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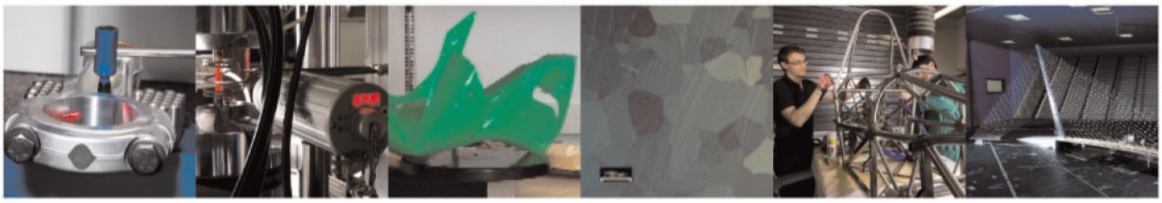

An extended form of the reciprocal-power function for tolerance allocation

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Abstract

The optimization of dimensional tolerances requires that a cost-tolerance function is evaluated consistently for all the part features involved in a given functional requirement. This is difficult because the parameters of commonly used functions are set using cost data from various sources and on possibly different scales. As an alternative, the paper proposes a revised form of one of the available cost-tolerance functions (reciprocal power), which expresses its parameters in empirical relationship with a set of design specifications on the toleranced features. These include the nominal dimension, the shape, the surface area, and the material. Following a previous study based on cost data available in literature, the values and expressions of the parameters are validated and refined using a feature-based method for the estimation of machining cost. The properties of the extended function allow to develop a simplified method for tolerance allocation that avoids the task of solving the optimization problem; it is a modified version of proportional scaling where the initial solution satisfies optimal ratios between tolerances. The discussion of the results and an application example help to justify the proposed function on grounds of correctness, convenience, and reference value.

Keywords

Tolerancing; tolerance optimization; cost-tolerance function; machining cost; cost estimation.

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1 Introduction

In the design of a mechanical assembly, tolerances must be specified for the functional dimensions of parts. Each dimension corresponds to a geometric feature that is created with a machining process (turning, milling, drilling, etc.). Traditionally, tolerances are set according to the expected geometrical variation of the process, and then possibly adjusted if the stackup of dimensional errors on a set of connected part features (tolerance chain) exceeds the allowable variation on a given assembly requirement. As a more advanced approach, tolerance allocation seeks an optimal set of tolerances satisfying one or more assembly requirements [1-4]. Most definitions proposed for the allocation problem are based on the minimization of the manufacturing cost with stackup constraints. The objective function of the optimization problem is the sum of the costs of the features in the tolerance chain; these are evaluated using a cost-tolerance function for each feature.
Despite possible cost reductions, allocation methods have found limited application in design practice. This is mainly due the limited availability of cost data that can dependably support the optimization of tolerances. The cost-tolerance functions proposed in literature include parameters whose values are not usually related to the properties of the toleranced features. Therefore, it is not guaranteed that costs are estimated on the same scale for features with different size and shape, or for parts made of different materials.
A previous study [5] outlined a possible approach to overcome this difficulty. It consists of finding extended formulations of cost-tolerance functions taking into account those design specifications that are likely to influence manufacturing costs. Such a formulation was proposed for the reciprocal power function, which is used in many allocation methods. Orientative values for the parameters of the function were calculated using cost-tolerance data collected from various sources and normalized on a common scale.
The objective of this paper is to validate and refine the above formulation, in order to get a cost-tolerance function that can be used on different types of mechanical assemblies without the need to collect domain-dependent cost data. The limitations related to the heterogeneous cost sources analyzed in the previous study are now overcome by using an existing method for the estimation of machining cost. The method is applied to the most common cases of toleranced features to generate consistent cost data, which allow a statistical estimation of the parameters of the extended cost-tolerance function. As will be shown below, the validation confirms the previously proposed form of the function and derives different values for some parameters.
The remainder of the paper is organized as follows. Section 2 reviews the literature on the topics related to the work (cost-tolerance functions, cost estimation). Section 3 recalls the previous results and describes the method used in the development of the cost-tolerance function. Section 4 describes the implementation of the method and presents the proposed form for the function. Section 5 highlights some properties of the function and discusses how they can be exploited in a scaling procedure that makes optimal allocation easier to deploy in design practice; this is also demonstrated on an example. Section 6 summarizes the contribution and limitations of the work.

2 Background

In most studies on tolerance allocation, cost-tolerance functions are used in either the objective function or the constraints of the optimization problem. As reported in some surveys [6-8], several cost-tolerance functions have been proposed. Each of them includes a set of parameters, which are evaluated from cost data published in textbooks or applying to specific cases. The choice of the function is usually driven by the solution methods proposed for the optimization problem. Earlier studies mostly adopted simple two-parameter functions such as the linear [9], the reciprocal [10] and the reciprocal squared [11]. The increasing complexity of allocation methods (from closed-form solutions to deterministic and stochastic algorithms) has led authors to prefer three-parameter functions such as the reciprocal power [12] and the exponential [13], which allow a more accurate approximation of available cost data. More complex expressions, such as the Michael-Siddall function [14] and the combined and polynomial functions [15] are potentially more accurate but require a larger amount of cost data for a reliable estimation of their parameters.
Comparative studies on cost-tolerance functions [16-20] focus on the choice of the correct function as a compromise between accuracy and suitability to specific allocation methods. As highlighted in [5], an often

neglected task is the selection of function parameters, which is seldom related to the properties of toleranced features. In some cases, the values set for the parameters are referred to published datasets. The one provided in [21] includes cost-tolerance graphs for several manufacturing processes; costs are given in relation to a baseline cost for the same process, making it difficult to compare data from different processes.

To overcome such limitation, some authors use cost-tolerance curves for combined processes, which were first proposed in [22]; they span a wide range of tolerance values, which correspond to different possible process sequences to machine a feature. Datasets for combined processes are also provided in [8, 15, 23]. A chart given in [1] uses a correction factor related to the type of machining process (e.g. for prismatic or rotational workpieces), and has a further advantage: the cost is related to a quality index that takes into account the relationship between tolerances and nominal dimensions. This seems one of the first attempts to relate cost data with design specifications, thus improving the consistency between the cost estimates of the different features in a tolerance chain. Trying to cover a wider set of specifications, [5] normalizes data from different sources and analyzes their statistical correlation with the IT tolerance grade of the ISO system of limits and fits [24]. This results in an equation for the selection of the parameters of the reciprocal power function, which will be used as a starting point for the present work.

Several methods have been used to estimate the parameters of cost-tolerance functions from available cost data. Linear regression allows a statistical estimation of the accuracy of the function, and has been mostly used on data relating to single processes [15, 25-27]. For combined processes, the cost-tolerance curve is less regular in shape due to the occurrence of break-even points between alternative process sequences. Therefore, additional assumptions are needed in the regression model; in [28], regression is used to evaluate discrete points of the cost-tolerance curve, which are then linearly interpolated. As an alternative to regression, neural networks have been trained with cost-tolerance data for both single [29] and combined processes [30]; the resulting cost-tolerance models are said to allow a more accurate approximation of actual costs. This drives allocation away from the use of an explicit cost-tolerance function, as it has also been attempted using fuzzy methods [31] and the iterative solution of an equation based on process constraints [32].

As opposed to using published cost data, some studies propose procedures to allow the collection and maintenance of cost data at manufacturing companies. In [33], cost-tolerance functions are constructed by choosing the least cost alternatives from data collected for elementary operations (roughing, semi-finishing, finishing). In [34, 35], a coding related to part geometry is used to extract cost data from a process database. In [36-38], cost-tolerance functions are built from corporate data using activity-based costing. In [39, 40], objective cost estimations are replaced with the judgement of corporate experts on how cost would react to alternative tolerance choices in specific situations.

The estimation of machining cost [41, 42] is a critical task when developing cost-tolerance functions accounting for the influence of design specifications. Creating a feature on a part requires a sequence of machining operations that depends on the tolerance specification. The cost of each operation includes the use of manufacturing resources (machine tool, direct labor) and some overheads (indirect labor, indirect materials, jigs and fixtures, setup, programming, etc.). Under some assumptions about equipment and production volume, which will also apply to the present work, the cost can be estimated by applying an appropriate shop rate to the cycle time of the machining process.

Among the available approaches to the estimation of machining time, detailed engineering methods select the cutting parameters for each operation (e.g. cutting speed, feed, depth of cut) and use them to calculate cutting times [43-45]; non-productive times between operations are taken from charts relating to different manufacturing processes [46, 47]. As an alternative, feature-based methods do not require the choice of cutting parameters, and calculate cutting times using material removal rates depending on the work material and the type of feature. Existing feature-based procedures [48-50] provide charts with typical removal rates, possibly collected through detailed estimation on wide sets of cases. They allow a quick estimation with an acceptable compromise on accuracy with respect to engineering methods. Software tools have been proposed to streamline feature-based estimation by either CAD integration [51], evaluation of removal rates from design variables [52], or retrieval of cost data by group technology coding [53].

In this work, an existing feature-based method for the estimation of machining time will be used to develop an extended cost-tolerance function with the formulation suggested in [5]. This is necessary in order to overcome two main limitations of prior results. The first one is that the data used for the evaluation of function parameters came from different sources relying on possibly different assumptions; time estimation will help to verify if the function can actually fit cost data obtained with explicit and consistent assumptions. The second one is that the influence of some design specifications (e.g. the nominal dimension and the type of feature) was modeled only at a first approximation; more accurate expressions and parameter values will now be drawn from the estimation method.

3 Methodology

The development of the cost-tolerance function will be described in three main steps. First, the assumptions of the problem are recalled along with some prior results. Second, a regression model is defined to express cost as a function of tolerance and nominal dimension. Third, cost is estimated over a set of possible design specifications using an existing feature-based method.

3.1 Assumptions and prior results

The work will deal with the solution of the tolerance allocation problem under the following assumptions:

- Only tolerances on linear and angular dimensions are considered.
- The tolerances are regarded as design specifications, i.e. they are allocated to part dimensions without a prior selection of the machining process.
- The functional requirements (or key characteristics) are expressed in geometric terms as single or independent assembly dimensions, which are related to part dimensions through linear equations.
- The objective function to be minimized is the manufacturing cost, which increases as the tolerance decreases because the material has to be machined more slowly or in multiple phases to reduce geometric errors.
- The constraints of the optimization problem include only the allowable variation on the assembly dimension, which is compared with the stackup of part tolerances calculated using the root-sum-square (RSS) equation.
- The optimal solution is calculated analytically using the method of Lagrange multipliers [11].

Let Y be a functional requirement, i.e. a dimension involving features on two distinct parts in an assembly. In general, Y depends on a set of dimensions X_i (i = 1, ... n) on features of individual parts, which are connected in a chain of geometric relations (tolerance chain). It is assumed that the relationship between Y and the X_i is linear or, for 2D or 3D tolerance chains, linearized by first-order Taylor approximation:

$$Y = \sum_{i=1}^{n} S_i X_i \qquad (1)$$

where $S_i = \partial Y/\partial X_i$ is the sensitivity of Y with respect to X_i. The variation T_Y allowed on the functional requirement, i.e. the difference between the upper and lower specification limits on Y, is assumed to be known. The values of the tolerances T_i on dimensions X_i are the results of the following optimization problem:

$$\begin{aligned} &\min\ C = \sum_{i=1}^{n} C_i(T_i) \\ &\text{s.t. } T_Y = c\sum_{i=1}^{n} S_i^2 T_i^2 \end{aligned} \qquad (2)$$

where $C_i(T_i)$ is the cost-tolerance function for dimension X_i, and the stackup of tolerances is expressed by the corrected RSS equation; c is an inflation factor that accounts for possible violations of the statistical assumptions on the X_i (independence, normal distribution, no bias, equal process capabilities), and is typically set at about c = 1.5 [54, 55].

It is also assumed that the cost-tolerance functions are of the reciprocal-power type:

$$C_i = a_i + \frac{b_i}{T_i^k} \qquad (3)$$

where:

- a_i is a fixed cost that will not be considered in the following as it does not influence the result of tolerance allocation;
- b_i is the factor of the variable cost (with respect to tolerance), and is different for each of the X_i;
- k is the exponent of the variable cost, and is assumed to be equal for all the X_i.

Under the above assumptions, the optimal values of the T_i can be calculated analytically with the method of Lagrange multipliers [11]. Once the T_i are known, the designer will be able to choose the correct sequence of machining operations for the different features (e.g. roughing, finishing, or grinding). This implies that each of the $C_i(T_i)$ covers a wide range of tolerances, which corresponds to different process choices (cost-tolerance function for combined processes).

The objective of this work is to evaluate the parameters b_i and k consistently for dimensions on features with different properties. In detail, a typical value must be found for k, verifying that it holds with a good approximation for features of different types; furthermore, b_i should be expressed as a function of feature properties. As the machining cost is roughly proportional to the cutting time, the factors that may have an effect on b_i include the following:

- the nominal dimension X_i, because a larger dimension will require a longer cutting time;
- the surface area of the feature, for a similar reason as above;
- the type of feature (rotational surface, planar surface, hole, etc.), because it requires different machining processes that will have different cutting times for the same feature size;
- the material of the workpiece, because its machinability will require different cutting times for the same feature type and size.

In [5], the two parameters were preliminarily estimated from cost-tolerance data from different sources, obtaining the following expressions:

$$\begin{aligned} k &= 0.55 \\ b_i &= f_M\, f_F\, f_A \beta \cdot X_i^{k/3} \end{aligned} \qquad (4)$$

where β is a constant, while the coefficients f_M , f_F and f_A depend respectively on the material, type of feature, and surface area of the feature. The main property expressed in (4) is that the exponent k of the reciprocal power function also determines the influence of the nominal dimension on the cost; in [5], this result was derived from empirical relationships between dimensional tolerances and nominal dimensions. The values of the coefficients were set according to assumptions that will be recalled and revised in the present work. Special attention will be paid to the coefficient f_F which relates the cost to the type of machined feature; the values given in [5] were preliminarily obtained by comparing cost data available in literature and referring to different machining processes.

In [56], equations (4) were found to apply also to angular dimensions if the nominal angle θ is replaced with the equivalent nominal dimension

$$X_i = \frac{\sin^3 \theta}{l_1 l_2} \qquad (5)$$

where l_1 and l_2 are the lengths of the adjacent edges.

3.2 Regression model

It must first be verified that the expressions (4) are consistent with the properties of the involved variables (material, type of feature, surface area, nominal dimension). For this purpose, some choices already discussed in [5] will be recalled below with a few changes deriving from released assumptions and approximations.

For two of the variables (surface area and material) it is reasonable to confirm the previous results. As mentioned before, it is assumed that the cost-tolerance function spans a wide tolerance range covering machining processes with increasing accuracy (combined processes). Furthermore, the parts are assumed to be manufactured in medium to large quantities on multipurpose CNC machines with automatic tool change. This implies that the variable cost includes only the cost of the cutting time and neglects the costs of non-productive, handling and setup times. For a given choice of machining operations and cutting parameters, the cutting time is approximately proportional to the area of the machined surface (actually, the overtravel of feed motion may take proportionally longer for smaller features). Therefore:

- the coefficient f_A is properly set to the surface area of the feature in cm^2;
- the coefficient f_M is the reciprocal of the machinability rating of the material, i.e. the ratio between the removal rates of the work material and a reference material (low-carbon steel). Tab. 1 gives orientative values of the coefficient for some categories of engineering materials (e.g. [57]).

Tab. 1: Evaluation of the coefficient related to material

Material	f_M
Aluminum alloys	0.3
Copper alloys	0.5
Low-carbon steel	1
Cast iron	1.3
Mid-carbon steel	1.3
Stainless steel	1.5
Alloy steel	2

In [5], the assumed values for the coefficient f_F related to the type of feature were obtained by comparing cost-tolerance data available in literature for different processes. In the following section, new values of f_F will be found by estimating cutting time for different possible sequences of machining operations, which correspond to different types of machined features. By definition, the factor will be set to a unit value for a reference type of feature, namely an external cylindrical or flat surface on a rotational workpiece.
Leaving aside the three factors discussed above, the effect of the nominal dimension remains to be verified. Replacing (4) into (3) and setting coefficients f_M, f_F and f_A to unity gives

$$C_0 = \beta \frac{X^{k/3}}{T^k} \qquad (6)$$

which is the expression proposed for the cost of machining a surface area of 1 cm^2 by external cylindrical or face turning on low-carbon steel. If equation (6) is correct, it must satisfy the typical relationship between tolerance and nominal dimension for a given accuracy level of the manufacturing process. In detail, the ISO system of limits and fits [24] calculates the tolerance T in mm as

$$T = \frac{nI}{1000} \qquad (7)$$

where I is the standard tolerance factor in μm, which grows approximately with the power 1/3 of the nominal dimension:

$$I \approx 0.45 X^{1/3} \qquad (8)$$

and n is a factor that grows exponentially with the IT tolerance grade g (the expression is more accurate than the regression equation used in [5]):

$$n = 10^{0.2(g-1)} \qquad (9)$$

Replacing (8) and (9) in (7) gives

$$\frac{T}{X^{1/3}} = \frac{0.45}{1000} 10^{0.2(g-1)} \qquad (10)$$

Comparing (10) with (6), the cost can be expressed as a function of g:

$$C_0 = \beta \left(\frac{1000}{0.45} \right)^k 10^{-0.2k(g-1)} \qquad (11)$$

This equation can be written as

$$C_0 = p \cdot 10^{-q(g-1)} \qquad (12)$$

where

$$p = \beta \left(\frac{1000}{0.45} \right)^k, \quad q = 0.2k \qquad (13)$$

In the following validation by machining time estimation, equation (12) will be used as a linear regression model of C_0 for different types of features. The same literature data used in [5], in a different scale from that assumed here, would fit the model with parameters p = 6.66 and q = 0.110; the approximation is statistically acceptable with coefficient of determination R^2 = 0.93 and normal residuals. Based on these data, equations (13) would provide the following estimates for the parameters of the reciprocal power function in the proposed formulation:

$$k = \frac{0.110}{0.2} = 0.55, \quad \beta = 6.66 \left(\frac{0.45}{1000} \right)^{0.55} = 0.10 \qquad (14)$$

These values will be compared with those obtained by the cost estimation method described below.

3.3 Machining time estimation

The choice of validating the expressions (4) by machining cost estimation aims to improve the accuracy of those parameters that rely on the consistency of cost data across different processes, such as the exponent k and the coefficient f_F depending on the type of feature. Besides, it wants to provide further support to the observation that the effect of the nominal dimension depends on the same exponent k of the tolerance (reduced in a 1/3 ratio).

The variable cost to be estimated is the cost of the operations needed to machine a feature, excluding those that do not depend on the specified tolerance. According to the above assumptions, such cost is approximately equal to the cutting time of those operations multiplied by the hourly cost of the process. The latter will be regarded as a constant, which is reasonable if workpieces do not require machine tools with special sizes. Therefore the cost is proportional to the total cutting time of the tolerance-depending operations for a given type of feature.

The procedure that will be used for the estimation of cutting time is a part of the Boothroyd method for machining cost estimation [50]. It is a feature-based method that uses aggregate design data about the workpiece and the machined features. Compared to engineering methods, it does not require a detailed planning of the machining process or the selection of cutting parameters (cutting speed, feed, depth of cut). The method defines three types of removal rates (machined volume, area, or length per minute), and provides typical values for them under different possible assumptions (minimization of machining time or cost).

The estimation of the parameters of the cost-tolerance function will use the removal rates of the original method [50] for most machining operations. The cutting time of each feature will be estimated on a limited number of machining operations with increasing accuracy: roughing, finishing, and grinding. Each operation will be associated with an IT grade assumed to be obtained in average cutting conditions, i.e. with a negligible scrap rate and a nearly optimal balance between cutting costs and tool replacement costs.

For all machining operations, the cutting time t [min] for a feature with area A_m [cm^2] is estimated as

$$t = \frac{A_m}{Q_A} \qquad (15)$$

where Q_A [cm^2/min] is the area removal rate, which depends on the type of operation.

Tab. 2 shows the removal rates assumed for the machining operations corresponding to the main types of part features. The source is indicated for each piece of data: some rates coincide with those suggested by Boothroyd for low-carbon steel in optimal cutting conditions, possibly with additional assumptions. Other rates result from more detailed evaluations as discussed below.

Tab. 2: Removal rates assumed for cutting time estimation

Part type	*Feature*	*Operation*	*Tolerance*	Q_A [cm^2/min]	*Source*
Rotational	Ext. plane/cyl.	Rough turning	IT10	400	Avg. cutting data
		Finish turning	IT7	240	Avg. cutting data
		Ext. cyl. grinding	IT5	275	[50] w/ assumption
Both	Int. plane/cyl.	Rough boring	IT10	320	Assumption
		Finish boring	IT7	190	Assumption
		Int. cyl. grinding	IT5	190	[50] w/ assumption
Prismatic	Plane	Rough side/face milling	IT11	320	[50]
		Finish side/face milling	IT8	280	[50]
		Surface grinding	IT6	220	[50] w/ assumption
Prismatic	Step/groove	Rough end milling	IT11	70	[50]
		Finish end milling	IT8	65	[50]
Both	Hole (D, L)	Drilling	IT10	90 k_D k_L	[50]
		Boring/reaming	IT7	90 k_D k_L	[50]

k_D = 0.2, 0.35, 0.6, 1, 1.5 for D = 3, 6, 12, 25, 50 mm
k_L = 1, 0.8, 0.7, 0.55, 0.5 for L/D <2, 3, 4, 5, 6

For external cylindrical and face turning, [50] suggests very high removal rates (700 and 400 cm^2/min for roughing and finishing). These have been revised using recommended cutting parameters for commercial tools. For rough turning with coated carbide tools, [58] suggests cutting speeds in the range v_C = 110-160 m/min, feeds f = 0.2-0.4 mm/rev, and depths of cut a_P = 2-4 mm. Assuming average values of the parameters in the respective ranges, the corresponding removal rate is

$$Q_A = v_C\, f = 135\,\mathrm{m/min} \cdot 0.3\,\mathrm{mm/rev} \approx 400\,\mathrm{cm^2/min} \qquad (16)$$

Similarly, for finish turning with coated carbide or cermet tools, the cutting parameters v_C = 160-210 m/min, f = 0.05-0.2 mm/rev, and a_P = 0.5-2 mm give:

$$Q_A = v_C\, f = 185\,\mathrm{m/min} \cdot 0.13\,\mathrm{mm/rev} \approx 240\,\mathrm{cm^2/min} \qquad (17)$$

For internal cylindrical and face turning, [50] reports the same removal rates as for external turning. However, it is reasonable to assume that less severe cutting conditions are chosen in internal turning to avoid additional geometric errors due to the flexibility of the boring bar. Removal rates for lathe boring will thus be reduced by 20% with respect to external turning, resulting in 320 and 190 cm^2/min for roughing and finishing. The same removal rates are also assumed for cylindrical and face boring on prismatic workpieces.
For grinding, [50] estimates the cutting time t_g [min] as

$$t_g = \frac{V_m}{Q_V} \qquad (18)$$

where V_m [in^3] is the volume removed by the grinding wheel and Q_V [in^3/min] is the volume removal rate. The latter parameter depends on the width w_g of the wheel [in]:

$$Q_V = Q_g\, w_g\, f_g \qquad (19)$$

where Q_g = 0.68 in^3/min is a reference removal rate, and f_g is a coefficient depending on the type of grinding process (1.24 for external cylindrical grinding, 1.15 for internal cylindrical grinding, 1 for surface grinding). With a further assumption on the width of the wheel (1” for surface and external cylindrical grinding, 3/4” for internal cylindrical grinding), the above equations and SI conversion yield a volume removal rate Q_V = 13.8 cm^3/min for external cylindrical grinding, 9.6 cm^3/min for internal cylindrical grinding, and 11.1 cm^3/min for surface grinding. The equivalent area removal rate is

$$Q_A = \frac{Q_V}{a} \qquad (20)$$

Assuming a total grinding allowance a = 0.5 mm, this gives the approximate removal rates Q_A listed in Tab. 2: 275 cm^2/min for external cylindrical grinding, 190 cm^2/min for internal cylindrical grinding, 220 cm^2/min for surface grinding.

4 Implementation

The above methods will now be used to estimate the parameters of the cost-tolerance function.
Any type of feature requires a sequence of machining operations depending on the specified tolerance. The variable cost of the feature corresponds to the cutting time associated with the tolerance grade of the last operation. For example, an external cylindrical feature on a rotational workpiece with IT5 tolerance is obtained with the following sequence: rough turning, finish turning, external cylindrical grinding. For an IT7 tolerance the sequence includes only the first two operations, and for an IT10 tolerance it is limited to the first operation. In the estimation of cutting time, each operation is assumed to be done in a single pass. This is what actually occurs for a finishing operation, while in a roughing operation only the last pass will be considered (as any previous pass is meant to remove excess material regardless of tolerance); moreover, as discussed above, the multiple passes of grinding can be treated as a single-pass operation with an equivalent removal rate. Fig. 1 illustrates the above assumptions.

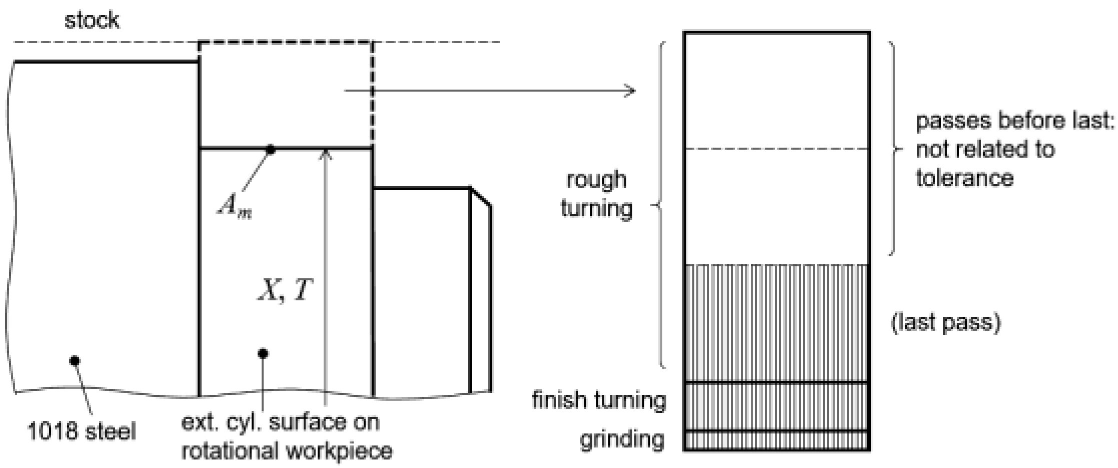

Fig. 1: Sequence of machining operations for a feature

The method described in subsection 3.3 is first used to estimate the cutting time needed to machine a 1-cm^2 feature on a low-carbon steel workpiece. In equation (15) is A_m = 1 cm^2, while Q_A has the values listed in Tab. 2. The time estimates are given in Tab. 3 for features of different types.
The hypothesis to be tested is that the above estimates are in accordance with equation (12), where C_0 is equal to the estimated time t of the sequence corresponding to tolerance grade g for the type of feature considered. Fig. 2 shows the results in a semilogarithmic graph of t as a function of g. Equation (12) is equivalent to

$$\log t = \log p - q(g-1) \qquad (21)$$

Therefore the proposed formulation is correct if the estimated times for each sequence are approximated by straight lines with equal decreasing slope for all types of features. The graph shows that such condition is satisfied with good approximation. For the sequences with three operations, the coefficients of determination R^2 of the linear regression are in the 0.98-0.99 range. Besides, all regression lines are very close to being parallel. For holes machined by drilling and boring/reaming sequences, the estimated cutting times shown in the graph refer to the case $k_D = k_L = 1$ as per Tab. 2 (i.e. a 25-mm hole with depth lower than twice the diameter); for different values of the two coefficients, the estimated times are multiplied by the same factor, thus the slope of the corresponding line does not change.

Tab. 3: Estimation of machining time

Part type	*Feature*	*Tol.*	*Sequence*	*Calculation*	t [10^{-3} min]
Rot.	Ext. cyl./plane	IT10	R-turn	1/400	2.5
		IT7	R-turn, F-turn	1/400+1/240	6.7
		IT5	R-turn, F-turn, ext. cyl G	1/400+1/240+1/275	10.3
Both	Int. cyl./plane	IT10	R-bore	1/320	3.1
		IT7	R-bore, F-bore	1/320+1/190	8.3
		IT5	R-bore, F-bore, int. cyl. G	1/320+1/190+1/190	13.7
Prism.	Plane	IT11	R-mill (face/side)	1/320	3.1
		IT8	R-mill, F-mill	1/320+1/280	6.7
		IT6	R-mill, F-mill, surface G	1/320+1/280+1/220	11.2
Prism.	Step/groove	IT11	R-mill (end)	1/70	14.3
		IT8	R-mill, F-mill	1/70+1/65	29.7
Both	Hole	IT10	Drill	$1/90 \cdot 1/k_D k_L$	$11.1 / k_D k_L$
		IT7	Drill, bore/ream	$(1/90+1/90) \cdot 1/k_D k_L$	$22.2 / k_D k_L$

R = rough, F = finish, G = grind

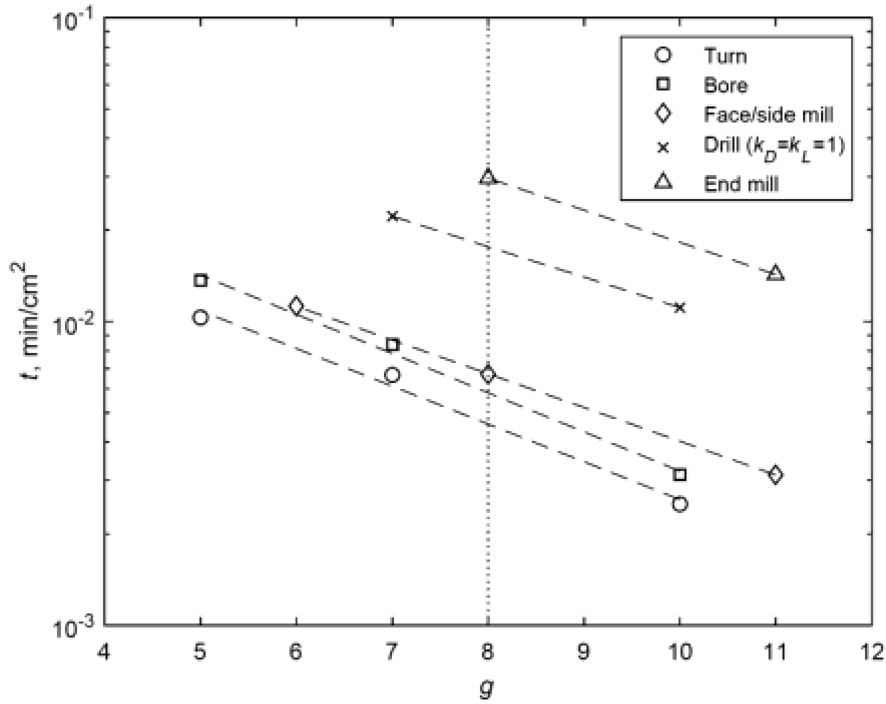

Fig. 2: Estimated machining time versus IT tolerance grade for different types of features

Within its limitations, the above analysis seems to support the correctness of equation (6) with exponent k approximately constant for features of different types. This is confirmed by the estimates of parameters p and q shown in Tab. 4 for all the regression lines. According to (13), k can be estimated as

$$k = \frac{q}{0.2} \qquad (22)$$

It results that the exponent of the reciprocal power function can be chosen in the 0.5-0.6 range, with an approximate average value

$$k \approx 0.55 \qquad (23)$$

corresponding to $q = 0.11$ and equal to the previous estimate in (14). The deviations from the average value of k are within about 10% for all the feature types. As can be easily found from equation (11), such variation of k determines a variation of cost in the order of 30% (assuming g = IT8), which is reasonable compared to other sources of uncertainty on cost estimation.

Tab. 4: Regression parameters for machining time

Part type.	*Feature*	*p*	*q*	*k*	t_{IT8} [min]
Rotational	Ext. cyl./plane	0.0342	0.1245	0.62	0.0046
Both	Int. cyl./plane	0.0467	0.1292	0.65	0.0058
Prismatic	Plane	0.0403	0.1111	0.56	0.0067
Prismatic	Step/groove	0.1633	0.1058	0.53	0.0297
Both	Hole	0.0889 / $k_D k_L$	0.1003	0.50	0.0176 / $k_D k_L$

The same results might also be used to estimate the factor β of (6) from (13). However, due to the slight differences in slopes, the intercepts of the regression lines with the line g = 1 would not be consistent with the actual proportions of costs in the useful range of tolerance grades. As shown in Fig. 2, more correct proportions among the costs of different types of features can be obtained by calculating the regression value t_{IT8} of the estimated time for g = 8. These are listed in the last column of Tab. 4.
To set the remaining parameters of the cost-tolerance function, the cost will be expressed as a time in minutes. Based on (21), the t_{IT8} for external rotational features and the average slope give an alternative estimate of the intercept

$$\log\, p = t_{\mathrm{IT8}} + 7q = -1.57 \;\rightarrow\; p = 0.0271 \qquad (24)$$

hence

$$\beta = 0.0271\left(\frac{0.45}{1000}\right)^{0.55} \approx 0.4\cdot 10^{-3} \qquad (25)$$

It can be noted that β has a different value from (14), because the literature data used in [5] were normalized in a different way and not expressed in machining minutes. The coefficient f_F for each type of feature is finally evaluated from the corresponding value of t_{IT8} (last column of Tab. 4) divided by the one related to external rotational features (first entry of the same column, t_{IT8} = 0.0046). According to the results in Fig. 2, the same ratios between the costs of machining operations for different features are assumed to apply over the whole range of IT tolerance grades. Approximate values are listed in Tab. 5.

Tab. 5: Evaluation of the coefficient related to feature type

Part type.	*Feature*	f_F
Rotational	Ext. cyl./plane	1
Both	Int. cyl./plane	1.25
Prismatic	Plane	1.5
Prismatic	Step/groove	6
Both	Hole	4 / $k_D k_L$

5 Results and discussion

Some properties of the function will be discussed below with the aim of further simplifying the solution of the allocation problem. Next, the results of the work will be demonstrated on an example.

5.1 Properties and scaling procedure

The proposed form of the reciprocal power cost-tolerance function can be used in any tolerance allocation method that is based on the combined-process assumption, i.e. on the use of a single cost function for the entire allowable tolerance range on a part feature (as opposed to the use of different functions for machining processes with increasing accuracy). Like the original reciprocal power function with constant k, it is suitable for an analytical solution of the allocation problem. Such opportunity will now be exploited to highlight some properties of the lowest-cost allocation.
According to the method of Lagrange multipliers, the constrained optimization problem in (2) has the same solution of the following unconstrained optimization problem:

$$\min C_L = \sum_{i=1}^{n} C_i(T_i) + \lambda\left(T_Y^2 - c\sum_{i=1}^{n} S_i^2 T_i^2\right) \qquad (26)$$

The condition $\partial C_L/\partial T_i = 0$ provides the expression of the multiplier λ:

$$\lambda = \frac{\partial C_i / \partial T_i}{2cS_i^2 T_i} \qquad (27)$$

As λ is a constant, equation (27) means that the optimal tolerances are in constant proportions (within their allowable ranges) regardless of the specified assembly variation T_Y:

$$T_i \propto \frac{\partial C_i / \partial T_i}{S_i^2} \qquad (28)$$

The derivative of the reciprocal power function in (3) is

$$\partial C_i / \partial T_i = -kb_i T_i^{-(k+1)} \qquad (29)$$

As k is assumed equal for all dimensions, the proportionality condition becomes

$$T_i \propto \left(b_i S_i^{-2}\right)^{\frac{1}{k+2}} \qquad (30)$$

According to the results of this work, the expression of factor b_i in (4) gives the following condition on the optimal tolerances:

$$T_i \propto \left(f_{Mi} f_{Fi} f_{Ai} X_i^{k/3} S_i^{-2}\right)^{\frac{1}{k+2}} \qquad (31)$$

Once the coefficients f_M, f_F and f_A have been evaluated for each feature, the allocation problem is solved by setting the tolerances to initial values in the proportions of equation (31), and then scaling them uniformly according to the value specified for T_Y.
Based on the estimated values for k, the exponent $1/(k+2)$ that appears in (31) is fairly small. Therefore the influence of each of the coefficients related to feature properties is much less than linear. This adds robustness to the results of this work, because any uncertainty on the parameters (especially the exponent k and the coefficients related to material and feature type) is unlikely to have a strong impact on the optimal proportions between the tolerances.
If $k = 0.55$, the tolerances can be set proportionally to the following factor:

$$F = f_M^{0.39} f_F^{0.39} f_A^{0.39} X^{0.072} S^{-0.78} \qquad (32)$$

which can be simplified into:

$$F = \varphi_M \varphi_F \varphi_A \varphi_X \varphi_S \qquad (33)$$

In detail, the coefficients in (33) can be evaluated as follows:

- φ_M depends on the work material (Tab. 6) with values ranging between –35% and +30% with respect to low-carbon steel;
- φ_F depends on the feature type (Tab. 7) with values ranging from 1 (external rotational surfaces) to about 4 (holes with small diameters and high depth/diameter ratios);
- φ_A depends on the ratio between the surface area A of the feature and the area A_{max} of the largest feature in the same tolerance chain (Tab. 8), with values ranging down from 1 ($A = A_{max}$) to about 0.1 ($A = 0.001\ A_{max}$);
- φ_X depends on the ratio between the nominal dimension X of the feature and the maximum nominal dimension X_{max} in the same tolerance chain (Tab. 8), with values ranging down from 1 ($X = X_{max}$) to about 0.6 ($X = 0.001\ X_{max}$);

- φ_S depends on the absolute value of the sensitivity S of the assembly dimension Y with respect to the nominal dimension X of the feature (Tab. 9), with values ranging between –40% and +70% with respect to the most common case when $|S| = 1$ for sensitivities between ±2 e ±0.5.

Tab. 6: Effect of the material on the optimal tolerance

Material	φ_M
Al alloy	0.63
Cu alloy	0.76
Low-C steel	1
Cast iron, mid-C steel	1.11
Stainless steel	1.17
Alloy steel	1.31

Tab. 7: Effect of the type of feature on the optimal tolerance

Feature	φ_F
Rotational, external	1
Rotational, internal	1.09
Prismatic, plane	1.17
Step/groove	2.01
Hole (D, L)	1.72 $\varphi_D \varphi_L$

D [mm]	φ_D
3	1.87
6	1.51
12	1.22
25	1
50	0.85

L/D	φ_L
< 2	1
3	1.09
4	1.15
5	1.26
6	1.31

Tab. 8: Effects of the feature area and of the nominal dimension on the optimal tolerance

A/A_{max}	φ_A
1	1
0.8	0.92
0.5	0.76
0.2	0.53
0.1	0.41
0.05	0.31
0.01	0.17
0.005	0.13
0.001	0.07

X/X_{max}	φ_X
1	1
0.8	0.98
0.5	0.95
0.2	0.89
0.1	0.85
0.05	0.81
0.01	0.72
0.005	0.68
0.001	0.61

Tab. 9: Effect of the sensitivity on the optimal tolerance

S	φ_S
0.2	3.5
0.5	1.7
1	1
1.5	0.73
2	0.58
3	0.42
5	0.28

5.2 Example

Fig. 3a shows a wheel assembly where the pin 1 is the fixed axle for the hub 6 of a wheel with a rubber rim; the roller bearings 5 are kept at a distance from the supports 3 by means of the spacers 4. The pin is axially restrained on the supports by two circlips 2. The functional requirement to be controlled is the axial clearance of the pin. As shown in the sketch of Fig. 3b, the clearance corresponds to the assembly dimension Y, which is the result of a tolerance chain involving the axial dimensions X_i (i = 1, ... 6) on the corresponding parts. The variation allowed on the clearance is T_Y = 0.4 mm.

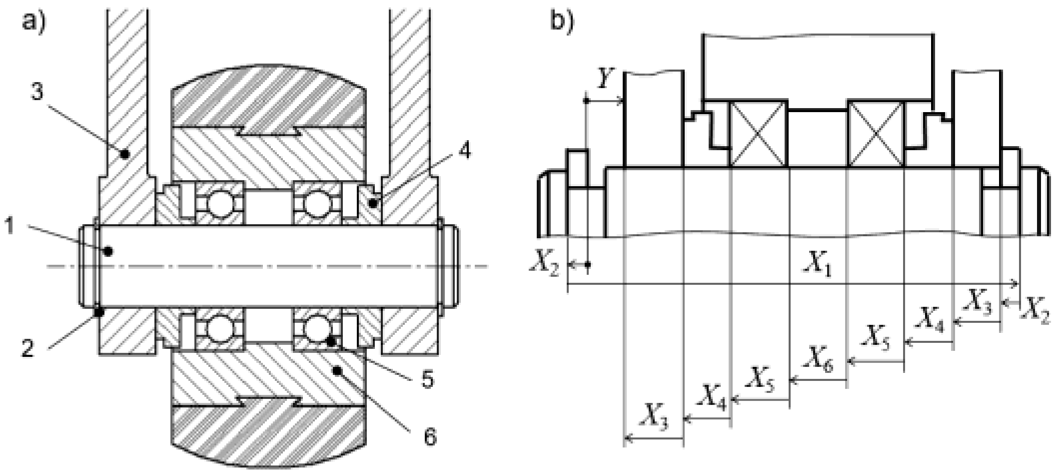

Fig. 3: Example: a) wheel assembly, b) tolerance chain

The tolerance chain is expressed by the linear equation

$$Y = X_1 - X_2 - X_2 - X_3 - X_3 - X_4 - X_4 - X_5 - X_5 - X_6 \qquad (34)$$

where dimensions X_2, X_3, X_4 and X_5 appear twice as distinct random variables related to equal parts on the left and right sides of the assembly. The stackup equation of the chain is

$$T_Y = c\sqrt{T_1^2 + 2T_2^2 + 2T_3^2 + 2T_4^2 + 2T_5^2 + T_6^2} \qquad (35)$$

The value c = 1.5 is assumed for the inflation factor. The circlips and the bearings are stock components with tolerances T_2 = 0.1 mm (IT12 on the 1.2-mm thickness) and T_5 = 0.011 mm (IT6 on the 12-mm width). These are removed from the stackup equation in order to highlight the unknown tolerances T_1, T_3, T_4 and T_6:

$$T_Z = \sqrt{T_Y^2 - 2c^2\left(T_2^2 + T_5^2\right)} = c\sqrt{T_1^2 + 2T_3^2 + 2T_4^2 + T_6^2} \qquad (36)$$

where the residual assembly variation is T_Z = 0.339 mm. Tab. 10 lists the data relating to the four tolerances to be allocated. The materials, areas and nominal dimensions X_i are derived from part drawings, while the sensitivities S_i of the dimensions are consistent with the RSS equation taking into account the double occurrence for parts 3 and 4.

In the following, the allocation of tolerances is done in two ways. The first one is the analytical solution of the optimization problem (2) by means of the cost-tolerance function in the proposed form (4). The second one is the scaling of an initial solution deriving from the proportionality factor (33).

Tab. 10: Data for tolerance allocation on the example

i	*Material*	*Feature*	*Area* [cm^2]	X_i [mm]	S_i
1	low-C steel	rot. ext.	0.91	86.4	1
3	Al alloy	plane	24.13	14	1.414
4	Cu alloy	rot. ext.	8.42	10	1.414
6	cast iron	rot. int.	4.40	12	1

To allow an analytical solution, the parameters of the cost-tolerance functions for dimensions X_1, X_3, X_4 and X_6 are determined according to the results in Section 4. Tab. 11 shows the evaluation of factor b_i. With the help of Tab. 1 and Tab. 5, the data in Tab. 10 allow to evaluate the coefficients f_{Mi}, f_{Fi} and f_{Ai}. Finally, the b_i of the four functions are calculated from equations (4), (23) and (25).

Tab. 11: Calculation of the factors of cost-tolerance functions for the example

i	f_{Mi}	f_{Fi}	f_{Ai}	$b_i \cdot 10^3$ [min]
1	1	1	0.91	0.82
3	0.3	1.5	24.13	7.05
4	0.5	1	8.42	2.57
6	1.3	1.25	4.40	4.51

The optimization problem is thus defined as

$$\begin{aligned}&\min C = \frac{b_1}{T_1^k} + \frac{b_3}{T_3^k} + \frac{b_4}{T_4^k} + \frac{b_6}{T_6^k}\\ &\text{s.t. } T_Z = c\sqrt{T_1^2 + 2T_3^2 + 2T_4^2 + T_6^2}\end{aligned} \tag{37}$$

For an analytical solution, all the tolerances are expressed as functions of T_1 in the stackup equation:

$$T_Z = cT_1\sqrt{\left(\frac{T_1}{T_1}\right)^2 + 2\left(\frac{T_3}{T_1}\right)^2 + 2\left(\frac{T_4}{T_1}\right)^2 + \left(\frac{T_6}{T_1}\right)^2} \tag{38}$$

which allows to find T_1 once the ratios between tolerances are known. The condition (30) on optimal tolerances gives

$$\frac{T_i}{T_1} = \left(\frac{b_i}{b_1}\frac{S_1^2}{S_i^2}\right)^{\frac{1}{k+2}} \tag{39}$$

From the data in Tab. 10 and Tab. 11, the optimal tolerances are derived as shown in Tab. 12. To keep machining costs to a minimum, tighter tolerances should be specified on two dimensions (the distance between the grooves in pin 1 and the width of spacer 4). It would obviously be impossible to evaluate the merit of this allocation with respect to those that could be obtained with existing methods. However, the result has the advantage of representing an evaluation of "ease of machining" resulting from a set of factors (material, type of feature, nominal size, influence on the assembly requirement) that had not been considered together in previous approaches to the allocation problem.

Tab. 12: Allocated tolerances for the example (from analytical optimization)

i	T_i/T_1	Ti [mm]
1	1	0.06
3	1.77	0.11
4	1.19	0.07
6	1.95	0.12

An alternative route for the solution of the allocation problem is the optimal scaling procedure deriving from the properties of the cost-tolerance function. Tabs. 6-9 allow a direct evaluation of the proportionality factors F_i between the optimal tolerances according to (33). The results are shown in Tab. 13, and help to understand the criteria that lead to coarser or tighter tolerances on the different dimensions. In detail, X_1 and X_4 call for especially tight tolerances because the corresponding features are relatively inexpensive to machine accurately. For the pin 1, this is due to the small area of the machined features (the grooves for the circlips). For the spacers 4, the relevant factors are the small size of the features (the two sides) and the good machinability of the material; besides, the width has an amplified effect on the assembly dimension due to the double occurrence. The other two dimensions have looser tolerances mainly for reasons related to the material (wheel hub 6) and to the type of feature (width of the supports 3), which tend to increase their machining cost.

Tab. 13: Evaluation of the proportionality factor for the example

i	φ_{Mi}	φ_{Fi}	φ_{Ai}	φ_{Xi}	φ_{Si}	F_i
1	1	1	0.28	1	1	0.28
3	0.63	1.17	1	0.87	0.75	0.48
4	0.76	1	0.65	0.86	0.75	0.32
6	1.11	1.09	0.50	0.87	1	0.53

If the F_i are taken as the initial values of the tolerances in mm, the optimal values are calculated from them by applying the scaling factor

$$s = \frac{T_Z}{c\sqrt{F_1^2 + 2F_3^2 + 2F_4^2 + F_6^2}} = 0.223 \qquad (40)$$

Tab. 14 shows the results of the calculation. The result is the same as that obtained by analytical optimization (Tab. 12); this was fully expected because the use of equation (33) and the coefficients in Tab. 6-9 are just a different way of applying equation (31) with the average value (23) assumed for the exponent *k*. On a practical side, this is easier and quicker than explicitly solving the optimization problem (2) with the extended reciprocal-power cost-tolerance function proposed in the paper.

Tab. 14: Allocated tolerances for the example (from optimal scaling)

i	$s \cdot F_i$	Ti [mm]
1	0.223·0.28	0.06
3	0.223·0.48	0.11
4	0.223·0.32	0.07
6	0.223·0.53	0.12

6 Conclusions

The paper has proposed an extended form of the reciprocal power cost-tolerance function, which evaluates cost parameters according to a set of design specifications (material, type of feature, area, and nominal dimension). Compared to a previous feasibility study, the work presents two main contributions. First, better estimates of some parameters are obtained by the use of a feature-based method for the estimation of machining costs. Second, the properties of the cost-tolerance functions are exploited in a scaling procedure for tolerance allocation, which allows to minimize cost without having to explicitly solve an optimization problem.
The results could give the following advantages for the solution of tolerance allocation problems:

- The extended form of the function allows to evaluate the costs of all dimensions of a tolerance chain in the correct proportions. This is a necessary condition to ensure that the optimal allocation approaches the actual minimum cost achievable at production stage.
- The parameters of the function can be readily set for all the features of a tolerance chain without the need to collect additional information (expert judgment, cost data from literature or industry). This improves the convenience of the procedure and thus removes one of the obstacles that seem to be preventing the deployment of allocation methods in design practice.
- A ready-to-use cost-tolerance function can be of help to future research on tolerance allocation. New cost-based optimization methods will be able to benefit from the easy estimation of the parameters of cost-tolerance functions. The scaling procedure will possibly serve as a benchmark for more sophisticated methods that do not use cost-tolerance functions for combined processes, so that their additional benefits could be assessed.

Future developments will have to overcome some limitations of the proposed approach:

- The attempt to cover a wide diversity of machined parts and features may affect the accuracy of the cost-tolerance function. Such a compromise will have to be avoided by introducing further detail and company expertise in both the classification of machining features and the estimation of machining costs.

- The scope of the cost-tolerance function is still limited to the allocation of dimensional tolerances. An extension to geometric tolerances according to ASME-GD&T and ISO-GPS standards will be a further step toward a full acceptance of allocation methods in industry, where the adoption of modern tolerancing criteria is increasingly required.

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