# A Youla-Kucera Formulation of the Controller Design From Data Problem

Freddy Valderrama<sup>a</sup>, Fredy Ruiz<sup>b,\*</sup>

 <sup>a</sup> Escuela de Educación Industrial, Universidad Pedagógica y Tecnológica de Colombia, Duitama, Colombia
 <sup>b</sup> Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano,

Via Ponzio 34/5, 20133 Milan, Italy

# Abstract

The Youla-Kucera parametrization is a fundamental result in system theory, very useful when designing model-based controllers. In this paper, this formulation is employed to solve the controller design from data problem, without requiring a process model. It is shown that, given a set of input-output data generated by the plant and a desired closed-loop reference model, it is possible to estimate an stable filter that parametrizes the controller that minimizes the norm between the closed-loop dynamics and the requested behavior. The employed parametrization gives more degrees of freedom in the controller design than previous works in literature, allowing to achieve more stringent closed-loop performances. The proposed design methodology does not imply a plant identification step and it provides an estimate of the model-matching error between the requested and the resulting model as indicator of stability and performance of the derived control loop. The proposed solution is evaluated in regulation problems for non-minimum phase systems through Monte Carlo simulations and in experimental conditions for the regulation of temperature in an ohmic assisted hydrodistillation process.

*Keywords:* Data-driven control, Identification for control, Uncertain systems, Controller parametrization, Youla-Kucera parametrization.

# 1. Introduction

Currently, data acquisition technology permits to collect a large amount of measurements from industrial plants. When enough plant data are available in order to design a controller, there exist two main approaches in the scientific literature. In a model-based controller design procedure, a plant model is estimated from data, possibly constructing also an uncertainty model, and then,

<sup>\*</sup>Corresponding author

*Email addresses:* freddy.valderrama@uptc.edu.co (Freddy Valderrama), fredy.ruiz@polimi.it (Fredy Ruiz)

such a model is employed to design a controller, resulting in a two-steps procedure. This method can lead to sub-optimal controllers, due to the modelling errors. On the other hand, the available data can be employed to directly design a controller, avoiding the plant model identification. This approach has been named Direct Data-Driven Controller (DDC) tuning. The interested reader can referred to [1], where an interesting comparison between Model-based controller design and DDC tuning is presented.

There exist adaptive (iterative) and non-iterative DDC tuning techniques. In the first ones the controller parameters are adjusted at periodic intervals, while in the second, the controller is designed based on the information contained in one batch of experimental data (one-shot). In this work, we are interested in oneshot techniques for two reasons: First, one disadvantage of adaptive schemes is that they require several experiments to update the controller parameters, this can lead to excessive costs in industrial applications. On the other hand, standard model-based controller design methods are not-iterative, they use a single mode or data set, thus non-iterative DDC are a direct replacement of model-based controller design techniques.

Within a linear framework, [2] presents a comparison of non-iterative DDC tuning methods, considering correlation approach (CbT), periodic errors in variables (EiV), inverse controller (IC) and prediction error methods (PEM). All of them are constructed within a stochastic framework. A more recent review work of DDC methods can be found in [3], where the authors perform a qualitative comparison and briefly explain each technique. A different approach to solve the DDC problem follows a deterministic formulation using Set-membership techniques. Recent results on this approach can be found in [4], [5] and [6].

In all the methods mentioned above, the main ingredients of the DDC problem are a set of input-output data generated by the plant to be controlled, a closed-loop reference model where performance specifications are embedded, and a given controller structure, usually parametrized by a set of fixed basis functions. When the set of bases is not consistent with the reference model, the resulting controller can yield to closed-loop instability. In this sense, in [7] a set of conditions are established to define the behaviors that the closedloop can reach, in order to select an achievable reference model. An approach to automatically select the basis functions that allow to achieve a requested closed-loop behavior is proposed in [8], while [9] proposes a two degrees of freedom parametrization to better approximate the sensitivity function and the input-output transfer function.

One of the main challenges in the context of DDC methods is guaranteeing stability. Considering that no plant model is available in this framework, standard stability tests cannot be performed. A possibility is to test the controller before actual implementation [10] (i.e. a-posteriori tests). In [11] some aposteriori stability estimators are proposed for an iterative DDC tuning scheme. In [12] an invalidation test, based on the available data, is employed in order to detect if the controller may led to unstable closed-loops. This test requires the accurate identification of a possibly unstable system in an errors-in-variables framework. Some attempts to incorporate a stability condition at the design step in non-iterative DDC can be found in [13] and [14]. Both methods consider an extended PID controller structure (i.e. FIR filter plus integrator) leading to convex optimization problems. However, such methods do not offer acceptable performances when the desired reference model is not achievable by the selected controller structure. Moreover, the approach in [14] requires to estimate the  $\mathcal{H}_{\infty}$  norm of a loop subsystem with the number of samples of the experiment going to infinity to test stability. [15] uses the unfalsified control theory to derive relations between the choice of the performance criterion to be optimized and closed-loop stability conditions. In the case the controller is not linearly parametrized (i.e. it is possible to modify zeros and poles of the controller) such degree of freedom leads to a non-convex optimization problem. In this approach, unlike the other methods, it is necessary to define a closed-loop reference model and also a target input sensitivity transfer function to find the parameters of a stabilizing controller.

The Youla-Kucera parametrization is a fundamental result in system theory that allows to parametrize all the controllers that stabilize a given plant. It has been extensively applied in optimal and robust control when designing modelbased controllers, see e.g. [16, 17]. However, in its original form it is not applicable when the plant model is not available. In this paper, this parametrization is employed to solve the controller design from data problem, without requiring a process model. The proposed framework does not require the selection of a fixed controller structure, avoiding the task of controller parametrization definition. The main contributions of this paper can be summarized as:

- A novel approach to controller tuning from data, relying on the Youla Kucera parametrization, is proposed, avoiding the plant identification.
- The controller structure is not fixed *a priori* but is a consequence of the tuning procedure. This allows to achieve more stringent reference models than those previously proposed in literature, while maintaining a convex optimization problem to tune the controller parameters.
- The problem to tune a stabilizing controller is cast into an identification problem with additive noise affecting both the input and the output. Data sets can be generated in open- or closed-loop operation.
- Our approach is a non-iterative solution that relies on errors-in variables identification. Thus, the controller tuning procedure does not require iterations or multiple experiments.
- An *a posteriori* test is derived to estimate the deviation between the desired closed-loop dynamics and the resulting control loop, without requiring the plant transfer function. The test provides information about the stability of the derived control system and the feasibility of the requested performance.

The outline of the paper is as follows. In Section 2, the problem formulation is presented. In Section 3, the data-driven controller tuning approach, based on the Youla-Kucera parametrization, is derived. Section 4 summarizes the controller design methodology. In Section 5 the proposed solution is illustrated in simulation, through the design of a controller for a non-minimum phase system. Finally, in Section 6, a controller for the regulation of temperature in an ohmicassisted hydrodistillation process is designed and experimentally evaluated. The conclusions end the paper in Section 7.

# 2. Statement of the problem

In this section the data-driven controller (DDC) tuning problem is formulated. First, the setting and main assumptions are presented.



Figure 1: Assumed feedback control structure

Consider a discrete-time linear-time invariant (LTI) single-input single-output (SISO) feedback control scheme, as depicted in Fig. 1, P(z) is a stable (possibly non-minimum phase) plant,  $C(z, \theta)$  is the controller,  $\theta$  is a vector of controller parameters, r(k) is the reference signal, v(k) is output noise/disturbances, u(k) and y(k) are the plant input and output signals, respectively.

For the system interconnection in Fig. 1, the aim of the controller tuning procedure is to select an optimal controller  $C^o(z, \theta^o)$  minimizing some performance criterion and guaranteeing internal stability. For example, an optimization problem can be stated as:

$$C^{o}(z,\theta) = \operatorname{argmin} J_{RM}(\theta)$$
 (1)  
s.t.

Loop internally stable

For the reference model based cost function

$$J_{RM}(\theta) = \left\| M(z) - \frac{P(z)C(z,\theta)}{1 + P(z)C(z,\theta)} \right\|_2^2$$
(2)

Being M(z) a strictly proper reference model for the closed-loop system (i.e.  $M(z) \neq 1$ ), where performance specifications are embedded.

If system P(z) is unknown, Problem (1) can not be solved directly. The common controller design procedure for unknown plants is to follow a twostep procedure where first a system model  $\hat{P}(z)$  is derived form data, possibly including some information on uncertain dynamics  $\Delta_P(z)$ , and then, a controller is obtained solving Problem (1) for  $\hat{P}(z)$ .

The following assumptions define the framework of the data-driven stabilizing controller tuning problem.

**Assumption 1.** P(z) is unknown. The available information on the plant is a set of input-output data generated by the system interconnection given in Fig. 1, with the plant initially at rest, that is,

$$u(k) = T_u(z)r(k) + H_u(z)v(k),$$
  

$$y(k) = T_y(z)r(z) + H_y(z)v(k),$$
  

$$\mathcal{D} = \{r(k), u(k), y(k), k = 1, 2, ..., N\},$$
(3)

Where r(k) is the reference input, v(k) is the process noise, u(k) is the process input and y(k) is the noisy process output.

Two operation conditions are considered:

1. **Open-loop operation:** In this condition, there is not controller C(z) and

$$\begin{array}{rcl} T_u(z) &=& 1, \\ H_u(z) &=& 0, \\ T_y(z) &=& P(z), \\ H_y(z) &=& 1. \end{array}$$

2. Closed-loop operation: In this condition, an stabilizing controller C'(z) is available and

$$T_u(z) = C'(z)(1 + P(z)C'(z))^{-1},$$
  

$$H_u(z) = -C'(z)(1 + P(z)C'(z))^{-1},$$
  

$$T_y(z) = P(z)C'(z)(1 + P(z)C'(z))^{-1}P(z)$$
  

$$H_y(z) = (1 + P(z)C'(z))^{-1}.$$

Assumption 2. Reference signal r(k) is a Wide-Sense Stationary (WSS) signal, persistently exciting of any order. That is, its power spectral density (PSD) satisfies  $\Phi_r(j\omega) > 0$ ,  $\forall \omega$ , see e.g. [18].

**Assumption 3.** The output noise v(k) is a WSS signal independent of the reference signal r(k), i.e., the cross-correlation between them is zero:

$$R_{vr}(\tau) = \mathbb{E}[v(k)r(k-\tau)] = 0; \forall \tau.$$

Considering the previous assumption, the controller design from data problem can be stated as follows:

**Problem 1.** Data-Driven Stabilizing Controller Tuning: Given a data-set  $\mathcal{D}$  generated as in Assumptions 1, 2 and 3, and a reference model M(z), find a controller  $\hat{C}(z, \theta)$  that solves (1).

The assumption on system stability can be relaxed following the approach proposed in [15], where it is assumed that an stabilizing controller is available *a priori*.

# 3. A stabilizing controller structure

Let us recall that the set of all the stabilizing controllers  $C(z, \theta)$  for the loop in Fig. 1, given a stable plant P(z), can be expressed as

$$\mathcal{C}^{S} = \left\{ C(z,\theta) = \frac{Q(z,\theta)}{1 - P(z)Q(z,\theta)} : Q(z) \in \mathcal{H}_{\infty} \right\}$$
(4)

where  $Q(z, \theta)$  is any stable and proper transfer function. The previous result is known as the Youla-Kucera parametrization for a stable plant, [19].

When the *Youla-Kucera* parametrization is adopted to find an optimal controller solving (1), the cost function (2) becomes

$$J_{RM}(\theta) = \|M(z) - Q(z,\theta)P(z)\|_{2}^{2}$$
(5)

That is, the complementary sensitivity function of the loop becomes  $Q(z, \theta)P(z)$ .

**Definition 1.** For any stable closed-loop reference model M(z) and stable filter Q(z), the model matching error transfer function is:

$$\Delta_M(z,\theta) = M(z) - Q(z,\theta)P(z) \tag{6}$$

Given the previous analysis, from now on we focus in the problem to estimate a filter  $Q^*(z, \theta^*)$ , such that the cost function (5) is minimized.

#### 3.1. A data-driven cost function

Notice that to estimate a filter  $Q^*(z, \theta^*)$  minimizing (5) it is required the knowledge of the plant P(z). But, under the assumptions of the framework, the plant is unknown. The following Lemma allows to relate the model-based cost function with a signal-based one.

**Lemma 1.** Let x(k) be a set of instrumental variables generated as

$$x(k) = W(z)r(k),$$

where W(z) is a Bounded-Input Bounded-Output (BIBO) stable filter. Given an asymptotically stable system P(z) and a data set  $\mathcal{D}$  generated as in Assumptions

1 and 3, for any stable filter  $Q(z,\theta) \in \mathcal{Q}$  it holds that the cross-correlation function

$$R_{ex}(\tau, \theta) = \mathbb{E}[e(k, \theta)x(k-\tau)]$$

between the noisy output of the model matching error transfer function

$$e(k,\theta) = M(z)u(k) - Q(z,\theta)$$
(7)

$$= [M(z) - Q(z,\theta)P(z)]u(k) + Q(z,\theta)H_y(z)v(k)$$
(8)

and the instrumental variable x(k), satisfies:

$$||R_{ex}(\tau)||_{2}^{2} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \left[ M(e^{j\omega}) - Q(e^{j\omega}, \theta) P(e^{j\omega}) \right] T_{u}(e^{j\omega}) W(e^{j\omega}) \right|^{2} \Phi_{r}^{2}(j\omega) d\omega(9)$$

Moreover, if the reference signal r(k) satisfies Assumption 2,  $T_u(z)$  has no zeros on the unitary circle, and

$$|W(e^{j\omega})| = |T_u(e^{j\omega})\Phi_r(j\omega)|^{-1},$$
(10)

 $it \ holds \ that$ 

$$J_{MR}(\theta) = ||R_{ex}(\tau,\theta)||_2^2$$

**Proof:** See Appendix A.

**Remark 1.** Note that the filter W(z) required to equalize the model-based cost function (5) and the correlation function norm in (9) depends on the transfer function between the reference r(k) and the plant input u(k). For open-loop operation, this is not a problem, while for closed-loop conditions  $|T_u(z)|$  must be estimated from data. For example, considering that r(k) and v(k) are independent, from standard spectral analysis it is known that

$$\Phi_{ur}(e^{j\omega}) = T_u(e^{j\omega})\Phi_r(e^{j\omega}),$$

where the input spectrum  $\Phi_r(e^{j\omega})$ , and the cross-spectrum  $\Phi_{ur}(e^{j\omega})$  can be derived from the available data set.

From the previous Lemma, the controller design problem can be transformed into a system identification one, where the unknown system is  $Q(z, \theta)$  and the plant model is not involved. Fig. 2 shows a block diagram of the interconnection that describes the model matching error system. Note that e(k) is obtained as the difference between the plant input signal u(k) filtered by the reference model M(z) and the noisy plant output signal y(k) filtered by  $Q(z, \theta)$ , that is

$$e(k,\theta) = M(z)u(k) - Q(z,\theta)y(k)$$

Given the signals

$$y_Q(k) = M(z)u(k), \ u_Q(k) = y(k)$$



Figure 2: Block diagram representing the controller tuning problem. The subsystem inside the dashed line constitutes the setup for the estimation of  $Q(z, \theta)$ .

the design of a controller/filter  $Q(z, \theta)$  has been posed as an identification problem where the input  $u_Q(k)$  is noisy and the output  $y_Q(k)$  is noise-free but correlated with the input noise if the data set is generated in closed-loop, that is, an Errors In Variables (EIV) problem, solved through a specific selection of instrumental variables.

The Instrumentals Variables (IV) method is a well know procedure to deal with EIV identification problems in stochastic settings, see e.g., [20]. In DDC frameworks, this approach has been employed in [21], requiring a second experiment where the plant is subject to the same input u(k) or the estimation of a plant transfer function. These solutions are not properly aligned with the non-iterative DDC tuning techniques.

**Remark 2.** In most approaches to DDC tuning (i.e. CbT, VRFT,...) it is required to approximate the sensitivity function generated by any controller  $C(z, \theta)$  to the sensitivity function of the optimal loop, i.e.,  $1/(1 + P(z)C^0(z, \theta)) \approx 1/(1 + P(z)C(z, \theta))$ , to obtain a time-domain expression that approximates the cost function (2). Note that such approximation is not required in our approach.

# 3.2. A structure for Q.

Several structures can be assumed to design the filter  $Q(z, \theta)$ . For example, recursive polynomial structures such as ARX or ARMAX can be employed. However, the formulation requires that  $Q(z, \theta) \in \mathcal{H}_{\infty}$ . Imposing stability constraints in autoregressive structures leads to complex non-linear constraints, turning the estimation problem into a highly non-convex optimization program, see e.g. [22]. On the other hand, Finite Impulse Response (FIR) models guarantee stability without additional constraints. Therefore, a FIR structure is adopted for  $Q(z, \theta)$  as follows,

$$Q(z,\theta) = \sum_{i=1}^{m_q} \theta_i z^{-(i-1)},$$
(11)

where  $m_q$  is the filter impulse response length.

Then, the controller design problem becomes a parametric estimation problem, where the filter parameters are selected from the set:

$$\mathcal{Q} = \{Q(z,\theta) : \theta \in \mathcal{R}^{m_q}\}$$

Note that for FIR filters, the only parameter that defines the system structure is the length of the impulse response  $m_q$ . Now we analyze the set of equations that define the filter impulse response in a model-based setting, in order to get insights about the selection of  $m_q$ .

From the cost function in Eq.(5), the impulse response of the model-matching error system is:

$$h^{\Delta M}(k,\theta) = h^{M}(k) - h^{P}(k) * h^{Q}(k;\theta).$$
(12)

where  $h^M(k)$ ,  $h^P(k)$  and  $h^Q(k;\theta)$  are the reference model, plant and filter impulse responses, respectively, and (\*) is the convolution operator. In matrix form it becomes

$$h^{\Delta M}(k,\theta) = \begin{bmatrix} h^{M}(0) \\ h^{M}(1) \\ h^{M}(2) \\ \vdots \\ h^{M}(m_{q}-1) \\ \vdots \end{bmatrix} - \begin{bmatrix} h^{P}(0) & 0 & 0 & 0 & \dots & 0 \\ h^{P}(1) & h^{P}(0) & 0 & 0 & \dots & 0 \\ h^{P}(2) & h^{P}(1) & h^{P}(0) & 0 & \dots & 0 \\ \vdots & \ddots & & & & \\ h^{P}(m_{q}-1) & h^{P}(m_{q}-2) & \dots & & h^{P}(0) \\ \vdots & & & & \vdots \end{bmatrix} \begin{bmatrix} h^{Q}(0) \\ h^{Q}(1) \\ h^{Q}(2) \\ \vdots \\ h^{Q}(m_{q}-1) \end{bmatrix}.$$
(13)

Eq. (13) shows that in order to minimize the  $\ell_2$  norm of the model matching error system, the euclidean norm of the  $h^{\Delta M}(k,\theta)$  vector must be minimized. Moreover, Eq. (13) shows also that the model matching error can be driven to zero if and only if the relative order (input-output delay) of the reference model M(z) is greater or equal to the relative order of the plant P(z), and  $h^M(k)$  belongs to the column space of the matrix formed by shifted versions of  $h^P(k)$ . Finally, recall that M(z) is a stable system, therefore, its impulse response  $h^M(k)$  decays exponentially and can be bounded as  $|h^M(k)| \leq L\rho^k$ ,  $k \geq 0$ , for some finite bound L > 0 and decay rate  $\rho \in (0, 1)$ . Therefore,  $h^M(k)$ becomes negligible for some  $k \geq L_M$  for a reasonable  $L_M$  value. The same statement can be made for P(z). Therefore, a necessary condition to achieve the minimum of  $h^{\Delta M}(k; \theta)$  is that  $m_q \geq L_M$ .

#### 3.3. Data-driven approach to tune $Q(z, \theta)$

The previous section described a transformation of the controller design problem into a signal correlation minimization problem, where the plant model is not implied. However, infinite length signals are considered. In practice, the estimation of  $Q(z, \theta)$  must be performed with a finite length data set. Let  $\zeta(k)$  be the vector of instrumental variables well correlated with u(k)and uncorrelated with v(k) at time k, given by,

$$\zeta(k) = [x(k+l), x(k+l-1), \cdots x(k), x(k-1), \cdots, x(k-l)]^T$$
(14)

where l is a proper integer. Details for the selection of l can be found in [23].

The model matching error signal  $e(k; \theta)$  for a FIR filter can be expressed as

$$e(k,\theta) = y_Q(k) - \phi(k)\theta \tag{15}$$

for the regressor

$$\phi(k) = [u_Q(k), \ u_Q(k-1), \ \dots, \ u_Q(k-m_q+1)]$$

Given N samples of instrumental variables  $\zeta(k)$ , signal  $y_Q(k)$  and regressor  $\phi(k)$ , the sample correlation function becomes

$$f_N(\theta) = \frac{1}{N} \sum_{k=1}^{N} \zeta(k) \left[ y_Q(k) - \phi(k)\theta \right]$$
(16)

and the finite sample cost function for the controller design results

$$J_{DD}(\theta) = f_N^T(\theta) f_N(\theta) \approx \sum_{\tau=-l}^l R_{ex}^2(\tau)$$
(17)

The parameter  $\hat{\theta}$  defining the data-driven optimal filter  $\hat{Q}(z,\hat{\theta})$  is selected as

$$\hat{\theta} = \arg\min_{\theta} f_N^T(\theta) f_N(\theta), \tag{18}$$

resulting in a quadratic optimization problem, whose solution can be obtained by least-squares. Notice that Assumption 2 guarantees that the optimization problem in (18) is well posed.

#### 3.4. Deriving a controller from data

Once an optimal filter  $\hat{Q}(z,\hat{\theta})$  has been estimated, the optimal controller  $\hat{C}(z,\hat{\theta})$ , which solves Problem 1 is

$$\hat{C}(z,\hat{\theta}) = \hat{Q}(z,\hat{\theta})(1 - \hat{Q}(z,\hat{\theta})P(z))^{-1}$$
(19)

Then, in order to recover the controller it is required to know the process model P(z). However, in the data-driven setting, P(z) is unknown. Note that if the minimum of the model-based cost function (2) is 0, i.e.,  $\Delta M(z, \theta^*) = 0$ , then the optimal controller is given by

$$C(z,\theta^*) = Q(z,\theta^*)(1-M(z))^{-1}$$
(20)

that does not depend on P(z).

Therefore, under the assumption that  $\Delta M(z, \theta^*)$  is "small", the proposed data driven controller is obtained as

$$C_{DD}(z,\hat{\theta}) = Q(z,\hat{\theta})(1 - M(z))^{-1}$$
(21)

**Remark 3.** It must be highlighted that the structure of the controller derived in (21) is a function of the reference model M(z) and the estimated filter  $Q(z, \theta^*)$ .

**Remark 4.** When a reference model M(z) with unitary DC gain is requested, i.e. M(1) = 1, and the resulting filter Q(z) is such that  $\sum_{i=1}^{m_q} \theta_i \neq 0$ , the resulting controller  $C_{DD}(z, \theta)$  exhibits integral action, it has a pole in z = 1.

Note that any stable filter  $Q(z, \theta)$  guarantees a stable loop if the controller is derived as in Eq. (20). However, when a controller is derived as in Eq. (21), it is necessary to verify whether it guarantees an internally stable loop when applied in closed-loop.

**Result 1.** For any FIR filter  $Q(z, \theta)$ , the controller derived as

$$C_{DD}(z,\theta) = Q(z,\theta)(1 - M(z))^{-1}$$
(22)

leads to a loop with complementary sensitivity function

$$T_{DD} = QP(1 - \Delta M)^{-1} \tag{23}$$

that is asymptotically stable if

$$||\Delta M(e^{j\omega})||_{\infty} \le 1$$

The result follows from the evaluation of the loop transfer function, recalling that  $\Delta M = M - PQ$  is a stable system.

Using the available data, it is possible to estimate the frequency response of  $\Delta M$ . Note that when the input u(k) is applied to  $\Delta M$ , the resulting inputoutput relation can be expressed as:

$$e(k) = \Delta M(z)T_u(z)r(k) + (H_u(z) - Q(z)H_y(z))v(k)$$

where the signal e(k) can be reconstructed from data as

$$e(k) = y_Q(k) - Q(z)y(k).$$

Then, for fixed Q(z), the output signal e(k) is available and, recalling that r(k) and v(k) are uncorrelated, from standard spectral analysis results, an unbiased estimator of the frequency response of  $\Delta M$  is:

$$|\widehat{\Delta M}(e^{j\omega})| = |\Phi_{er}(j\omega)\Phi_{rr}^{-1}(j\omega)||T_u(e^{j\omega})|^{-1}$$
(24)

When open-loop data is available, the estimate reduces to

$$\Delta M(e^{j\omega})| = |\Phi_{eu}(j\omega)\Phi_{uu}^{-1}(j\omega)|.$$
<sup>(25)</sup>

When using Result 1 to determine the stability of the loop it must be recalled that it derives form the Small Gain theorem, then  $||\Delta M(e^{j\omega})||_{\infty} \leq 1$  is a sufficient but not necessary condition for closed-loop stability. Moreover,  $\widehat{\Delta M}(e^{j\omega})$ is an estimate of the system response with an associated variance, therefore when  $||\widehat{\Delta M}(e^{j\omega})||_{\infty}$  is close to 1 for a given data set  $\mathcal{D}$  and reference model M(z) it does not necessarily imply that the resulting control loop is unstable.

# 4. Summary Procedure to tune $\hat{Q}$

The following algorithm summarizes the proposed data-driven controller design procedure.

Given a data set generated as in Assumptions 1, 2 and 3, a reference model M(z) and a filter length  $m_q$ , properly selected, the following procedure allows to obtain a controller that approximately minimizes (2).

## Algorithm 1. Youla-Kucera data-driven controller tuning algorithm

1. Using the N input-output samples in  $\mathcal{D}$  obtain the signals:

$$y_Q(k) = M(z)u(k)$$
$$u_Q(k) = y(k)$$
$$x(k) = W(z)r(k)$$

2. Form the vectors

$$\phi(k) = [u_Q(k), \ u_Q(k-1), \ \dots, \ u_Q(k-m_q+1)]$$
$$\zeta(k) = [x(k+l), \ \dots x(k), \ \dots, \ x(k-l)]^T$$

 $and \ matrices$ 

$$X = \frac{1}{N} \sum_{t=1}^{N} \zeta(k) \phi^{T}(k),$$
(26)

$$Z = \frac{1}{N} \sum_{t=1}^{N} \zeta(k) y_Q(k)$$
 (27)

3. Obtain the impulse response coefficients of the optimal data-driven filter  $\hat{Q}(z,\theta)$  as:

$$\hat{\theta} = (X^T X)^{-1} X^T Z \tag{28}$$

4. Derive the data-driven controller as

$$C_{DD}(z,\hat{\theta}) = Q(z,\hat{\theta})(1 - M(z))^{-1}$$
(29)

5. Form the signal

$$e(k) = y_Q(k) - Q(z)y(k).$$

and estimate the model matching error transfer function as

$$|\widehat{\Delta M}(e^{j\omega})| = |\Phi_{er}(j\omega)\Phi_{uu}^{-1}(j\omega)||T_u(e^{j\omega})|^{-1}$$
(30)

6. If  $|\widehat{\Delta M}(e^{j\omega})| < 1$  for all  $\omega$ ,  $C_{DD}(z, \hat{\theta})$  is accepted and can be applied to the actual plant P(z).

#### 5. Numerical example

In order to test the approach on a demanding condition, we take the next example from [15]. Consider the problem of controlling a non-minimum phase and stable, but unknown, linear system with continuous-time transfer function

$$P(s) = \frac{s - 0.5}{s^3 + 2s^2 + 0.65s + 0.175}$$

The control design requirements are given by the following second-order model:

$$M_C(s) = \frac{w_n^2}{s^2 + 2\zeta s w_n + w_n^2}$$

where  $\zeta = 0.5$  and  $\omega_n \in [0.3, 1]$ , resulting in a model whose step response shows an overshoot close to 13% and settling time between 7s and 20s. The reference model M(z) is obtained as a zero order hold discrete time equivalent of  $M_C(s)$  with sampling time  $T_s = 2.5s$ , and a pure-delay of d samples, such that  $M(z) = z^{-d}M_C(z)$ . Note that this desired closed-loop transfer function cannot be obtained by the feedback control structure in Fig. 1 with a stable controller.

As first step, a data set  $\mathcal{D}$  is collected applying to system P(s) an input u(t) in open-loop, generated as a Pseudo-Random Binary (PRB) signal with sampling time  $T_s = 2.5s$ , length N = 512, clock period 2 samples and range  $u(t) \in \{-1, 1\}$ , filtered by a zero-order hold. The output signal y(t) = P(s)u(t) is sampled with period  $T_s = 2.5s$  to obtain the discrete-time signal y(k), corrupted by white noise v(k), resulting in a Signal to Noise Ratio,  $SNR \approx 20dB$ .

In Algorithm 1, once a reference model M(z) has been selected, the only parameter required to solve the data-driven controller design problem is the FIR filter length  $m_q$ . On the other hand, Result 1 allows to estimate the  $\mathcal{H}$ -infinity norm of the model-matching error transfer function given a model M(z) and a filter length  $m_q$ . The estimate of  $||\Delta M(e^{j\omega})||_{\infty}$  for different delay times (d = 0 and d = 1), increasing values of filter length  $m_q$  and reference model bandwidth

 $w_n$  is depicted in Fig. 3, allowing to test the feasibility of reproducing several reference models. Note that in the case d = 0 the estimate of the unmodelled dynamics norm is larger than 0.4 for all the considered filter lengths and model bandwidths, while for d = 1 it is lower than 0.3, excepting when  $m_q = 2$ , i.e., for a very short filter. Therefore, a reference model M(z) with d = 1 is selected to continue with the controller design procedure.

It is observed that the estimated  $\mathcal{H}$ -infinity norm of the model-match error is an increasing function of the reference model bandwidth. It can be noted that the norm of  $\Delta M(e^{j\omega})$  is lower than 0.35 and almost insensitive to the filter length for  $m_q \geq 5$  indicating that adding more degrees of freedom to the Youla-Kucera parametrization does not necessarily lead to better results. From the previous analysis, a model with  $\omega_n = 0.6$  results in a good trade-off between bandwidth and uncertainty, and  $m_q = 5$  is a reasonable length for the filter Q(z), providing a model with  $||\Delta M(e^{j\omega})||_{\infty} = 0.22$ .



Figure 3:  $||\Delta M(e^{j\omega})||_{\infty}$  estimates for different values of filter length  $m_q$  and reference model natural frequency  $w_n$ . Left: Reference model delay d = 0. Right: Reference model delay d = 1.

As a second step, a Monte Carlo experiment is performed. 100 realizations of u(k) are generated and applied to system P(s), each one elapsing 1280 s and using  $T_s = 2.5 \ s$ , thus N = 512. The output signal y(k) is corrupted by noise v(k) maintaining a  $SNR \approx 20 dB$ . The same reference model and filter length employed in the first test are considered. The proposed controller design strategy is applied to each data set resulting in 100 controllers. The performance of the obtained control systems is compared with those of a set of controllers obtained with the same data sets and reference model, using the Correlationbased Tuning (CbT) method proposed in [24]. In this case, the controllers are parametrized as extended PID transfer functions, i.e.,

$$C(\theta, z) = \sum_{i=1}^{m} \frac{\theta_i z^{1-i}}{1 - z^{-1}}.$$
(31)

Based on the resulting performance, m = 8 basis functions are employed.

The performance of the estimated controllers for the 100 input and noise realizations, in terms of frequency response, is illustrated in the Fig. 4 for both methods. Fig. 5 shows the corresponding closed-loop step responses.



Figure 4: Results of Monte Carlo experiment. Closed-loop frequency response of controllers tuned via the Youla-kucera parametrization (dashed blue lines) and via CbT (dashed red lines). The Reference model response is shown (black line) and the response of a controller parametrized by optimal Q obtained assuming that P is known is depicted in the green line.

Figures 4 and 5 show that performance is comparable for both procedures in high frequency, while in low frequencies the YK parametrized controllers achieve better results and the CbT controllers show some bias. Regarding closed-loop step response, the average Root mean squared error (RMSE) for the controllers designed with the YK parametrization is 0.064, while for those designed with the CbT approach it is 0.134. Also, the maximum error is lower for the YK-parametrized controllers ( $E_{max} = 0.224$ ) than for the CbT method ( $E_{max} = 0.293$ ).

In order to illustrate the controller parametrization obtained via the *Algorithm* 1, one of the controller is

$$C(\hat{\theta}, z) = \frac{-0.5804 + 0.4291z^{-1} - 0.05096z^{-2} - 0.1186z^{-3} + 0.2284z^{-4} - 0.0044z^{-5} - 0.0008z^{-6} + 0.0047z^{-7}}{1 - 0.04604z^{-1} - 0.5629z^{-2} - 0.3911z^{-3}}$$
(32)



Figure 5: Results of Monte Carlo experiment. Closed-loop step response of controllers tuned via the Youla-kucera parametrization (dashed blue lines) and via CbT (dashed red lines). The Reference model response is shown (thin black line), and the response of a controller parametrized by optimal Q obtained assuming that P is known is depicted in the green line.

Next, for all the controllers tuned via the YK parametrization  $||\Delta M(e^{j\omega})||_{\infty}$  is estimated via (24). The worst case for  $||\Delta M(j\omega)||_{\infty}$  is 0.24 and the average is 0.22. i.e., there exist a low risk of obtaining an unstable control loop for any of the 100 controllers. Fig. 6 shows a histogram of the estimated error norm.

For the case of the CbT controllers, in [25] it is shown that

 $||M(z) - P(z)(1 - M(z))C(z)||_{\infty} < 1$  is a sufficient condition of closed-loop stability. Then, such a norm is calculated for the 100 controllers estimated in the Monte Carlo experiment, taking into account that the plant is known for this example. Results are depicted in Fig. 6. It is to highlight that, 86 controllers do not satisfy the stability criterion, however, all the CbT controllers achieve closed-loop stability when interconnected with the actual plant.

Finally, we illustrate the behavior of the methodology employing closed-loop data. The system is operated in the feedback connection shown in Fig. 1, with the PI controller

$$C(z) = -0.15 \frac{z - 0.7}{z - 1}$$

This controller guarantees an asymptotically stable loop, but the performance is not adequate. The resulting closed-loop step response is shown in Fig. 8. A Monte Carlo experiment is performed. 100 realizations of signals r(k) and v(k) are generated and applied to the closed-loop system, with N = 512 and maintaining a  $SNR \approx 20 dB$  in the system output. The same reference model and filter length employed in the first test are considered.

Figures 8 and 7 show the step and frequency responses of the tuned loops compared to the reference model, the optimal model-based design and the de-



Figure 6: Stability criterion histogram for the Monte Carlo experiment.  $||\Delta \hat{M}(j\omega)||_{\infty}$  for our method in blue, and  $||M - P(1 - M)C||_{\infty}$  for CbT method in red.

parting closed-loop system. The average RMS error of the step responses is 0.071, while the maximum error is 0.248. It can be concluded that performance is comparable to the results obtained using open-loop data and better than those of controllers tuned with the CbT methodology.

# 6. Application in an essential oil extraction process

In this Section the proposed controller design methodology is evaluated on an experimental setting. A temperature controller for an essential oil extraction system has been designed from data.

Essential oils from aromatic plants are very appreciated in different industries. Ohmic-assisted hydrodistillation (OAHD) is a novel oil extraction technique, where steam is generated by heat produced applying electric current to a mixture of vegetable material, water and salt. According to [26] its main advantage over conventional distillation is that shorter extraction times and lower energy consumption are obtained. Thereby, it is environmentally friendly.

The short extraction times in OAHD are a consequence of the rapid increase in temperature caused by the internal heating and the electroporation phenomenon. A non-thermal effect facilitates the extraction of essential oil via the breakdown of cell membranes produced by the electrical current passing through the material, [27]. However, the effect of electroporation on the extraction time has not been properly quantified and it is required to derive accurate dynamic



Figure 7: Results of Monte Carlo experiment with closed-loop data. Closed-loop frequency response with initial controller C(z) (dashed red line). Closed-loop frequency responses of controllers tuned via the Youla-kucera parametrization (thin blue lines). The Reference model response is shown (black line) and the response of a controller parametrized by optimal Q obtained assuming that P is known is depicted in the green line.

models of the OAHD process.

To quantify the electroporation impact on oil extraction, independently of the heating power, it is required to regulate the temperature of the mixture into the camera during the *heating time period* properly manipulating the applied power, for later evaluating the resulting kinetics of extraction.

## 6.1. Process description

Fig. 9 shows the main parts of the OAHD process employed in this work. In this equipment, a mixture of water-salt-vegetable material is heated in an ohmic-heated camera (OH camera) by an electric current. Salt is added to the mixture in order to increase conductivity since, in general, the conductivity of the vegetable material is not enough to allow electrons to flow. Essentially, the camera is a vessel equipped with two electrodes. The mixture temperature is increased up to the water boil point. At this point, steam starts to flow, dragging the oil molecules that are released from the plants. The oil-steam mixture rises through the camera and reaches the condenser, this device causes the mixture to cool and change to liquid phase. Finally, the liquid mixture reaches the Clevenger apparatus, in which oil and water are separated. The Clevenger allows the refilling of the OH camera with condensed water, maintaining an adequate water level in the camera.



Figure 8: Results of Monte Carlo experiment with closed-loop data. Closed-loop step response with initial controller C(z) (dashed red line). Closed-loop step responses of controllers tuned via the Youla-kucera parametrization (thin blue lines). The Reference model response is shown (thin black line), and the response of a controller parametrized by optimal Q obtained assuming that P is known is depicted in the green line.

The plant, instrumentation and actuator are depicted in Fig. 9. Its main characteristics are (i) Volumetric capacity 1 [L] for the mixture water-salt-vegetable material, and 1 [L] of empty space for steam flow. (ii) Maximum working power 1 [kW] to guarantee the efficiency of the condenser. To vary the extraction power a digitally controlled dimmer is employed. Such a dimmer module also includes a Wattmeter. An electrically isolated thermocouple (kC2198JG120A120 by MINCO) is employed to register the temperature of the mixture.

Employing the previous experimental setup and considering that no first-principles model of the process is available, two data-driven controller tuning strategies are evaluated experimentally. The first strategy is the Youla-Kucera parametrization data-driven controller tuning (YK-DDC) proposed in this work, and the second strategy is Correlation-based tuning (CbT) presented in [24]. The latter is selected in order to do a fair comparison since both strategies employ the same input information.

## 6.2. Controller design tuning problem

A tracking problem is posed, where the aim is to follow a given temperature trajectory. The duty cycle of the input voltage waveform (provided by the dimmer) is the manipulated variable (u(k)), measured in percentage. The output variable is the mixture temperature T(k), measured in (°C). The reaction curve for u(k) = 100% and initial temperature  $T(0) = 25^{\circ}C$  is shown in Fig. 10.



Figure 9: Experimental setup

The loop performance requirements are defined as the first-order reference model

$$M(s) = \frac{1}{s\tau_c + 1},\tag{33}$$

and the time constant of the reference model is selected during the tuning procedure.

Experimental data are obtained around an operating temperature of  $\overline{T} = 90^{\circ}C$ . The data set  $\mathcal{D}$  for the controller tuning is registered with a sampling time Ts = 2[s]. A Pseudo-random Binary Sequence (PRBS) with N = 512 samples is used as input u(k). The applied duty cycle and the resulting mixture temperature are depicted in Fig. 11.

The first step in the tuning procedure is to define the parametrization of the controllers. To do that, and at the same time to select a proper time constant  $\tau_c$  for the reference model, the  $\mathcal{H}$ -infinity norm of the model-matching error transfer function is estimated from data employing *Result 1*. Results are depicted in Fig. 12 for increasing values of the FIR filter length  $m_q$  and reference model time constant  $\tau_c$ . It can be seen that filter lengths lower than 5 lead to system with high model-matching errors, while for  $m_q = 5$  the resulting norm of the error is lower than 0.5 for time constants of the reference model higher than 8. From the previous analysis  $m_q = 5$  and  $\tau_c = 8[s]$  are selected to tune the temperature controller. In this case  $||\Delta \hat{M}(j\omega)||_{\infty} = 0.44$ , indicating that the resulting closed-loop is stable.

For the CbT controller the extended PID parametrization in Eq. (31) is employed. Different numbers of basis functions were evaluated and the best results were obtained with m = 7.



Figure 10: Reaction curve in OAHD for u(k) = 100% and  $T(0) = 22^{\circ}C$ 

Both methods are applied to the data set shown in Fig. 11. The resulting controllers provided by the proposed YK-DDC approach and by the CbT method are shown in Eqs. (34) and (35), respectively. Note that both controllers have the same structure and are of the same order.

Each controller is evaluated experimentally in closed-loop, considering a piecewise constant temperature reference signal. The results for the YK-DDC controller are shown in Fig 13, while the response with the CbT controller are depicted in Fig. 14. In both figures, the duty cycle u(k) is also shown.

Fig. 15 shows simultaneously the step-response of both controllers for a temperature change of 1°C with different initial temperatures  $\{89^{\circ}C, 90^{\circ}C, 91^{\circ}C\}$ . It can be highlighted that the rise time with the YK-DDC controller is  $t_r = 35[s]$ while the response with the CbT controller shows a rise time  $t_r > 100[s]$ . The results indicate that the performance obtained via the YK-DDC method is faster than the one achieved with the CbT approach, even though both methods use the same reference models, data sets and lead to the same controller structure.

# 7. Conclusions

In this work we have presented a solution to the controller design from data problem, based on a Youla-Kucera parametrization of the controller. Departing from a set of input-output data measured from a stable, linear, time-invariant, SISO system, we have proposed a procedure to estimate a Finite Impulse Response filter that parametrizes a controller without requiring the plant model.



Figure 11: Duty cycle (blue) and mixture temperature (red) data employed in the temperature controller design.

$$K_{YK} = \frac{64.2 - 71.9z^{-1} + 4.2z^{-2} + 2.35z^{-3} + 10.3z^{-4} - 43.7z^{-5} + 31.4z^{-6}}{1 - z^{-1}}$$
(34)

$$K_{cbt} = \frac{104.9 - 131z^{-1} + 34.2z^{-2} - 22.7z^{-3} + 36.9z^{-4} - 32.4z^{-5} + 12.67z^{-6}}{1 - z^{-1}} \quad (35)$$

Open and closed loop data can be handled by the methodology using instrumental variables for the estimation of the filter. The proposed parametrization allows to impose reference models more stringent that those achievable with extended PID controller structures, usually employed in controller design from data. The presented method translates the controller design process into an errors-in-variables identification problem and the solution is obtained by leastsquares estimation.

An *a-posteriori* test has been proposed to estimate the size of the modelmatching error of the resulting control loop. It allows to properly select the only tuning parameter of the method, the FIR filter length, and also to establish if the requested performance can be achieved by the closed-loop system given the available data set. It also provides an estimation of the risk of obtaining an unstable loop.

The performance of the solution has been illustrated by means of a Monte Carlo simulation, showing that the proposed solution allows to obtain better performance and stability margins than the CbT approach. The YK-DDC method has been employed also to design a temperature controller in the camera of a Ohmic distiller. The methodology allowed to obtain a stabilizing controller with faster response than a controller derived with the CbT approach, considering the same reference model, experimental data and controller structure. In particular, the rise time is three times lesser, maintaining null overshot and null



Figure 12:  $||\Delta M(e^{j\omega})||_{\infty}$  for increasing values of filter length  $m_q$  and reference model time constant  $\tau_c$ .

steady-state error.

Current research is focused to extend the method to nonlinear and multi-variable systems, to derive less conservative stability conditions and to improve noise handling.

# **CRediT** authorship contribution statement

**F. Valderrama**: Conceptualization, Methodology, Software, Investigation, Writing. **F. Ruiz**: Conceptualization, Methodology, Validation, Writing, Supervision.

# Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgments

The work of F. Valderrama was supported by a doctoral scholarship from Gobernación de Boyacá-Colombia, call 733-Colciencias). The authors thank Pontificia Universidad Javeriana, Bogotá, Colombia, were F. Valderrama was PhD student while part of this work was developed.



Figure 13: Experimental results with the YK-DDC controller in the OAHD process. Reference temperature (Thick Black line). Mixture temperature (Dashed green line). Duty cycle (Thin orange line).

# Appendix A

# Proof of Lemma 1:

The model matching error signal is

$$\begin{aligned} e(k,\theta) &= & M(z)u(k) - Q(z,\theta)y(k) \\ &= & M(z)(T_u(z)r(k) + H_u(z)v(k)) - Q(z,\theta)(T_y(z)r(k) + H_y(z)v(k)) \end{aligned}$$

Then, the cross-correlation function between e(k) and x(k) becomes:

$$R_{ex}(\tau) = \mathbb{E}\left[\left(\left(M(z)T_u(z) - Q(z,\theta)T_y(z)\right)r(k) + \dots + \left(M(z)H_u(z) - Q(z,\theta)H_y(z)\right)v(k)\right)W(z)r(k-\tau)\right]$$

From Assumption 3, v(k) and r(k) are independent, then,

$$Rex(\tau) = \mathbb{E}\left[\left(M(z)T_u(z) - Q(z,\theta)T_y(z)\right)W(z)r(k)r(k-\tau)\right]$$

From the definitions of  $T_u(z)$  and  $T_y(z)$ , it follows that,

$$R_{ex}(\tau) = \mathbb{E}\left[\left(M(z) - Q(z,\theta)P(z)\right)T_u(z)W(z)r(k)r(k-\tau)\right]$$

From Parseval's theorem, the 2-norm of the correlation function is

$$||Rex(\tau)||_{2}^{2} = \sum_{\tau=-\infty}^{\infty} R_{ex}^{2}(\tau)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \left[ M(e^{j\omega}) - Q(e^{j\omega}, \theta) P(e^{j\omega}) \right] T_{u}(e^{j\omega}) W(e^{j\omega}) \right|^{2} \Phi_{r}^{2}(j\omega) d\omega$$
(36)



Figure 14: Experimental results with the CbT controller in the OAHD process. Reference temperature (Thick Black line). Mixture temperature (Dashed green line). Duty cycle (Thin orange line).

proving the first part of the Lemma.

Finally, for  $|W(j\omega)| = |T_u(e^{j\omega})\Phi_r(j\omega)|^{-1}$ , it follows that

$$||R_{ex}(\tau)||_{2}^{2} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |M(e^{j\omega}) - Q(e^{j\omega}, \theta)P(e^{j\omega})|^{2} d\omega = J_{MR}(\theta)$$

## References

- S. Formentin, K. van Heusden, A. Karimi, A comparison of model-based and data-driven controller tuning, International Journal of Adaptive Control and Signal Processing 28 (10) (2014) 882-897. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1002/acs.2415, doi:10.1002/acs.2415.
   URL https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.2415
- [2] K. Van Heusden, A. Karimi, T. Söderström, On identification methods for direct data-driven controller tuning, International Journal of Adaptive Control and Signal Processing 25 (5) (2011) 448–465.
- [3] Z.-S. Hou, Z. Wang, From model-based control to data-driven control: Survey, classification and perspective, Inf. Sci. (Ny). 235 (2013) 3–35. doi:10.1016/j.ins.2012.07.014.



Figure 15: Closed-loop step responses. YK-DDC controller (Green dashed line). CbT controller (Red thin line). Reference model (M) (Blue dashed line with circle markers). Temperature reference (Black thick line).

- [4] F. Valderrama, F. Ruiz, Controller design from data under UBB noise, Proc. Am. Control Conf. (2014) 5103–5108doi:10.1109/ACC.2014.6858626.
- [5] F. Valderrama, F. Ruiz, A comparative study of vrft and set membership data-driven controller design techniques: Active suspension tuning case, in: Data-Driven Modeling, Filtering and Control: Methods and applications, IET, 2019, Ch. 10, pp. 266–290.
- [6] V. Cerone, D. Regruto, M. Abuabiah, Direct data-driven control design through set-membership errors-in-variables identification techniques, in: Proc. Am. Control Conf., 2017, pp. 388–393. doi:10.23919/ACC.2017.7962984.
- [7] P. Kergus, M. Olivi, C. Poussot-Vassal, F. Demourant, From reference model selection to controller validation: Application to loewner datadriven control, IEEE Control Systems Letters 3 (4) (2019) 1008–1013. doi:10.1109/LCSYS.2019.2920208.

- [8] F. Valderrama, F. Ruiz, Limited-complexity controller tuning: A set membership data-driven approach, European Journal of Control (2020). doi:https://doi.org/10.1016/j.ejcon.2020.07.002.
- [9] F. Valderrama, F. Ruiz, D. Patino, Evaluation of set-membership approaches for data-driven tuning of two-degree-of-freedom controllers, in: 2019 American Control Conference (ACC), 2019, pp. 5668–5673. doi:10.23919/ACC.2019.8814372.
- [10] K. van Heusden, A. Karimi, D. Bonvin, Data-driven controller validation, IFAC Proceedings Volumes 42 (10) (2009) 1050 – 1055, 15th IFAC Symposium on System Identification. doi:https://doi.org/10.3182/20090706-3-FR-2004.00174.
- [11] L. C. Kammer, R. R. Bitmead, P. L. Bartlett, Direct iterative tuning via spectral analysis, Automatica 36 (9) (2000) 1301 – 1307. doi:https://doi.org/10.1016/S0005-1098(00)00040-6.
- [12] A. Sala, A. Esparza, Extensions to "virtual reference feedback tuning: A direct method for the design of feedback controllers", Automatica 41 (8) (2005) 1473–1476. doi:10.1016/j.automatica.2005.02.008.
- [13] A. Lanzon, A. Lecchini, A. Dehghani, B. D. O. Anderson, Checking if controllers are stabilizing using closed-loop data, in: Proceedings of the 45th IEEE Conference on Decision and Control, 2006, pp. 3660–3665. doi:10.1109/CDC.2006.377549.
- [14] K. van Heusden, A. Karimi, D. Bonvin, Data-driven model reference control with asymptotically guaranteed stability, International Journal of Adaptive Control and Signal Processing 25 (4) (2011) 331–351. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1002/acs.1212, doi:10.1002/acs.1212.
- [15] G. Battistelli, D. Mari, D. Selvi, P. Tesi, Direct control deunfalsification, International Journal sign via controller of Robust and Nonlinear Control 28(12)(2018)3694-3712. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1002/rnc.3778, doi:10.1002/rnc.3778.
- [16] V. Kučera, A method to teach the parameterization of all stabilizing controllers, IFAC Proceedings Volumes 44 (1) (2011) 6355 – 6360, 18th IFAC World Congress. doi:https://doi.org/10.3182/20110828-6-IT-1002.01148.
- [17] F. Blanchini, P. Colaneri, Y. Fujisaki, S. Miani, F. A. Pellegrino, A youla-kučera parameterization approach to output feedback relatively optimal control, Systems and Control Letters 81 (2015) 14 – 23. doi:https://doi.org/10.1016/j.sysconle.2015.04.006.
- [18] L. Ljung, System Identification: Theory for the User, Prentice Hall, 1987.

- [19] J. C. Doyle, B. A. Francis, A. R. Tannenbaum, Feedback Control Theory, Prentice Hall Professional Technical Reference, 1991.
- [20] T. Soderstrom, P. Stoica, Instrumental variable methods for system identification., Berlin:Springer, 1983.
- [21] M. C. Campi, S. M. Savaresi, Direct nonlinear control design: the virtual reference feedback tuning (VRFT) approach, IEEE Transactions on Automatic Control 51 (1) (2006) 14–27.
- [22] L. Ljung, T. Chen, Convexity issues in system identification, in: 2013 10th IEEE International Conference on Control and Automation (ICCA), 2013, pp. 1–9. doi:10.1109/ICCA.2013.6565206.
- [23] K. van Heusden, A. Karimi, D. Bonvin, Data-driven estimation of the infinity norm of a dynamical system, in: 2007 46th IEEE Conference on Decision and Control, 2007, pp. 4889–4894. doi:10.1109/CDC.2007.4434184.
- [24] A. Karimi, K. van Heusden, D. Bonvin, Non-iterative data-driven controller tuning using the correlation approach, in: 2007 European Control Conference (ECC), 2007, pp. 5189–5195. doi:10.23919/ECC.2007.7068802.
- [25] K. van Heusden, A. Karimi, D. Bonvin, Data-driven controller validation, IFAC Proceedings Volumes 42 (10) (2009) 1050 – 1055, 15th IFAC Symposium on System Identification. doi:https://doi.org/10.3182/20090706-3-FR-2004.00174.
- [26] M. Seidi Damyeh, M. Niakousari, M. T. Golmakani, M. J. Saharkhiz, Microwave and ohmic heating impact on the in situ hydrodistillation and selective extraction of satureja macrosiphonia essential oil, Journal of Food Processing and Preservation 40 (4) (2016) 647–656. doi:https://doi.org/10.1111/jfpp.12644.
- [27] M. Seidi Damyeh, M. Niakousari, Impact of ohmic-assisted hydrodistillation on kinetics data, physicochemical and biological properties of prangos ferulacea lindle. essential oil: Comparison with conventional hydrodistillation, Innovative Food Science and Emerging Technologies 33 (2016) 387 – 396. doi:https://doi.org/10.1016/j.ifset.2015.12.009.