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Shrouded by time and tile: a structural investigation for preservation of Cuba's historic School of Ballet classroom and theatre domes

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Abstract

Vittorio Garatti's School of Ballet in Havana, Cuba, is internationally acknowledged as a significant architectural and cultural icon whose survival is even now jeopardized by degradation such as moisture encroachment and corrosion, excessive deflections, and cracking of primary elements. While Garatti's vaulted forms symbolize the idealistic egalitarianism and free expression of their time of origin (1961-1964), their current state bears the story of the ensuing decades, in which practical, social, and environmental mechanisms have each played a part. This work comprised a multi-stage structural investigation aiming to elucidate the theatre and classroom domes' authentic structural condition and assess the behaviour and efficiency of these unique structural systems. Archival research and site investigations supplied the domes' construction techniques, geometry, and properties, which informed geometric and mechanical models. Detailed inspection disclosed their condition and behaviour. Then, non-linear finite element analysis with a concrete damaged-plasticity model simulated structural damage, yielding conclusions about their current condition. These conclusions, particularly that the theatre dome is stable while the classroom domes are significantly damaged and at-risk, informed conservation recommendations for future campaigns, with the goal of securing these irreplaceable icons against continued degradation which could soon become irreversible and even catastrophic.

Keywords: historic preservation, reinforced concrete masonry, non-linear finite element analysis, dome cracking, thin-tile vaulting, masonry formwork

1. Introduction

Winding down and through a shallow valley next to the meandering Rio Quibu in Havana, Cuba, the tile passageways of the Ballet School achieve a reflective unity with the park-like landscape of the ISA campus. And soaring above the vaulted halls, the three classroom domes and the magnificent choreography theatre dome repeat the graceful, canopied forms of the neighbouring forest.

Commissioned and designed in a burst of idealism and freedom in the early days of the post-revolutionary Cuban state, the ISA project enjoyed short-lived favour. Construction on the Ballet School was stopped just weeks before its completion due to lack of funding, and in response to governmental criticism of the project as too individualistic and untrue to the now-communist state's ideals. Unoccupied for several decades, the School suffered jungle and moisture encroachment and deteriorated significantly [1].

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Following its rediscovery in 1999 and ensuing international acclaim, the Ballet School has been well-studied architecturally [2], [3]. But earnest structural investigation started only in 2017, when Douglas [4], [5] rediscovered the constructive secret of the domes: a reinforced concrete endoskeleton lying within the tiled faces. The damaged condition of the Ballet School domes, plus the aging condition of other schools within the ISA complex, has prompted international conservation efforts [6], culminating in a Getty Foundation grant in 2018 [7] to an international consortium to study the campus and create a comprehensive conservation plan.

This structural investigation campaign was performed in this context, and aims to provide clarity about the following features of the Ballet School domes:

- 1) The authentic structural system as evidenced by and affected by the construction sequence, including the historical context for its selection.
- 2) The current condition of the domes as evidenced by observed damage and correlated by the numerical analysis.

To this end, the Choreography Theatre dome and one representative Classroom dome (of the three) were studied. The investigation process is starts with the archival and literature research, followed by the site investigations, which together inform the structural analysis campaign. Finally, the results of the holistic investigation inform the diagnosis and prognosis of the structures and the ensuing intervention recommendations.

While this project presents and validates a numerical analysis method for the ISA vaults, the hybrid structural system is not unique to ISA. An analysis of the historic-architectural context reveals that it was used more broadly across Latin America, as well as in North America and the Mediterranean. Consequently, the procedure detailed herein may provide a readily implementable modelling approach for diagnosis of such 20th century bending-active vaults.

2. Background investigation

Archival and literature research were conducted to elucidate the original design and the as-built structures. The parts of the Choreography Theatre, as they are discussed in this paper, are presented in Figure 1. The Classroom structure assembly is described by Figure 2.

2.1 Archival investigation

Photographs from the ISA archives taken by of builder Jose Mosquera and architect Vittorio Garatti reveal the original construction system of the Ballet School domes. As seen in Figure 3, the inner tile layers provided a sort of "formwork" in which the reinforcing steel and then concrete were placed. A final layer of tile was then grouted on the exterior face of the concrete to complete the structures' tile "skin." The amount of formwork and scaffolding used for the placement of the shells' inner tile layers and the concrete is unclear, but it does appear that complete formwork was only required for the ribs and rings of the domes' upper cap.

Several extent construction drawings show the design for the pendentive reinforcing, which, from the construction photographs (Figure 3, left), appears to have been realized. These also indicate a layer of reinforcing steel within the thin concrete shell covering the ribs in the uppermost, spherical dome shells.

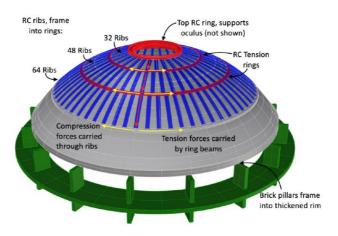


Figure 1. Choreography Theater components. Ribs (blue) support dome shell (grey) and frame into "crown" (gray, lower portion). This upper dome sits on pillars and a two-story mezzanine (green).

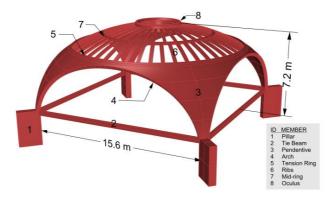


Figure 2. Structural components of Practical Classrooms. Dome shell covering ribs omitted for clarity.

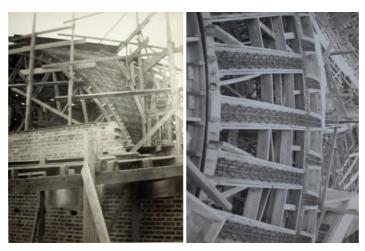


Figure 3. Rebar in the Classroom pendentives (left) and the domes' ribs and rings (right). Courtesy of ISA archives.

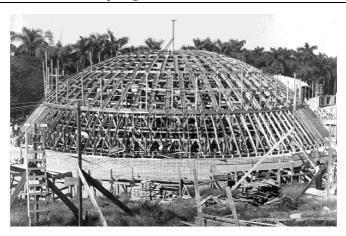


Figure 4. Theatre dome under construction, with scaffolding and wooden formwork visible. Courtesy of ISA archives.

2.2. Literature investigation

2.2.1. Selection of tile vaulting as Schools' primary construction system

The selection of the tile vaulting system, the "boveda tabicada", for the Schools is well discussed by authors such as Loomis [1], Juanas and Jiminez [3], and others. The method facilitated the organic and free-form buildings envisioned by the three architects, allowing them to create complexes which seemed to grow naturally from the park-like landscape of the campus-formerly golf course. Further, tile, as a locally sourced material, was preferred above steel and concrete for economic and political reasons in response to the recently-introduced U.S. embargo.

The initial inspiration for using thin-tile vaulting reportedly came when the architects crossed paths with a Catalan-vaulting trained mason, previously from Barcelona, whose father had worked as a mason on Gaudi's thin tile vaults. This mason, Gumersindo, trained local craftsmen in the method [1].

Thus, the people's Schools were constructed by local craftsman using a traditional method and primarily local materials: it truly was an architecture of the Cuban revolution.

2.2.2. History of reinforced and unreinforced concrete-masonry shells

Due to the prevailing misidentification of the system as true thin-tile vaulting, the context and inspiration for the implementation of the reinforced concrete-tile vault has been less discussed. Two features bear investigation: 1) The use of concrete inside the tile vault and the corresponding treatment of the tile vault as "stay-in-place formwork" for the concrete. 2) The inclusion of reinforcing steel within the vault.

The use of tile as formwork for a concrete shell was not original to Garatti and the other ISA architects, but has origins as early as the Roman Empire. Lopez et al. [8] traces the development of this system and give the earliest recorded form as a Roman-era vaults. Both free-standing or supported tile "formwork" may have been used in these early examples.

In the nineteenth and twentieth centuries, other architects and builders used various iterations of reinforced and unreinforced tile-concrete shells. These included the Guastavinos, Dieste, and Le Corbusier. Just prior to the construction of the Schools, Sanchez del Rio published a treatment of innovative reinforced tile constructions, including shells and vaults, which he completed in Venezuela. This publication (in IASS) was circulated in Latin America and is known to have reached Cuba by around 1960 [9].

These architects and their works provided the contemporary context for the ISA architects' training and practice both in the Mediterranean and in Latin America. Since it is apparent that they not only modeled the use of Catalan vaulting or concrete thin shell structures, as detailed by Loomis, but also, as addressed by Lopez, exhibited varying levels of knowledge of and use of the tile-as-formwork method and a reinforced-hybrid system, the three ISA architects also had significant opportunities for exposure to these constructs. Consequently, it is likely that their decision to implement the hybrid reinforced concrete-tile system was based on past familiarity rather than an independent, original development of the concept.

Since this structural system is not unique to ISA, the methods of analysis and diagnosis explored in this case study may be applicable to similar systems (reinforced, concrete-tile, bending active shells) worldwide.

2.3. Site investigation: NDT

Project partners, including Zenith Ingegneria and Università di Parma, performed non-destructive evaluation on the Ballet School structures. Sonic rebound tests were used to characterize the concrete strength and indicated excellent quality concrete with a compressive strength of about 40-50 MPa. A laser scan of the dome captured the current geometry and provided accurate dimensions for the numerical model.

2.4. Site investigation: visual inspection

The damage patterns of the Theatre and Classroom domes were visually surveyed to illuminate the authentic structure and its behavior and condition and allow validation of the structural analysis models. For the Practical Classrooms, select phenomena are presented below:

- Cracking at the arch apex and third-points, both on the bottom (intrados) and top (extrados).
- Loss of tile across the pendentives and some in the upper dome.
- Horizontal cracking in the tile or (where tile loss has exposed it) concrete of the pendentive extrados. In some locations they extend through the side tile layers on the edges of the pendentives. No cracking was evidenced on the pendentive intrados.

Crack maps were produced which detail the visible tile and concrete cracks on the extrados of the pendentives. The comprehensive crack map for all surveyed pendentives may be seen in Figure 10, left. This study led the hypothesis that these cracks, though apparently stabilized by the pendentive rebar, were a significant damage occurrence and might lead to a structural failure if allowed to develop.

For the Choreography Theatre, more minor structural concerns were observed, as summarized below:

- Efflorescence due to moisture encroachment diffusely on the dome intrados.
- Increased efflorescence at ribs-crown connection, indicating internal section damage.
- Cracks and efflorescence at pillar-dome connection presenting as shear cracking due to radial deformation of dome.
- Slight microcracking in ring beam consistent with bending (tensile) stress.
- Seasonal flooding of the dome, which could cause soil deterioration and differential settlement of the domes' pillars.

3. Structural analysis

3.1. Geometric model and boundary conditions

In order to analyze the structures through nonlinear finite element analyses, suitable geometric and material representations had to be made.

For the Choreography Theater, the 3D model was created from 2D drawings provided by project partners based on site and archival data. The Classroom model was created from a laser scan of the structure also performed during the site investigation. For both structures, rotational symmetry was assumed to simplify the models, and representative cross-sections and profiles were defined from the provided drawings to create the solids and shells, respectively. All members were modeled as homogeneous solids, except for the upper dome shell, which was modeled as a shell element. Reinforcing elements were modeled as typical for Abaqus: wire or shell elements were assigned the properties of the reinforcing steel and then embedded in the solids. The resulting models are shown in Figure 5 and Figure 6.

The major modeling simplification was the treatment of the hybrid cross-sections in the homogeneous solid elements: in order to simplify the model and analysis by circumventing complex interactions between discrete layers (the tile layers and the inner concrete layer), one "effective" cross-section was defined that took into account the reinforced concrete layer and the tile layers. The extrados tile layer (less than two centimeters thick) was assumed to contribute little resistance in comparison to the total cross-section, an assumption supported by its widespread spalling or absence in the Classroom domes. The intrados tile layers are considered part of the stress-resisting cross-section and were therefore lumped into the effective solid cross-section. The ability of this idealized model to reproduce the behavior of the real structure will be evidenced by similar damage patterns in the model and the structure and will be evaluated before any conclusions are drawn from the analysis results.

The boundary conditions for the finite element models of the two structures are shown in Figure 5 and Figure 6. For the Choreography theater, the pillars and mezzanine were omitted from the model for computational efficiency and pins defined around the perimeter in place of the pillars. The pillars were included in the Classroom model (the full analysis included studies of the efficacy of the tie beams and the pillar demand), and their bottoms were supported with pins.

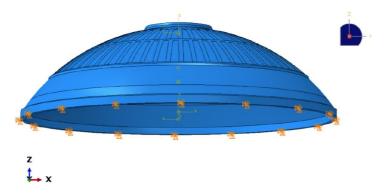


Figure 5. Geometric model of Choreography Theater showing pinned supports around perimeter. Outlines of ribs are visible through dome shell.

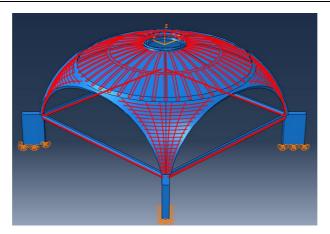


Figure 6. Geometric model of Classroom showing pinned supports on pillars. Embedded reinforcing elements are highlighted red.

For the complete investigation, applied loads included self-weight, wind loads, and differential settlement. Preliminary numerical simulations on the Practical Classroom revealed that the self-weight was the primary structural demand and most likely to produce the real structure damage. Consequently, numerical analysis results for the Practical Classroom are presented mainly for the dead load cases. The Choreography Theatre results are discussed for all three loading cases.

Various mesh geometries and properties were explored and the best mesh, balancing convergence and computational efficiency, was identified. The results from the refined mesh cases are presented herein.

3.2. Material mechanical model: concrete damaged plasticity

In order to capture the damaged condition of the structure, a nonlinear constitutive model was defined using the Concrete Damaged Plasticity approach. This material mechanical model defines the plastic behavior of concrete as the degradation of an initially linear elastic curve. The damage variable quantifies the ratio of total damaged area to total initial area in an element cross-section, and varies from 0 (undamaged cross-section) to 1 (fully damaged cross-section). The CDP model is available in commercial FEM software, including Abaqus, and is easily implementable by specifying damage parameters and the compressive and tensile damage data. It has been used for modeling historic structures [10] and for concrete structures in general [11]. For a given grade of concrete, typical compressive damage curves and tensile damage curves are given in literature [12], [13]. The ultimate tensile strength for the damage curve definition was calculated from the given compressive strength using ACI 318 [14]. Four different concrete grades, ranging from 40 to 50 MPa compressive strength and 3.9 to 4.4 MPa tensile strength, were analyzed and the damage results compared to study the effect of concrete strength on the dome response and resilience.

4. Results and discussion

4.1. Choreography theatre

Figure 7 demonstrates how the ribs transfer loads from the oculus to the crown via bending, as shown by the presence of tensile principal stresses (von Mises stresses) that are minimum at the oculus and maximal at the connection to the crown. These tensile stresses increase under the wind load (Figure 7, right) and differential support settlement cases.

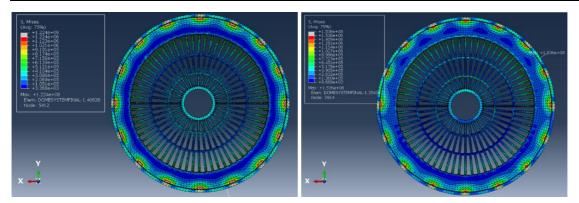


Figure 7. Von Mises stress results in Choreography Theater dome, bottom view. For self-weight (left) and wind load (right).

The primary tensile damage phenomena is cracking at the interface where the ribs terminate into the reinforced crown, which occurs under self-weight loading and increases under the successive demands of wind loading and differential support settlement. This correlates well to the location of efflorescence noted in the site investigations and indicates this efflorescence is due to stress-initiated microcracking.

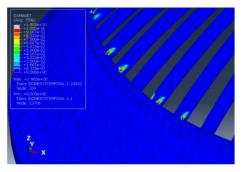


Figure 8. Tensile damage at connection of ribs to crown. Visible in all loading cases.

Under the differential settlement case, the lower ring beam sustained slight tensile damage, but this and the rib-base cracking were the extent of damage sustained and therefore showed consistency with the authentic structure. The global stability of the structure was maintained under all loading conditions.

4.2. Classroom domes

The plot of the dome principal stresses clearly indicates that the structure is not funicular. Rather, tensile stresses exist at the arch apexes and near the mid-height of the pendentives, as shown in Figure 9. The spherical dome cap functions under low-level bending stresses (not visible) primarily induced by the arched openings between the pendentives which the dome spans.

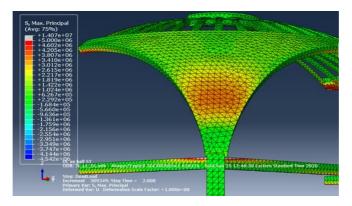


Figure 9. Maximum principal stress under self-weight (linear elastic model).

To validate the stress results, the principal stress plot, which indicates the tensile stresses due to bending, was compared with the real domes' cracking. Figure 10 shows how the majority of the recorded cracks occur within the high to moderate stress region on the pendentive extrados.

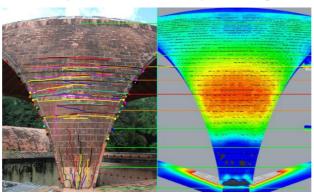


Figure 10. Comparison of cracking behavior of classroom pendentives with model principal stress.

Comparison of the damage results of the four concrete grades indicates that concrete strength is a significant factor in the strength and resilience of the structure. The 40 MPa-grade model fails completely under self-weight loading. The 43 to 50 MPa-grade models do not fail under self-weight, and the 50 MPa model is even able to sustain 700-year wind load levels with minimal damage. Consequently, a 4% increase in tensile strength separates failure from survival under self-weight loading, and a 12% increase in tensile strength separates failure under self-weight from resilience under hurricane-level wind loads.

For the 40 MPa-grade model, the tensile damage plots demonstrate agreement with the real structure damage and illuminate the failure progression and collapse mechanism of the structure. The failure progression is summarized below and shown in the following figures:

- 1) Tensile cracks occur at the arch apex and the tie beam-pillar connection.
- 2) The pendentive cracks horizontally in response to the significant bending demand. This crack initiates on the extrados and propagates immediately to the tensile reinforcing. Since the tensile reinforcing is located near the compression face (the intrados) of the pendentive, the crack soon also migrates across the whole cross-section.
- 3) In response to the continued development of the hinges and commensurate rotational displacements at those locations, compatibility or "flattening" cracks occur radially in the

pendentives. These initiate inside the pendentive or near its arched edge and propagate upward toward the tension ring.

4) The flattening cracks and plastic hinges continue to develop until the structure fails due to excessive deflection, excessive section loss, or total collapse.

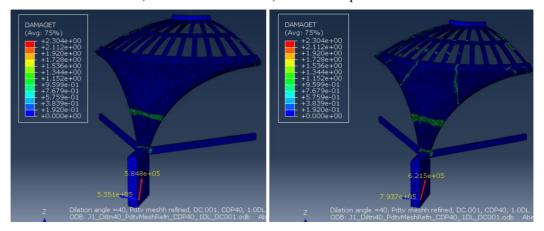


Figure 11. Tensile damage variable plot for stage 3 (left), showing initiation of pendentive hinge, and stage 4 (right), showing "flattening cracks".

If the dome is viewed as a series of two (or four) intersecting arches, then damage stages one and two comprise the initiation of five hinges within each arch: one at the base of each pendentive, one at the apex of the arch, and one at the mid-height of each pendentive. The final collapse mechanism of the dome can then be understood as the completion of the five-hinge mechanism in the arch.

To understand the extent of damage in the real structure, as well as validate the model against it, the location and extent of cracking is compared between the real structure comprehensive crack map and the model tensile damage plots.

These plots show significant agreement between the model and the structure. All five hinges are evidenced in the real structure, as well as "flattening cracks" within the arches. This indicates the dome is currently operating somewhere between mechanism initiation and completion: the question is how much further the hinges must develop before the plastic hinges complete and failure occurs.

One notable difference between the real structure and the model is the propagation of the pendentive tensile crack across the cross-section. While this occurs fairly quickly in the model, even before the flattening cracks initiate, it does not appear to have occurred in the real structure as no horizontal cracking is observed on the intrados of the pendentives. Possibly due to its under-reinforcing, or perhaps due to the FE model limitations (whether mesh size or damage model precision), the model quickly destabilizes and fails once the pendentive tensile damage has initiated. The plasticity of the real structure appears to exceed that of the model, particularly since it has stood in its present damaged condition for some years, which is encouraging for the structural prognosis.

However, due to the unknown residual capacity in the dome and its state of initiated mechanism formation, additional loading on the structure should be avoided. Also, deterioration of the structure via moisture incursion, rebar corrosion, and concrete spalling and section loss, must be remediated. Further degradation of critical cross-sections, particularly of the rebar in the cracked pendentives or arches, could compromise the dome without the addition of loads.

4. Conclusions

While the School of Ballet vaults exclusively comprise a reinforced concrete-tile hybrid section rather than the pure-tile vault traditionally attributed to it, Architect Vittorio Garatti was nevertheless able to

achieve magnificent forms both aesthetically or symbolically, and structurally. The Choreography Theatre dome has a span of 31 meters, and comprises ribs of about 38 centimeters thickness supporting a reinforced concrete-tile vault only 10 centimeters thick.

The Practical Classroom vaults are even more daring, placing a traditional spherical dome on an unheard-of support structure: four freestanding pendentives. This form would only be achievable through the hybrid system employed, and not through pure tile vaulting. While achieving a reasonably efficient span to thickness ratio for a bending-active structure of about 1:100, the structure's primary weakness lies in a simple oversight. The reinforcement is placed on what is the compression side of the pendentive in the highest-demand region (at the pendentive mid-height). This lack of tension-stiffening, both against creep and instantaneous strain, is likely the primary contributor to the current damaged condition of the structure.

Not only did the hybrid system allow for more expressive and daring forms by providing bending resistance, it also expedited the construction process. Rather than requiring complete formwork for the whole concrete dome, only the ribs required complete wooden formwork and support. For the pendentive shells and the spherical shells over the ribs, the tile vault layers functioned as stay-in-place formwork for the reinforced concrete. And the tiled layers could be constructed using reduced or no support or centering.

The diagnosis approach for the structures was informed by an holistic investigation campaign. Though funds and site access /equipment availability limited data team could gather, the numerical models demonstrated reasonable agreement with the real structure and may therefore illuminate the behavior, condition, and past actions of these structures.

The selected method of analysis – finite element methods using a concrete damaged plasticity mechanical model on a homogeneous solids geometric model with embedded reinforcing – was compared with the real condition of the structure and showed satisfactory agreement in both stress and damage results. Consequently, this FE application holds some promise for the assessment of other bending-active and reinforced shell structures constructed in the 20th century worldwide, such as those of Sanchez del Rio or Eladio Dieste.

4.1. Diagnosis of choreography theatre

For the Choreography theatre, the visual inspection and the model each indicated that the dome was well-designed for both self-weight and wind loads. No significant concerns, either in model damage or recorded real structural damage, exist. Therefore, interventions can be addressed to such issues as moisture incursion or tile spalling due to environmental agents and no structural interventions are urgently indicated.

4.2. Diagnosis of practical classrooms

For the Practical Classrooms, the visual inspection and FE model both indicated that the pendentives are overloaded and under excessive bending stress that, whether due to instantaneous effects or concrete creep, has caused significant concrete cracking in the pendentive extrados. Other areas of concern are the cracking, section loss, and potential rebar corrosion at the arch apex and within the arch. The continued deterioration of the dome, merely from environmental exposure of the crack cross-sections but also from increased loads, could have catastrophic implications. Therefore, structural interventions that will alleviate the over-loaded condition of the pendentives are recommended.

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