An investigation on the magnetic interaction for frequency up-converting piezoelectric vibration energy harvesters

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***Abstract*—This work presents an investigation of different approaches for modelling the magnetic force between permanent magnets for realizing the frequency up-conversion (FuC) in piezoelectric vibration energy harvesters (PVEH). Different analytical models are compared with finite element analyses (FEA). After the investigation, the FuC mechanism is applied on a meso-scale case study and dynamic analyses in the time domain are performed in case of harmonic monochromatic acceleration signal on the device at low-frequency. Both the repulsive and the attractive layouts of the magnets are considered and a larger amount of power is recovered in case of repulsive configuration.**

Keywords — piezoelectric energy harvesting, frequency up-conversion nonlinear dynamics, MEMS

I. Introduction

The increasing necessity of Internet of things (IoT) in the last years, encouraged the scientific community to develop MEMS autonomous sensors for creating smart networks of communicating objects. To this aim, the exploiting of environmental kinetic energy for self-powering sensors through piezoelectric vibration energy harvesting systems is a widely adopted strategy [1], [2]. Linear resonant energy harvesters have been largely investigated in the past [3]. Unfortunately, they are not very effective due to the typical high frequency of piezoelectric transducer (i.e. hundreds of Hertz) in comparison to the low-frequency content of the environmental vibrations (i.e. 0-50 Hz). Without a good dynamic amplification, they guarantee a low amount of energy as result. With this motivation, nowadays the research focuses to solve this problem through frequency up-conversion (FuC) techniques. Among different possibilities, a promising solution is the magnetic FuC via permanent magnets. It exploits the strong nonlinearity of the magnetic force between permanent magnets for creating impulsive dynamic phenomena on a piezoelectric transducer. Many works have been developed in recent years on rotational mechanisms. Pillastch et al. designed an energy harvester with a rotating proof mass as in mechanical watches [4], [5]. Pozzi [6] proposed a knee-joint piezoelectric energy harvester magnetically actuated. Xue and Roundy [7], [8] presented other works related to rotational mechanism with an in-depth analysis on different magnets configurations. In the framework of translational mechanisms, the work of Li et al [9] put in evidence the effect of bistability if the moving magnet is connected to a spring. An improvement of the operational bandwidth in case of two oscillators is presented in the work by Kim et al [10] in 2020. In the previous cited papers, both analytical and numerical approaches are used to describe the magnetic forces but the experimental comparisons are often presented directly on the harvester. In this work, we present an investigation on the magnetic forces between permanent magnets in section II. In section III, the magnetic interaction is applied on a concept of harvesting system composed of a piezoelectric cantilever and a low-frequency mass (LFM) by presenting also numerical analyses in time domain. Closing remarks are proposed in section IV.

II. Magnetic interaction

The computation of the magnetic force between two permanent magnets is a complex operation due to its nonlinear dependence from many variables (e.g. shape of the magnets, entity of magnetization, spatial orientation of the magnets). In this work, we present a comparison between analytical formulas and finite element analyses (FEA).

A. Analytical formulas

Akoun and Yonnet proposed [11] an approach based on the interaction energy in which the force computation is derived through the gradient operator. The validity of the formula is guaranteed in case of cuboidal magnets with parallel magnetization vectors. The generic component of the force 3D space is the following:

$$F_i = \frac{J \cdot J'}{4\pi\mu_0} \sum_{m,n,p,q,r,s} (-1)^{m+n+p+q+r+s} \cdot \phi_i \tag{1}$$

J and ***J'*** in (1) are the magnetization vectors, μ_0 is the magnetic permeability of vacuum. The parameters *m, n, p, q, r, s* are related to the corners of the magnets, They can be equal to 0 or 1. The combinations of them identify the corners of the magnets (*m, p, r* for one magnet and *q, r, s* for the other). The coefficients Φ_i depend on the geometry of the magnets. As an alternative, it can be assumed an inverse square approximation with reference to the figure (1):

$$F_{mag}=F_0\, h^2/r_{mag}^2 \qquad (2)$$

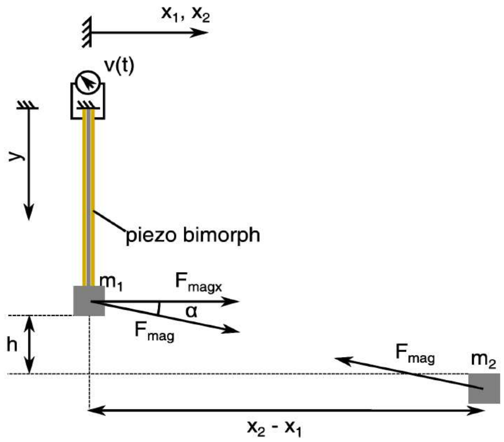

Fig. 1. Interaction between permanent magnets

In (2), F_{mag} is the magnetic force value as a function of the peak F_0 when the magnets are put at the gap distance h, r_{mag} is the separation distance. F_y and F_z, in accordance to figure (2) can be derived through simple trigonometry. Another recent approach has been proposed by Schomburg et al [12]. The y-component can be computed with the following set of equations:

$$F_y=\frac{F_0}{(h+d_e)^2} \qquad if\ |x_1-x_2|\leq A-a \qquad (3)$$

$$F_y=\frac{F_0}{(h+d_e)^2}\left[\frac{a+A}{2A}-\frac{x_2-x_1}{2a}\right] \qquad if\ a-A\leq|x_1-x_2|\leq A+a \qquad (4)$$

$$F_y=0 \qquad if\ |x_1-x_2|\geq A+a \qquad (5)$$

where F_0 is the forced exterted when the magnets are put in contact and de is the gap distance at which the peak force reduces to ¼ of F_0. A and a are the half-lengths of the magnets facing to the direct interaction. The x-component is computed as:

$$F_x= F_{x,m}\left[\frac{a^4}{a^4+(x-x_1-A)^4}-\frac{a^4}{a^4+(x_2-x_1+A)^4}\right] \qquad (6)$$

In (6), $F_{x,m}$ contains the parameters F_0 and d_e and depends on the gap h. Since d_e is unknown, this approach can be used after the experimental characterization.

B. Finite element simulations

Numerical solution has been determined with finite element analyses through the commercial software COMSOL Multiphysics®. A full 3D model has been built with tetrahedral quadratic finite elements. The computation leads to the evaluation of the magnetic field in the space around two magnets, in different positions. As a consequence, the Maxwell stress tensor is computed and, by means of a suitable integration procedure, the resultant force between the magnets is obtained. It is important to highlight that FEAs require two hours and 10 min of computing time while the previously described analytical approaches a few milliseconds on a simple MATLAB© code. An example of the result of FEA is reported in figure (2).

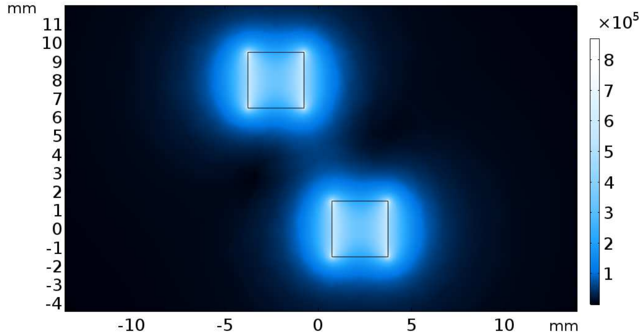

Fig. 2. Contour plot of the magnetic field (in A/m) obtained via FEA.

C. Critical comparison

The comparison between the different analytical formulas and the FEA results confirms that the best fit, for various gaps, is obtained by adopting the Yonnet model. For the sake of brevity, the comparison is reported only for Yonnet model vs FEA, in the specific case of h = 2 mm. The smaller the gap, the higher the improvement achieved by Yonnet approach with respect to the other models.

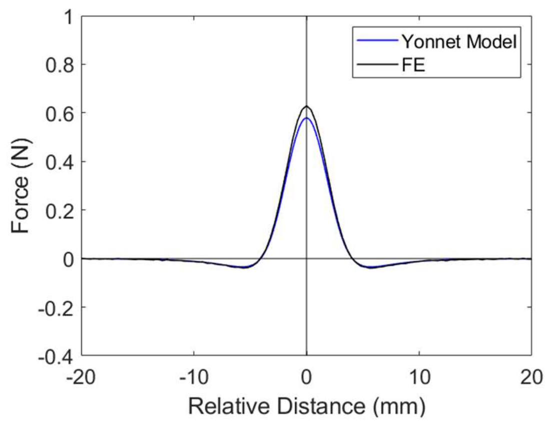

Fig. 3. F_y component for h = 2 mm

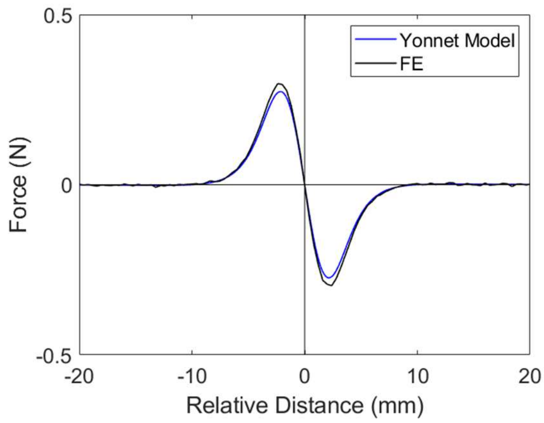

Fig. 4. F_x component for h = 2 mm

In all cases the peak force is well represented in comparison to FEA. However, this is not sufficient for a good evaluation of the magnetic energy. In fact, the whole curve on the analyzed path of ± 20 mm must be taken into account for a reasonable evaluation. For this reason, we considered a dimensionless L^2 norm of the discrepancy as the following:

$$err=\frac{\int_{-20\,mm}^{+20\,mm}(F_i-F_{sper})^2\,dz}{\int_{-20\,mm}^{+20\,mm}(F_{sper})^2\,dz} \quad (7)$$

where F_i is the generic simulations data. The quantitative examination of such a discrepancy indicator, confirms that the Yonnet model is generally in agreement with FEA results, while the inverse square relation is reasonable only for large values of *h*. The Gaussian fit provides very accurate results but it cannot be used as a predictive approach.

III. Modelling and Numerical simulations

We consider here a system composed of a low-frequency mass and a piezoelectric cantilever harvester both equipped with a permanent magnet (see figure 5). When the dynamics of the LFM is activated by an external environmental low-frequency vibration, a magnetic interaction arises with the harvester and since the interaction force is nonlinear as shown in the figures (3) and (4), the result is a train of free vibrations of the piezoelectric transducer.

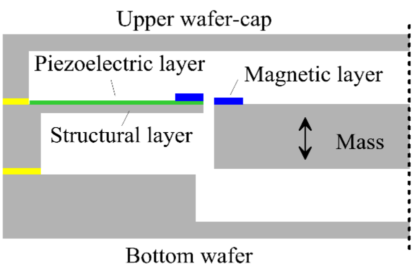

Fig. 5. Concept of the proposed device

The model of the piezoelectric transducer has been developed in previous works by Ardito et al [13] and Gafforelli et al [14] taking also into account three-dimensional effects. The lumped parameters model is developed for the cantilevered harvester, via Rayleigh-Ritz method with one dof for each physics, in case of a resistive circuit connected to the harvester:

$$\begin{cases} m\ddot{U}+c_m\dot{U}+k_lU-\theta V=-m_y\ddot{Y}+F_x(U,\,U_s) \\ C_e\dot{V}+\theta\dot{U}+V/R=0 \end{cases} \quad (8)$$

The differential system (8) is highly nonlinear for the presence of the magnetic force. Only the F_x component is used since the magnetization vector is parallel to the beam axis and the axial force on the cantilever is such that it does not affect the bending response. The system has been implemented in a MATLAB© code for carrying out the time domain analysis. It is presented in the following a mesoscale example both in the case of repulsive and the attractive configurations between magnets and all the data are summarized in table 1.

TABLE I. DATA FOR THE SIMULATIONS

Parameter	***Value***	***Description***
b	3 mm	Cantilever width
L	15 mm	Cantilever length
t_{PZT}	150 µm	PZT thickness
t_{sil}	200 µm	Silicon thickness
ρ_{PZT}	7.70 g/cm^3	PZT unit mass
ρ_{sil}	2.33 g/cm^3	Silicon unit mass
E_{PZT}	100 GPa	PZT Young's modulus
E_{sil}	148 GPa	Silicon Young's modulus
e_{31}	-12 N/m/V	31 piezoelectric constant
e_{33}	+20 N/m/V	33 piezoelectric constant
ε_{33}^s (ε_0)	2000	Relative dielectric constant
m_s	0.05 kg	Low-frequency mass (LFM)
k_s	60 N/m	Stiffness of the LFM system
Q_m	250	Cantilever quality factor
Q_M	40	LFM quality factor
h	1.0 mm	Gap between magnets
R	100 kΩ	Load resistance

A harmonic motion is introduced for the driving mass (figure 6). The amplitude of the applied displacement is 30 mm and the frequency is 7.96 Hz. Consider that the frequency of the cantilever is 577.3 Hz. The attached circuity is characterized by the optimal load resistance, computed at the resonance frequency of the cantilever.

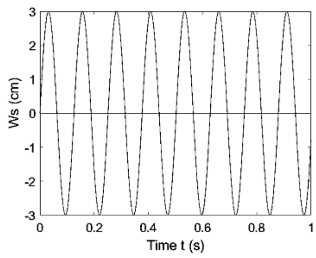

Fig. 6. Imposed displacement of the driving mass (black line)

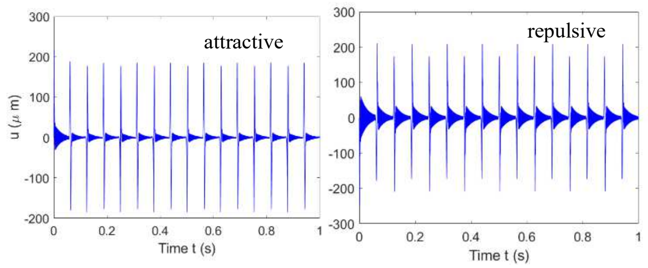

Fig. 7. Displacement of the piezoelectric cantilever (blue line) for attractive and repulsive magnetic interaction

Figure (7) shows that the system behavior depends on the poles orientation. When the displacement of the driving mass is around 0, the tip of the piezoelectric beam follows the movement in veiw of the magnetic interaction. At a certain instant, the elastic rstoring force of the cantilever is larger than the magnetic force, so the beam is suddenly released and start to vibrate (mainly) according to its first eigenmode. The examination of figure 7 shows that the release happens at about 200 µm for both cases. On the other hand, the free vibration is endowed with larger amplitude for the repulsive case: as a consequence, the average power production is 0.235 mW for the repulsive configuration and 0.176 mW for the attractive configuration.

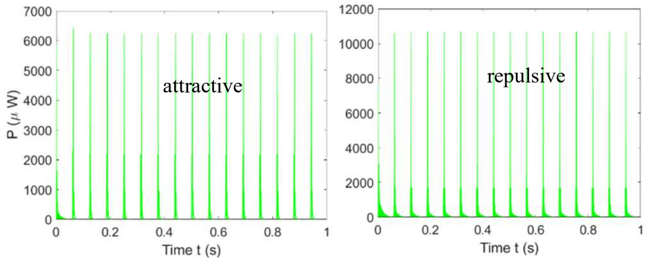

Fig. 8. Comparison of power production between attractive and repulsive configurations

IV. Conclusion

This work is focused on the investigation of different approaches for modelling the magnetic force for frequency up-conversion. A comparison between analytical formulas and finite element based solutions is presented. The study showed that the Yonnet formula is a robust tool in terms of discrepancy with respect to the accurate simulations via FEA and it works in a few milliseconds. Furthermore, it requires only the magnetization value as input parameter. The inverse square formula can reproduce the peak force but it fails in energy terms. The approach of Schomburg requires two fitting parameters (F_0 and d_c) and is pretty rough for the longitudinal interaction. The application of the magnetic interaction on a mesoscale case study in the section III, shows interesting nonlinear effects on the LFM that highlight different dynamic behavior depending on the poles orientation. The time domain analyses provide a more promising power generation of the repulsive scheme with 0.235 mW in comparison to 0.176 mW of the attractive one with a resistive load of 100 kΩ. The achieved results are the basis to develop an experimental setup, that is shown in figure 9 and that is currently adopted for tests. A piezoelectric beam is clamped in a polymeric case and is equipped with a cubic magnet. The driving mass is located in front of the cantilever's tip and embeds an identical magnet, positioned in repulsive or attractive configurations. The motion of the driving mass is guided by the polymeric case and can be imposed by means of a dynamic actuator (not shown in the figure). The tests can be carried out at different frequency of the imposed motion, that correspond to different velocity of the driving mass when pass in front of the cantilever. The preliminary experimental results are encouraging. In particular, the effect of frequency is clearly identified: the greater the velocity of the driving mass, the more abrupt the plucking effect, with the consequence of larger oscillations of the piezoelectric beams.

The achieved results pave the way for the application of the magnetic plucking at the MEMS scale. As a matter of fact, both piezoelectric and magnetic materials can be introduced in the fabrication process of MEMS. The plucking effect, possibly coupled to the introduction of suitable designed metamaterials [15] may boost the performance of microscale piezoelectric harvesters.

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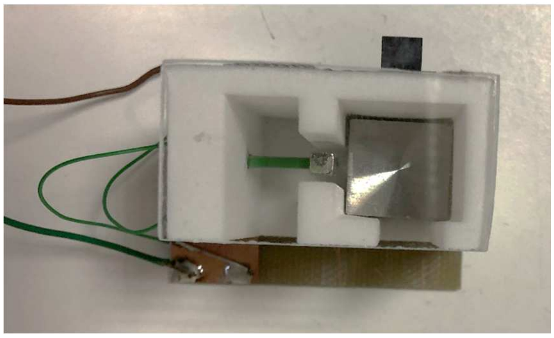

Fig. 9. Experimental setup adopted for the tests

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