Freeway Traffic Control via Second-Order Sliding Modes Generation

Giulia Piacentini, Gian Paolo Incremona and Antonella Ferrara

Abstract— **The paper deals with the design of a Suboptimal Second-Order Sliding Mode (SSOSM) control algorithm for local ramp metering of freeway systems. Indeed, sliding mode control is well-known for its robustness in front of uncertain terms and it perfectly fits to solve the control problem in case of traffic systems. Moreover, the proposed control law is able to steer the so-called sliding variable, chosen as the error between the density of the cell in the vicinity of the ramp and its reference value, to zero in a finite time. The traffic flow is modeled by means of the macroscopic second-order METANET model and the approach is finally assessed in simulation with satisfactory results.**

I. Introduction

Nowadays congestion on freeways is a major problem that strongly affects efficiency of the system and also has a great socio-economical impact. The development of efficient freeway traffic control and management tools is absolutely important in order to reduce travel times, fuel consumption and pollutants emissions. Highways were originally designed to have a sufficient capacity for virtually unlimited mobility, but, with the increasing number of traveling vehicles, recurrent and non-recurrent congestion has started to strongly degrade the system. Ramp metering is a well-known traffic control method that regulates the number of vehicles that can access the mainstream from an on-ramp, depending on the current traffic situation [1]. Its implementation is usually accomplished by traffic lights, either by allowing the entrance of one car at a time (control via red phase duration), or via traffic cycles. Ramp metering has proved to be really effective for congestion dissipation since it directly affects the density of the highway and it can be either local or coordinated. The local one is implemented in the vicinity of each ramp considering only the local value of the density, while the coordinated ramp-metering controller collects the values of all the densities and decides the control input for each ramp. One of the first simple ramp metering strategy is the well-known ALINEA [2], a simple local feedback control structure that has been successfully applied worldwide, and for which several field results are also available [3]. ALINEA was then extended by assuming a proportional integral version called PI-ALINEA [4] that has shown better performances in the case of a bottleneck located downstream of the metered on-ramp. Besides these simple control structures, more sophisticated control approaches for ramp metering have successfully been studied, often based on optimal control approaches [5]. For example, in [6] the problem of coordinated ramp metering is formulated as a constrained discrete-time nonlinear optimal control problem. A distributed model predictive control (MPC) scheme for freeway systems has been considered in [7]. The drawback of optimal control approaches is the computational burden that derives from the solution of large and often nonlinear optimization problems. In order to reduce the computational effort of the standard MPC that computes the control law at each time step, an event-triggered MPC scheme is proposed in [8], where the control law is updated only when a given set of conditions is verified. In [9] a first-order model is formulated in a switched version and a switched controller is applied where different control laws are adopted depending on the current mode of the system.

Motivated by the uncertain nature of the system at hand and by the necessity to adopt a computationally light easy-to-implement solution, in this paper we propose a sliding mode control approach to solve the ramp metering problem. Sliding Mode Control (SMC) [10]–[12] is well-known to be a robust control approach that deals well with systems with uncertainties, as the traffic system is. SMC is based on the idea to have a variable structure controller depending on the changing state of the system. In the literature, there are few examples in which SMC has been applied to the context of traffic control with ramp-metering. In [13], for instance, a first-order traffic model is adopted for a stretch of highway with several on-ramps and a so-called drift algorithm is applied for the sliding mode control. In this case, a coordinated Multi-Input Multi-Output (MIMO) sliding mode controller is applied, using both a first-order and a second-order algorithm. In [14] differential flatness is combined with a first-order SMC in order to keep the density close to its prescribed value by modeling the traffic with the well known first-order Lighthill-Whitham-Richards (LWR) model [15] [16], without taking into consideration the queue length dynamics. The same approach has then be extended by applying a Super-Twisting sliding mode approach in [17]. In the present paper we introduce a Suboptimal Second-Order Sliding Mode (SSOSM) control [18] to ramp metering in order to minimize the error between a chosen reference value, i.e., the critical density, and the value of the density in the vicinity of the on-ramps. In this way congestion gets reduced and the throughput is increased, with benefit for the travel time of drivers.

In the following, Section II introduces the main notation used in the paper and recalls some preliminary issues on second-order sliding modes. In Section III the adopted macroscopic METANET model is introduced and ramp metering problem is formulated. In Section IV the proposed SSOSM

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algorithm is described together with two modifications in order to improve its performance. Simulation results are illustrated in Section V, while some conclusions are gathered in Section VI.

II. PRELIMINARIES

In this section, some preliminary elements will be introduced: the main notation used in the paper is reported, and basics of sliding mode control are recalled.

A. Notation

The notation used in the paper is mostly standard. Let $\mathbb{R}$ denote the set of real numbers, while $\mathbb{N}$ is the set of natural numbers. Given a vector x, then x_i are its entries and $x^\top$ is the transpose. Let $x^{[j]}$ be the vector associated to the index j. Given a scalar function $s(x) : \mathcal{S} \to \mathbb{R}$, then $\mathrm{sgn}(s) = 1$ if $s > 0$, $\mathrm{sgn}(s) \in [-1, 1]$ if $s = 0$ and $\mathrm{sgn}(s) = -1$ if $s < 0$.

B. Second-order sliding mode control

In order to formulate the control problem at hand, it is convenient to make reference to a canonical form frequently used in the development of sliding mode control laws. Consider a Single-Input-Single-Output (SISO) system affine in the control variable as follows

$$\begin{cases} \dot{x}(t) = f(x(t)) + b(x(t))u(t) \\ y(t) = \sigma_1(x(t)) \end{cases} \tag{1}$$

where $x \in \mathcal{X}$ ($\mathcal{X} \subset \mathbb{R}^n$ bounded) is the state vector, with initial conditions $x(t_0) = x_0$, t_0 being the initial time instant and $u \in \mathcal{U} \subset \mathbb{R}$ is the control input such that $\mathcal{U} := [-\alpha, \alpha]$ with $\alpha > 0$, while $f(x(t)) : \mathcal{X} \to \mathbb{R}^n$ and $b(x(t)) : \mathcal{X} \to \mathbb{R}^n$ are uncertain functions of class $\mathrm{C}^1(\mathcal{X})$. The output function $\sigma_1(x) : \mathcal{X} \to \mathbb{R}$ is of class $\mathrm{C}^2(\mathcal{X})$. The latter will play the role of the so-called "sliding variable". Furthermore, the following assumptions hold.

Assumption 1: If $u(t)$ in (1) is designed so that, in a finite time t_r (the so-called "reaching time"), $\sigma(x(t_\mathrm{r})) = 0 \; \forall x_0 \in \mathcal{X}$ and $\sigma(x(t)) = 0 \; \forall t > t_\mathrm{r}$, then $\forall t \geq t_\mathrm{r}$, there exists a point $\bar{x}$ such that it is an asymptotically stable equilibrium point of (1) constrained to $\sigma(x(t)) = 0$. □

Assumption 2: System (1) has an uniform and time invariant relative degree equal to 1. □

The control law can be designed either as a discontinuous control law, or as the output of an integrator having in input the discontinuous signal, i.e., $w(t) = \dot{u}(t)$. This second strategy is recognized as Higher-Order Sliding Mode (HOSM), typically aimed at chattering reduction. Having in mind to design a Second-Order Sliding Mode (SOSM) control, letting $\sigma = [\sigma_1, \dot{\sigma}_1]^\top = [\sigma_1, \sigma_2]^\top$ be the vector of the sliding variable and its time derivative, the relative degree of system (1) is artificially increased to 2 [19]. Given the system dynamics (1), the so-called "auxiliary system" is written as

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 \\ \dot{\sigma}_2 = h(\sigma) + g(\sigma)w \\ w = \dot{u} \\ \sigma_1(t_0) = \sigma_{1_0} \end{cases} \tag{2}$$

Assumption 3: The continuous functions $h(\cdot)$, $g(\cdot)$ are such that

$$\exists \; \beta > 0 : |h(\sigma)| \leq \beta \tag{3}$$
$$\exists \; \varepsilon > 0 : g(\sigma) \leq \varepsilon \tag{4}$$
$$\exists \; \gamma > 0 : g(\sigma) \geq \gamma \, , \tag{5}$$

with β, γ and ε being known positive constants. □

This assumption means that the uncertainties affecting the systems are bounded. In our case study it is reasonable due to the nature of the involved variables. Relying on (2)-(5), we are now in a position to formulate the control problem.

Control Problem 1: Design a feedback control law Ψ as

$$u(t) = \Psi(\sigma_1, \sigma_2)$$

such that $u \in \mathcal{U}$ and $\forall x_0 \in \mathcal{X}$, $\exists \; t_\mathrm{r} > 0 : \sigma_1(x(t)) = 0$, $\forall t \geq t_\mathrm{r}$, in spite of the uncertainties. □

III. PROBLEM SETTING

In this work we study the ramp metering problem in freeway systems, where the aim is to regulate the on-ramp flow by using traffic lights in order to make the traffic density at the mainsteram segment as close as possible (ideally equal) to a critical density denoted as ρ_cr. The model adopted to describe the highway system is a continuous version of the macroscopic second-order METANET model [20]. In the following the adopted model and the ramp metering principle will be discussed in detail.

A. Freeway model

We consider a stretch of freeway divided in N cells of equal length L_i, $i = 1, \ldots, N$. For each section i, a sketch of which is reported in Figure 1, the following variables are defined: vehicle (veh) density (ρ_i, veh km^{-1} lane^{-1}), mean speed of the traffic flow (v_i, $\mathrm{km\,h}^{-1}$) and the traffic flow (q_i, veh h^{-1}). Furthermore, having in mind to apply a sliding mode control, typically designed in the continuous time framework, a continuous version of the METANET model is hereafter adopted.

1) Conservation law: The dynamics of the system is captured by the conservation law, given by the following differential equation

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\rho_i(t) = m_{\mathrm{m}i}(t) = \frac{1}{L_i \lambda_i}\left(q_{i-1}(t) - q_i(t) + q_{\mathrm{r}i}(t)\right) \tag{6}$$

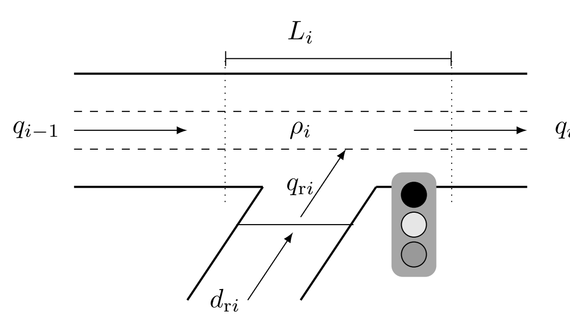

Fig. 1. Freeway segment

where $m_{\mathrm{m}i}$ denotes the density change rate, $\lambda_i \in \mathbb{N}$ is the number of lanes, while $q_{\mathrm{r}i}$ is the metered on-ramp flow.

2) Velocity model: The dynamics of the mean speed of the traffic flow in the ith cell is given by

$$\begin{aligned}\tfrac{\mathrm{d}}{\mathrm{d}t}v_i(t) = a_i(t) = \frac{1}{\tau}\left(V(\rho_i(t) - v_i(t)\right) + \\ + \frac{1}{L_i}v_i(t)\left(v_{i-1}(t) - v_i(t)\right) + \\ - \frac{\nu}{\tau L_i}\frac{\rho_{i+1}(t) - \rho_i(t)}{\rho_i(t) + \kappa} - \frac{\delta}{L_i\lambda_i}\frac{q_{\mathrm{r}i}v_i(t)}{\rho_i(t) + \kappa}\end{aligned} \quad (7)$$

where a_i denotes the acceleration, τ is the time constant, ν is the anticipation constant, δ is on-ramp constant and κ is a correction factor expressed in vehicles per kilometer per lane. Moreover, the steady-state speed $V(\rho_i(t))$ is expressed as a function of the free-flow speed v_{f}, and of the critical density ρ_{cr}, i.e.,

$$V(\rho_i(t)) = v_{\mathrm{f}} \exp\left(-\frac{1}{p}\left(\frac{\rho_i(t)}{\rho_{\mathrm{cr}}}\right)^p\right) \quad (8)$$

where p is an empirical correction factor to take into account the maximum flow, given the features of the considered cell.

3) Ramp model: For each metered ramp, let $\omega_{\mathrm{r}j}$, $j \in \mathbb{N}$, be the number of vehicles in queue on the jth ramp. Note that when the jth ramp corresponds to the ith cell, then $i = j$. Then, the queue model is given by

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{r}j}(t) = m_{\mathrm{r}j}(t) = d_{\mathrm{r}j}(t) - q_{\mathrm{r}j}(t) \quad (9)$$

where $d_{\mathrm{r}j}$ is the traffic demand at the on-ramp origin. The output of the ramp depends on the traffic flow on the main segment and on the flow rate input $r_j(t) \in [r_{\min},\, 1]$, where $r_j = 1$ is the case when the on-ramp is unmetered and $r_{\min} \geq 0$. More specifically, one has that

$$q_{\mathrm{r}j}(t) = r_j(t)\hat{q}_{\mathrm{r}j}(t) \quad (10)$$

where $\hat{q}_{\mathrm{r}j}(t) = \min\{\hat{q}_{1\mathrm{r}j}\,;\hat{q}_{2\mathrm{r}j}\}$ and

$$\hat{q}_{1\mathrm{r}j} = d_{\mathrm{r}j}(t) + \frac{\omega_{\mathrm{r}j}(t)}{T} \quad (11)$$

$$\hat{q}_{2\mathrm{r}j} = Q_{\mathrm{sat}} \min\left\{1\,;\frac{\rho_{\max} - \rho_j(t)}{\rho_{\max} - \rho_{\mathrm{cr}}}\right\} \quad (12)$$

with T being a sampling time, Q_{sat} being the on-ramp capacity under free-flow conditions and $\rho_{\max}$ being the maximum density of the mainstream segment.

B. The ramp metering problem

We define the following traffic control problem

$$\begin{aligned}&\min_{r_i,\, i=j} \int_{t_0}^{t_{\mathrm{f}}} \left(\sum_{i=1}^{N} \rho_i\, L_i\, \lambda_i + \sum_{j=1}^{M} \omega_{\mathrm{r}j}\right) \mathrm{d}t \\ \text{s.t.}\quad & \tfrac{\mathrm{d}}{\mathrm{d}t}\rho_i = m_{\mathrm{m}i} \quad \tfrac{\mathrm{d}}{\mathrm{d}t}v_i = a_i \\ & \tfrac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{r}j} = m_{\mathrm{r}j} \\ & \rho_i \leq \rho_{\max} \quad r_j \in [r_{\min},\, 1],\end{aligned} \quad (\mathrm{TfcCP})$$

with M being the number of ramps. More specifically, the control objective is to minimize the so-called Total Time Spent (TTS, veh h), which is the sum of the Total Travel Time (TTT, veh h), that is the total time spent by the drivers in the mainstream, and of the Total Waiting Time (TWT, veh h), that is the waiting time of the drivers on the ramps. Furthermore, the minimization problem is subject to previously described dynamics, to the maximum density of the cell and to input constraints. Since it is possible to show that the solution to (TfcCP) implies the critical density, the previous control problem is reformulated as a ramp metering problem of regulating the on-ramp flow via traffic lights in order to track the reference traffic density, equal to the critical one, at the mainstream segment, that is

$$\rho_i(t) = \rho_{\mathrm{cr}},\ \forall\, t \geq t_0,\ \forall\, i \in \mathbb{N}\,, \quad (\mathrm{RMtrCP})$$

with the metered on-ramp flow such that $r_i \in [r_{\min},\, 1]$.

IV. The Proposed Sliding Mode Based Ramp Metering Algorithm

In this section the previous ramp metering problem (RMtrCP) is recast according to the SOSM control formulation introduced in Section II-B. Therefore, starting from the freeway cell dynamics (6) and (7), the sliding variable, according to Assumption 1, is chosen as the error

$$\sigma_{1i} = \rho_{\mathrm{cr}} - \rho_i\,, \quad (13)$$

while $x^{[i]} = [\rho_i,\, v_i, \omega_{\mathrm{r}i}]^\top$ and $u_i = r_i$, with $\mathcal{U} := [r_{\min},\, 1]$, in order to fit system (1). Moreover, by virtue of the choice of the sliding variable, it is possible to write the auxiliary system as in (2), while Assumptions 2 and 3 hold due to the physical nature of the system at hand. Thus, we are now able to solve Control Problem 1, by using a second order sliding mode control law.

A. The classical SSOSM control algorithm

A particular case of SOSM approach is the SSOSM control [19]. Starting from the Bang-Bang principle [21], if bounded uncertain terms are present, with bound as in (3)–(5), it is possible to generate a "suboptimal" state trajectory with respect to that obtained with the Bang-Bang minimum time optimal control law. Specifically, computing the local minimum or maximum of the sliding variable, referred to as $\sigma_{1i\,\max}$, instead of its first time derivative, the SSOSM control law can be defined as

$$w_i(t) = -\eta_i \cdot \overline{\alpha}_i \operatorname{sgn}\left(\sigma_{1i}(t) - \frac{1}{2}\sigma_{1i\,\max}(t)\right) \quad (14)$$

where $\dot{w}_i = r_i$ in our case, while $\eta_i = \eta^*$ and $\overline{\alpha}_i$ are chosen such that

$$\overline{\alpha}_i > \max\left(\frac{\beta_i}{\eta^*\gamma_i},\, \frac{4\beta_i}{3\gamma_i - \eta^*\varepsilon_i}\right) \quad (15)$$

$$\eta^* \in (0, 1] \cap \left(0,\, \frac{3\gamma_i}{\varepsilon_i}\right)\,. \quad (16)$$

As for the stability analysis, according to [19], one can prove that, under sufficient conditions (15) and (16), the control law (14) implies a contraction property of the extremal values of the sliding variable in time, so that the sliding variable and its first time derivative are steered to zero in a finite time t_{r}.

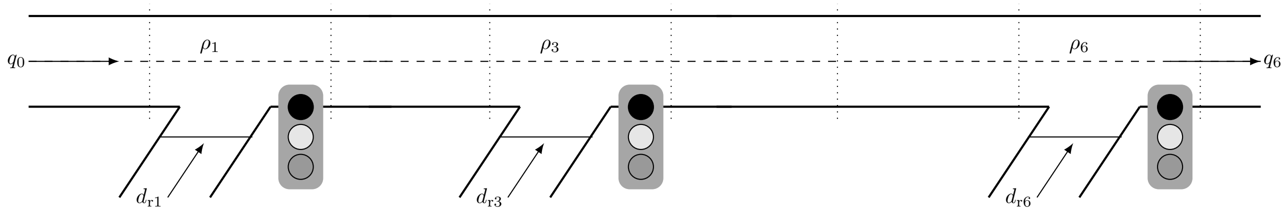

Fig. 2. Sketch of the considered highway portion

B. Extension to the classical SSOSM control

In our proposed solution we improve the SSOSM control performance through two modifications. A saturation strategy is introduced to take into account the limit of the on-ramp flow r_i, and a supervision mechanism is introduced in order to avoid to block the ramp flow for a long interval of time and to periodically guarantee the access to the mainstream segment.

1) Saturation of the control law: Since the ramp signal r_i is constrained to assume values between $r_{\min}$ and 1, while $\overline{\alpha}_i$ has to be set according (15), it is needed to modify the proposed control (14) in order to take into account these bounds. Inspired by [18], the SSOSM control law is modified adding an additional law depending on the sign of the input signal $r_i(t)$. More specifically, the control law (14) becomes

$$w_i(t) = \begin{cases} -\eta_i \cdot \overline{\alpha}_i \operatorname{sgn}\left(\sigma_{1i}(t) - \mu\sigma_{1i\max}(t)\right) & r_{\min} < r_i(t) < 1 \\ -\overline{\alpha}_i \operatorname{sgn}\left(r_i(t)\right) & \text{otherwise}\ , \end{cases} \tag{17}$$

where $\mu \in \left[\frac{1}{2}, \bar{\mu}\right]$ and $\bar{\mu}(t) = \frac{\sigma_{1i}(t)}{\sigma_{1i\max}(t)}$, $\forall\, t \in \mathcal{T} := \{t_k\}$, $k \in \mathbb{N}$ and $\mathcal{T}$ being the sequence of the time instants t_k when the control law switches.

2) Ramp supervision: As is evident in the ramp model, a sampling time T related to the traffic lights is needed in the definition of the ramp metering algorithm. Since this value can sensibly affect the performance of the controller, a supervision mechanism is introduced as follows: if the flow rate input $r_i(t) = r_{\min}$ for a time interval greater than cT, $c \in \mathbb{N}$, then set $r_i > r_{\min}$; if instead $r_i(t) = 1$ for less than $\frac{cT}{2}$, maintain $r_i = 1$ up to $\frac{cT}{2}$. This mechanism has the aim to avoid the generation of long queues on the ramps, thus avoiding too high waiting time on the ramps.

V. Case Study

The proposed control scheme has been assessed in simulation, modeling a stretch of freeway by means of the METANET model. To this aim we have considered a stretch of highway of $N = 7$ cells with $M = 3$ ramps located in the first, third and sixth cell ($j \in \{1, 3, 6\}$). The sketch of the highway is depicted in Figure 2. The inflow to the highway, i.e., the demand coming from cell $i = 0$ and from the on-ramps, is assumed to have a trapezoidal shape and it is reported in Figure 3. The parameters used for the model in the simulations are reported in Table I, following the case study in [17]. Figure 4 depicts the trend of the density along the highway when the ramp metering control is not applied. At the beginning of the simulation, the demand of the on-ramp of the cell 6 shows a peak. Furthermore, the sum of the ramp demand and the mainstream inflow overcomes the highway capacity value and this causes a congestion that starting from cell 6 propagates for several cells upstream. The same happens around the first hour of simulation when a peak of demand appears at ramp 3 and again at 2 hours of the

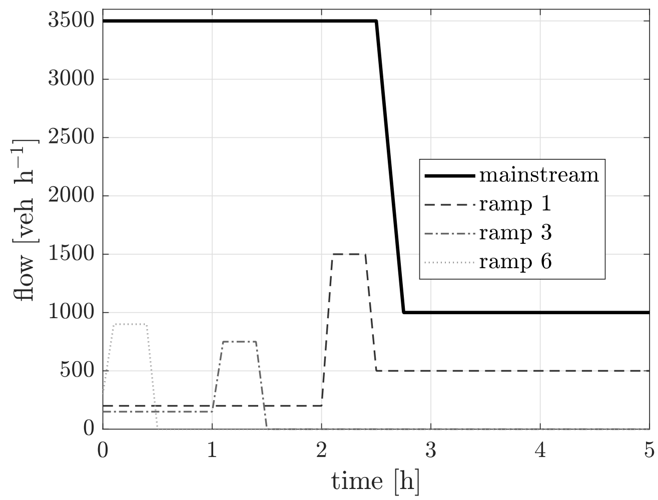

Fig. 3. Inflow to the highway

TABLE I
SIMULATION PARAMETERS

N	7
M	3
t_{f}	5 h
ρ_{cr}	33.5 veh km^{-1}lane^{-1}
$\rho_{\max}$	180 veh km^{-1}lane^{-1}
v_{f}	102 km h^{-1}
p	1.867
τ	0.005 h
T	0.0028 h
ν	60 km h^{-2}
κ	40 veh km^{-1}lane^{-1}
δ	0.0122
L_i	1 km
λ_i	2 lane
$\overline{\alpha}_i$	10
η_i	0.9
c	4
$r_{\min}$	0

TABLE II
PERFORMANCE INDEXES

Strategy	TTT	TWT	TTS	RMSE
Unmetered	1769	311	2080	26.33
ALINEA	1689	337.9	2027	20.99
FOSM	1690	323.5	2014	20.94
SSOSM	1453	396	1849	18.02

simulation when a severe congestion is caused by the peak of demand coming from the first ramp. In the uncontrolled scenario, the access from the on-ramps is not regulated, so all the incoming vehicles can enter the highway and there is not queue forming at the ramps. Consider now the SSOSM control action introduced in Section IV for which the critical density, i.e., the value for which the traffic throughput is maximum, has been chosen as reference value. The trend of the density in this controlled case is reported in Figure 5. It is possible to notice that by applying the ramp metering control, congestion in the mainstream is completely avoided. The control variables r_1, r_3 and r_6 of the three controlled ramps are reported in Figure 6. As remarked in Section III, the control variable r_i is the metering rate, that represents the portion of incoming flow that can access the highway from the on-ramp. The first control that is activated is the one of ramp 6. According to its demand, depicted in Figure 3, a peak at the beginning of the simulation can be noticed. In this case the controller strongly reduces the input variable for the whole duration of the demand peak, so that metering rate takes value $r_{\min}$, i.e., ramp closed, for around an hour, but without undesired effect since the demand of the ramps is zero. The same scenario happens for the other two ramps when their demands increase causing a congestion. The trend of the sliding variables is instead reported in Figure 7. As it can be observed, the control successfully steers the error to zero when the demand coming from the on-ramp is high, thus making the density be practically equal to its set-point value. When there is not enough demand to get the density stay close to the critical one, the error becomes high but negative. As highlighted before, this behavior does not represent a problem in the traffic control since the traffic is in free-flow condition.

The drawback that occurs by controlling the on-ramps inflows is obviously the formation of queues at the ramps. Their trends are depicted in Figure 8. The control effectiveness is evaluated considering the value assumed by the TTS, that is the time spent by all the drivers on the highway both in the mainstream and waiting at the queue, as remarked in Section III-B. Finally, in order to quantitatively verify the performance of the proposal, the SSOSM control algorithm presented in the present paper has been compared with a First-Order Sliding Mode (FOSM) control algorithm, with the ALINEA strategy and with the unmetered case. The achieved results are reported in Table II, in terms of TTT, TWT, TTS and Root Mean Square Error (RMSE) of the density. The TTS shows a 11.1% reduction by applying the proposed SSOSM with respect to the unmetered case, given

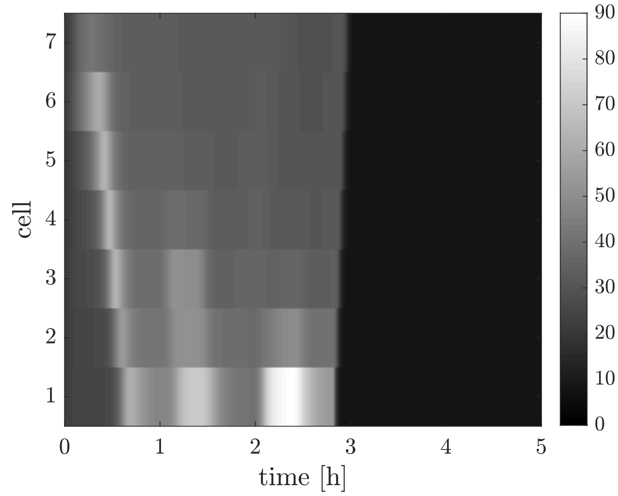

Fig. 4. Density trend in the uncontrolled case

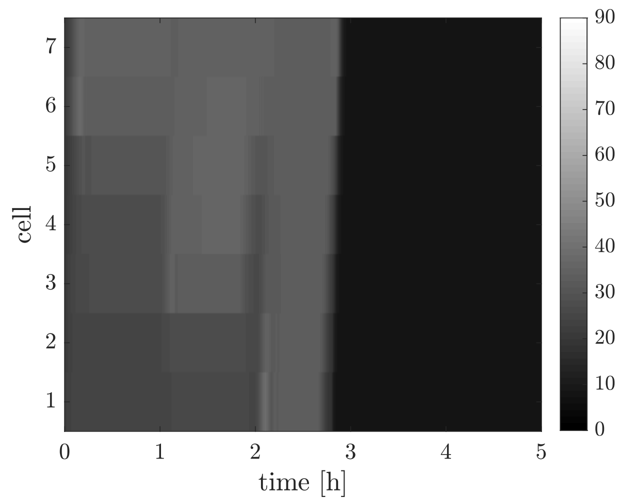

Fig. 5. Density trend in the controlled case

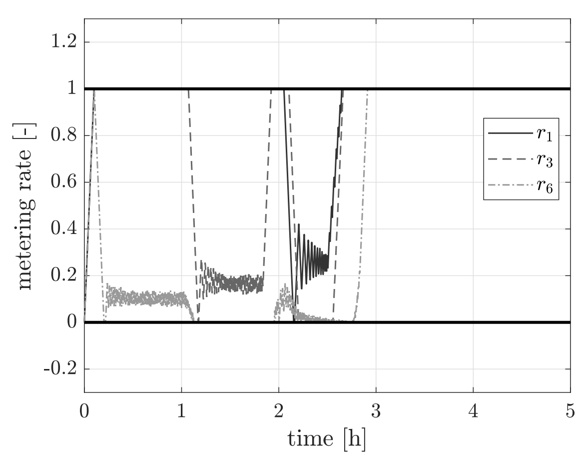

Fig. 6. Controlled metering rate for the three ramps

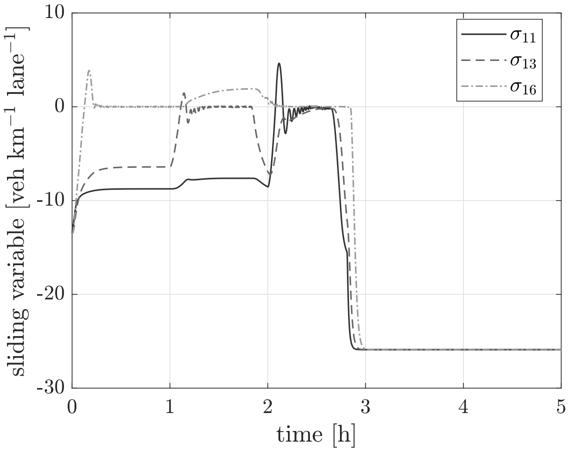

Fig. 7. Trend of the sliding variables

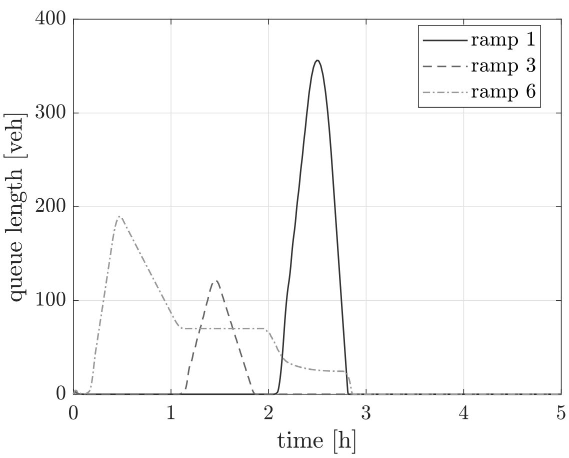

Fig. 8. Queue length at the ramps

by an increase of the TWT due to the ramps closure and a strong reduction of the TTT. The latter results outperform the improvements that are achieved via ALINEA strategy. The FOSM algorithm succeeds as well in reducing the TTS with respect to the unmetered case (3.2%) but its performance is lower than the proposed SSOSM. Moreover, the latter outperforms all the other strategies in terms of RMSE.

VI. Conclusions

In this paper a Suboptimal Second-Order Sliding Mode (SSOSM) control has been proposed to solve a ramp metering problem in freeway systems. After the introduction of the system according to the so-called METANET model, it has been recast to fit into the typical class of systems of sliding mode control theory. The SSOSM controller has been designed taking into account all the requirements in terms of input constraints on the on-ramp flow rate and in terms of sampling time of the traffic lights. Therefore, a saturated and supervised SSOSM algorithm has been finally designed and assessed in simulation on a quite realistic case study, even in comparison with a first-order sliding mode control, with ALINEA strategy and with the unmetered case, achieving satisfactory results. Future works will be devoted to further investigate the robustness features of the proposal in front of additional uncertainties and delays affecting the system.

VII. Acknowledgments

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