

# Flexible Pricing Strategies in Electric Free-Floating Bicycle Sharing

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This work was supported in part by Zehus SpA, and in part by the European Union under the Funding Scheme SME-2—SME Instrument Phase 2 under Grant 756656.


**ABSTRACT** Bike sharing is an important tool to reduce congestion and pollution in urban areas. Electrically Power Assisted Bicycles (EPAC's) make cycling possible also for sedentary people. Standard EPAC's are difficultly integrable into a free-floating sharing system because the battery pack requires frequent recharging. This paper studies the challenges, opportunities and solutions of implementing a free-floating bike sharing system based on electric bicycles. The analysis revolves around the charge sustaining paradigm. The idea of charge sustaining leverages the metabolic efficiency gaps to reduce the overall physical effort required without determining a net discharge of the battery. Already validated in private bicycles, the idea needs to be modified and adapted to the challenges of a shared fleet. The paper analyzes two approaches to the fleet level energy management and assistance control of a fleet of charge sustaining bicycles. Specifically, we compare a fixed price approach against a flexible pricing approach where the user can select the cost based on the pedaling effort they are willing to exercise. A simulation framework (calibrated on data collected during a large trial in Milan, Italy) assesses the operational costs and revenues of the two approaches quantifying how they depend on the design and environmental parameters. We provide and validate a lower bound in terms of usage rate that guarantees economic sustainability, additionally showing that a flexible pricing strategy can lower this bound and grant more degrees of freedom to the users.

**INDEX TERMS** Bicycles, electric vehicles, intelligent vehicles, public transportation, hybrid power systems.

## I. INTRODUCTION

The decarbonization process our society needs to undertake requires a deep change in how we understand mobility. The current mobility model, based on privately owned fossil fueled vehicles, is not sustainable. A possible approach is that of optimizing mobility adopting a systems engineering approach. This requires a multi-modal approach where users have access to different vehicles for different trips. Bicycles will play an important role in this scenario.

Bicycles are eco-friendly, efficient, have a small footprint and promote a healthy lifestyle. One could argue that bicycles are close to the ideal means of transportation in urban contexts. The main limitation of bicycles is that they require a certain level of physical prowess: many people may not be willing or capable to exert the required effort. Electrically Power Assisted Bicycles (EPAC) overcome this limitation.

The associate editor coordinating the review of this manuscript and approving it for publication was Chandan Kumar .

In EPAC's, an electric motor amplifies the rider's pedaling torque with the additional advantage of a transparent assistance: they are operated exactly as a normal bicycle without requiring additional training nor license. See, for example, [1]–[5] for more information on EPAC's.

A truly multi-modal mobility approach cannot rely on users privately owning all the vehicles they may need to operate. Shared mobility is thus posed to play a central role in the future. In the context of this paper, we are particularly interested in bike sharing (as discussed in [6]–[9]). Bike sharing systems are either station-based or free floating. In station-based services, the users pick up and return the bicycles at designated stations; in free-floating systems, users can pick up and release the bicycles anywhere within a region of operation. Free-floating services are more convenient for the users, but pose more challenges for the operators (see for example [10]–[12]). In this field, most research focuses on the analysis of the bikes spatial distribution in order to plan efficient rebalancing operations and maximize the usage

rate. Among the most recent works in this field, we mention [10], [13]–[17] for the analysis of usage patterns data of bike sharing services in different cities and [18]–[21] for studies on bike repositioning strategies. See for example [22], [23] for an exhaustive review of the issues that the literature considered up-to-now when designing and managing a bike sharing service.

Given the picture painted above, a free-floating EPAC's sharing system represents a great incentive for a greener and more efficient urban mobility. Recently, a number of companies (Uber, Jump and Lime to name three) launched free floating EPAC sharing systems. Operating these services is challenging: in addition to the complexities of managing a bicycle sharing system, the operators need to make sure that the batteries are always charged. Usually technicians, whose job is to maintain the fleet, drive around the city in vans. These interventions have a direct economic impact, and indirect effects on pollution and traffic. The benefits associated with reducing these interventions are thus multifaceted.

This paper investigates the challenges and opportunities of operating a free-floating EPAC's sharing systems and proposes to apply a charge sustaining paradigm to electric free-floating bike sharing. According to the charge sustaining principle, one can exploit the variability of the human pedaling efficiency and obtain a reduction of the cycling effort while preventing battery depletion. This concept was first applied to privately owned bicycles in [24], [25]. More recently, in [26], we redesigned the energy management system to account for bike sharing. In the latter work, we focused on the bicycle-level energy management. The present paper investigates the features of the complete fleet-level energy and assistance management system. We study two philosophies of fleet-level management: a fixed price approach that considers a fixed cost per bicycle rental and a flexible pricing approach. The flexible pricing approach, inspired by strategies developed for the energy market [27], lets the users decide the level of assistance they want and charge them accordingly. Flexible pricing introduces an additional control variable with which it is possible to influence the energy dynamics of the entire fleet.

We quantitatively assess and compare the two strategies using a complete model of the energy stored on board of each bicycle, and operation dynamics of the entire fleet. The main contributions of the paper can be summarized as:

- we propose a parametric model of the fleet and user dynamics. The model is calibrated on the data collected during a trial on an electric bike sharing system (under the name of *Bitride*) in Milan, Italy.
- We propose an economic model that accounts for the operational costs, profits and the user price elasticity. The goal of the model is to enable a comparative analysis of different fleet control strategies.
- We study the feasibility and potentials of free floating electric bike sharing comparing two fleet-level energy management systems and the corresponding pricing strategies.

To the best of the authors' knowledge, no work in the open scientific literature addresses the control of the state of charge dynamics of an entire fleet of shared bicycles and its associated costs. Some works [27], [28] explore the possibility of using flexible pricing to improve the availability of shared transportation systems with [29], [30] focusing on bicycle sharing systems, but the idea of reducing the charging costs by coordinating a vehicle level energy management system with a flexible pricing strategy is unexplored.

The paper is structured as follows: Section II describes the characteristic of the free-floating bike-sharing service and the *Bitride* trial used as the basis for the work. Section III proposes an energy-oriented model of the entire fleet. Section IV details the two fleet management strategies. Section V studies the quantitative results. Section VI draws the conclusions and discusses their applicability.

## II. SYSTEM DESCRIPTION

A bike sharing system is determined by the type of bicycle and the environment. This section describes both.

The reference bicycle (see Fig. 1) is a city bicycle equipped with an All-in-One (AiO) powertrain (see [24], [25]). The powertrain contains:

- a 250 W brushless motor,
- a 160 Wh Lithium Cobalt Oxide battery pack,
- an Inertial Measurement Unit (IMU),
- an Electric Control Unit (ECU),
- sensors for pedaling cadence and pedaling direction.



**FIGURE 1.** Some of the bicycles employed in the electric bike sharing pilot.

The AiO was primarily designed for privately owned bicycles. Sharing applications require some modifications; in particular the bicycle needs additional electronics for location tracking (mainly through a Global Navigation Satellite System), connection with the back-end servers that manage the rental process (through the cellular network) and to lock and unlock the bicycle. This leads to a continuous power drainage also when the bicycle is not being used.

The second aspect characterizing a bike sharing service is the environment where it operates with its users and geographic characteristics. We consider the *Bitride* trial. *Bitride* was a European Union supported pilot testing the feasibility

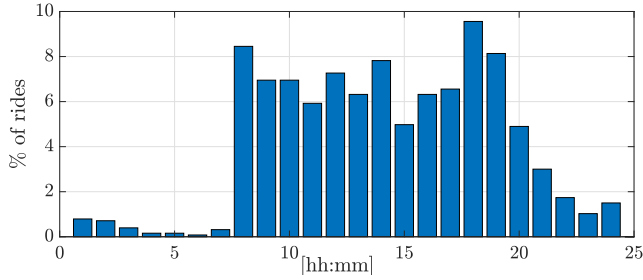


FIGURE 2. Hourly distribution of the number of rides.

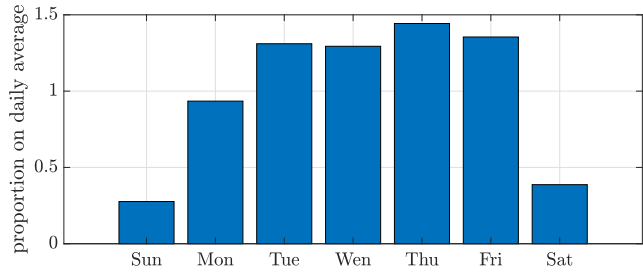


FIGURE 3. Weekly distribution of the number of rides.

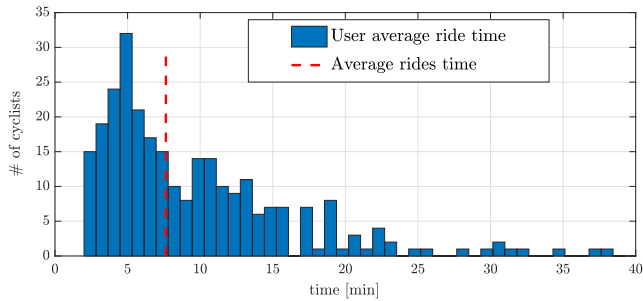


FIGURE 4. Cyclist average rides time distribution.

of an electric bike sharing system in Milan. The system deployed 300 bicycles and recruited a user basin of 280 riders. Over a three months period, there were 1300 rides. For each ride, we recorded the cyclist ID, the bicycle ID, the traveled distance, duration and time of day. For a limited number of trials also the velocity and slope profiles were recorded.

During this pilot, a maintenance team intervened whenever a battery pack was depleted. This intervention guaranteed that the bicycle availability was almost at 100% all the time. This type of intervention is expensive; limiting the cost of these interventions is the main goal of the charge sustaining paradigm when applied to bike sharing.

The data confirm that bicycle rentals are not uniformly distributed throughout the day and the week: rentals are more frequent during working hours and on business days. Figure 2 and 3 plot these distributions.

Each ride also reveals essential information on how the users employ this type of bicycle. Fig. 4 plots the distribution of rides duration. Most of the cyclists use the bicycle for very short rides, about 5 minutes. The average of all rides duration is about 8 minutes. This is a very different type of use with respect to personal bicycles that are usually employed for longer commutes.

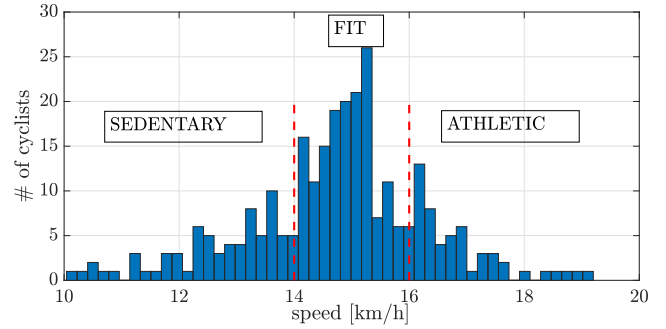


FIGURE 5. Average ride speed distribution and cyclist type classification.

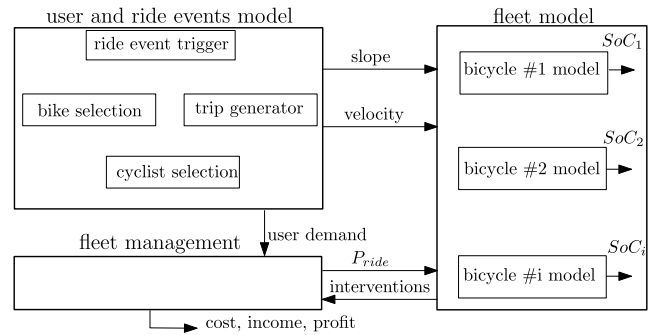


FIGURE 6. Bike sharing system model block diagram.

Fig. 5 plots the distribution of the ride average speed.

From the average velocity distribution, we classify riders as Sedentary (10 – 14 [km/h]), Fit (14 – 16 [km/h]) and Athletic (16 – 20 [km/h]).

### III. BIKE SHARING SYSTEM MODELING

In this section, we derive a model that describes the dynamics of the complete bicycle sharing system. Many aspects determine the behavior of the electric bicycle sharing system; in particular one has to consider the energy dynamics of each bicycle, the user renting behavior, the cost of each maintenance intervention and the price of the rentals. Fig. 6 schematically represents the complete system. In the following subsections, we describe the main elements separately.

#### A. FLEET MODEL

This component describes each bicycle state of charge (*SoC*) dynamics. Two factors determine the *SoC* dynamics: the bicycle dynamics and its energy and assistance management system.

##### 1) BICYCLE DYNAMICS

The power requirements of the bicycle assistance system derive from the longitudinal force balance (see [31]):

$$\begin{aligned}
 M\dot{v} &= F_g(\theta) + F_\mu(v) + (T_c + T_m + T_b)r \\
 F_g &= -Mg \sin(\theta) \\
 F_\mu &= -\frac{1}{2}\rho C_x A v^2 - D_v v - C_r \cos(\theta).
 \end{aligned}$$

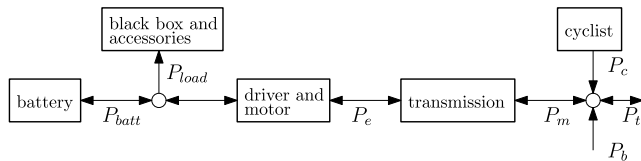


FIGURE 7. Schematic representation of the power flows during the ride.

where  $M$  is the total equivalent mass of the bicycle and of the cyclist,  $v$  is the longitudinal bicycle speed,  $\rho$  is the air density,  $A$  is the front area of the bicycle and of the cyclist,  $C_x$  is the drag coefficient,  $D_v$  is the mechanical friction coefficient,  $\theta$  is the road slope (positive uphill),  $r$  is the wheel rolling radius,  $C_r$  is the rolling friction coefficient,  $T_c$  is the traction torque delivered by the cyclist at the wheel (only positive),  $T_b$  is the braking torque delivered via mechanical brakes (only negative), and  $T_m$  the wheel torque generated by the motor (that can be both positive -if assistive; and negative - if regenerative).

Fig. 7 summarizes the power flows. The battery power,  $P_{batt}$ , is the sum of the power consumed by the loads  $P_{load}$  and the power absorbed (or generated) by the motor driver that controls the motor.  $P_e$  represents the electrical power of the motor, with  $P_m$  the mechanical power at the wheel.  $P_c$  is the power applied by the cyclist. The sum of  $P_m$ ,  $P_c$  and the friction braking power gives the resulting traction power,  $P_t$ , defined at the wheel.

The power absorbed by the auxiliary loads is  $P_{load} = 2.5$  [W] when the bicycle is used and 0.3 [W] when the bicycle is in stand-by; when the bicycle is standing-by, it goes in a power saving mode reducing the communication frequency with the servers.

We model the State of Charge (SoC) of the battery, *i.e.* the ratio between the residual energy and the battery capacity, with a Coulomb counting approach:

$$SoC(t) = SoC(0) - \frac{100}{3600} \frac{\int_{t_0}^t I_{batt}(\tau) d\tau}{Q_0}$$

where  $Q_0$  is the total battery capacity in Ampere hour and  $I_{batt}$  is the battery current.

The reader is referred to [26], [32] for a more detailed discussion and validation of this model. The model is complemented by a closed-loop velocity controller that makes it possible to simulate the power flow for a mission profile characterized by a velocity and slope profiles.

## 2) BICYCLE-LEVEL ENERGY AND ASSISTANCE MANAGEMENT SYSTEM

The bicycle-level energy and assistance management system (EMS) determines the instantaneous motor torque based on the cyclist's action and a requested average battery target power  $P_{ride}$ . It implements the logic detailed in [26], [32] as shown in Fig. 8.

The control logic implements a nested architecture. The maneuver assistance module recognizes different types of maneuvers and guarantees that assistance (and regeneration)

is active only when compatible with the condition (for example assistance is disabled when going downhill). The power control loop manages the level of assistance (and regeneration) to reach a desired battery power level; the outermost SoC control loop avoids slow drifts in the battery charge.

The EMS is capable of tracking a desired battery power while providing a repeatable and pleasant riding experience. The choice of the desired battery power determines the level of assistance the rider experiences. A positive  $P_{ride}$  yields a net assistance and can considerably reduce the physical effort required for a single trip. Previous works (see [25], [26], [32], [33]) show that it is possible to reduce the cycling effort even for negative values of  $P_{ride}$ , *i.e.* values that lead to a net recharging of the battery on the single trip. The control logic, similarly to what happens in electric - internal combustion engine hybrids, exploits differences in the pedalling efficiency. In particular, [25] shows that it is possible to achieve a  $P_{ride} = 0$  [W] and a reduction of up to 25% of equivalent expended metabolic energy (with respect to a traditional bicycle). More recently, [26] considers negative  $P_{ride}$  finding that, on average, the effort reduction effect is present up to  $P_{ride} = -22$  [W]. This means that the EMS can harvest up to 22 [W] without negatively affecting the overall riding experience with respect to a traditional bicycle.

At initialization, the state of charge of all bikes is set to 50%; this allows for negative  $P_{ride}$  also at fleet initialization.

## B. USER AND RIDE EVENT MODEL

We model the interaction with the users through the concept of ride events. A ride event corresponds to a user unlocking a bicycle, cycling to their destination and finally locking the bicycle. Each ride event is parametrized by the time it happens (ride trigger), the selected bicycle (bike selection), and the trip profile (trip generator). We model these aspects stochastically. The proposed model focuses mainly of the time dynamics, rather than on spatial dynamics. The model assumes that the bicycles are uniformly distributed in space. The assumption is valid for two reasons: 1) as it will become clear later on, the energy management issue is challenging when the demand is low with respect to the number of available bicycles. When this happens, the bicycle geographical scarcity is not going to play a mayor role: users will always find a bicycle nearby. 2) The data on which the model is calibrated were collected in a relative small geographic area where the uniformity assumption was always valid.

### 1) RIDE TRIGGER

The model determines the time of the day of a ride based on the usage distribution recorded during the *Bitride* pilot. We consider the probability of starting a ride event as function of the hour in the day and day in the week:  $f_{n, hour}(hour)$  is the probability density describing the distribution of number of rides for each hour of the day. Figure 9, for example, plots the rides distribution for the hour starting at 11 AM. The model also considers that the use of the system is not uniform throughout the week; we assign to each week day

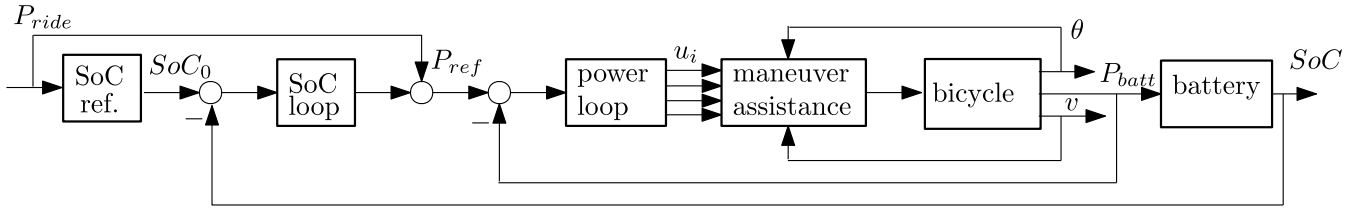


FIGURE 8. Schematic of the bicycle EMS.

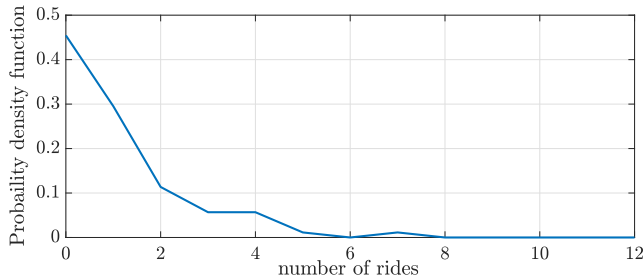


FIGURE 9. Hourly ride density function for 11 AM.

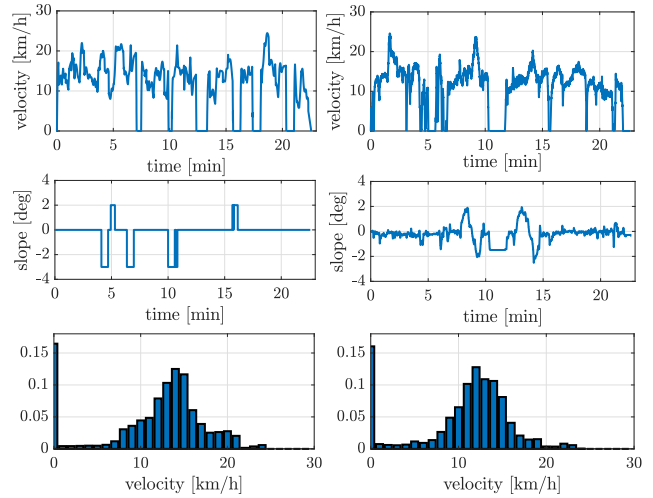


FIGURE 10. Comparison between a generated trip (on the left) and a recorded trip (on the right). From top to bottom: bicycle speed, slope and velocity distribution.

a weight  $w_{n,day}$ , which is the proportion of number of rides observed on the week day with respect to the average. The product of the hourly rides distribution  $f_n(hour)$  and the daily rides distribution  $w_{n,day}$  generates  $f_n(hour, day)$ , which is the density function of the number of rides given a week day and day hour.

We calibrated the nominal model on the data collected during the *Bitride* pilot, but we introduce the possibility of scaling the model to quantify how different rental frequencies affect the fleet dynamics. We introduce the parameter  $\nu$  that is the bike average daily usage (expressed in rentals per day per bicycle). The original data refers to a value  $\nu_0$ .

2) BIKE SELECTION

Each ride event is assigned a bicycle in the fleet according to a uniform distribution. Recall that each bike is characterized by its *SoC* at the beginning of the ride.

3) TRIP GENERATOR

The trip generator returns a speed  $v(t)$  and road slope profile  $\theta(t)$  starting from information on trip duration and a cyclist type.

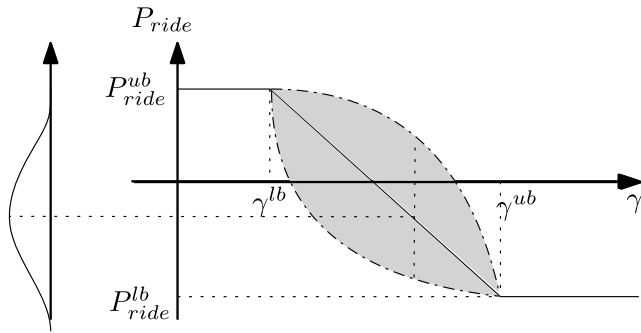
Any trip is a combination of 9 primitive maneuvers:  $\{hard\ start, soft\ start, constant\ speed, coasting\ down, stop, hard\ braking, soft\ braking, hard\ sprint, soft\ sprint\}$ . Each maneuver, with the exception of *start* and *coasting down*, is parametrized by a constant acceleration and a range of final speeds. The *coasting down* maneuver represents the bicycle slowing under its own friction; we parametrize the *start* maneuver as a time-varying acceleration profile that starts with a peak and returns to zero with a settling time of 4 seconds. The trip generation algorithm executes the following steps:

- 1) Sequence of primitives generation. The primitive maneuvers are the nodes of a Markov Chain. Each

transition from a maneuver to another has a probability of happening. This mechanism, tuned on recorded data, allows only feasible transitions (e.g. the transition from *stop* to a braking maneuver is meaningless) (see [34] for more information on mechanisms to tune this type of Markov Chains).

- 2) Duration of each maneuver choice. For the maneuvers with a non null acceleration, the algorithm samples the final speed from a uniform distribution and consequently determines the duration. For the maneuvers with null acceleration, the duration is directly sampled from a uniform distribution.
- 3) Sequence composition. The algorithm adds maneuvers until the trip time matches the randomly generated duration (see Fig. 4) is reached; at that point a final *soft braking* maneuver brings the bicycle to a full stop.
- 4) Road slope generation. The algorithm computes a road slope for each *constant speed* maneuver. The road slope is sampled from a distribution that matches the orography of the city.

The random distributions used in the trip generation are parametrized according to the type of user: (sedentary, fit and athletic). Fig. 10 shows the comparison between the velocity and the slope recorded during one of the trips of the trial and one of the trips generated by the trip generation algorithm. One can see that the two profiles have similar



**FIGURE 11.** User nominal demand model as function of  $\gamma$ . The gray area represents the randomization of the curve.

characteristics. The goal of the trip generation algorithm is not that of replicating a recorded profile, but rather that of generating realistic and ever varying trips.

The time evolution is discretized based on a time step  $\Delta T = 1 [h]$ . We assume that at each time step, each cyclist can start at most one ride. At each time step, the number of ride events is drawn from the probability density  $\bar{f}_n(hour, day)$  and multiplied by  $v/v_0$  to obtain the number of rides for that hour.

#### 4) USER DEMAND MODEL

In most bike sharing services, the users can only decide whether they want to rent a bicycle or not. The proposed EMS opens an additional degree of freedom: the assistance level  $P_{ride}$ . Assuming a rational user (or *homo economicus* hypothesis - see [35]), if given complete freedom, users would inevitably choose the highest level of assistance. The innate laziness of the *homo economicus* can however be counteracted by their willingness to minimize monetary expenses.

We model these characteristics with an elasticity to price variations, or in other words, how much the user is willing to pay for a given level of assistance. Our model is inspired by the theory of utilities flexible pricing as discussed in [36]–[38]. We assume that the rider is primarily sensitive to price variations; in other words, if given the possibility of choosing the power level, cyclist would be glad to exert a higher physical effort for a price discount. Let us parametrize the discount as  $\gamma$ , that is the price variation associated to a change in the level of assistance. For example,  $\gamma = 0$  means that changing the level of assistance does not entail any change of price. A  $\gamma = 0.1\text{€}/\text{W}$  means that the price of the ride varies of  $0.1\text{€}$  for every W of change of assistance that the user selects. We model the nominal relation between the desired assistance level and the discount with a piece-wise linear model as depicted in Fig. 11. From figure, the following comments are due:

- The lower the discount, the higher the level of assistance preferred by the users.
- Conversely, as  $\gamma$  increases, the users will be incentivized to reduce the level of assistance. If  $\gamma$  is sufficiently high, some users may choose a negative assistance level meaning that they will pedal to recharge the bicycle.

- The model has two parameters  $\gamma^{lb}$  and  $\gamma^{ub}$ . Each user is assigned a value  $\gamma^{lb} \in [0, 0.1]$  and  $\gamma^{ub} \in [0.15, 0.25]$ . In this way, the model considers different types of users with different sensitivities.

Despite the lack of specific studies on price elasticity in electric bike sharing systems, the saturation effect shown in figure has been measured in other applications, for example [39] considers utility pricing.

Each time a bicycle is rented, the model determines the preferred  $P_{ride}$  from a gaussian probability distribution centered on the nominal  $P_{ride}(\gamma)$  and whose standard deviation varies with  $\gamma$ , as shown by the gray area in Fig. 11. The randomization accounts for time variability in the user price elasticity. Note that, according to this model, with the right incentive, a user could accept to increase the pedaling effort beyond the  $-22\text{ W}$  battery power limit identified as the maximum value where a pedaling efficiency benefit is present.

In modeling the user demand, we assume that the users only change their preferred level of assistance without changing the decision of renting the bicycle or not. This assumption is valid only if the final price is close to the prices that are currently practiced. To put the pricing results into perspective, it is convenient to consider the cost of similar services at the time of writing. In Milan, the rental price of station based electric bicycles ranges from  $0.25\text{€}$  to  $4\text{€}$  (based on duration, maximum 2 hours) on top of the yearly subscription fee of  $36\text{€}$ . The free floating electric bike sharing managed by Uber charges  $1\text{€}$  per rental plus  $0.23\text{€}$  per minute, to about  $4\text{€}$  per rental. In [14], the authors considered a free floating service with prices around  $5\text{€}$  for a 20 minutes ride.

#### C. FLEET MANAGEMENT

The fleet management component models the fleet battery charge control strategy along with pricing strategies and economic aspects.

##### 1) PRICING STRATEGY

The main task of the pricing strategy is that of determining  $P_{ride}$  and the price for each ride. These values can be fixed a priori according to some considerations or could be the result of the interaction between the strategy and the users. The control and pricing strategies will be detailed in the next section.

##### 2) ECONOMIC MODEL

The economic aspects of the service are determined by revenues ( $R$ ) and costs ( $C$ ), whose difference yields the net profit ( $P$ ) and thus eventually the economic sustainability.

There are two types of service costs: fixed costs, which are related to the initial investments and service logistics; and, variable costs, which are mainly due to fleet maintenance. Our analysis focuses on the variable costs as these are the ones that are directly affected by the energy and assistance management system and the pricing strategy. In particular, the costs that we are mainly interested are the recharge interventions. All other costs, while extremely important and

impactful are not a direct consequence of the fleet electrification which is our main scope.

A bicycle requires an intervention each time its state of charge drops below a critical threshold. Every time this happens, the bicycle is removed from the pool of the available bicycles and returned re-initialized after two hours. We assume each intervention costs  $C_{IN} = 80 \text{ €}$  (mainly due to wages and amortization of the vans employed to transport the bicycles).

There exist many revenue streams associated to bike sharing: advertisement space rental, subsidies, subscription fees, and single trip cost. In our model, we neglect advertisement space rental and subsidies because we are interested in studying the factors that are directly affected by the fleet electrification. We thus consider that each single trip generates a revenue based on the pricing strategies described later.

The combination of the above mentioned elements yields a model of the complete fleet.

#### IV. PRICING AND FLEET CONTROL STRATEGIES

The economic sustainability of the electric bike sharing scheme depends on the balance between costs and revenues. If seen as a control problem, one has two control variables:  $P_{ride}$  and the ride cost  $\epsilon$ . In this section, we propose two control and pricing strategies: a fixed approach, and a flexible price approach that tries to leverages the users' elasticity.

##### A. FIXED PRICE STRATEGY

In this approach,  $P_{ride}$  is determined only based on energy considerations and does not allow for any degree of freedom. In this context, the only available control variable is  $P_{ride}$  and it is used to minimize the recharging interventions.

The most trivial approach to guarantee charge sustaining is to require that each bicycle harvests, during the rental, the power consumed since the previous rental. Such a bike-level approach would transform the rental process in a lottery: since the battery pack is depleted also when the bicycle is not being used, bicycles that have just been used would yield a better cycling experience than bicycles that have been standing-by for a long period of time. Service level unpredictability is generally not well received by users and must be avoided.

The proposed fixed price strategy overcomes this problem by implementing a model based control of the average state of charge of the entire fleet. In this way, it offers the cyclists the same ride experience independently from the bike current state of charge, bike history, and ride duration.

The strategy imposes the same average battery power  $P_{ride}$  to all bicycles and rides.  $P_{ride}$  is derived from the model of the average battery of the fleet:

$$\begin{aligned} \overline{SoC}(t + \Delta T) &= \overline{SoC}(t) + \Delta \overline{SoC}(t), \\ \Delta \overline{SoC}(t) &= \frac{\sum_{i=1}^{N_{bike}} \int_t^{t+\Delta T} \frac{-100}{Q} [P_{inact}(1 - \sigma_i(t)) + P_{ride}\sigma_i(t)] dt}{N_{bike}} \end{aligned} \quad (1)$$

$\overline{SoC}(t)$  [%] is the average among bikes of the battery charge at instant  $t$  [h],  $\Delta \overline{SoC}(t)$  is the average among bikes of the variations of state of charge with respect to the initial condition at time  $t$  [h],  $Q$  [Wh] is the total battery capacity,  $\sigma_i$  is the activation variable indicating whether bike  $i$  is being rented at time  $t$  and  $P_{inact}$  [W] is the power dispersion during stand-by.

Average charge maintaining is equivalent to imposing  $\Delta \overline{SoC}(t) = 0$ , that is

$$P_{ride}^0 = - \frac{P_{inact} \left( \frac{1}{N_{bike}} \sum_{i=1}^{N_{bike}} \int_0^t (1 - \sigma_i(t)) dt \right)}{\left( \frac{1}{N_{bike}} \sum_{i=1}^{N_{bike}} \int_0^t \sigma_i(t) dt \right)} \text{ [W]}. \quad (2)$$

Equation (2) can also be formulated as function of the daily rides per bike  $\nu$  and average ride duration  $\bar{t}_{ride}$  [h]:

$$P_{ride}^0(\nu) = - \frac{P_{inact} \left( 1 - \frac{\nu}{24} \bar{t}_{ride} \right)}{\frac{\nu}{24} \bar{t}_{ride}}. \quad (3)$$

From the formulation of  $P_{ride}^0(\nu)$  in (3), we observe that

- $P_{ride}^0$  linearly depends on  $P_{inact}$ .
- $P_{ride}^0$  hyperbolically depends on  $\nu$  and on  $t_{ride}$  with a diminishing return as the parameter grows and with a very rapid growth of the required power as the parameter decreases.
- The approach focuses on the average fleet SoC, not the battery levels of each bike. The proposed solution gives priority to the homogeneity of the service at the expense of a higher risk of having bicycles that require maintenance.

In the fixed price strategy, the price is constant and equal to  $\epsilon_C$ .

##### B. FLEXIBLE PRICE STRATEGY

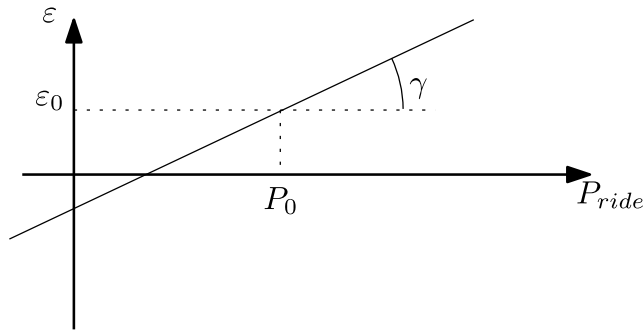
The flexible price strategy attempts to leverage the users price sensitivity to reduce maintenance costs. As noted in Section III, users may be incentivized to reduce the assistance level if offered a high enough discount on the rental price.

The flexible price strategy thus employs both available control variables offering the freedom (at a price) to each user to choose the level of assistance they prefer. This idea results in the proposed flexible pricing strategy

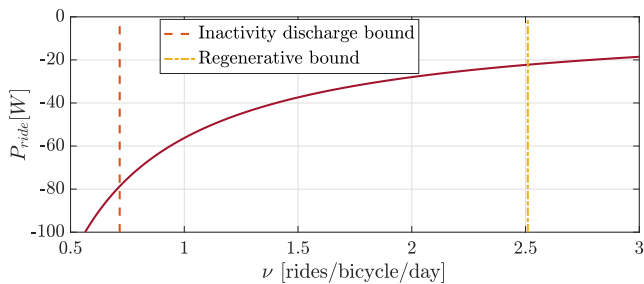
$$\epsilon = \epsilon_0 + \gamma(P_{ride} - P_0) \quad (4)$$

where  $\gamma$  is the discount rate from the nominal case,  $\epsilon_0$  is the nominal price and  $P_0$  the nominal power. Fig. 12 graphically represents the idea. From figure, a few comments are due:

- The users can choose the assistance level that they are most comfortable with. The strategy will determine the price.
- The strategy is not saturated to positive values, this effectively opens the possibility of paying the users to recharge a bicycle.
- Given the consideration on the limitation and validity of the user demand model, we are mostly interested in studying the impact of  $\gamma$ . Generally speaking, increasing



**FIGURE 12. Flexible pricing strategy: the price is determined based on the user defined assistance level.**



**FIGURE 13. Power goal as function of the average daily rents per bike.**

$\gamma$  increases the reward for recharging the bicycle (or, from another point of view, the penalty for more assistance). We will need to fix values of  $\varepsilon_0$  and  $P_0$  in order to compare the two strategies.

- We opted for a linear pricing strategy as a proof-of-concept. More complex strategies are also possible. The only requirement is that curves are monotonically increasing.

## V. FLEET DYNAMIC ANALYSIS

This section analyzes the dynamics of the bike sharing service; we first consider the two strategies separately and then we compare their features.

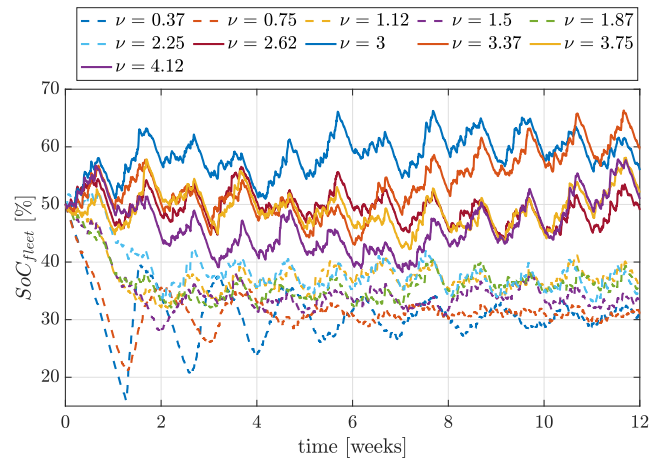
### A. FIXED PRICE STRATEGY ANALYSIS

As described above, the fixed price strategy computes  $P_{ride}$  according to (3). Fig. 13 plots the target power as a function of the pick up rate for the characteristics of the bicycle used in the trial.

The figure seems to indicate that charge sustaining at the fleet level is always achievable; in reality the sustainability of the fixed price strategy is determined by two additional factors: a) without a discount, users will only accept to regenerate so much power and b) no matter how much energy the cyclists regenerate at each trip, if a bicycle is not rented for a long period of time, its battery will be depleted. These factors can be analytically quantified.

The first limit derives from plugging the bound  $P_{ride}^{lb} = -22 [W]$  in (3):

$$v_{lb}^{reg} = \frac{-24}{\bar{t}_{ride}} \frac{1}{1 + P_{ride}^{lb}/P_{inact}} [\text{rides/bicycle/day}]. \quad (5)$$



**FIGURE 14. Average state of charge of the fleet over time as a function of the usage rate. Continuous lines correspond  $\nu > v_{lb}$ , dashed lines to  $\nu < v_{lb}$ .**

For usage rate below  $v_{lb}^{reg}$ , it is impossible to guarantee average charge sustaining because the effort required of the cyclists is too high.

The second limit accounts for the discharge rate of a bicycle in stand-by. If we define  $T_{steady}$  as the time required for the battery to discharge from the initial charge to the intervention level  $SoC_{min}$ , when the bike is left inactive,

$$T_{steady} = \frac{SoC_{init} - SoC_{min}}{P_{inact}} [h], \quad (6)$$

the average required daily usage rate per bike  $v_{lb}^{inact}$  necessary to guarantee with confidence level  $\bar{p}$  that each bike is used at least once before intervention is:

$$v_{lb}^{inact} = 24 \left( 1 - \left( \frac{1 - \bar{p}/100}{N_{bike}} \right)^{\frac{1}{T_{steady}}} \right) [\text{rides/bicycle/day}].$$

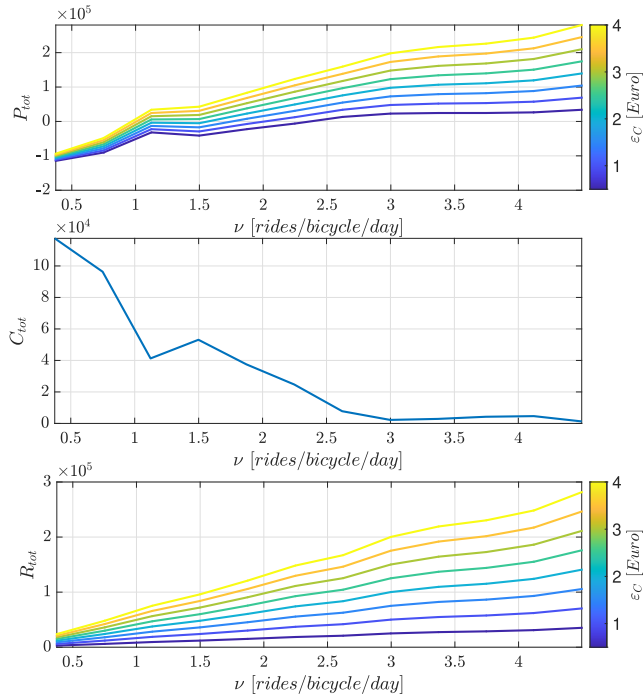
Fig. 13 also plots the regeneration limit and the inactivity bound (with a confidence of level 70%). For the fleet at hand, the regenerative bound  $v_{lb} = v_{lb}^{reg} = 2.5 [\text{rides/bicycle/day}]$  is the limiting factor.

The model described in Section III allows us to deepen the analysis of the fleet dynamics considering also transient phenomena and economic aspects.

Figure 14 plots the behavior of the average  $SoC$  for different values of  $\nu$  over a 12-weeks period. From figure, one can conclude that

- When  $\nu > v_{lb}$ , as predicted by the analysis, the average fleet state of charge is maintained around the initialization one.
- When  $\nu < v_{lb}$ , the average battery level (dashed lines) quickly drops below the initial value and converges to a lower value. The lower equilibrium  $SoC$  is maintained due to the recharging interventions.
- The average  $SoC$  occasionally exhibits a slow drift; this is due to the fact that the control logic operates in open loop with respect to the average fleet State of Charge.





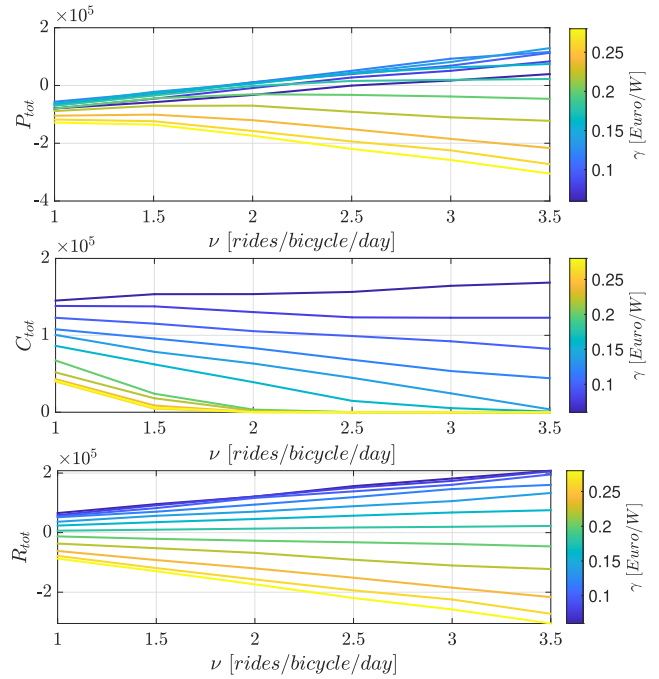
**FIGURE 15.** Sensitivity of the economic states (profit  $P_{tot}$ , costs,  $C_{tot}$ , and revenues  $R_{tot}$ ) to the usage rate  $\nu$ , evaluated at the end of a 12 weeks simulation of the fleet with CSS for different value of  $\epsilon_C$ .

Fig. 15 plots the economic states at the end of the evaluation period. The figure considers different usage rates and fixed costs ranging from 0.75€ to 4€ a range where the demand model is considered reliable, based on the comparison outlined in Section III-B4. From these results, we propose the following considerations:

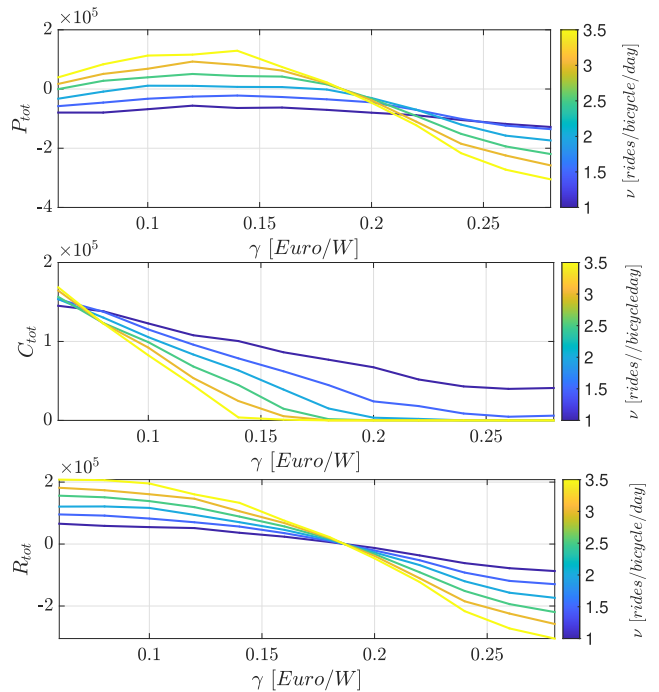
- In the fixed price strategy, revenues are directly proportional to  $\nu$  and  $\epsilon_C$ .
- Maintenance costs are independent from  $\epsilon_C$ . They decrease as  $\nu$  increases and converge for  $\nu > \nu_{lb}$  to a minimum value of 50 interventions every 3 months (recall that the fleet is composed of 300 bicycles). Reaching a steady-state average state of charge (as shown in Fig. 14) does not guarantee the absence of interventions. Occasionally bicycles may undergo battery depletion because of particularly challenging rental patterns, as also noted in [19].
- When usage rate is low the total profit suffers from both a high interventions cost and a low profit, which results in a net loss, while for  $\nu > \nu_{lb}$  the income grows linearly on  $\nu$ .
- Depending on the ride price, the compensation of the maintenance costs occurs at  $\nu$  between 1 [ride/bicycle/day] and 2.5 [ride/bicycle/day].

**B. FLEXIBLE PRICE STRATEGY**

The flexible price strategy offers the option to chose the preferred assistance level. Figure 16 plots the economic state variables at the end of the 12-week period as a function of  $\nu$  for different values of the discount rate ( $\gamma$ ). Fig. 17 completes



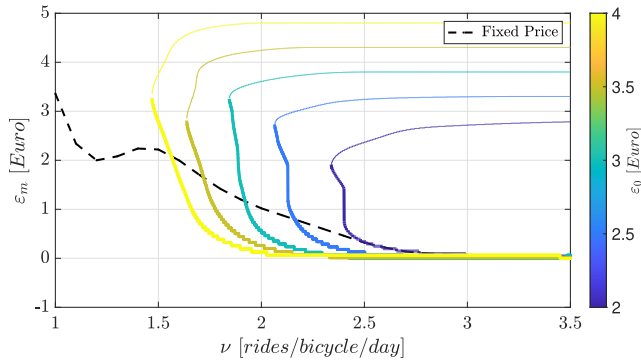
**FIGURE 16.** Economic states sensitivity to  $\nu$  for different values of  $\gamma$ . The simulation considers  $\epsilon_0 = 3 \text{ €}$  and  $P_0 = -10 \text{ W}$ .



**FIGURE 17.** Economic states sensitivity to  $\gamma$  for different values of  $\nu$ . The simulation considers  $\epsilon_0 = 3 \text{ €}$  and  $P_0 = -10 \text{ W}$ .

the analysis from the dual perspective, by showing the states as a function of the discount rate for several values of usage rate. from figure, we observe that:

- The revenues depend linearly on  $\nu$ . If  $\gamma$  is above 0.17 [€/W], the slope is negative. In these cases, the discount rate is so high that users will, on average,



**FIGURE 18.** Break even curves as a function of usage rate  $\nu$  and average ride cost  $\epsilon_m$ .

choose a high level of regeneration in order to benefit from the discount.

- The flexible price strategy allows the service managers to also influence the maintenance costs. The costs depend linearly on  $\nu$  (until saturation at zero); and  $\gamma$  has a quite strong effect on the cost also for  $\gamma$  below 0.17 [€/W].
- The profit linearly depends on the usage rate. The dependency of the profit on the discount rate is complex. The profit has an optimum for intermediate discount rates and the optimal discount rate weakly depends on the current usage rate. If too high a discount rate is used, the profit will decrease: in this case the revenues are dropping faster than the reduction in the costs.
- When employing a flexible price scheme, as the discount rate decreases, the fleet behaves more and more as a non-charge sustaining fleet.
- Since the economic model decouples the pick up frequency dynamics and the price, the average ride price  $\epsilon_m$  monotonically depends on  $\gamma$ , as also shown by dividing  $R_{tot}$  by the number of rentals in Fig. 17.

### C. STRATEGIES COMPARISON

The two strategies depend on design parameters as well as environmental characteristics. The easiest way to compare the two is by looking at the break even point<sup>1</sup> and its dependency on several parameters. The break even point is a proxy of the conditions in which the service is sustainable from the economic point of view.

Fig. 18 plots the break even curves as a function of the usage rate and the average ride price for different values of  $\epsilon_m$  (varying in a range in which the demand model is considered reliable). Each point on a continuous curve for the flexible price case corresponds to a different  $\gamma$ . Recall that the fixed price strategy has only one design parameter, that is the cost of the single ride. The flexible price strategy has two parameters that define the price curve:  $(\gamma, \epsilon_0)$ . The following conclusions are in order:

<sup>1</sup>Here break even is employed loosely as we are considering only one source of revenue and one type of cost.

- The most impactful variable in determining the service sustainability is the usage rate. Unsurprisingly, the higher the usage rate, the easier is to reach break even.
- In the flexible pricing case, there exist two break even values of  $\gamma$  for each values of  $\nu$ . The two points correspond to two different philosophies. The  $\gamma$  that yields a lower average ride price incentivizes more the cyclists to recharge the bicycles. The other break even point exploits the fact that cyclists are willing to pay more to have more assistance. The extra revenue is used to compensate the additional costs of recharging. In this latter case, the system forfeits the charge sustaining idea and is more similar to a traditional electric bike sharing. The maximum profit lays in between these two extreme cases.
- There exists a threshold (1.6 [rides/bicycle/day] for  $\epsilon_0 = 4\text{€}$ ) above which it is possible to reach break even with a lower average ride cost than that of the fixed price strategy; these break even points correspond to the philosophy where the fleet partially relies on the cyclists to recharge the batteries.
- By exploiting the lower half of the break even curve of the flexible price case, an increase of  $\epsilon_0$  leads to a decrease of the average price to reach sustainability.
- As  $\nu$  increases, all solutions tend to reach break even with an average ride of price tending to 0. If  $\nu$  is sufficiently high the charge sustaining strategy guarantees low maintenance costs.

## VI. DISCUSSION AND CONCLUSION

This paper extends the idea of charge sustaining electric bicycles to free-floating electric bike sharing. We propose two different pricing models that serve as fleet level management algorithms: the fixed price strategy aims at the minimization of the maintenance interventions on the fleet of bicycles; the flexible pricing strategy incentivizes the user to recharge the bicycle in order to reduce the maintenance cost.

We characterize the dynamic and economic properties through analytical analysis and simulations. The simulations are based on a stochastic model calibrated on data collected during the pilot phase of *Bitride*. The model accounts for different types of users and mirrors the actual rental dynamics recorded during the pilot.

This work proposes a methodological framework to study the feasibility of electric free floating bike sharing. Furthermore, we quantitatively study an instance of a charge sustaining system. These results to be interpreted in light of the main features of the proposed analysis framework:

- the model, being calibrated on real data, accurately describes and predicts the State of Charge dynamics of the fleet. This allows us to draw strong conclusions on the fixed price strategy. In particular, our framework precisely quantifies the impact of the usage rate on maintenance. In this sense, we show that a usage rate of 2.5 [rides/bicycle/day] is the minimum value for

which genuine charge sustaining and assistance can be provided.

- For values of  $\nu$  lower than 2.5 [rides/bicycle/day], users contribution is needed to cover the operational costs. This contribution takes the form of higher service cost (fixed price case and upper break even for flexible pricing) or lower assistance at a lower cost (lower break even point of the flexible price case).
- The economic model is a differential model with respect to a traditional bike sharing service; in fact, it accounts only for one stream of revenue (rentals) and one cost (maintenance). This limits the capability of the model to predict the actual profit of the service. It however provides a useful tool to preliminarily and quantitatively assess the sustainability of transitioning to an electric bike sharing system. The analysis of the model shows that, based on the usage rate, either the fixed or flexible price models yield the lowest average price to the public. This can also be interpreted from the accessibility point of view. Once a minimum level of usage rate is reached, the flexible pricing strategy yields a more accessible service. Some users will be able to pay more for a better service, effectively helping subsidizing the service for other users who, by charging the bicycles, can still access the service at a lower cost.
- Our framework assumes that the price will not affect the demand (but only the service quality); this is a strong assumption which prevents the extrapolation of our results beyond the price range that is currently being employed in bike sharing services. One could safely expect that by increasing the price the usage rate will also be affected, but no data on this are currently available.
- Our analysis approach is general, but for the sake of concreteness, it has been applied to a specific case. The numerical results depend on the specific characteristics of the sharing and bicycles at hand. In particular, our analysis shows that the stand-by consumption is the most important characteristic of the bicycle. By reducing the stand-by consumption, one can skew the results toward a lower usage rate limit.

This research, while proving the advantages of the charge-sustaining paradigm when applied to a bike sharing scenario, opens up a number of interesting questions mainly regarding the validation of the hypotheses at the basis of the user demand model and the design of possible business models that would leverage these features.

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