Ejecta analysis for an asteroid impact event in the perturbed circular restricted three-body problem

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In a context of future asteroid exploration missions, within the Collecting Asteroid-Orbiting Samples - CRADLE project in collaboration between Politecnico di Milano and the Japan Aerospace Exploration Agency (JAXA), we envision the possibility to perform in-orbit collection. In this work, the dynamical behaviour of ejecta following the impact of a small kinetic impactor is analysed in the context of the circular restricted three body problem. The effect of the impact location is considered, alongside a statistical distribution of the initial ejecta plume. A preliminary mission concept that exploits the passage of particles through the L_2 gap is investigated. Specific attention is given to the fate of those ejecta that pass through the L_2 gap with conditions that allow their capture. This condition is coupled with the characteristics of the impact and of the ejecta model to study the feasibility of such a solution. A specific application to the case of asteroid Ryugu is presented.

I. Introduction

Asteroids carry fundamental information on the evolution of our Solar System and they are rich in valuable resources such as metals, silicates, and water. Therefore, improving our knowledge of these small bodies is of interest from both a scientific and economical point of view. The resources they contain can be exploited, via future asteroid mining, to extract rare metals and water to enable the self-sustainability of long-duration missions. To enable such missions, it is important to have a better knowledge of the asteroids' composition; in fact, their physical composition is varied and, in most cases, poorly understood. However, it can be significantly improved collecting and studying their samples. Improving our knowledge, we can better target asteroids to be exploited and increase the efficiency of asteroid deflection missions. Several missions have visited asteroids and other small bodies; however, only few have orbited, landed, or impacted on them. Examples are JAXA missions Hayabusa and Hayabusa2 [1–3], ESA Rosetta, and NASA OSIRIS-REx. One of the most challenging aspects of such missions is to collect and sample asteroids material by means of an on-ground collection, involving landing (or touchdown) and mining. In a context of future asteroid exploration missions, within the Collecting Asteroid-Orbiting Samples - CRADLE project, we envision the possibility to perform in-orbit collection as an alternative to landing or touchdown operations [4]. Such a collection mechanism relies on the knowledge of the dynamical behaviour of small particles orbiting the asteroid, which is influenced by the third body effect, solar radiation pressure and the gravitational potential of the asteroid. This work performs a preliminary feasibility analysis fur such particle collection missions, focusing on a collection mechanism. Specifically, the collection mechanism is based on intercepting the ejected particles at the Sun-asteroid L_2 gap [5]. This type of mechanism has the advantage of knowing in advance where the particles will be and what size of particle can be collected depending on the distance from the asteroid surface. We show a preliminary feasibility analysis as a function of the target asteroid properties. Then we study the effect of the impact location and impact physics on the possibility of collecting the particles at L_2, estimating the amount of collectable particles. Finally, we analyse the effect of the asteroid rotation on the ejection conditions and, consequently, on the possibility to collect the particles at L_2.

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II. Methodology

A. Collection scenario

This section describes one option for the collection strategy examined in this work. Specifically, the it is proposed to analyse the possible collection of particles at the L_2 gap of the Sun-asteroid system. This region is considered favourable for collection because the particles reach this area with a small speed and are "confined" within a specified region that is determined by the energy of the particles that we aim to collect. Additionally, the L_2 positions serves as a "mass spectrometer" that is different $L2$ locations are associated to different particle sizes and collection times. Therefore, by moving along the Sun-asteroid line, we can target the collection of particles of different sizes based on the distance from the asteroid's surface.

B. Dynamical model

The dynamical model used in this work is the Circular Restricted Three-Body Problem (CR3BP) perturbed by solar radiation pressure (SRP) [6]. The choice of neglecting the gravitational perturbations, mainly J_2, directly follows the objectives of this work that is to have a general understanding of the phenomenon and of possible suitable targets. As the gravitational potential is accurately known only for few asteroids, this was considered a suitable assumption.

The effect of SRP is considered using a "cannon ball" model, where the ejected particles are considered spheres. In this case, the effect of SRP is conservative and the magnitude of its acceleration can be expressed as follows:

$$a_{\text{SRP}} = P_{\text{SRP}} \frac{S}{m} c_{\text{R}} = \frac{\beta \mu_{Sun}}{r_{\text{sp}}^2} \hat{\mathbf{r}} \tag{1}$$

where S is the particle cross-section exposed to the Sun, m is the mass and c_{R} is the reflectivity coefficient (0 for translucent particles, 1 for black bodies, and 2 for reflective particles), μ_{Sun} is the gravitational parameter of the Sun, r_{sp} is the Sun-to-particle distance and $\hat{\mathbf{r}}$ the Sun-to-particle direction.. The lightness parameter, β, is a non-dimensional parameter related to the magnitude of the SRP acceleration, and can be expressed as follows:

$$\beta = \frac{P_0}{c} \frac{AU^2}{\mu_{Sub}} \frac{3 c_{\text{R}}}{2 \rho_{\text{p}} d_{\text{p}}} \tag{2}$$

Where $P_0 = 1367 \text{ W/m}^2$ is the solar flux at 1 AU, c is the speed of light, AU is the astronomical unit, ρ_{p} is the particle density and d_{p} the particle diameter.

The equations of motion are expressed in non-dimensional form in a synodic reference frame centred in the asteroid.

$$\begin{cases} \ddot{x} - 2\bar{n}\dot{y} = V_{/x} \\ \ddot{y} + 2\bar{n}\dot{x} = V_{/y} \\ \ddot{z} = V_{/z} \end{cases} \tag{3}$$

where x, y, and z are the non-dimensional particle positions with respect to the centre of the asteroid in the rotating frame, and $\bar{n}$ is the non-dimensional mean motion, equal to unity in this case. The potential V is expressed as follows:

$$V = \frac{1}{2}\left(x^2 + y^2\right) + (1-\mu)\,x + \frac{(1-\beta)\,(1-\mu)}{r_{\text{sp}}} + \frac{\mu}{r_{\text{ap}}} + \frac{1}{2}\,(1-\mu)^2 \tag{4}$$

with r_{sp} and r_{ap} the distances between the Sun and the particle and the asteroid and the particle, respectively.

C. Ejecta model

The ejecta model describes the characteristics of the ejected particles after a kinetic impact. The ejected particles are defined by their size, ejection speed, and launch direction. These quantities represent the initial conditions for the orbit propagation in the CR3BP. Specifically, we will use the ejecta model to describe the effect of an impact with a small kinetic impactor, comparable to the one of the Hayabusa2 mission. The ejecta model is defined using a density function of the form [7]:

$$\phi(s, u, \xi, \psi) = A s^{-1-\alpha} u^{-1-\gamma} f(\xi) g(\psi) \tag{5}$$

where s is the particle radius, u the ejection velocity, ξ the in-plane launch angle, and ψ the out-of-plane launch angle (measured with respect to the local horizontal frame centred at the impact location). The exponents α and γ regulate the slope of the distributions and they can be computed exploiting experimental correlations [8–10]. The multiplicative factor, A, is a scaling constant used for mass conservation. For further details on the adopted ejecta model, the reader is referred to [4].

III. Results

A. Preliminary reachability and collectability analysis

In this section, we consider the population of Near-Earth Asteroids (NEAs) and analyse, in a simplified fashion, the possibility of carrying out the collection scenario described in Section II.A. Two main characteristics are considered to evaluate the preliminary feasibility of the scenario given the target asteroids:

1. The required ΔV to reach the asteroid.
2. The altitude of the L_2 point with respect to the asteroid surface, computed for a test particle of 1 mm in diameter.

For the first point, the method used to compute the mission ΔV is the one outlined by Shoemaker and Helin [11]. We used the NASA Small-Body Database to retrieve asteroids' orbital, dimensions, and composition information. Density information is derived from the spectral class of the asteroid according to [12].

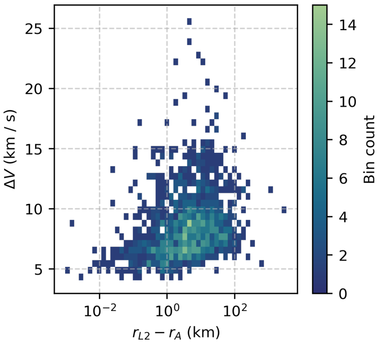


Figure 1 Target NEAs distribution as function of required ΔV and L_2 altitude.

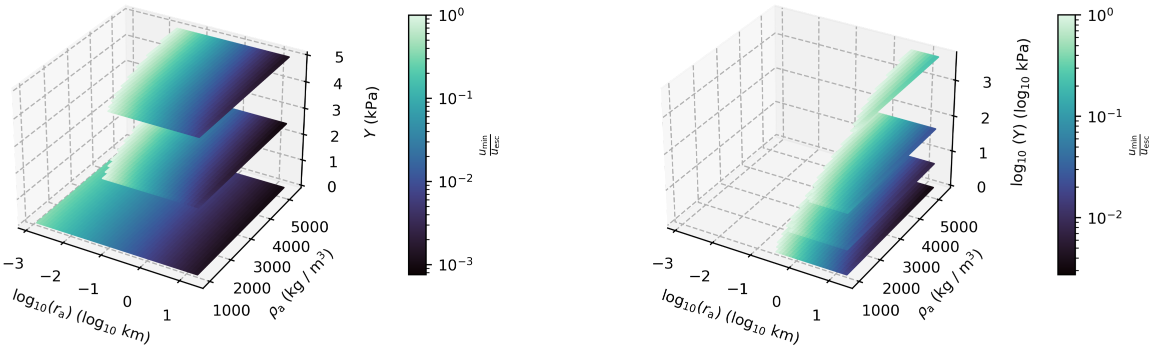
Fig. 1 shows the computed required ΔV and L_2 point altitudes for all NEAs. It is possible to observe that the majority of the targets have ΔV requirements below 10 $\mathrm{km\,s^{-1}}$, and several (about 40%) also below 6 $\mathrm{km\,s^{-1}}$, which is comparable to a Mars mission. Among these targets, the majority have a L_2 altitude between 1 and 10 km, which is considered a reasonable altitude for mission operations connected to sample collections. The low end of the possible L_2 altitude, comprised between 10 and 100 m may instead be to demanding for a spacecraft, without compromising its safety. From this preliminary analysis we can conclude that the L_2 collection strategy outlined in Section II.A can be a viable option for several targets, both in terms of required ΔV and potential feasibility of the collection scenario.

B. Collectability vs. target properties

The collectability of the particles is not only influenced by the location of the L_2 as discussed in Section III.A. It is necessary to consider the impact physics to properly assess the collectability; in fact, depending on the impact, different ejection conditions characterise the ejecta. Naturally, different ejection conditions lead to a diverse dynamical behaviour of the particles. Specifically, in this work, we decided to examine the minimum ejection velocity generated by a kinetic impactor. We consider the minimum velocity to be a relevant parameter that gives useful insight about the collectability of the ejected particles. In fact, by comparing the minimum ejection speed with the escape velocity of the asteroid we can immediately understand if the particles can remain in the neighbourhood of the asteroid for a sufficient time. The minimum ejection velocity can be estimated as follows:

$$u_{\min} = U C_1 \left[\frac{n_2 R}{a} \left(\frac{\rho}{\delta} \right)^{\nu} \right]^{-1/\mu} \tag{6}$$

where U is the impact velocity of the kinetic impactor, R is the crater radius, computed following [8], and ρ and δ are the target and impactor densities, respectively. The coefficients C_1, n_2, ν, and μ are a function of the material. For the characteristics of the materials considered in this work the reader is referred to [8].



(a) Sand-like material.

(b) Weakly cemented basalt material.

Figure 2 Fraction between minimum ejection velocity and escape velocity as function of the asteroid radius, density, and strength.

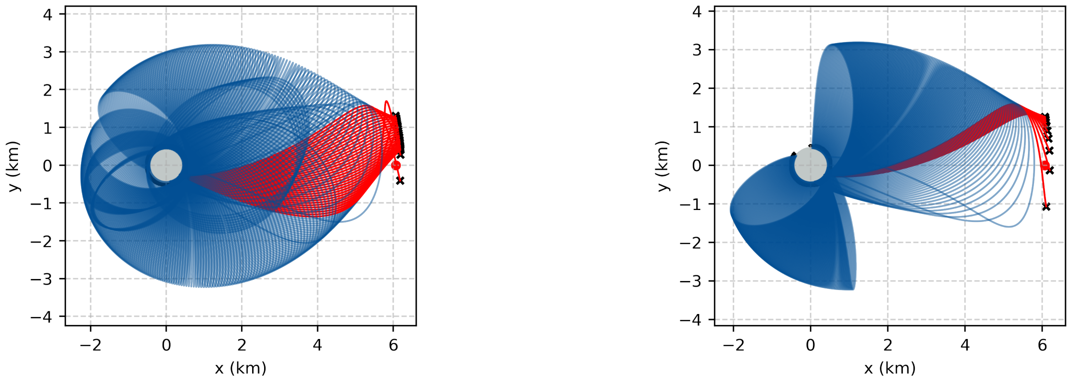
In this work, we study the ratio between the ejection and the escape velocity as a function of the target asteroid radius, density, and strength (Y). Fig. 2 shows this analysis for two type of materials: sand and weakly cemented basalt (WCB). The impactor is characterised by an impact velocity, $U = 2$ km s^{-1}, radius, $a = 0.075$ m, and density, $\delta = 2.7$ g cm^{-3}. It is possible to observe from Fig. 2 that the ensemble of possible targets is influenced by the targets' properties, i.e., radius and density, with larger and denser asteroids more likely to have particles not escaping. In addition, the material type and its strength greatly influence the target availability. Specifically, more cohesive materials (WCB is more cohesive than sand-like materials) tend to have higher ejection velocities, thus leading to a lower probability of having collectable particles. Similarly, the greater the strength is assumed for a material, the lower target availability. This is an important aspect as the material strength is a highly unknown parameter and must be carefully considered for a robust collection scenario [4].

C. Collectability vs. impact location

An interesting aspect to consider when evaluating the collectability of the particles is the impact location on the asteroid's surface. In this work, we study as an example an impact onto an asteroid of the dimensions of Ryugu [13], characterised by a sand-like material. The procedure is the following:

- Select a particle diameter
- Fix the ejection velocity to "open" a gap at L_2: $v = v_{C_2} + 0.02 \cdot (v_{\mathrm{esc}} - v_{C_2})$
- Compute the Jacobi constant and, with it, the ejection velocity at the asteroid's surface
- Perform a grid search in grid location with 5 deg bin in right ascension and declination (with respect to a synodic reference frame centred at the asteroid), and in the ejection angles (in-plane and out-of-plane) with 5 deg bins
- Check what conditions lead to an escape (see Fig. 3).

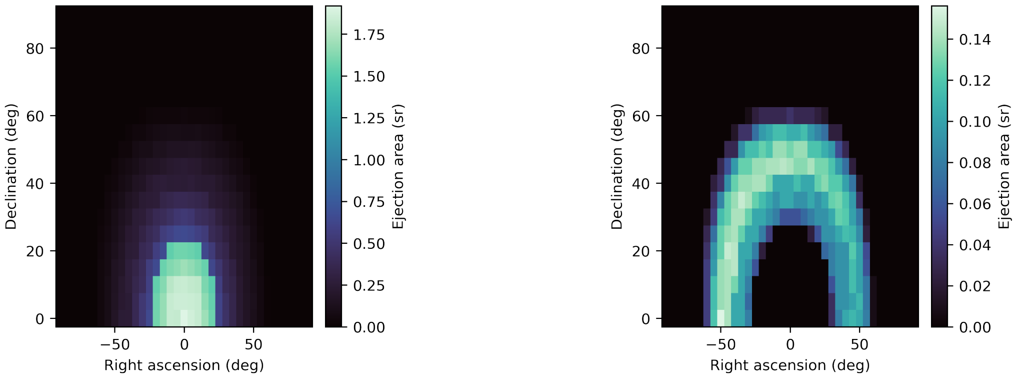
The two plots of Fig. 3 show the difference in the escape conditions when adding the constraint on the ejecta model about the ejection directions. Fig. 3a assume that all directions are allowed; instead; Fig. 3b considers launch angles limited between 25 and 65 deg [14]. Following this procedure and with some post-processing, we can evaluate the ejection area associated to the escape trajectories as a function of the impact location. This area can be interpreted as follows: considering the the hemisphere characterising all the possible ejection location, the ejection area is the portion of this hemisphere leading to escapes via L_2. Fig. 4 show the different available ejection areas without and with constraints on the ejection direction. It is interesting to notice that the ejection area is greatly reduced and that the "best" impact location are completely different in the two cases. It is also interesting to notice the symmetry of the computed ejection areas. This is because we did not consider the asteroid rotation in this analysis.



(a) Escape trajectories without limitations on the ejection angles.

(b) Escape trajectories limiting ejection angles between 25 and 65 degrees.

Figure 3 Example trajectories escaping via the L_2 gap (in red).



(a) Without limitations on the ejection angles.

(b) Limiting ejection angles between 25 and 65 degrees.

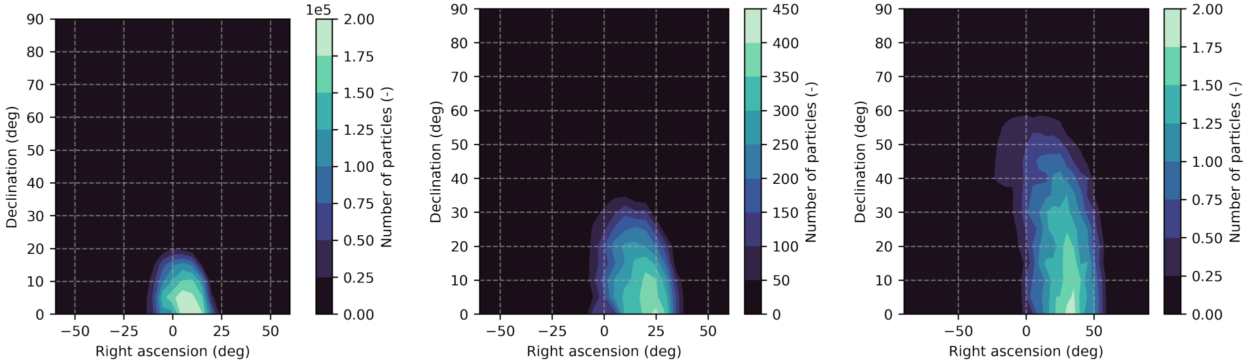
Figure 4 Ejection area leading to escapes via the L_2 gap as a function of the impact location.

1. Asteroid rotation effect

Finally, combining the trajectory propagation, the computation of the ejection area, and the ejecta model of Eq. (5), we can estimate the number of particles passing through L_2 as a function of the impact location. In this example we also included the effect of the asteroid rotation, with a rotation around the z-axis of the asteroid. Fig. 5 show this analysis for an impact onto a Ryugu-like asteroid, considering ejection angles between 25 and 65 degrees and three different particle diameters. We first observe that the rotation introduces a strong asymmetry in the results, shifting the best impact location towards positive right ascensions. In addition, we notice that bigger diameters can be collected with impacts on more locations with respect to smaller ones; however, as expected, the allow less particles to be collected. In general, for this example with notice a preferred impact location to maximise particles passing through the gap along the equatorial line of the asteroid, with a right ascension between 15 and 30 degrees. Of course, different asteroid characteristics and, in particular, rotation axis and speed, will modify the results.

IV. Conclusions and future work

The work has presented a preliminary analysis for a concept of in-orbit particle collection missions exploiting the dynamical peculiarities of the Sun-asteroid L_2 point for collection. Analysis showed that limitation on the available targets for such missions may arise when considering the target properties, such as radius, density, material type and strength. In addition, we analysed the impact of the ejection model and of the asteroid rotation on the collectability of the particles at L_2. We noticed that the limitation on the ejection angle reduces the available impact location, while the asteroid rotation contributes in shifting the possible impact locations.



(a) Particle diameter: 0.2 mm.

(b) Particle diameter: 1 mm.

(c) Particle diameter: 5 mm.

Figure 5 Estimated number of particles passing through the L_2 gap for three particle diameters.

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