supplier.

Overall, under simultaneous higher p, DU, and SU, the retailer's and the main supplier's profits are minimum. On the contrary, the backup supplier's profit is maximum since the retailer mainly relies on the backup supplier to face the risks and uncertainties. On the other hand, under simultaneous lower p, DU, and SU, the retailer's and the main supplier's profits are maximum, whereas the backup supplier's profit is minimum.

4.2 The buyback contract mechanism between the retailer and the backup supplier

In this section, we analyze the mechanism of the buyback contract between the retailer and the backup supplier, and how the decision parameters influence the retailer's ordering policy and the profits.

4.2.1 The return price w_r

We investigate the impact of the buyback price w_r on the backup supplier's order quantity I* (Figure 2) and on the backup supplier's expected profit (Figure 3) to support him determining the optimal return price.

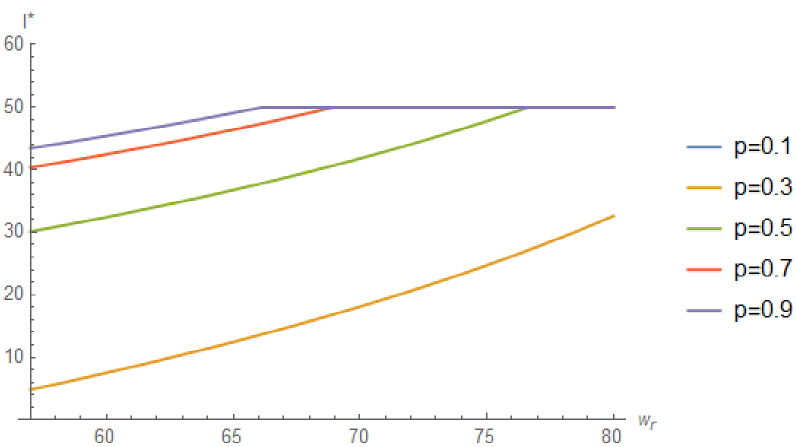

Figure 2: The impact of the return price w_r on the order quantity I*

Figure 2 shows that the backup supplier's ordering quantity is an increasing function of w_r. In particular, for a very low probability of disruption p, the order to the backup supplier remains negligible. At moderate values of p, the order quantity starts increasing significantly with w_r, and, for high values of p and w_r, the order quantity reaches its maximum (represented by the average demand).

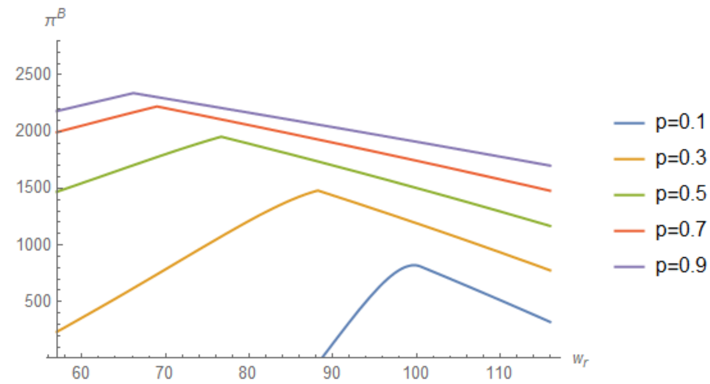

Figure 3: The impact of the return price w_r on the backup supplier's profit

Figure 3 shows that the backup supplier's expected profit is a concave function in w_r: as the return price increases, the expected profit also increases, up to a maximum value and after that, it decreases again. This is because a higher return price increases the backup supplier's order from the retailer (as shown in Figure 2) and, consequently, his profit. However, after a certain point, it becomes prohibitive for the backup supplier to further raise w_r because the increased margin is not able to compensate anymore for the higher price to pay back to the retailer. Moreover, we notice that such behavior is more evident for higher values of the disruption probability: as the probability of disruption increases, the optimal return price decreases. Since, in the case of high disruption, the retailer mainly relies on the backup supplier, there is no need for the backup supplier to offer a very high return price to become more attractive. We can conclude that, depending on the domain in which the business is operating, the backup supplier can strategically set the value of the buyback price that maximizes his profit.

4.2.2 The buyback supplier's wholesale price w_b

We now consider the effect of the wholesale price on the retailer's ordering policy. Figure 4 indicates that, as the buyback supplier's wholesale price decreases, the optimal order quantity to the backup (main) supplier increases (decreases).

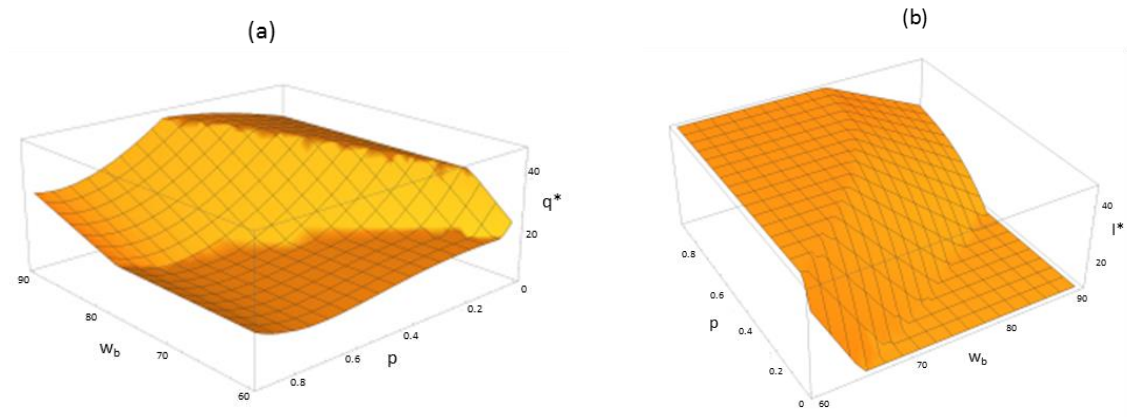

Figure 4: The impact of the main supplier's wholesale price w_b on the main supplier's order quantity q* (a) and on the backup supplier's order quantity I* (b).

However, it can be observed that, if the disruption probability is very low, then the effect of w_b on the optimal quantities is negligible since, in this case, the retailer relies on the main supplier. For high values of p and low values of w_b, the order quantity to the backup (main) supplier reaches its maximum (minimum). It is worth noting that the order quantity to the main supplier, although minimal, never reaches zero. We can conclude that, even when the disruption probability is very high, the retailer may still find it convenient to place an order with the main supplier. In this case, one of the driving components is the tradeoff between the backup supplier's wholesale price and the main supplier's wholesale price.

4.2.3 The buyback quantity λ

Next, we study the effect of the buyback quantity λ on the retailer and backup supplier. Figure 5 indicates that the backup supplier's order quantity is increasing with λ: the more quantity the retailer is allowed to return, the more quantity she orders from the backup supplier.

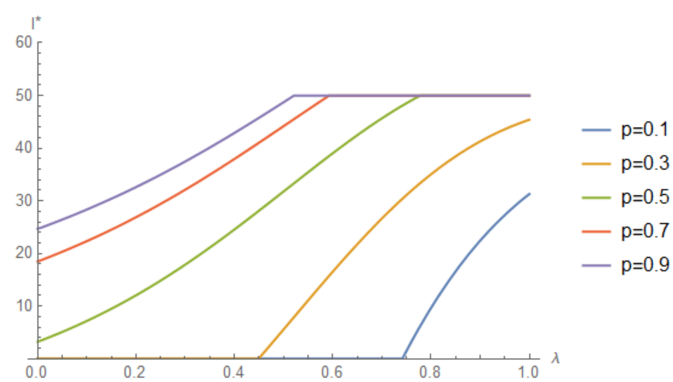

Figure 5: The impact of the buyback quantity λ on the backup supplier's order quantity

However, we also notice that, in the case of low disruption probability, the retailer only places an order to the backup supplier if the buyback quantity is extremely high: if disruption is most likely not going to happen, the retailer has no incentive to place an order with the much more expensive backup supplier, unless this can guarantee to buy back most of the ordered quantities. Figure 6a shows that the backup supplier's profit is a concave function in the buyback quantity λ: when λ increases, the backup supplier's order quantity also increases (as already observed in Figure 5), which consequently increases his profit.

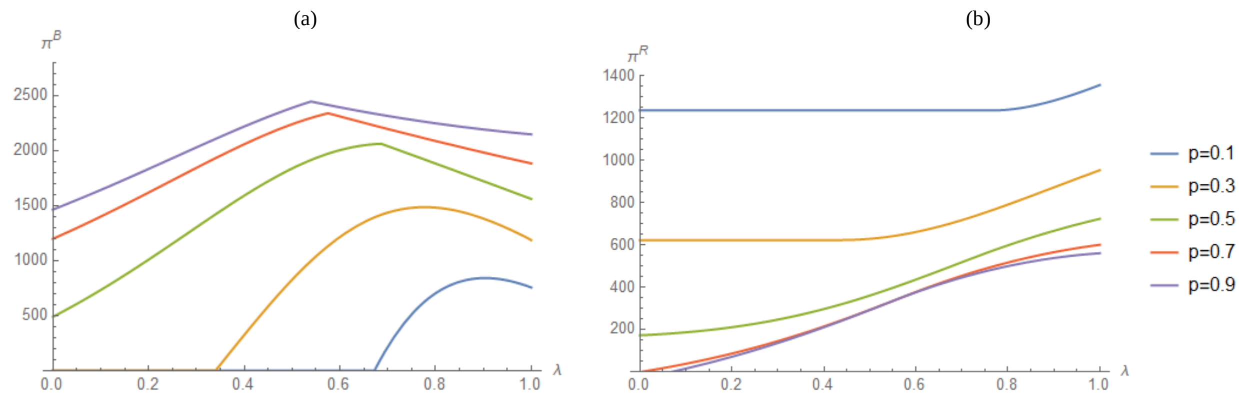

Figure 6: The impact of the buyback quantity on the backup supplier's (a) and the retailer's (b) profits

It means that the backup supplier can easily increase his profit offering a larger buyback quantity. However, after a specific threshold value of λ, the cost bared by the backup supplier (to pay back the high quantity of returned products) becomes prohibitive and, therefore, his profit starts to decrease again. We can observe that such threshold point decreases for higher values of disruption probability: as previously discussed in section 4.2.1, when the disruption probability is high, the backup supplier's order quantity increases anyway, and therefore there is no need for the backup supplier to offer a high buyback quantity to become more attractive. For low values of the disruption probability, the retailer's profit is slightly sensitive to the variation of λ (Figure 6b), and only starts to increase for high values of p. We also observe that the retailer's profit decreases with increasing values of p since, in case of disruption also, the retailer's probability to fulfill the final customer's demand decreases.

4.3 The risk sharing contract mechanism between the retailer and the main supplier

In this section, we analyze the mechanism of the risk sharing contract between the retailer and the main supplier, and how the decision parameters influence the retailer's ordering policy and the profits. We recall that the retailer and the main supplier share the risk due to under-production by sharing the emergency production cost c_e. The retailer pays a percentage β of c_e, and the main supplier the remaining 1-β. Similarly, the retailer shares the risk due to over-production with the main supplier by purchasing the extra units at a cost w_2 (lower than the wholesale price w). Since our model does not allow for a closed-form expression of the optimal β and the optimal w_2, we recur to the sensitivity analysis and study the impact of β and w_2 on the optimal order policy and profits. Before proceeding with the risk sharing parameter analysis, we first present the results for the main supplier's wholesale price w.

4.3.1 The main supplier's wholesale price w

We study the effects of the main supplier's wholesale price w on the main supplier's order quantity, the main supplier's profit and the retailer's profit. We observe that the order quantity is a decreasing function of w (Figure 7): a higher wholesale price leads to a lower order quantity (we notice that the values of q* set at zero correspond to a negative backup supplier's profit).

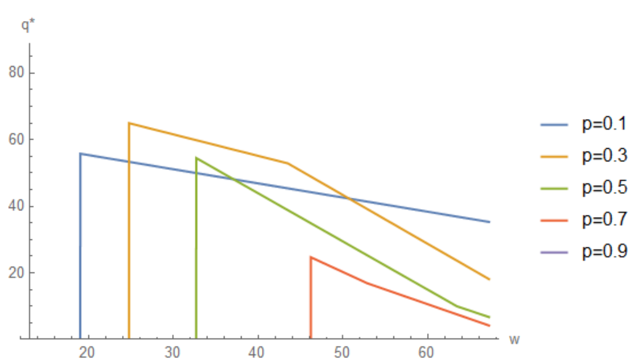

Figure 7: the impact of the main supplier's wholesale price (w) on the main supplier's order quantity

While the retailer (Figure 8a) always benefits from a lower w (except for p=0.9, when it is not affected by w since the main supplier's order quantity is null), the main supplier's profit is a concave function in w (Figure 8b): if the wholesale price increases, the supplier gets a higher margin from selling his products to the retailer. However, after a certain value of w, the profit starts to decrease because the order placed from the retailer also decreases. As expected, we also notice that the main supplier's profit is decreasing with the disruption probability since, as p increases, the retailer places an increasing portion of the order with the backup supplier.

(a) (b)

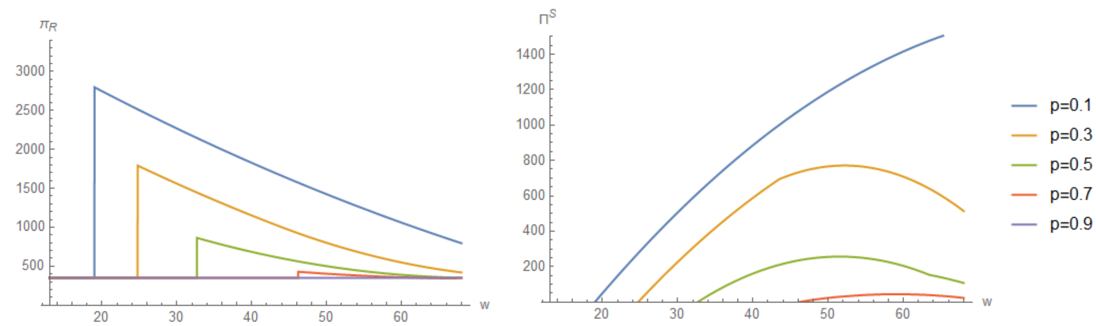

Figure 8: The impact of the main supplier's wholesale price (w) on the retailer's (a) and the main supplier's (b) profits

4.3.2 The effect of sharing under-production and over-production risk

In this section, we study the variation of the ordering policy in accordance with the emergency production cost-sharing factor β and the over-production cost w_2. As per the definition of risk sharing contract given in section 3.1, the higher β, the higher the percentage of emergency cost incurred by the retailer. Furthermore, from the assumptions defined in section 3.1 and the parameters' values set at the beginning of this section, it follows that $\beta < 0.74$. Figure 9 shows that, as β increases, the main supplier's order size decreases (this is always true except for very low values of disruption probability) and the order quantity to the backup supplier increases—the more expensive the emergency cost is for the retailer, the more she recurs to the backup supplier.

(a) (b)

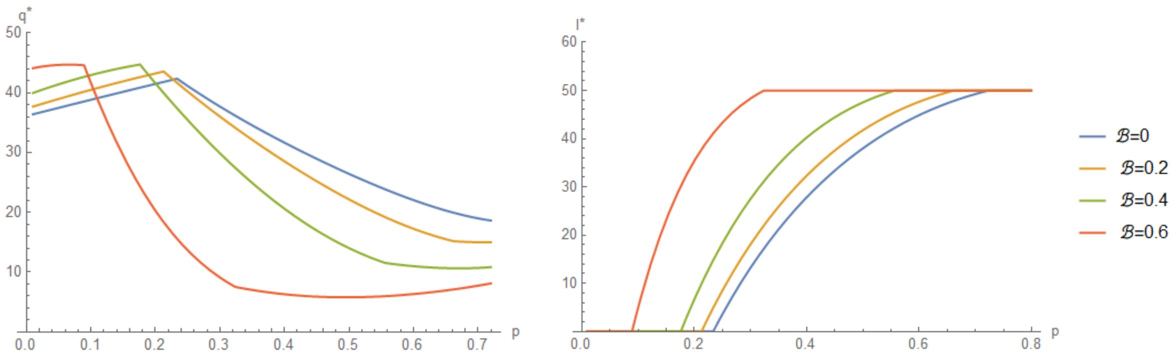

Figure 9: The impact of sharing the emergency production cost on the ordering policy

The retailer's profit decreases with increasing β and becomes constant as the maximum portion of the order is placed with the backup supplier (Figure 10a). On the other hand, the main supplier's profit increases with a higher value of β for low disruption probabilities and stabilizes for very high disruption probabilities (Figure 10b).

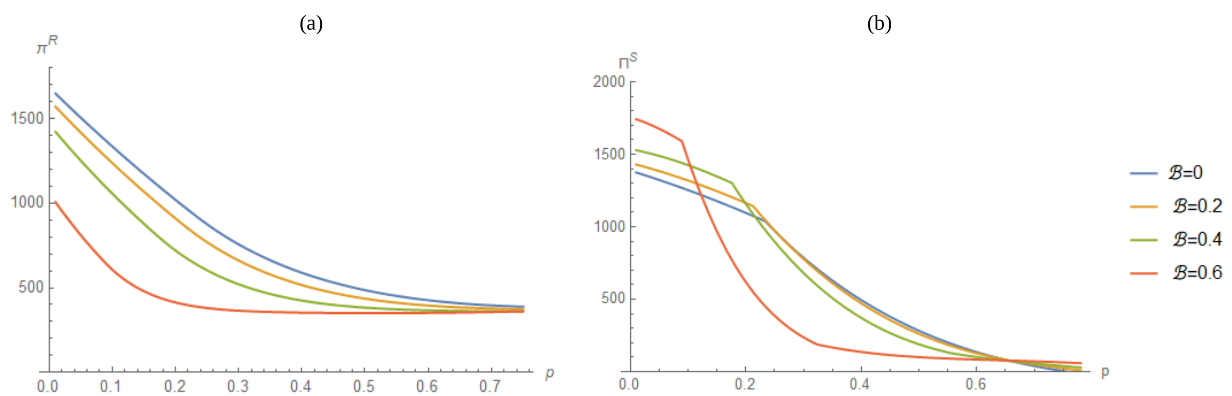

Figure 10: the impact of sharing the emergency production cost on the retailer's profit (a) and on the main supplier's profit (b)

We also investigate the ordering policy with respect to the over-production cost w_2, i.e., the discounted price at which the retailer purchases the overproduced quantities. From the assumptions defined in section 3.1 and the parameters' values defined at the beginning of this section, it follows that $w_2 < 15$. In Figure 11, we see that the ordering policy shows little sensitivity to variations of w_2, except for low values of the disruption probability: in such cases, order quantities of both the main supplier and the backup supplier are decreasing with w_2. That is because, in the case of low disruption, the retailer mostly relies on the main supplier, and therefore is more affected by the price for over-production.

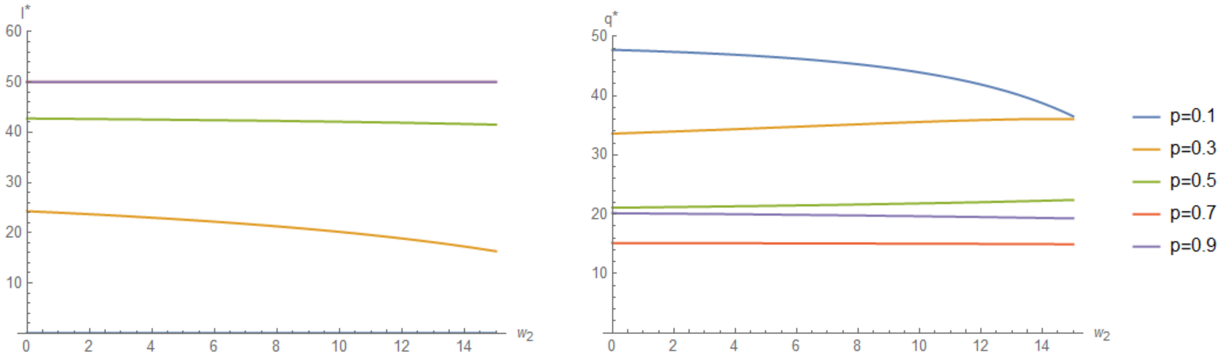

Figure 11: The impact of w_2 on the ordering policy

Similarly, the profit functions only show significant sensitivity to w_2 for low values of the disruption probability (Figure 12). In this case, the main supplier's profit decreases as w_2 approaches the production cost, while the retailer's profit increases with w_2.

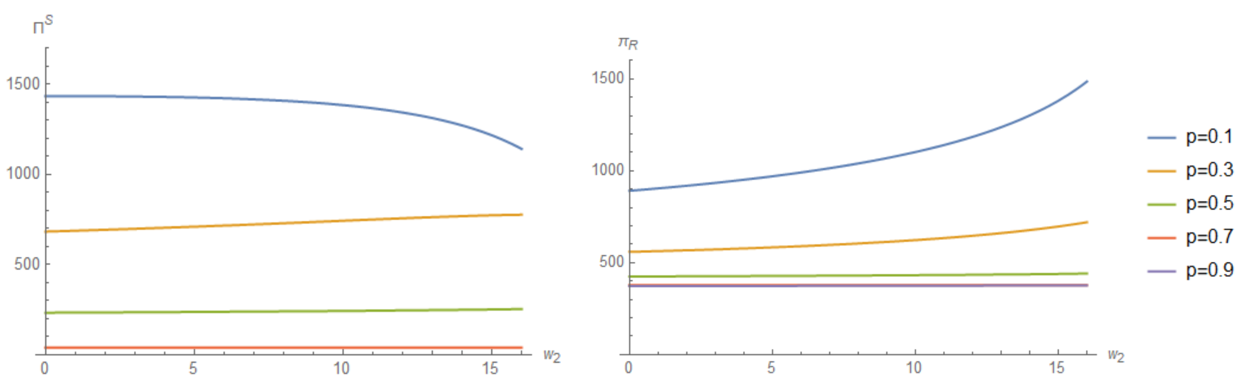

Figure 12: The impact of w_2 on the main supplier's profit (a) and on the retailer's profit (b)

5. Conclusions and managerial insights

In this paper, we have investigated two typologies of contract mechanisms—a risk sharing contract between the retailer and the main supplier — and a buyback contract between the retailer and the backup supplier—when supply disruption risk, uncertain demand, and random yield are considered simultaneously. While the existing research has compared the optimal strategies under demand uncertainty and supply disruption only, we investigate the effect of the contract parameters under the combination of supply uncertainty, demand uncertainty, and supply disruption. In particular, we consider SU in the form of random yield, and DU in the form of uncertain demand. This paper's results reveal that the combination of these risks leads to different strategic decisions for the retailer (in terms of ordering policy) and for the main supplier (in terms of quantity produced). Furthermore, a set of important managerial parameters are negotiated between the retailer and the main supplier (the wholesale price, the discounted price for over-production, and the percentage of emergency cost-shared) and between the retailer and the backup supplier (the buyback price and the maximum portion of buyback items).

This paper presents the conditions for optimality for the retailer's order quantities to the main and backup suppliers, and the main supplier's production quantity. Furthermore, our results show a linear relationship between the main supplier's optimal production quantity and the optimal order quantity to the main supplier. Our analysis also indicates that, as the disruption probability increases, then the backup supplier's profit increases, while the main supplier's and retailer's profits decrease. This follows from the fact that, as p increases, the main supplier's order quantity decreases and the backup supplier's order quantity increases. Even when the disruption probability is high, the retailer may still find it convenient to place a small order with the main supplier. In this case, one of the driving components results to be the wholesale price of the backup supplier versus the wholesale price of the main supplier. Considering SU and DU, the result highlights that (i) the main supplier's profit is maximum when the disruption probability is low and the DU is high or the SU is low; (ii) the retailer's profit is maximum when the disruption probability is low and the DU is moderate or the SU is moderate; and (iii) the backup supplier's profit is maximum when the disruption probability is high and the DU is low or the SU is high.

Concerning the buyback contract between the retailer and the buyback supplier, our results show how the buyback supplier can strategically set the return price and return quantity to maximize his profit. When the disruption probability is very low, then the backup supplier needs to offer a very high return price and/or return quantity to become attractive for the retailer, since, in this situation, the retailer normally relies on the main supplier, which is economically more convenient. Also, the higher the disruption probability is, the more negotiation power the backup supplier has, which implies that he doesn't need to offer a very high return price and/or return quantity to optimize his profit. With respect to the risk sharing contract between the retailer and the main supplier, our results determine the wholesale price that the main supplier has to offer the retailer to maximize his profit.

For the emergency cost-sharing, the main supplier needs to consider that the more portion of the cost is allocated to the retailer, the more this latter finds the backup supplier attractive (except for very low values of the disruption probability). Similarly, an increase in the over-production cost drives the retailer to increase the order to the backup supplier. However, it is also noticed that the retailer is sensitive to the over-production cost only in the case of low disruption probability since, in the case of higher disruption probability, the main supplier becomes unreliable.

This work can be extended in the following ways. First, it will be interesting to include in the analysis the lead times of the two suppliers and investigate whether the difference in delivery times of the main supplier and the backup supplier plays a significant role for the retailer. Second, one could extend the existing model to a multi-period, multi-product setting. Third, in the numerical analysis, demand and supply uncertainty could be modeled according to different probability distributions. Fourth, a detailed analytical study on the trade-off between the buyer and the supplier is an interesting scope for future research. Lastly, it would be of interest to consider the possibility for the retailer to choose from multiple primary suppliers.

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Appendix

A1. Proof of Proposition 1

The proof follows He and Zhang (2008).

(i) For optimality, we apply the first-order conditions to expression (4):

$$\frac{\partial \Pi^s}{\partial Q} = (1-p)\left[w_2 \int_{\frac{q}{Q}}^{\infty} uf(u)du + c_e(1-\beta)\int_0^{\frac{q}{Q}} uf(u)du\right] - c \quad \text{(A1)}$$

$$= (1-p)\left[w_2 \int_0^{\infty} uf(u)du - w_2 \int_0^{\frac{q}{Q}} uf(u)du + c_e(1-\beta)\int_0^{\frac{q}{Q}} uf(u)du\right] - c = 0.$$

Rearranging the terms in equation (A1) and recalling that $\mu = \int_0^{\infty} uf(u)du$, it follows that

$$\int_0^{\frac{q}{Q}} uf(u)du = \frac{c-\mu w_2(1-p)}{(c_e(1-\beta)-w_2)\,(1-p)} \quad \text{(A2)}$$

We then apply the second-order conditions for optimality to expression (4):

$$\frac{\partial^2 \Pi^s}{\partial Q^2} = -(1-p)(c_e(1-\beta)-w_2)\frac{q^2}{Q^3} f\left(\frac{q}{Q}\right) < 0, \quad \text{(A3)}$$

which implies $c_e(1-\beta) > w_2$, i.e., the over-production cost paid by the retailer should be less than the under-production emergency cost. This makes sense in reality since, otherwise, the retailer would prefer the emergency source versus the main supplier.

(ii) We show that $Z(y) = \int_0^y uf(u)du$ is a monotone function of y. Since $\frac{\partial Z(y)}{\partial y} = yf(y) > 0$, it follows that equation (5) has a unique solution and q/Q is a constant.
(iii) The proof follows from (1), (2), and from taking the first derivative (with respect to w_2) to the right-hand side of expression (5), resulting in $\frac{-c+(\beta-1)c_e\mu(p-1)}{(p-1)((\beta-1)c_e+w_2)^2}$. This latter is positive when $\mu < \frac{c}{(c_e(1-p)(1-\beta))}$.

A2. Proof of Proposition 2

We apply the first-order condition to expression (8) and take the first derivatives to the retailer's profit function with respect to I and q and setting them at zero:

$$\frac{\partial \Pi^R}{\partial q} = s(1-p) *$$

$$* \begin{bmatrix} \int_{\frac{1}{k}}^{\infty} f(u)\left(\frac{d(ukq+I)}{dq}(ukq+I)g(ukq+I) + \int_{ukq+I}^{\infty} ukg(x)dx - \frac{d(ukq+I)}{dq}(ukq+I)g(ukq+I)\right)du \\ + \int_0^{\frac{1}{k}} f(u)\left(\frac{d(q+I)}{dq}(q+I)g(q+I) + \int_{q+I}^{\infty} g(x)dx - \frac{d(q+I)}{dq}(q+I)g(q+I)\right)du \end{bmatrix}$$

$$-w_2(1-p)\left[\int_{\frac{1}{k}}^{\infty}(uk-1)f(u)du\right] - w(1-p) - c_e(1-p)\beta\left[\int_0^{\frac{1}{k}}(1-uk)f(u)du\right]$$

$$-(1-p)c_u * \left[\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} -ukg(x)f(u)dxdu + \int_0^{\frac{1}{k}}\int_{q+I}^{\infty} -qg(x)f(u)dxdu\right] \tag{A4}$$

$$+(w_r-c_r)(1-p)\left[\int_{\frac{1}{k}}^{\infty} uk\lambda If(u)\, g(ukq+(1-\lambda)I)du + \int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} ukf(u)g(x)dxdu \right.$$

$$- \int_{1/k}^{\infty} uk\lambda If(u)\, g(ukq+(1-\lambda)I)du + \frac{d(q+(1-\lambda)I)}{dq}\int_0^{1/k}\lambda If(u)\, g(q)du$$

$$\left. + \int_0^{1/k}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu - \frac{d(q+(1-\lambda)I)}{dq}\int_0^{1/k}\lambda If(u)\, g(ukq)du\right] = 0,$$

which gives

$$(s+c_u)\left[\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} ukf(u)g(x)dxdu + \int_0^{\frac{1}{k}}\int_{q+I}^{\infty} f(u)g(x)dxdu\right] +$$

$$(w_r-c_r)\left[\int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} ukf(u)g(x)dxdu + \int_0^{\frac{1}{k}}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu\right] \tag{A5}$$

$$= w + w_2\left[\int_{\frac{1}{k}}^{\infty}(uk-1)f(u)du\right] + c_e\beta\left[\int_0^{\frac{1}{k}}(1-uk)f(u)du\right].$$

$$
\begin{aligned}
\frac{\partial \Pi^R}{\partial I} = -w_b + p\Bigg[& s\left(Ig(I) + \int_I^{\infty} g(x)dx - Ig(I)\right) + c_u \int_I^{\infty} g(x)dx \\
& + (w_r - c_r)\left(\int_0^{(1-\lambda)I} \lambda g(x)dx + \int_{(1-\lambda)I}^{I} g(x)dx\right)\Bigg] \\
& + (1 \\
& - p)s\Bigg[\int_{\frac{1}{k}}^{\infty} f(u)\Bigg(\frac{d(ukq+I)}{dI}(ukq+I)g(ukq+I) \\
& + \int_{ukq+I}^{\infty} g(x)dx \qquad (A6) \\
& - \frac{d(ukq+I)}{dI}(ukq+I)g(ukq+I)\Bigg)du \\
& + \int_0^{\frac{1}{k}} f(u)\left(\frac{d(q+I)}{dI}(q+I)g(q+I) + \int_{q+I}^{\infty} g(x)dx - \frac{d(q+I)}{dI}(q+I)g(q+I)\right)du\Bigg] \\
& - (1-p)c_u\left[\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} -f(u)g(x)dxdu + \int_0^{\frac{1}{k}}\int_{q+I}^{\infty} -f(u)g(x)dxdu\right] \\
& + (1-p)(w_r \\
& - c_r)\Bigg[\int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} f(u)g(x)dxdu \int_{\frac{1}{k}}^{\infty}\int_0^{ukq+(1-\lambda)I} \lambda f(u)g(x)dxdu \\
& + \int_0^{\frac{1}{k}}\int_0^{q+(1-\lambda)I} \lambda f(u)g(x)dxdu + \int_0^{\frac{1}{k}}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu\Bigg] = 0
\end{aligned}
$$

which gives:

$$
\begin{aligned}
(1-p)\Bigg[& (s+c_u)\left(\int_{\frac{1}{k}}^{\infty}\int_{ukq+I}^{\infty} f(u)g(x)dxdu + \int_0^{\frac{1}{k}}\int_{q+I}^{\infty} f(u)g(x)dxdu\right) \\
& + (w_r - c_r)\Bigg(\int_{\frac{1}{k}}^{\infty}\int_{ukq+(1-\lambda)I}^{ukq+I} f(u)g(x)dxdu + \int_{\frac{1}{k}}^{\infty}\int_0^{ukq+(1-\lambda)I} \lambda f(u)g(x)dxdu \\
& + \int_0^{\frac{1}{k}}\int_{q+(1-\lambda)I}^{q+I} f(u)g(x)dxdu\Bigg) \\
& + \int_0^{\frac{1}{k}}\int_0^{q+(1-\lambda)I} \lambda f(u)g(x)dxdu\Bigg] \qquad (A7) \\
& + p\left[(s+c_u)\int_I^{\infty} g(x)dx + (w_r - c_r)\left(\int_0^{(1-\lambda)I} \lambda g(x)dx + \int_{(1-\lambda)I}^{I} g(x)dx\right)\right] = w_b.
\end{aligned}
$$

We then apply the second-order conditions for optimality and take the second-order derivatives:

$$\frac{\partial^2 \Pi^R}{\partial q^2} = -(s + c_u - w_r + c_r)(1 - p)\left(\int_{\frac{1}{k}}^{\infty} u^2k^2 f(u)g(ukq + I)du + \int_0^{\frac{1}{k}} f(u)g(q + I)du\right) \quad (A8)$$

$$-(w_r - c_r)(1 - p)\left(\int_{\frac{1}{k}}^{\infty} u^2k^2 f(u)g(ukq + (1 - \lambda)I)du + \int_0^{\frac{1}{k}} f(u)g(q + (1 - \lambda)I)du\right) < 0,$$

$$\frac{\partial^2 \Pi^R}{\partial I^2} = -p[(s + c_u - w_r + c_r)g(I) + (w_r - c_r)(1 - \lambda)^2 g((1 - \lambda)I)] -$$

$$(1 - p)\left[\begin{array}{l}(s + c_u - w_r + c_r)\left(\int_{\frac{1}{k}}^{\infty} f(u)g(ukq + I)du + \int_0^{\frac{1}{k}} f(u)g(q + I)du\right) \\ +(w_r - c_r)(1 - \lambda)^2\left(\int_{\frac{1}{k}}^{\infty} f(u)g(ukq + (1 - \lambda)I)du + \int_0^{\frac{1}{k}} f(u)g(q + (1 - \lambda)I)du\right)\end{array}\right] < 0,$$

$$(A9)$$

and

$$\frac{\partial^2 \Pi^R}{\partial q \partial I} = -(s + c_u - w_r + c_r)(1 - p)\left[\int_{1/k}^{\infty} ukg(ukq + I)f(u)du + \int_0^{1/k} g(q + I)f(u)du\right] \text{ (A10)}$$

$$-(w_r - c_r)(1 - p)(1 - \lambda)\left[\int_{\frac{1}{k}}^{\infty} ukg(ukq + (1 - \lambda)I)f(u)du + \int_0^{\frac{1}{k}} g(q + (1 - \lambda)I)f(u)du\right].$$

We still need to verify that $\Delta = \frac{\partial^2 \Pi^R}{\partial I^2} * \frac{\partial^2 \Pi^R}{\partial q^2} - \left(\frac{\partial^2 \Pi^R}{\partial I\, \partial q}\right)^2 > 0$.

$$
\begin{aligned}
\Delta = {} & p[(s + c_u - w_r + c_r)g(I) + (w_r - c_r)(1-\lambda)^2 g((1 \\
& -\lambda)I)]\Bigg[(s + c_u - w_r + c_r)(1 \\
& -p)\left(\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + I)f(u)du + \int_0^{\frac{1}{k}} g(q+I)f(u)du\right) \\
& + (w_r - c_r)(1 \\
& -p)\left(\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + (1-\lambda)I)f(u)du + \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du\right)\Bigg] + \\
& (s + c_u - w_r + c_r)^2(1 \\
& -p)^2\Bigg[\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + I)f(u)du * \int_{\frac{1}{k}}^{\infty} g(ukq + I)f(u)du \\
& + \left(\int_0^{\frac{1}{k}} g(q+I)f(u)du\right)^2 \\
& + \int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + I)f(u)du * \int_0^{\frac{1}{k}} g(q+I)f(u)du + \int_{\frac{1}{k}}^{\infty} g(ukq + I)f(u)du \\
& * \int_0^{\frac{1}{k}} g(q+I)f(u)du\Bigg] + (s + c_u - w_r + c_r)(w_r - c_r)(1 \\
& -p)^2\Bigg[\left(\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + (1-\lambda)I)f(u)du + \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du\right) \\
& * \left(\int_{\frac{1}{k}}^{\infty} g(ukq + I)f(u)du + \int_0^{\frac{1}{k}} g(q+I)f(u)du\right) + (1 \\
& -\lambda)^2\left(\int_{\frac{1}{k}}^{\infty} g(ukq + (1-\lambda)I)f(u)du + \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du\right) \\
& * \left(\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq + I)f(u)du + \int_0^{\frac{1}{k}} g(q+I)f(u)du\right)\Bigg] \\
& + (1-p)^2(1-\lambda)^2(w_r -
\end{aligned}
$$

$$-c_r)^2 * \begin{bmatrix} \int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq+(1-\lambda)I)f(u)du * \int_{\frac{1}{k}}^{\infty} g(ukq+(1-\lambda)I)f(u)du + \left(\int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du\right)^2 \\ \left(\int_{\frac{1}{k}}^{\infty} u^2k^2 g(ukq+(1-\lambda)I)f(u)du + \int_{\frac{1}{k}}^{\infty} g(ukq+(1-\lambda)I)f(u)du\right) * \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du \end{bmatrix}$$

$$-(s+c_u-w_r+c_r)^2(1-p)^2\left[\left(\int_0^{\frac{1}{k}} g(q+I)f(u)du\right)^2 + \left(\int_{\frac{1}{k}}^{\infty} ukg(ukq+I)f(u)du\right)^2 + 2\int_{\frac{1}{k}}^{\infty} ukg(ukq+I)f(u)du * \int_0^{\frac{1}{k}} g(q+I)f(u)du\right]$$

$$-(w_r-c_r)^2(1-p)^2(1-\lambda)^2\begin{bmatrix} \left(\int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du\right)^2 + \left(\int_{\frac{1}{k}}^{\infty} ukg(ukq+(1-\lambda)I)f(u)du\right)^2 \\ +2\int_{\frac{1}{k}}^{\infty} ukg(ukq+(1-\lambda)I)f(u)du * \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du \end{bmatrix}$$

$$-2(s+c_u-w_r+c_r)(w_r-c_r)(1-p)(1-\lambda)\begin{bmatrix} \int_0^{\frac{1}{k}} g(q+I)f(u)du * \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du + \int_{\frac{1}{k}}^{\infty} ukg(ukq+I)f(u)du * \int_{\frac{1}{k}}^{\infty} ukg(ukq+(1-\lambda)I)f(u)du \\ + \int_0^{\frac{1}{k}} g(q+I)f(u)du * \int_{\frac{1}{k}}^{\infty} ukg(ukq+(1-\lambda)I)f(u)du + \int_0^{\frac{1}{k}} g(q+(1-\lambda)I)f(u)du * \int_{\frac{1}{k}}^{\infty} ukg(ukq+I)f(u)du \end{bmatrix}.$$

$$\Delta = p\{(s+c_u-w_r+c_r)g(I) + (w_r-c_r)(1-\lambda)^2 g((1-\lambda)I)\} * \begin{bmatrix} (s+c_u-w_r+c_r)(1-p)\begin{bmatrix} \int_{1/k}^{\infty} u^2k^2 g(ukq+I)f(u)du \\ + \int_0^{1/k} g(q+I)f(u)du \end{bmatrix} \\ +(w_r-c_r)(1-p)\begin{bmatrix} \int_{1/k}^{\infty} u^2k^2 g(ukq+(1-\lambda)I)f(u)du \\ + \int_0^{1/k} g(q+(1-\lambda)I)f(u)du \end{bmatrix} \end{bmatrix}$$

$$+(s+c_u-w_r+c_r)^2(1-p)^2\begin{bmatrix} \int_{1/k}^{\infty}(ukg(ukq+I))^2 f(u)du - \left(\int_{1/k}^{\infty} ukg(ukq+I)f(u)du\right)^2 \\ + \int_{1/k}^{\infty}(uk-1)^2 g(ukq+I)f(u)du * \int_0^{1/k} g(q+I)f(u)du \end{bmatrix}$$

$$+(1-p)^2(1-\lambda)^2(w_r - c_r)^2\left[\begin{array}{l}\int_{1/k}^{\infty}(ukg(ukq+(1-\lambda)I))^2 f(u)du-\left(\int_{1/k}^{\infty}ukg(ukq+(1-\lambda)I)f(u)du\right)^2\\ +\int_{1/k}^{\infty}(uk-1)^2 g(ukq+(1-\lambda)I)f(u)du*\int_0^{1/k}g(q+(1-\lambda)I)f(u)du\end{array}\right]$$

$$+(w_r-c_r)(s+c_u-w_r+c_r)(1-p)^2\left[\begin{array}{l}\left\{\int_{\frac{1}{k}}^{\infty}(uk-1+\lambda)^2 g(ukq+(1-\lambda)I)f(u)du*\int_0^{\frac{1}{k}}g(q+I)f(u)du\right\}\\ +\left\{\int_0^{\frac{1}{k}}\lambda^2 g(q+I)f(u)du*\int_0^{\frac{1}{k}}g(q+(1-\lambda)I)f(u)du\right\}\\ +\left\{\begin{array}{l}\int_{\frac{1}{k}}^{\infty}u^2k^2 g(ukq+(1-\lambda)I)f(u)du*\int_{\frac{1}{k}}^{\infty}g(ukq+I)f(u)du\\ +(1-\lambda)^2\int_{\frac{1}{k}}^{\infty}g(ukq+(1-\lambda)I)f(u)du*\int_{\frac{1}{k}}^{\infty}u^2k^2 g(ukq+I)f(u)du\\ +2(1-\lambda)\int_{\frac{1}{k}}^{\infty}ukg(ukq+(1-\lambda)I)f(u)du*\int_{\frac{1}{k}}^{\infty}ukg(ukq+I)f(u)du\end{array}\right\}\\ +\left\{\int_{\frac{1}{k}}^{\infty}(uk-1+\lambda)^2 g(ukq+I)f(u)du*\int_0^{\frac{1}{k}}g(q+(1-\lambda)I)f(u)du\right\}\end{array}\right].$$

We recall that $w_r>c_r$, $s+c_u-w_r+c_r>0$ and, by Jensen's inequality, it results:

$$\int_{\frac{1}{k}}^{\infty}\{ukg(ukq+(1-\lambda)I)\}^2 f(u)du \ge \left(\int_{\frac{1}{k}}^{\infty}ukg(ukq+(1-\lambda)I)f(u)du\right)^2 \text{ and}$$

$$\int_{\frac{1}{k}}^{\infty}\{ukg(ukq+I)\}^2 f(u)du \ge \left(\int_{\frac{1}{k}}^{\infty}ukg(ukq+I)f(u)du\right)^2.$$

Since all the remaining terms are positive, it results that $\Delta > 0$ and we can conclude that the solution to (9a) and (9b) corresponds to the optimal main supplier's order quantity and backup supplier's order quantity. If the solution is out of feasible space, then the variable value which is infeasible is set at the domain boundary and the other variable is calculated from equation (2).