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ON THE OPTIMAL PERIOD OF SPANWISE FORCING FOR TURBULENT DRAG REDUCTION

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INTRODUCTION

Efforts to reduce turbulent skin-friction drag are important for both environmental and economic reasons. Various methods have been proposed over the years, and among these, techniques that use active predetermined wall-based actuation without feedback are particularly noteworthy for their simplicity and effectiveness.

This study examines spanwise forcing [3], which has been proven effective at high Reynolds and Mach numbers and offers significant energy savings. The simplest and earliest variant of spanwise forcing is the spanwise oscillation of a plane wall [2]. Although the spatially uniform oscillation is not among the most efficient implementations, it is considered here as the prototypical form of spanwise forcing, because its working principle is shared by the other variants. The wall periodically oscillates in the spanwise direction as a function of time t according to a prescribed harmonic law

$$w_w(t) = A \sin\left(\frac{2\pi}{T} t\right), \tag{1}$$

where w_w is the spanwise velocity component of the wall (the other components are set to zero), and A and T indicate the amplitude and period of the oscillation.

The harmonic oscillation of the wall generates a spanwise cross-flow that is periodic after space- and phase-averaging, and that superimposes to and interacts with the turbulent flow. The phase-averaged spanwise flow coincides (small deviations are present for large T) with the analytical laminar solution $w_{SL}(y, t)$ of the second Stokes problem, hereafter referred to as the Stokes layer or SL, which reads:

$$w_{SL}(y, t) = A \exp\left(-\frac{y}{\delta}\right) \sin\left(\frac{2\pi}{T} t - \frac{y}{\delta}\right), \tag{2}$$

where δ is the SL thickness and y the wall-normal coordinate. Since the maximum amplitude A of the wall oscillation only appears as a multiplicative factor because of the linearity of the governing equations, the SL is shaped by the remaining two parameters T and δ. These two quantities are not independent, and once T is set, δ is determined as a function of T and the fluid kinematic viscosity by

$$\delta = \delta_{SL}(T, \nu) \equiv \sqrt{\frac{\nu T}{\pi}}, \tag{3}$$

where the SL thickness δ defined above is the wall distance where the maximum spanwise velocity during the oscillation reduces to $A \exp(-1)$.

Starting from the early numerical studies of [2] and [1], the available evidence indicate that there is an optimal oscillation period T_{opt}, which corresponds to an optimal Stokes Layer thickness $\delta_{opt} = \delta_{SL}(T_{opt}, \nu)$, for which drag reduction is maximized. There is general agreement that $T^+_{opt} \approx 100$, corresponding to a penetration depth of the SL of $\delta^+_{opt} \approx 5.7$.

Despite the evidence, there is no consensus on the physical interpretation of these optimal values which can be understood in more than one way. For instance, T_{opt} can be directly linked to other time scales in the flow, such as the characteristic lifetime of near-wall coherent structures. Additionally, due to the flow's convective nature, T_{opt} can be translated into a longitudinal length scale through a convection velocity. Furthermore, within the Stokes layer, the optimal period also defines the maximum lateral displacement of the moving wall, $D_{max} = AT$, which is another potentially significant length scale of the flow. The optimal period might also be seen as determining the optimal penetration depth δ_{opt} of the Stokes layer via equation 3, representing a diffusion length scale in the wall-normal direction and a measure of the near-wall mean spanwise shear. Our inability to distinguish among these various interpretations reflects our current limited understanding of the overall drag reduction mechanism associated with the oscillating wall setup.

The objective of this work is to advance this understanding by clarifying the significance of the (T_{opt}, δ_{opt}) optimum. Based on DNS, we move beyond the traditional concept of the oscillating wall and remove the constraint $\delta = \delta_{SL}(T, \nu)$: we explore the complete (T, δ) two-dimensional space of parameters and investigate the role of T and δ separately. In other words, instead of imposing the harmonic spanwise oscillation of the wall to generate the SL, we enforce a mean spanwise velocity profile of the form (2) at each time step, and vary δ and T independently.

METHODS

Direct numerical simulations (DNS) of the turbulent flow in an indefinite plane channel are carried out, to study the effect of the Stokes layer generated by the sinusoidal oscillations of the walls after its period T and thickness δ are decoupled. We remove the link (3) between T and δ_{SL} that exists when a true Stokes layer is created by the oscillation of the wall. An extended Stokes layer profile (ESL)

$$\langle w \rangle_h(y, t; \delta, T) = A \exp\left(-\frac{y}{\delta}\right) \sin\left(\frac{2\pi}{T} t - \frac{y}{\delta}\right) \tag{4}$$

is indeed enforced directly at each time step; the operator $\langle \cdot \rangle_h$ indicates spatial averaging along the homogeneous directions. While enforcing an arbitrary profile $\langle w \rangle_h(y, t)$ may suggest that the present numerical experiments are mere thought experiments that are possible with DNS only, it should be remarked that our procedure is equivalent to solving the Navier–Stokes equations with the boundary condition (1) and an additional volume forcing that is practically zero whenever the extended

Stokes layer (4) reduces to the standard Stokes layer. We measure that the two techniques almost provide the same results in terms of $\mathcal{R}$, with a small deviation only at large T (see figure 1, where for $T^+ = 200$ the relative discrepancy is of 6%); the ESL always provides a smaller $\mathcal{R}$ compared to the true SL. Here we define $\mathcal{R} = 100 \times (C_{f0} - C_f)/C_{f,0}$ with $C_{f,0}$ and C_f the skin friction coefficient of the uncontrolled and controlled case, respectively. The simulations are carried out at the bulk Reynolds number $Re_b = U_b h/\nu = 7000$ for all cases, which corresponds to a friction Reynolds number of $Re_\tau = u_\tau h/\nu \approx 400$ in the unforced case. The oscillating period is varied in the $10 \leq T^+ \leq 200$ range, while δ varies between $2 \leq \delta^+ \leq 20$. The amplitude of the forcing is set to $A^+ = 12$.

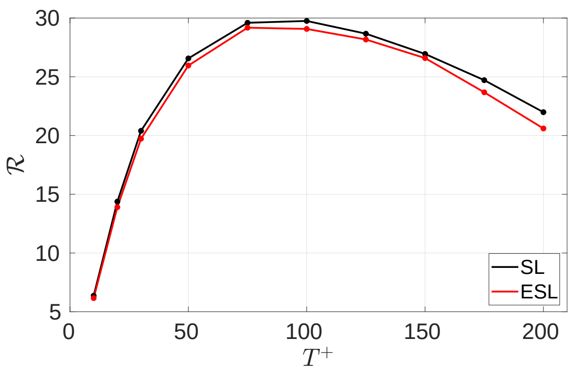

Figure 1: Drag reduction versus oscillation period for the oscillating wall (black) and the present approach with $\delta = \delta_{SL}$ (red).

RESULTS

Figure 2 shows that, once δ and T are made independent, the maximum drag reduction on the SL line is not particularly meaningful in view of the global $\mathcal{R}$ map. Along the SL line, a maximum $\mathcal{R}$ of $\approx 30\%$, shown by the black symbol, is indeed found at $(T^+_{opt}, \delta^+) \approx (100, 5.7)$, but the position of the actual maximum in the two-dimensional plane is larger and quite far from it. Indeed, the global maximum drag reduction obtained with the ESL is $\mathcal{R}_{max} \approx 40\%$, found for $(T^+, \delta^+) \approx (30, 14)$; see the red symbol in figure 2. Hence, the maximum drag reduction is significantly larger than that on the SL line, and is obtained by decreasing the oscillating period from $T^+ = 100$ to $T^+ = 30$, while at the same time increasing the SL thickness from $\delta^+ = 5.7$ to $\delta^+ = 14$. Note that, when moving along the SL line, it is impossible to change T and δ in opposite directions. The $\mathcal{R}$ map can be divided into different regions according to the behaviour of the drag reduction at varying parameters T and δ. The area of the global optimum is quite broad, spanning the region of $20 \leq T^+ \leq 50$ and $8 \leq \delta^+ \leq 14$; the values of δ correspond to the position of the buffer layer, where the near-wall cycle takes place, suggesting that the maximum $\mathcal{R}$ is gained for the ESL effectively interacting with the near-wall coherent structures of the wall. Where δ is very small the spanwise motion is confined in the viscous sublayer where the turbulent activity is weak. Similarly, for small oscillating periods $\mathcal{R}$ is relatively small and independent on δ since the oscillating period is too small compared to the flow time scales, and the resulting oscillating motion and the incoming flow are decoupled. As T increases above $T^+ > 30$, the local optimum thickness δ^+ moves towards smaller values, suggesting that with longer oscillating period the ESL is more effective when its influence remains confined closer to the wall. For large T, $\mathcal{R}$ degrades quickly at large δ. A possible explanation of the suboptimal $\mathcal{R}$ is the spanwise instantaneous velocity field being quite different from the laminar ESL due to the slow period of oscillation and large δ significantly interacting with the underling turbulence.

We conclude that the values $T^+ \approx 100$ and $\delta^+ \approx 6$ do not possess a special meaning and they are the optimum when we are constrained by the control actuator to lie on the black line of figure 2; instead designing a control which allows to decouple T and δ is able to provide a much higher $\mathcal{R}$.

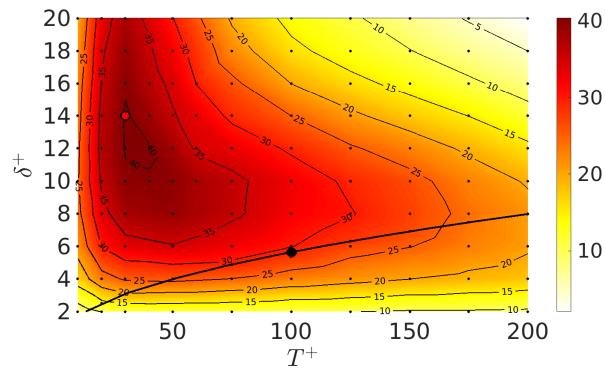

Figure 2: Drag reduction map in the (T, δ) space of parameters. The black thick line indicates the $\delta = \delta_{SL}$ constraint. The red dot identifies the point of maximum drag reduction, whereas the black dot indicates the maximum along the line $\delta = \delta_{SL}(T)$. The small black dots indicate each point of the detaset.

The drag reduction potential of the present control needs to be put in perspective accounting for both benefits, i.e. $\mathcal{R}$ and costs, i.e. energy spent to enforce the control. P_c is the control power required to create the ESL, when neglecting the losses of the actuation device. It is expressed as a fraction of the pumping power needed to maintain a constant flow rate. We measure that the power is a function of δ only, once A is fixed: the larger δ, the smaller P_c.

To compare benefits and costs of the control we define the net energy saving rate $P_{net} = \mathcal{R} - P_c$. For this A the oscillating wall leads to a negative P_{net} for all T, meaning that at $Re_\tau = 400$ the cost of the actuation overcomes the actual savings. The ESL, instead, guarantees the possibility of positive net benefits for some (T, δ) pairs, as the region of large $\mathcal{R}$ corresponds to the region of relatively small P_c due to the large δ. For $(T^+, \delta^+) = (30, 20)$ we measure a non-negligible positive $P_{net} \approx 17\%$, which is noteworthy when compared to the negative net balance of the oscillating wall at this value of A, whose maximum is $P_{net} \approx -30\%$. Also, note that in this work we are considering the simplest variant of the spanwise forcing.

The positive net power saving obtained when T and δ are decoupled paves the way for the implementation of alternative actuators able to enforce any velocity profile to the flow field in order to get the maximum net benefit. The control law does not need to be the result of the choice of the actuator as in the case of the wall oscillation, but more conveniently the actuator can be designed to induce the desired control law to the flow field. The profile of the resultant velocity is not constrained to follow the law of Eq.(4) anymore and a body force leading to a velocity profile able to target the mechanism which induces the reduction of drag can be designed.

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