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A Youla-Kucera Parametrization for Data-Driven Controllers Tuning ⋆

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Abstract: The Youla-Kucera parametrization is a fundamental result in system theory, very useful when designing model-based controllers. In this paper, this parametrization is employed to solve the controller design from data problem, without requiring a process model. It is shown that employing the proposed controller structure it is possible to achieve more stringent closed-loop performances than previous works in literature, maintaining a criterion to estimate the closed-loop stability. The developed design methodology does not imply a plant identification step and the solution can be obtained by least-squares algorithms in the case of stochastic additive noise. The designed solution is evaluated through Monte Carlo simulations for the regulation problem of an under-damped system.

Keywords: Data-based control, Identification for control, Uncertain systems, Parametric optimization.

1. INTRODUCTION

Most controller design procedures are model-based, where a plant model is derived from data and plant-knowledge and then employed to design a controller. The resulting controller is not necessarily optimal because the control loop performance is restricted by modelling errors. On the other hand, the available data can be employed to directly design a controller, avoiding the plant model estimation. This approach has been named Direct Data-Driven controllers (DDC) tuning. Some approaches to solve the DDC problem are correlation approach (CbT), periodic errors in variables (EiV), inverse controller (IV) and prediction error methods (PEM). A review of the existing methods can be found in Hou and Wang [2013]. A different approach to solve the DDC problem, following a deterministic formulation using Set-membership techniques, has been presented inValderrama and Ruiz [2014], Cerone et al. [2017].

The main ingredients of a DDC problem, specifically, in a model-reference control problem, are a set of input-output data generated by the plant to be controlled, a closed-loop reference model where performance specifications are embedded, and a given controller structure, usually parametrized by a fixed set of basis functions. When the set of bases is not consistent with the reference model, the resulting controller can yield to closed-loop instability.

One of the main challenges in DDC methods is guaranteeing stability. Considering that no plant model is available, standard stability test cannot be performed. A possibility is to test the controller before actual implementation, van Heusden et al. [2009]. In Kammer et al. [2000] some a-posteriori stability tests are proposed for an iterative DDC tuning scheme. In Sala and Esparza [2005] an invalidation test step, based on the available data, is employed for a non-iterative DDC scheme, in order to detect if the controller may led to unstable closed-loops. This test requires the accurate identification of a possibly unstable system in an errors-in-variables framework.

Some attempts to incorporate a stability constraint at the design step in non-iterative DDC can be found in Lanzon et al. [2006] and van Heusden et al. [2011]. Both methods consider an extended PID controller structure leading to convex optimization problems. However, such methods do not offer acceptable performances when the desired reference model is not achievable employing the selected controller structure. In Battistelli et al. [2018], the unfalsified control theory is employed to derive relations between the choice of the performance criterion to be optimized and the closed-loop stability conditions. However, the controller is non-linearly parametrized, leading to non-convex optimization problems.

The Youla-Kucera (Y-K) parametrization is a fundamental result in system theory that allows to parametrize all the controllers that stabilize a given plant. It has been extensively applied in optimal and robust control when designing model-based controllers, see e.g. Doyle et al. [1991]. However, in its original form has not been applied in the model-reference control problem when the plant model is no available. The Y-K parametrization has been employed previously in [Formentin and Karimi, 2013] to

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solve a mixed-sensitivity controller design from data problem. To the author's knowledge this parametrization has not been employed in a model reference setting.

In this paper, the Youla-Kucera parametrization is employed to solve the model-reference control problem, without requiring a process model. The proposed controller structure allows to reach more stringent reference models than those proposed previously in the literature, maintaining a convex optimization problem to tune the controller parameters. The approach is a non-iterative solution that exploits the CbT formulation. Thus, the controller tuning procedure does not require iterations or multiple experiments.

The outline of the paper is as follows. In Section 2, the problem formulation is presented. In Section 3, a stabilizing controller structure is comprehensively formulated. In section 4, a tuning scheme inspired by the CbT approach is developed, employing the proposed structure. Finally, in Section V a numerical example is developed. The conclusions end the paper in Section 6.

2. STATEMENT OF THE PROBLEM

In this section the data-driven controller (DDC) tuning problem is formulated. Firstly, the setting and main assumptions and presented.

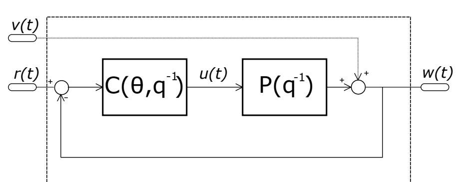

Fig. 1. Assumed feedback control structure

Consider a discrete-time linear-time invariant (LTI) single-input single-output (SISO) feedback control scheme, as depicted in Fig. 1, where q^{-1} denotes the backward shift operator, $P(q^{-1})$ is a stable plant transfer function, $C(\theta, q^{-1})$ is the controller transfer function, θ is a vector of controller parameters, $r(t)$ is the reference signal, $v(t)$ is output noise/disturbance, $u(t)$ and $w(t)$ are the plant input and output signals, respectively.

For the system interconnection in Fig. 1, the aim of the controller tuning procedure is to select an optimal controller $C^o(\theta^o)$ minimizing some performance criterion and guaranteeing internal stability. For example, an optimization problem can be stated as:

$$\begin{aligned} C^o(\theta, q^{-1}) \;&=\; \text{argmin} J(\theta) \\ &s.t. \\ &\text{Loop internally stable} \end{aligned} \tag{1}$$

For the cost function

$$J_{MR}(\theta) = \left\| M(q^{-1}) - \frac{P(q^{-1})C(\theta, q^{-1})}{1 + P(q^{-1})C(\theta, q^{-1})} \right\|_2^2 \tag{2}$$

Being $M(q^{-1})$ a strictly proper reference model for the closed-loop system (i.e. $M \neq 1$), where performance specifications are embedded. If system $P(q^{-1})$ is unknown, Problem (1) can not be solved directly.

The following assumption defines the framework of the data-driven stabilizing controller tuning problem.

Assumption 1. $P(q^{-1})$ is unknown. The available information on $P(q^{-1})$ is a set of input-output data generated by $P(q^{-1})$, initially at rest,

$$\mathcal{D} = \{w(t), u(t), t = 1, 2, ..., N\} \tag{3}$$

Where

$$w(t) = y(t) + v(t) = \sum_{j=0}^{t} h_j u(t-j) + v(t),$$

h_j are the impulse response coefficients of $P(q^{-1})$, $y(t) = \sum_{j=0}^{t} h_j u(t-j)$ is the noise-free plant output and $v(t)$ is the output noise/disturbance.

Considering the previous assumption, a data-driven stabilizing controllers tuning problem can be stated as follows:

Problem 1. Data-Driven Stabilizing Controller Tuning: Given a dataset $\mathcal{D}$ generated as in *Assumption* 1 and a reference model $M(q^{-1})$. Find a controller $\hat{C}(\theta)$ that solves (1).

3. A STABILIZING CONTROLLER STRUCTURE

Let us recall that the set of all the stabilizing controllers $C(\theta, q^{-1})$ for the loop in Fig. 1, given a stable plant $P(q^{-1})$ can be expressed as

$$\mathcal{C}^{sta} = \left\{ C(\theta, q^{-1}) = \frac{Q(\theta, q^{-1})}{1 - P(q^{-1})Q(\theta, q^{-1})} : Q(q^{-1}) \in \mathcal{H}_\infty \right\} \tag{4}$$

where $Q(\theta, q^{-1})$ is any stable and proper transfer function. The previous result is known as the Youla-Kucera parametrization for a stable plant, Doyle et al. [1991].

When the *Youla-Kucera* parametrization is adopted to find an optimal controller solving (1), the cost function (2) can be rewritten as

$$J_{MR}(\theta) = J_Q(\theta) = \left\| M(q^{-1}) - Q(\theta, q^{-1})P(q^{-1}) \right\|_2^2 \tag{5}$$

That is, the complementary sensitivity function of the loop becomes $Q(\theta, q^{-1})P(q^{-1})$.

Assumption 2. For the given closed-loop reference model $M(q^{-1})$, there exist an optimal filter $Q^*(\theta^*, q^{-1})$ such that,

$$M(q^{-1}) = Q^*(\theta^*, q^{-1})P(q^{-1}) \tag{6}$$

Remark 1. From the previous assumption, the optimal controller $C^*(\theta^*, q^{-1})$, which solves Problem 1 is,

$$C^*(\theta^*, q^{-1}) = Q^*(\theta^*, q^{-1})(1 - M(q^{-1}))^{-1} \tag{7}$$

It is worth noting that only the term $Q^*(\theta^*, q^{-1})$ is unknown, since $M(q^{-1})$ is proposed by the user.

Given the previous analysis, from now on we focus in the problem to estimate $Q^*(\theta^*, q^{-1})$, such that the cost function (5) is minimized.

3.1 A structure for Q.

Several structures can be assumed to design the filter $Q(\theta, q^{-1})$. For example, recursive polynomial structures such as ARX, ARMAX or OE, can be employed. The only requirement is that $Q(\theta, q^{-1}) \in \mathcal{H}_\infty$. However, imposing stability constraints in autoregressive structures, such as AR or ARMAX, leads to complex non-linear constraints, turning the tuning problem into a highly non-convex optimization program, see e.g. Ljung and Chen [2013]. On the other hand, Finite Impulse Response (FIR) models guarantee stability without additional constraints. Therefore, a FIR structure is adopted for Q as follows,

$$Q(\theta, q^{-1}) = \sum_{i=1}^{m_q} \theta_i q^{-(i-1)}, \tag{8}$$

where m_q is the filter impulse response length.

Then, the controller design problem becomes a parametric estimation problem, where the filter parameters are selected from the set:

$$\mathcal{Q} = \left\{Q(\theta, q^{-1}) : \theta \in \Theta \subseteq \mathcal{R}^{m_q}\right\}$$

3.2 The Q filter in terms of data.

Notice that to estimate a filter $\hat{Q}(\theta, q^{-1})$ minimizing (5) it is required the knowledge of the plant $P(q^{-1})$. But, under the assumptions of the framework, the plant is unknown. The following Lemma allows to relate the model-based cost function with a data-based error signal.

Lemma 1. Given an asymptotically stable system $P(q^{-1})$ and a data set $\mathcal{D}$ generated as in Assumption 1, any stable filter $Q(\theta, q^{-1}) \in \mathcal{Q}$ satisfies the time-domain relation:

$$e(\theta, t) = Mu(t) - Q(\theta)(w(t) - v(t)) \tag{9}$$

where $e(\theta, t)$ is the output of the model matching error transfer function (i.e., the argument of cost function in Eq. 2),

$$E_m(q^{-1}) = M(q^{-1}) - \frac{P(q^{-1})C(\theta, q^{-1})}{1 + P(q^{-1})C(\theta, q^{-1})}. \tag{10}$$

Moreover, if the reference model $M(q^{-1})$ satisfies Assumption 2, there exist an optimal filter $Q(\theta^*, q^{-1})$ such that:

$$e(\theta^*, t) = 0.$$

Then, for data set $\mathcal{D}$, the optimal filter $Q(\theta^*)$ satisfies,

$$Mu(t) = Q(\theta^*)(w(t) - v(t)) \tag{11}$$

Remark 2. In most approaches to DDC tuning (i.e. CbT,VRFT,..) it has been considered the approximation $1/(1 + P(q^{-1})C^0(\theta, q^{-1})) \approx 1/(1 + P(q^{-1})C(\theta, q^{-1}))$ to obtain a time expression which approximates the cost function (2). Note that such approximation is not required in our approach.

From the previous development, we are able to cast the problem to tune a filter $Q(q^{-1})$ into an identification problem as follows:

Problem 2. Given the signals

$$y_q(t) = M(q^{-1})u(t),\ u_q(t) = w(t)$$

Estimate from data an optimal filter $Q(\theta^*, q^{-1})$ that satisfies the relation:

$$y_q(t) = Q(\theta^*, q^{-1})u_q(t) \tag{12}$$

Note that the previous estimation problem is a system identification problem for system Q where the output $y_q(t)$ is measured without noise and the input $u_q(t)$ is noisy (see Figure 2).

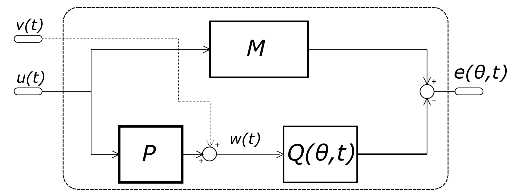

Fig. 2. Block diagram for Q tuning problem

4. $\hat{Q}$ TUNING SCHEME

In Problem 2, the estimation of Q has been posed as an identification problem where the input $u_q(t)$ is noisy and the output $y_q(t)$ is free of noise, i.e., an Errors In Variables (EIV) problem. In this work, we assume that $v(t)$ is i.i.d noise, however, it is possible to adapt the formulation to deal with Unknown but Bounded noises, following Set-Membership identification methods.

The instrumentals Variables (IV) method is a well know procedure to deal with EIV identification problems, Soderstrom and Stoica [1983]. In the following, the method proposed in van Heusden et al. [2007] is adapted to our framework, since it does not require a second experiment, neither the plant identification.

4.1 Correlation approach to tune $\hat{Q}$

Let the correlation function be defined as follows

$$f(\theta) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E\left\{\zeta_w(t) e(\theta, t)\right\} \tag{13}$$

Where $E\{\cdot\}$ indicates the mathematical expectation. $\zeta_w(t)$ is a vector of instrumental variables well correlated with $u(t)$ and uncorrelated with $v(t)$ given by,

$$\begin{aligned}\zeta_w(t) = [u_w(t+l), &u_w(t+l-1), \cdots u_w(t), \\ &u_w(t-1), \cdots, u_w(t-l)]^T\end{aligned} \tag{14}$$

where $u_w(t)$ is generated as a filtered version of the plant input, $u_w(t) = W(q^{-1})u(t)$, l is an proper integer and $e(\theta, t)$ is the model matching error defined in (9). Details for the selection of l can be found in van Heusden et al. [2007].

The optimal parameters defining filter Q are selected as

$$\hat{\theta} = arg \min_{\theta} f^T(\theta) f(\theta) = \sum_{\tau=-l}^{l} R^2_{eu_w}(\tau) \tag{15}$$

where $R_{eu_w}(\tau)$ is the cross-correlation function between $e(\theta, t)$ and $u_w(t)$, that is

$$R_{eu_w}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E\left\{e(\theta, t) u_w(t-\tau)\right\}$$

From (9), the previous equation can be rewritten as

$$R_{eu_w}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E\left\{[M - PQ(\theta)]u(t) W u(t-\tau)\right\}$$

Then, the cost function can also be represented in frequency domain, by means of the Parseval's theorem, as:

$$\lim_{l\to\infty} J_c(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} |[M - PQ(\theta)]\, W|^2\, \Phi_u^2(w)dw \quad (16)$$

where $\Phi_u^2(w)$ is the spectrum of the input signal. Finally, note that $|W| = (\Phi_u(w))^{-1}$ is required for criteria (5) and (16) being equal, that is, the magnitude of the filter frequency response equals the inverse of the signal spectrum. In this way, $\lim_{l\to\infty} J_c(\theta)$ is a good approximation of $J_{MR}(\theta)$.

Remark 3. Note that the main aim is minimizing the model matching error in (5) and in turn (10). But in order to apply the correlation method to tune the parameters of Q, a data error expression is required (i.e. $e(\theta, t)$ in (13)).Therefore, we have considered that the model matching error in (5) can be represented by means of a time expression in (9), this theoretical assumption can be approximated in practice when a signal $u(t)$ persistently exciting is employed, it means that its spectrum is rich enough to excite the dynamics of M.

4.2 Procedure to tune $\hat{Q}$

Given a data set generated as in Assumption 1, a reference model M and a filter length m_q, properly selected. The following procedure leads to a controller that approximately minimizes (2).

Given the structure for Q, it can be said that

$$Q(\theta, q^{-1}) = \beta^T(q^{-1})\theta \quad (17)$$

where

$$\beta(q^{-1}) = \left[1, q^{-1}, \cdots, q^{-(m_q-1)}\right] \quad (18)$$

Now, define the regressor as

$$\phi(t) = \beta(q^{-1})u_q(t) \quad (19)$$

Then, note that the error signal $e(\theta, t)$ can be expressed as

$$e(\theta, t) = y_q(t) - \phi(t)\theta \quad (20)$$

In terms of data, the correlation function is estimated as

$$f_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \zeta_w(t)\left[y_q(t) - \phi(t)\theta\right] \quad (21)$$

Recalling that $\zeta_w(t)$ is defined in Equation (14), it is possible to estimate the parameters of Q minimizing the criterion

$$J_N(\theta) = f_N^T(\theta) f_N(\theta). \quad (22)$$

the Least squares solution is:

$$\hat{\theta} = (X^T X)^{-1} X^T Z \quad (23)$$

where

$$X = \frac{1}{N}\sum_{t=1}^{N} \zeta_w(t)\phi^T(t), \quad (24)$$

$$Z = \frac{1}{N}\sum_{t=1}^{N} \zeta_w(t) y_q(t) \quad (25)$$

Finally, the controller to implement is given by

$$C(\hat{\theta}, q^{-1}) = Q(\hat{\theta}, q^{-1})(1 - M(q^{-1}))^{-1} \quad (26)$$

4.3 Stability margin estimation

Once a controller has been estimated by the previous procedure, it is necessary to estimate whether it guarantees an internally stable loop. A stability margin can be determined using the Small Gain Theorem, Doyle et al. [1991], considering the uncertainty associated to the estimated controller. The loop in Fig. 1 can be reformulated as shown in Fig. 3. From this scheme, the Small Gain Theorem leads to the following condition:

The controller given by

$$C(\hat{\theta}, q^{-1}) = Q(\hat{\theta}, q^{-1})(1 - M(q^{-1}))^{-1} \quad (27)$$

achieves a robustly stable loop if

$$\delta_Q(\theta) = \|\Delta_\theta\|_\infty = \|M(q^{-1}) - P(q^{-1})Q(\hat{\theta}, q^{-1})\|_\infty < 1 \quad (28)$$

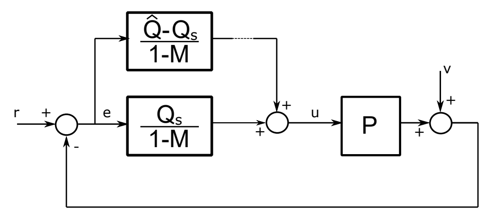

Fig. 3. Closed-loop with representation of the controller error $\frac{\hat{Q}-Q_s}{1-M}$. $Q_s = Q^*(\theta^*, q^{-1})$ as in Assumption 2.

Eq. (28) is a tool to estimate the stability of the loop.The approach in van Heusden et al. [2007] can be employed as it requires one data batch to estimate the norm, as in Assumption 1. Given that only an estimation of $\delta_Q(\theta)$ is available, the condition in (28) can be employed as a guide only, in other words, small $\hat{\delta}(\theta)$ values imply less risk of obtaining an unstable loop for a given controller, but it does not guarantee the closed-loop stability.

5. NUMERICAL EXAMPLES

In this Section, the Youla-Kucera data driven controllers tuning (YK-DDC) method proposed in the previous sections is evaluated in simulation. The performance of the solution is compared with the Correlation-based Tuning (CbT) method presented in van Heusden et al. [2011].

Consider the flexible transmission system introduced as a benchmark for digital control design by Landau et al. [1995]. The plant is

$$P(q^{-1}) = \frac{0.28261q^{-3} + 0.50666q^{-4}}{1 - 0.418q^{-1} + 1.589q^{-2} - 1.316q^{-3} + 0.886q^{-4}}$$

The control objective is given in terms of model-reference specifications. Two classes of reference models are tested,

$$M_1(q^{-1}) = \frac{(1-\alpha)^2 q^{-3}}{(1-\alpha q^{-1})^2},\ M_2(q^{-1}) = \frac{0.6q^{-5}}{1 - 0.75q^{-1} + 0.35q^{-2}}$$

considering under and over-damped required closed-loop behaviours. In CbT approach, the controller is parametrized as:

$$C(\theta, q^{-1}) = \sum_{i=1}^{m} \frac{\theta_i q^{-(i-1)}}{1 - q^{-1}} \quad (29)$$

Case I: Reference Model M_1**.** In this case, α indicates the location of the pole defining the desired loop speed and bandwidth (See Figure 4). $\alpha = 0.5$ is employed in the following, leading to a closed-loop bandwidth request much higher that previously reported in literature.

First, a data set is generated using a PRBS signal with $N = 512$ samples as plant input, then it is possible to assume $\Phi_u(\omega) \approx 1$. White noise is added to the plant output. The noise variance is selected such that the Signal to Noise Ratio (SNR) is approximately $20dB$. The SNR is calculated as

$$SNR = 10log\frac{\sum_{t=1}^{N} y(t)^2}{\sum_{t=1}^{N} v(t)^2}.$$

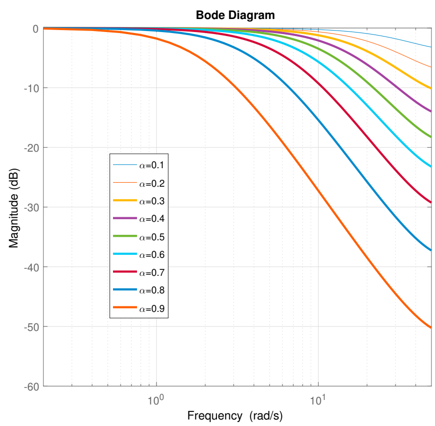

Fig. 4. Reference models M_1 class for diferrent α values

A Monte Carlo simulation is carried out, therefore, 1000 reference signals $u(t)$ are generated and applied to the plant P, maintaining $N = 512$. The set of output signals $y(t)$ is corrupted by independent noise realizations $v(t)$ maintaining a $SNR \approx 20dB$, leading to 1000 data sets $\mathcal{D}$. For each data set, a controller has been tuned via the procedure in subsection 4.2 employing $l = 20$, then each controller is tested in closed-loop with the actual plant P. For the *CbT* algorithm, it has been assumed $m = 6$ and also $l = 20$ as the instrumental variables length, the same values are assumed in van Heusden et al. [2011]. To YK-DDC $m_q = 12$ is fixed based on the impulse response of M_1. Results, in terms of step response, are depicted in Figure 5 for both methods.

For all controllers tuned via both approaches $\hat{\delta}(\theta)$ is estimated via the method in van Heusden et al. [2007], employing proper signals $e(\theta, t)$ for each approach. In CbT approach 16 controllers led to $\hat{\delta}(\theta) \geq 1$, while in our approach all controllers led to $\hat{\delta}(\theta) < 1$. Results for the stability criterion are reported in Table 1. In spite of such results, none of the controllers obtained via both approaches leads to unstable loops. It is worth noting that, according to Figure 5, the tracking of the reference model is better for the controllers obtained by the YK-DDC approach. The quality of the control loop is measured employing the maximum error E_{MAX} and the root mean squared error E_{RMS} of the closed-loop step response with respect to the reference model. The results are reported in Table 2.

Case II: Reference Model M_2. In this case, a more stringent reference model with underdamped behaviour

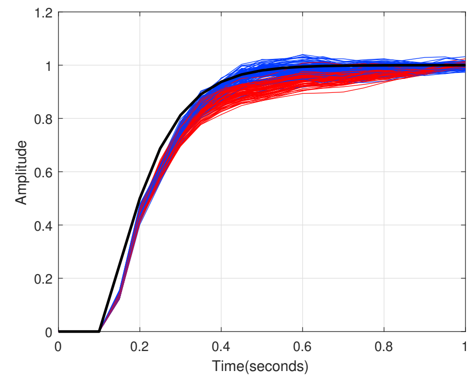

Fig. 5. Step response of M_1 (black line), results for 1000 controllers tuning via our approach (blue lines) and results for 1000 controllers tuning via CbT (red lines)
.

	Employed Method	$\hat{\delta}(\theta)$ Max	Min	Mean
Case I	CbT	1.433	0.193	0.393
	YK-DDC	0.531	0.179	0.241
Case II	CbT	2.610	0.970	1.101
	YK-DDC	0.510	0.171	0.261

Table 1. Stability results for infinity norm criterion for both approaches.

	Employed Method	Quality measures E_{MAX}	E_{RMS}
Case I	CbT	0.1819	0.0563
	YK-DDC	0.1489	0.0413
Case II	CbT	0.5184	0.1415
	YK-DDC	0.1091	0.0298

Table 2. Quality control results for both approaches in both cases.

is imposed. The aim is to evaluate the robustness of the design procedure when the reference model is complex. The step response of M_2 is shown in Figure 6 with a bold black line.

As in the first case, a data set with $N = 512$ and $SNR \approx 20dB$ is generated. A Monte Carlo experiment with 1000 data sets is performed. $m_q = 13$ is fixed based on the impulse response of M_2. The same parameters as in case I are selected for the CbT approach. The resulting closed-loop step responses are depicted in Figure 6. As can be observed, the performance obtained with the YK-DDC controllers (blue lines) is better than with CbT controllers.

For all controllers tuned via both methods $\hat{\delta}(\theta)$ is estimated via the method in van Heusden et al. [2007]. For the CbT approach, 77% of the controllers led to $\hat{\delta}(\theta) \geq 1$, while with the YK-DDC approach, all the controllers led to $\hat{\delta}(\theta) < 1$. Results for the stability criterion are reported in Table 1. Nevertheless, when evaluated on the actual plant, 7 of the controllers obtained via CbT lead to unstable loops and all the controllers obtained via YK-DDC yield stable loops. We can highlight that $\hat{\delta}(\theta)$ is a good indicator of closed-loop instability risk. Note that the YK-DDC method leads to controllers with $\hat{\delta}(\theta) << 1$ and no unstable loop was obtained.

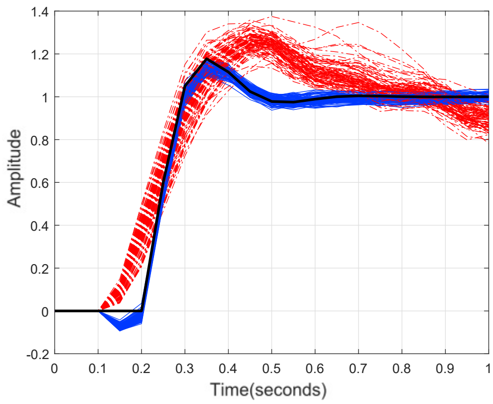

Fig. 6. Step response of M_2 (black line), results for 1000 controllers tuning via our approach (blue lines) and results for 1000 controllers tuning via CbT (red lines) .

6. CONCLUSIONS

In this work, we have presented a solution to the controller design from data problem, based on a Youla-Kucera parametrization of the controller. Departing from a set of input-output data measured from an stable, linear, time-invariant, SISO system, we have proposed a procedure to estimate a Finite Impulse Response filter that parametrizes a controller without requiring the plant model. The proposed parametrization allows to impose reference models more stringent that those achievable with extended PID controller structures, usually employed in controller design from data. The presented method translates the controller design process into an errors-in-variables identification problem and the solution is obtained by least-squares estimation. An *a-posteriori* stability test has been derived, allowing to assess from data the risk of obtaining an unstable loop for a given controller. The performance of the solution has been illustrated by means of Monte Carlo simulations, showing that the proposed solution allows to obtain better performance and stability margins than previous approaches. Further research is required to extend the method to multi-variable systems, state less conservative robust stability tests and to improve the noise handling.

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