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ORIGINAL RESEARCH PAPER

Robust design of grid shells

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ABSTRACT

Shell grids are vital in architecture and engineering for spanning large areas with minimal material while ensuring strength and aesthetics. However, local failures or missing members can lead to progressive collapse, threatening stability. This study analyzes shell grid failure using the finite particle method to simulate load redistribution and internal force evolution as components fail. By computing forces at each time-step, finite particle method provides a dynamic view of structural behavior under failure. The findings emphasize the role of load redistribution in collapse, aiding in designing more resilient shell grids.

KEYWORDS

shell grid, progressive collapse, finite particle method

1. INTRODUCTION

Shell grids are critical structures commonly used in architectural and engineering applications due to their ability to cover large spans with minimal material usage, while maintaining structural efficiency and aesthetic appeal. For these reasons grid shell structures continue to be a subject of active research, with ongoing studies exploring their structural efficiency, material innovations, and potential applications in modern architecture [1–3]. These structures are composed of interconnected bars or members, forming a grid-like pattern that provides stiffness and strength. However, the integrity of these kinds of structures can be compromised due to local failures or even the absence of some members during construction. These local issues can propagate and lead to progressive collapse, which poses a significant risk to the stability and safety of the entire structure. A progressive collapse initiates because of local structural damage (limit force of the elements or joints, buckling) develops, in a chain reaction mechanism, into a failure that is disproportionate to the initiating local damage [4–7].

Progressive collapse is gaining traction in research in recent years, in multiple important studies long-span spatial grid structures are investigated [8, 9]. Single-layer spatial grids, which are widely used in large-span stadiums and public structures, are especially vulnerable to collapse despite their high static indeterminacy. Regardless of various engineering achievements, methods for quantitatively evaluating the progressive collapse resistance of these structures remain limited. To address this gap, studies have introduced the Collapse Margin Ratio (CMR) as a measure to assess collapse resistance and applied it to real-world structures like the Shenzhen Universiade Sports Centre [10].

Another critical factor influencing the progressive collapse of spatial grids is seismic loading. Earthquake-induced failure mechanisms involve complex physical behaviors including yielding under tension, non-elastic buckling under compression, and degradation of structural capacity due to the Bauschinger effect [11]. Studies using force-displacement

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hysteresis models have demonstrated that the choice of fracture criteria significantly impacts the predicted collapse behavior of space truss structures under seismic conditions [12].

In this contribution, the progressive failure behavior of shell grids is analyzed using the Finite Particle Method (FPM) [13]. The method is implemented to simulate how structural components of shell grids, respond under stable loads and gradually failing elements until failure of the structure occurs. The primary focus of the study is on capturing the evolution of internal forces and the redistribution of stresses as individual components of the structure fail progressively. By using FPM, the study provides a detailed representation of the load transfer mechanisms that occur as members of the grid fail. The internal forces within the shell grid are computed at each time-step, allowing for an accurate simulation of the progressive damage and eventual collapse of the structure. These time-step analyses offer a dynamic perspective on how the structure's behavior evolves during the failure process. This enables a comprehensive understanding of how the specific characteristics of the grid affect its overall stability when subjected to local failures. The results of this study highlight the critical role of load redistribution and localized failures in contributing to the overall collapse of the structure. Understanding these mechanisms is essential for predicting the structural response to failures and for designing more resilient shell grids.

The insights gained from this study contribute to robust topology optimization strategies, which seek to improve the structural resilience of shell grids by accounting for uncertainties in loading conditions [14–16]. Understanding the mechanisms of progressive collapse is crucial for designing safer and more resilient large-span grid structures. By integrating numerical simulations, experimental validations, and engineering case studies, this research aims to enhance predictive models for structural failure and inform design practices that mitigate the risk of progressive collapse.

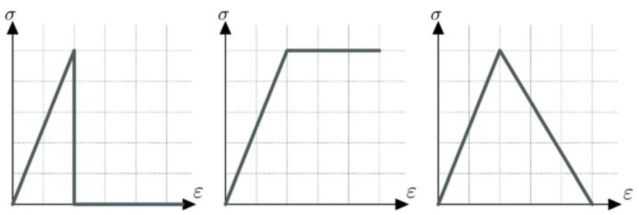

Fig. 1. Failure modes, respectively, Plastic, Elastic-softening, and Sudden (*Source*: on the basis of [17])

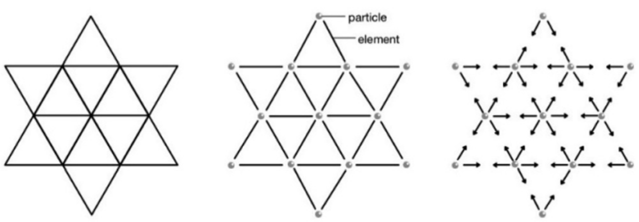

Fig. 2. Finite particle method

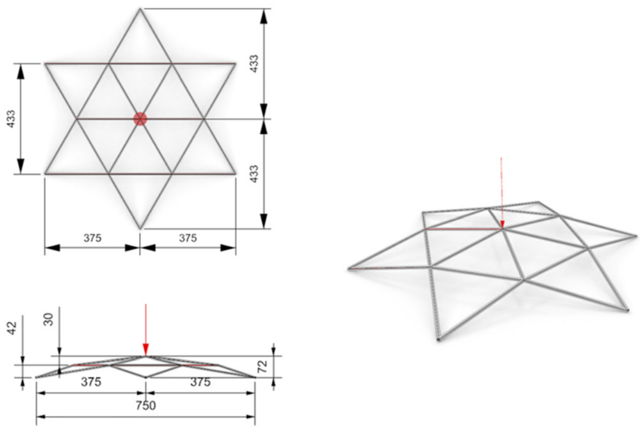

Fig. 3. Benchmark geometry (cm) (*Source*: Authors)

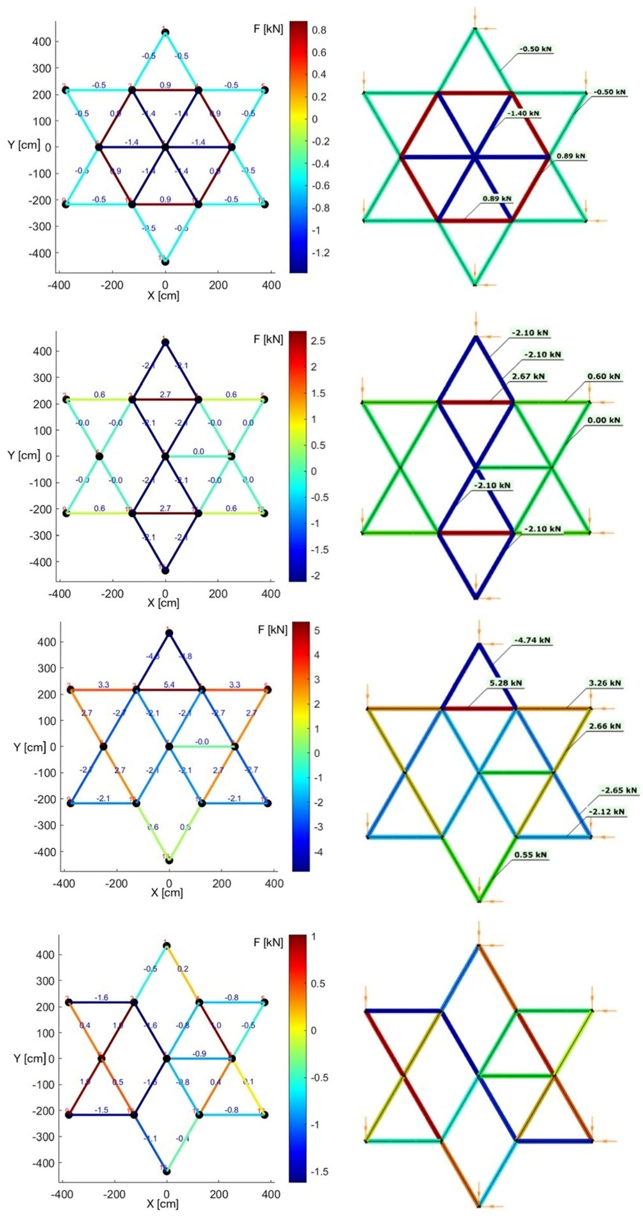

Fig. 4. Star structure load-bearing paths (*Source*: Authors)

2. MECHANICAL BACKGROUND

Robustness: a redundancy in the structure meaning more possible loadbearing ways [17, 18]. It is possible to describe it with a precise mathematical definition [19, 20], but in this article it is investigated through the progressive removal of elements.

Failure can occur in multiple scenarios, three main aspects to inspect are sudden-snap, plastic, and elastic-softening (Fig. 1) [21]. In this study only the last one among them is considered. It is assumed that joint and buckling failures are not dominant failure modes. The primary failure mode is assumed to be the loss of load-carrying capacity of the bar elements due to sudden rupture.

To study behavior some kind of simulation method is needed. One possibility is the Finite Element Method (FEM) [22, 23]. This method is capable of accurately calculating the mechanical behavior of the structure before failure, but it is not suited for a precise tracking of the redistribution of the forces in real time and falls short in case of a failing structure. A better option is FPM (Fig. 2), which is a time stepping dynamic analysis, letting us follow the structures behavior with an arbitrary time-step interval. FPM is a mesh-free computational approach allowing the investigation of large deformations. It is suitable to track real-time response of the structure under increasing loads or changing geometry. Thanks to the time-step analysis it makes it possible to follow collapse mechanisms, even though it is highly demanding from a computational standpoint.

FPM applies Newton's second law to each particle directly resulting in all nodes being in a locally equilibrated state opposed to the FEM, which enforces a global equilibrium on the structure. This makes it suitable for a situation where structural members will separate during the simulation. The particles in the discretized structure carry mass (m_α), translations (d), velocity ($\dot{d}$) and acceleration ($\ddot{d}$), while members define internal forces ($\boldsymbol{F}$). For a 3D truss structure, each particle α has three translational degrees of freedom, and its motion follows:

$$m_\alpha \ddot{\boldsymbol{d}} = \boldsymbol{F}^{ext} - \boldsymbol{F}^{int} - \boldsymbol{F}^{dmp}, \tag{1}$$

which in component form is:

$$m_\alpha \begin{bmatrix} \ddot{d}_x \\ \ddot{d}_y \\ \ddot{d}_z \end{bmatrix} = \begin{bmatrix} f_x^{ext} \\ f_y^{ext} \\ f_z^{ext} \end{bmatrix} - \begin{bmatrix} f_x^{int} \\ f_y^{int} \\ f_z^{int} \end{bmatrix} - \begin{bmatrix} f_x^{dmp} \\ f_y^{dmp} \\ f_z^{dmp} \end{bmatrix}. \tag{2}$$

The motion equations are solved using the explicit central difference method. Neglecting the details of FPM by using a

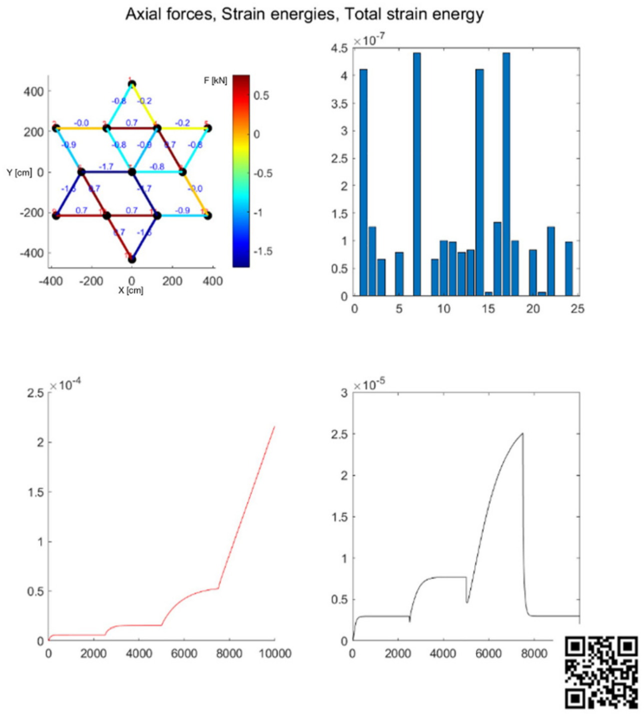

Fig. 5. Demonstration of a failed geometry (*Source*: Authors)

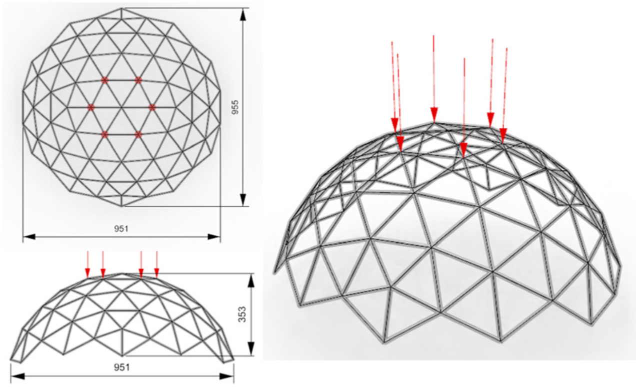

Fig. 6. Dome structure (cm) (*Source*: Authors)

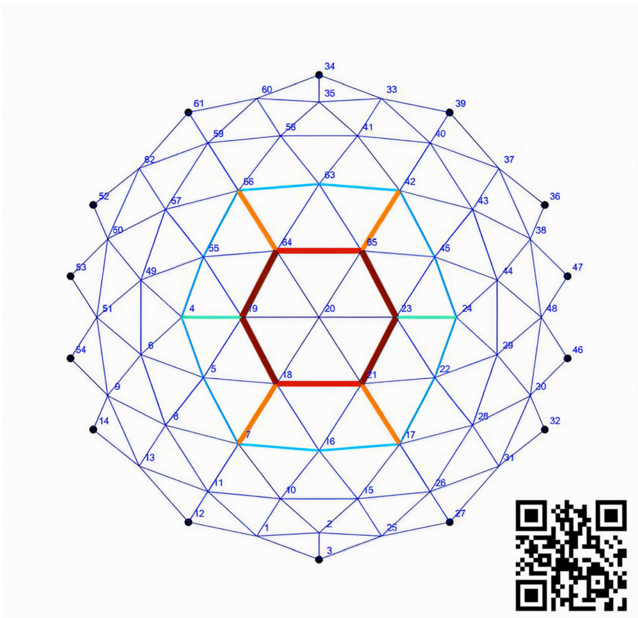

Fig. 7. Demonstration of the evolution of load-bearing paths (*Source*: Authors)

Δt timestep the velocities Eq. (3) and accelerations Eq. (4) of the particles can be defined iteratively,

$$\dot{\boldsymbol{d}}_n = \frac{1}{2\Delta t}(\boldsymbol{d}_{n+1} - \boldsymbol{d}_{n-1}), \tag{3}$$

$$\ddot{\boldsymbol{d}}_n = \frac{1}{\Delta t^2}(\boldsymbol{d}_{n+1} - 2\boldsymbol{d}_n + \boldsymbol{d}_{n-1}), \tag{4}$$

where $\boldsymbol{d}_{n+1}$, $\boldsymbol{d}_n$ and $\boldsymbol{d}_{n-1}$ equal the displacement of an arbitrary particle at n+1, n, $n-1$ timestep and Δt is a constant time increment. In addition, the translation at timestep n+1 can also be defined Eq. (5),

$$\boldsymbol{d}_{n+1} = \left(\frac{2}{2+\mu\Delta t}\right)\frac{\Delta t^2}{m_\alpha}\left(\boldsymbol{F}_n^{ext} - \boldsymbol{F}_n^{int}\right) + \left(\frac{4}{2+\mu\Delta t}\right)\boldsymbol{d}_n - \left(\frac{2-\mu\Delta t}{2+\mu\Delta t}\right)\boldsymbol{d}_{n-1}. \tag{5}$$

The total energy balance ensures numerical stability Eq. (6):

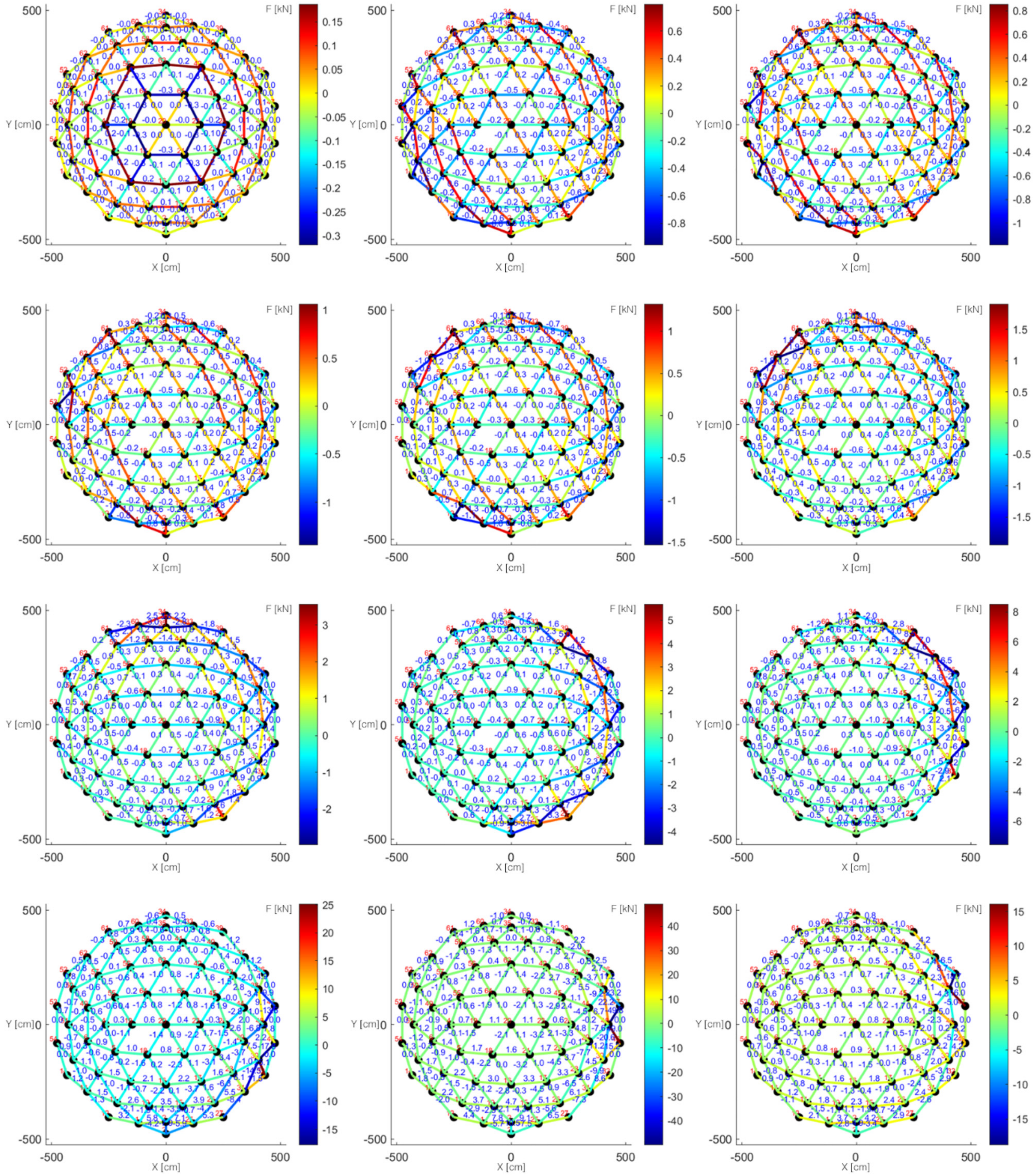

Fig. 8. Dome structure member removing steps (*Source*: Authors)

$$W_{ext} = W_s + W_k + W_d, \quad (6)$$

where W_{ext} is the external work, W_s is the strain energy stored in elements, W_k is the kinetic energy of particles and W_d is the damping work.

It is important to note that structural failure does not necessarily imply total collapse. Instead, failure may also be defined by the inability of a structure to fulfill its intended function, including cases when excessive displacements violate serviceability limit states EN 1990:2000 [24] EN 1993-1-1:2005 [25]. In this study, a single load case was considered without increasing the loads. Once the structure achieved equilibrium, the element exhibiting the highest strain energy was removed. If the removal of this element resulted in a rapid increase in external work, accompanied by a significant decrease in internal work that failed to recover, the structure was deemed unable to establish a load-bearing path near its previous equilibrium state. This condition was classified as structural failure.

3. PARAMETRIC STUDY

The parametric study was carried out using software developed by the authors. In this contribution, two structures were analyzed. The first structure was a well-known benchmark model, the Star Dome, which was utilized to validate the proposed methodology and compare the results with established finite element analysis programs [26]. This comparison ensured the accuracy of the computed internal forces.

The second structure is a geodesic dome to show how the path of forces changes as the most used elements are removed one after the other.

In both cases, all nodal connections possess three degrees of freedom, corresponding to displacements in the x, y, and z directions, with no rotational degrees of freedom considered. The displacements of the supporting nodes are constrained in all three spatial directions. The material properties were defined based on the typical characteristics of S235 structural steel, with a Young's modulus of 210 GPa and a yield strength of 235 MPa.

For the star dome, the supports are located at the six outermost vertices of the structure. A 1 kN load is applied at the highest point, which coincides with the geometric center of the structure (Fig. 3).

As elements are progressively removed, following the method described above, the structure attempts to redistribute the load and establish new load-bearing paths to support the external force (Fig. 4). According to the joint method equation ($m + r = 3j$), the structure in its final configuration still satisfies the necessary condition for being statically determinate.

However, despite obtaining an equilibrium solution through finite element analysis, the structure is classified as failed based on results from the FPM. These results indicate that the structure undergoes large displacements without successfully forming a stable load-bearing path over an extended period, suggesting a loss of stability and incipient collapse (Fig. 5).

The second structure analyzed was a geodesic dome. Like the first structure, a single load case was considered, consisting of six joint loads whose combined magnitude totals 1 kN. These loads were applied near the top central region of the structure (Fig. 6). The supporting nodes were positioned at the outermost vertices, as it is illustrated in Fig. 7.

The analysis followed the same method as before. After achieving an equilibrium state, the bar with the highest strain energy was removed, and the analysis proceeded until either failure criteria - as previously defined - was met, or the structure became statically over-determined based on the joint method equation. A demonstration video can be seen with the help of QR code in Fig. 7. Visualization of the entire problem can be seen in Fig. 8.

Unlike the previous case, the defined failure in this structure occurred well before it reached a statically over-determined state. According to the joint method equation, the structure remained 5 times statically indeterminate at the point where it experienced a significant rearrangement of load paths, which, in this study, was classified as failure.

4. CONCLUSIONS

This study investigated the progressive collapse behavior of two structural systems - a Star Dome and a Geodesic Dome - using a parametric approach. The methodology was validated through comparisons with finite element analysis results, ensuring the accuracy of internal force computations. The analyses focused on structural responses to sequential element removals, evaluating the ability of each structure to establish alternative load-bearing paths under static loading conditions.

The Star Dome served as a benchmark model, demonstrating the proposed method's reliability and highlighting its capacity to detect instability despite fulfilling the theoretical criteria for static determinacy. Although the final configuration satisfied the joint method equation ($m + r = 3j$), results from the Finite Particle Method indicated that the structure experienced large displacements and failed to re-establish a stable equilibrium, suggesting incipient collapse.

In contrast, the Geodesic Dome exhibited earlier failure based on the above-mentioned joint equation, occurring well before reaching a statically over-determined state. Despite remaining five times statically indeterminate based on theoretical calculations, the structure underwent significant load path rearrangements that were classified as failure due to its inability to redistribute forces effectively.

Both cases emphasize that static determinacy alone does not guarantee structural stability or robustness under progressive collapse scenarios. The results highlight the importance of integrating dynamic analyses and alternative evaluation methods, for example FPM, to capture critical failure mechanisms beyond traditional static criteria.

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