On Optimal Infrastructure Sharing Strategies in Mobile Radio Networks

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***Abstract*— The rapid evolution of mobile radio network technologies poses severe technical and economical challenges to mobile network operators (MNOs); on the economical side, the continuous roll-out of technology updates is highly expensive, which may lead to the extreme, where offering advanced mobile services becomes no longer affordable for MNOs which thus, are not incentivized to innovate. Mobile infrastructure sharing among MNOs becomes then an important building block to lower the required per-MNO investment cost involved in the technology roll-out and management phases. We focus on a radio access network (RAN) sharing situation where multiple MNOs with a consolidated network infrastructure coexist in a given set of geographical areas; the MNOs have then to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators. We address such strategic problems by giving a mathematical framework for the RAN infrastructure sharing problem which returns the "best" infrastructure sharing strategies for operators (coalitions and network configuration) when varying techno-economic parameters such as the achievable throughput in different sharing configurations and the pricing models for the service offered to the users. The proposed formulation is then leveraged to analyze the impact of the aforementioned parameters/input in a realistic mobile network environment based on LTE technology.**

***Index Terms*— RAN sharing, heterogenous networks, 4G, mathematical programming, game theory.**

I. Introduction

MOBILE telecommunication networks and services have been characterized by a dramatic uptake in the past two decades which is still to be over. According to [1], the penetration of mobile subscriptions has reached the amazing level of 96% worldwide in 2014, and the traffic delivered through mobile radio networks is expected to reach 49 Exabytes/month by 2021 [2] with a considerable share taken by bandwidth-eager services provided by aggressive Over The Top service providers.

To cope with such fast growing rate, the mobile networks have undergone, and are still undergoing, several technology migration phases cruising from the introduction of third generation (3G) and 3.5G wireless technologies on top of 2G networks to the standardization and deployment of the Long Term Evolution (LTE) with the recent launch of 5G initiatives [3]. The effect of such rapid evolution in the mobile networks technologies poses several technical and economical challenges to Mobile Network Operators (MNOs). On the technical side, the coexistence of multiple technologies in the Radio Access Network (RAN) calls for advanced radio resource orchestration procedures to cope with such heterogeneity. On the economical side, the combined effect of revenues of MNOs that tend to flatten and the network technology updates that are highly expensive may lead to the extreme where offering advanced mobile services becomes no longer affordable for MNOs which are not incentivized to innovate and migrate to new technologies [4].

In this context, the conventional model according to which each MNO retains complete control and ownership of its network is at odds with the large and frequent investments requested on the network infrastructure, and with the increased complexity in the management of the network components. Mobile infrastructure sharing among MNOs thus becomes an important building block to "break" such vertical and inflexible approach, by lowering the required per-MNO investment cost to cope in the technology roll-out and management phases.

Different forms of infrastructure sharing are already in place, ranging from basic unbundling and roaming, to site and spectrum sharing [5]. In these "classical" forms of sharing generally one MNO still retains ownership of the mobile network. On the other hand, we focus here on a RAN sharing scheme in which MNOs share a single radio infrastructure while maintaining separation and full control over the back hauling and respective core networks. In this work, we consider a scenario where multiple MNOs with a consolidated macro cells network infrastructure and consolidated market shares coexist in a given set of geographical areas; the MNOs have to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators.

We address such strategic problem by providing a mathematical framework for the analysis of the RAN infrastructure sharing problem that takes into account both technical and economical aspects and provides the optimal sharing strategies for MNOs, that include coalitions with other MNOs and network configuration. The proposed infrastructure sharing problem is first tackled from the perspective of a regulatory

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entity that can impose sharing configurations maximizing the quality of service perceived by all users and then from a single MNO perspective, in order to account for MNOs as profit-maximizing selfish entities. A Mixed Integer Linear Programming (MILP) formulation is proposed to determine sharing configurations maximizing the quality of service; this formulation includes techno-economic parameters such as the achievable throughput and the pricing models for the service offered to the users. For representing the MNO perspective, we propose a Non Transferable Utility (NTU) coalitional game model. The proposed mathematical framework is then leveraged to analyze the impact of the aforementioned parameters in a realistic mobile network environment based on LTE technology for which numerical values for technical and economic parameters are available. Note however that the proposed approach is general and can be easily applied to other scenarios with different small cell technologies.

The manuscript is organized as follows: Sec. II reviews the mainstream literature in the field of infrastructure sharing highlighting the main novelties of the proposed approach. In Sec. III, we introduce the reference scenario describing the techno-economic parameters involved in the infrastructure sharing problem and the proposed mathematical framework that allows to represent the problem from the two considered perspectives. Sec. IV describes the considered scenarios and cases while results and insights are reported in Sec. V, where the strategic behavior of MNOs in several different realistic scenarios is analyzed. Our concluding remarks are given in Sec. VI.

II. Related Work

The literature on infrastructure/resource sharing can be grouped in two main research tracks: (i) works dealing with techno-economic modeling of network sharing and (ii) works on practical algorithms for management and allocation of shared network resources. The first track mostly includes qualitative and quantitative studies of different sharing scenarios and models for estimating capital and operational expenditures. Particular attention is dedicated to the identification of drivers and barriers to network sharing or possible new organization of the mobile network value chain for sharing to be viable.

Meddour et al. [6] suggest guidelines for MNO involved in the sharing process and emphasize the need for subsidization and assistance from regulatory entities. Similarly, Beckman et. al [7] show that the role of regulatory entities is crucial to avoid the decline of market competition.

A recent work by Di Francesco *et al.* [8] introduces a competition-aware network sharing framework in the context of cellular network planning which allows to balance the cost benefit of sharing and the push toward next-generation technologies.

The authors of [9] model the capital and operational expenditures for different levels of sharing and suggest outsourcing as the solution to the challenges posed by network sharing. In [10], the authors propose a benchmark-based model that provides high-quality cost estimates for alternative delivery options of the MNO processes such as “regionalization”, “centralization” and “outsourcing”. Vaz *et al.* propose a framework to evaluate the performance of heterogeneous network deployment patterns in terms of net present value, capacity, coverage, and carbon footprint [11]. By means of a techno-economic analysis, the work in [12] addresses the cost/revenue viability of different WLAN value network configurations in the presence of MNOs and Service Application Providers and the use cases for which there is incentive to share.

In the field of strategic modeling of resource/infrastructure sharing, it is worth mentioning the works resorting to game theory. Malanchini *et al.* [13] resort to non-cooperative games to model the problems of network selection, when users can choose among multiple heterogeneous wireless access, and of resource allocation in which mobile network operators compete to capture users by properly allocating their radio resources. In [14], spectrum sharing among selfish MNOs in unlicensed bands is modeled as a non-cooperative game. The work in [15] and more extensively in [16] also use a non-cooperative game to model the strategic decision of a MNO regarding sharing its LTE infrastructure in a non–monopolistic telecom market. Another example of 4G infrastructure sharing is given in [17] which considers sharing LTE access network femtocells with other access technologies such as Wi–Fi. Cooperative game theory is used in [18] and [19]; in [18], the resource allocation problem in a shared network is formalized in a two step problem: resource sharing among the operators and resource bargaining among the users and Mobile Virtual Network Operators of each operator; the work in [19] considers not only sharing among MNOs but also among operators of different wireless access technologies.

The research track on practical aspect of resource/ infrastructure sharing focuses on algorithms and architectures for managing shared resources. The work in [20] suggests that radio resource management is handled by a third-party service provider or an inter-connection provider to preserve competition and reduce exposure. Anchora *et al.* ([21]) introduce a ns-3 implementation to assess the performance of spectrum sharing in a LTE multi-node/multi-MNO scenario, where a virtual central entity is responsible for applying the sharing policies to the common frequency pool. In [22], virtualization of the wireless medium (spectrum sharing) is proposed to exploit spectrum multiplexing and multi-user diversity while allowing MNOs to remain isolated. Instead, the authors in [23] introduce the Network without Borders concept as a pool of virtualized wireless resources with a shared radio resource manager. Along the same lines, Rahman *et al.* ([24]) introduce a novel architecture based on wireless access network virtualization, where the key tenet is to offload the baseband process from physical base station to backend devices; in this way, the physical base stations can be *sliced* into virtual base stations. In [25], instead, a 2-level radio resource scheduling (among MNOs and for each MNO among its user flows) BS virtualization scheme satisfying the 3GPP SA1 RSE ([26]) requirements has been proposed. The work in [27] proposes the necessary LTE architectural enhancements to adopt capacity, spectrum and hardware sharing, and provides a simulation-based comparative performance analysis of the proposed sharing scenarios and of no sharing case.

Johansson [28] provides an algorithm for fair allocation of the shared radio resource among multiple operators.

The aforementioned literature work either abstracts away technical aspects related to the mobile network performance to focus on more economic-oriented analysis and modeling, or the other way around. In our previous work [29], we focus on infrastructure sharing in a single and homogeneous geographical area. To the best of our knowledge, ours is one of the first attempts to strike a better balance between these two aspects of the sharing problem, by quantitatively modeling the relation between technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues) in mobile network infrastructure sharing. In this work we provide a more general framework which captures large-scale sharing scenarios featuring multiple geographical areas. Further, we consider two different perspectives: the single decision maker one, where the decision maker is a regulatory authority, and the multiple decision makers perspective, that accounts for the single MNO point of view.

III. Modeling the Problem of Mobile Network Infrastructure Sharing

We decided to explore two alternative infrastructure sharing configurations: *socially optimal* configurations providing the best service level for the users, which can be imposed by a regulatory authority[1] and *stable* configurations representing a setting where MNOs act as selfish entities aiming to maximize their profits from upgrading their network. While a centralized approach allows to model the problem of determining *socially optimal* configurations, cooperative game theory is more suitable to determine *stable* configurations. In Section III-A, we introduce the techno-economic parameters representing the considered scenario and provide an MILP formulation for the centralized approach. In Section III-B, we discuss how an NTU cooperative game is adopted to determine *stable* configurations. We remark that in Sections III-A and III-B, we use the term coalition with a slight abuse of terminology to represent a set of MNOs which build a unique shared network, both when they decide to join the coalition based on their profit and when the coalition is suggested as a socially optimal choice. In III-A, the *socially optimal* coalitional structure (partition of the set of MNOs) is selected according to the regulator point of view and each MNO is assigned to its corresponding coalition. Instead, in III-B, each MNO joins the coalition that maximizes its individual profit; in other words, a coalition is *stable* when none of its members has an incentive to leave the coalition.

A. Socially Optimal Coalitional Structures-an MILP Formulation

We consider a set $\mathcal{O}$ of MNOs who have up and running 3G/4G networks over a set $\mathcal{A}$ of dense urban areas: each area $a \in \mathcal{A}$ is populated by N_a users and has a size A_a. Parameter σ_i gives the share of users of MNO $i \in \mathcal{O}$ which is assumed to be equal in each area. The MNOs may consider investing to deploy additional LTE small-cells (HetNets) in some or all the areas. A MNO can either invest by itself or share the investment (and the deployed infrastructure) with a subset (or all) of the other MNOs. Let $\mathcal{S}$ denote the set of all possible coalitions that can be activated for the given set of MNOs (here we consider all possible non-empty subsets, thus $|\mathcal{S}|$ is equal to $2^{|\mathcal{O}|} - 1$). If a MNO invests by itself, the coalition is referred to as *singleton*. $\mathcal{S}_i$ is the set of coalitions MNO i can be part of. Each MNO inherits the customer base from its current network, assuming that users do not change their MNO but may subscribe to a new (LTE) data plan.

We consider the problem of determining the *socially optimal* sharing configurations, that is, how to partition MNOs in coalitions and how many small-cell base stations (BSs) each coalition of MNOs should activate in order to maximize the global service level provided to the users.

In each area a maximum number U_{max} of BSs can be activated by all coalitions.

Users are characterized by parameter δ that represents their willingness to pay for 1 Mbps of LTE rate on a monthly basis and therefore the monthly price of 1 Mbps.

We consider an investment lifetime D (in months). The investment costs are then calculated over the whole D period. Both capital (*e.g.*, site and BS acquisition) and operational (*e.g.*, hardware and software maintenance, land renting and power supply) expenditures contribute to the overall costs of the infrastructure [6].

Let g_{capex} and g^{α}_{opex} denote the fixed CAPEX and annual OPEX components, respectively. g^{α}_{opex} is calculated as a fixed percentage (ξ) of g_{capex}, i.e., $g^{\alpha}_{opex} = \xi g_{capex}$. We denote by g the cost of a single BS for the investment lifetime D which is determined as the sum of the fixed initial CAPEX and the OPEX accumulated during D, i.e.,

$$g = g_{capex} + \frac{1}{12} D g^{\alpha}_{opex}. \quad (1)$$

The BSs installation cost of a coalition is then divided among the coalition members based on their market shares.

We assume that the same coalitional structure will apply to all areas, that is, MNOs will be assigned to the same coalition in all areas, as it can be easier for MNOs to coordinate with the same set of MNOs in all the areas.[2] Table I recaps the problem's parameters notation.

The partitioning of the set of MNOs $\mathcal{O}$ into a *socially optimal* coalitional structure is modeled as follows. Binary variables y_s represent the coalition activation: y_s equals one if coalition s is activated in all the areas $a \in \mathcal{A}$ and it invests (deploys BSs) in at least one of them. The binary variable x_{is} is equal to one if MNO i is assigned to coalition $s \in \mathcal{S}_i$ and s

[1] It is usually the case that infrastructure sharing agreements are analyzed on a per-case basis by a regulatory authority aiming to assess the impact of such sharing agreements on the users; at the limit, regulators could impose configurations that provide the best service level for the users.

[2] In the case of *stable* sharing configurations, as MNOs decide by themselves which coalition to join, selecting the same coalition (set of collaborating MNOs) in all the areas might also require less time for the sharing agreements to be approved by regulators. Nevertheless, we have also investigated the case in which MNOs are assigned/select a different coalition in each area, which overall does not provide significant gains with respect to forcing the same one over all areas.

invests, it equals zero if i is assigned to any other coalition in $\mathcal{S}_i$ but s or s does not invest. Constraints (2) guarantee that each MNO i is assigned to at most one coalition from $\mathcal{S}_i$. Constraints (3) make sure that if s is activated ($y_s = 1$), all MNOs $i \in s$ are assigned to s.

$$\sum_{s \in \mathcal{S}_i} x_{is} \leq 1, \quad \forall\, i \in \mathcal{O}, \tag{2}$$

$$x_{is} = y_s, \quad \forall\, s \in \mathcal{S},\ \forall\, i \in s. \tag{3}$$

If coalition s is activated, it will deploy a certain number of BSs for each area $a \in \mathcal{A}$, represented by a non-negative integer variable u_s^a. If s is not activated or there is no investment ($y_s = 0$), the corresponding variables u_s^a, for each $a \in \mathcal{A}$, are forced to zero by means of Constraints (4). Conversely, a coalition is not active ($y_s = 0$) if it does not deploy any BS in any of the areas (Constraint (5)); Constraint (6) limits the overall number of BSs deployed by all coalitions in each area.

$$u_s^a \leq U_{max} y_s, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}, \tag{4}$$

$$y_s \leq \sum_{a \in \mathcal{A}} u_s^a, \quad \forall\, s \in \mathcal{S}, \tag{5}$$

$$\sum_{s \in \mathcal{S}} u_s^a \leq U_{max}, \quad \forall\, a \in \mathcal{A}. \tag{6}$$

We assess the quality of service provided by MNOs through the average rate perceived by the users, which is an important indicator of the users' level of satisfaction. This rate is different for each area $a \in \mathcal{A}$: firstly because we consider areas with different number of users (N_a) and size (A_a) and secondly because a different number of BSs (u_s^a) may be deployed in different areas. In the proposed model, we define two types of LTE user rate, namely nominal and average, for each coalition $s \in \mathcal{S}$. The nominal user rate is the maximum achievable LTE rate for a certain level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth[3] that a user perceives when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR depends on the number of BSs activated by the coalition the user belongs to since a larger number of BSs results in the user being on the average closer to its serving BS, and thus receiving a stronger signal, but also closer to the interfering BSs.[4] Thus, the nominal user rate of coalition s in area a, represented by a non-negative continuous variable $\rho_s^{a,nom}$, is a function of the number of deployed BSs u_s^a. The behavior of $\rho_s^{a,nom}$ as a function of u_s^a is investigated by simulating the deployment of the small cell BSs (see Subsec. IV-A).

Instead, the average user rate perceived by a user of coalition s in area a is represented by the continuous non-negative variables ρ_s^a and defined in terms of the nominal user rate ($\rho_s^{a,nom}$) and of the load of its serving BS as follows[5]:

$$\rho_s^a = \rho_s^{a,nom} (1-\eta)^{\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}}, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A},$$

where parameter η is the user activity factor, that is, the probability that a user is actually active in his/her serving BS, $\sum_{i \in s} \sigma_i N_a$ is the total number of users that are served by members of coalition s in area a, and the ratio $\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}$ is the average number of users served by one BS in area a. As a result, $\rho_s^{a,nom}$ is scaled down by the factor $(1-\eta)^{\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}}$ which accounts for the average congestion level at a serving BS in a.

In the MILP formulation, the nonlinearity of ρ_s^a in terms of u_s^a is handled by approximating ρ_s^a with a piecewise linear function described by the following constraints:

$$z_s^a \leq u_s^a, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}, \tag{7}$$

$$\rho_s^a \leq R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l}) + M(1 - z_s^a), \tag{8}$$

$$\forall\, s \in \mathcal{S}, \quad \forall\, a \in \mathcal{A},\ \forall\, l \in \{0, \ldots, L-1\},$$

$$\rho_s^a \leq R_s^{a,L} z_s^a, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}, \tag{9}$$

where z_s^a are binary variables that equal 0 if $u_s^a = 0$ (Constraints (7)) and therefore set to zero ρ_s^a when $u_s^a = 0$ (Constraints (9)). Constraints (8) model the piecewise linear functions approximating ρ_s^a, for any $s \in \mathcal{S}, a \in \mathcal{A}$, where L denotes the number of the linear pieces, $\alpha_s^{a,l}$ denotes the slope of the l-th linear piece, $U_s^{a,l}$ and $R_s^{a,l}$ are the coordinates (number of BSs and user rate, respectively) of the $(l+1)$ breakpoint whereas M is a big positive constant (see Appendix A for the details of the approximation).

Assuming that, in each area a, users of any member of coalition s can be served by any of the BSs activated by s in a, the average user rate provided by MNO i in area a, represented by continuous non-negative variable q_i^a, is equal to the average user rate of the coalition to which i is assigned, that is,

$$q_i^a = \sum_{s \in \mathcal{S}_i} \rho_s^a, \quad \forall\, i \in \mathcal{O},\ \forall\, a \in \mathcal{A}. \tag{10}$$

As for the investment cost and revenues for the MNOs, it is reasonable to model the revenue[6] per MNO i in area a as a continuous non-negative variable r_i^a which is linearly dependent on the MNO's user rate q_i^a in that area as shown in (11): δq_i^a is the monthly revenue obtained from one user, which is then multiplied by the investment lifetime D and the number of users $\sigma_i N_a$ of MNO i in area a:

$$r_i^a = \delta D \sigma_i N_a q_i^a, \quad \forall\, i \in \mathcal{O},\ \forall\, a \in \mathcal{A}. \tag{11}$$

The cost incurred by MNO i in area a, represented by non-negative continuous variable c_i^a, is a linear function of the number of BSs activated in a by the coalition to which i is

[3]We consider a 10 Mhz bandwidth in our simulations whether the BS is shared or not.

[4]Since we are considering a nominal rate, any other BS transmission will use a subset or all the resource blocks and therefore unavoidably interfere.

[5]We note that this equation is defined for $u_s^a > 0$, while we set $\rho_s^a = 0$ when $u_s^a = 0$.

[6]The price per unit of service (δ) represents the highest price all current users of each MNO are willing to pay for the new service. Therefore the number of users N is assumed independent of δ. Moreover, the proposed pricing model aims at translating the MNOs level of investment, which affects the service level perceived by users, into revenues. It is outside of the scope of the analysis we propose here to account for pricing models in line with those currently applied by MNOs which involve bundles of services, data caps etc. In the same lines, we do not account for the user migration among MNOs since it is generally determined by "non-technical" parameters such as special tariffs, bundle offers, brand fidelity and more in general marketing strategies.

TABLE I
SETS, PARAMETERS, AND CORRESPONDING VALUES

Symbol	Description	Value
$\mathcal{O}$	Set of MNOs	$\{$A,B,C$\}$, $\lvert\mathcal{O}\rvert$=3
$\mathcal{A}$	Set of Areas	$\{$Z1,Z2,Z3$\}$
$\mathcal{S}$	Set of coalitions	$\{$A,B,C,AB,AC,BC,ABC$\}$
$\mathcal{S}_i$	Set of coalitions MNO $i\in\mathcal{O}$ can join	$\{s\in\mathcal{S} \mid i\in s\}$
N_a	Number of users of area $a\in\mathcal{A}$	See Table IV
A_a	Size of area $a\in\mathcal{A}$	See Table IV
σ_i	Market share of MNO $i\in\mathcal{O}$	M_1:$\{$1/3,1/3,1/3$\}$, M_2:$\{$0.1,0.3,0.6$\}$
U_{max}	Max. number of BSs in the area	4000
δ	Monthly price of 1 Mbps	Equidistant values in [0.02,2]€/Mbps
D	Investment lifetime [months]	120 ([30],[28])
η	User activity factor	0.001
ξ	OPEX annual %	15% [31]
g_{capex}	CAPEX of BS cost	3000€
g	BS cost normalized for D	7500€

TABLE II
VARIABLE DOMAINS AND DESCRIPTION

Variable	Description
$x_{is}\in\{0,1\}$	1 if MNO $i\in\mathcal{O}$ joins coalition $s\in\mathcal{S}_i$ in all areas, 0 otherwise
$y_s\in\{0,1\}$	1 if coalition $s\in\mathcal{S}$ is created in all areas, 0 otherwise
$u_s^a\in\mathbb{Z}_+$	Number of BSs activated by coalition $s\in\mathcal{S}$ in area $a\in\mathcal{A}$
$z_s^a\in\{0,1\}$	1 if coalition $s\in\mathcal{S}$ activates at least one BS in area $a\in\mathcal{A}$, 0 otherwise
$\rho_s^{a,nom}\geq 0$	Nominal user rate for coalition $s\in\mathcal{S}$ in area $a\in\mathcal{A}$
$\rho_s^a\geq 0$	User rate for coalition $s\in\mathcal{S}$ in area $a\in\mathcal{A}$
$q_i^a\geq 0$	User rate for MNO $i\in\mathcal{O}$ in area $a\in\mathcal{A}$
$c_i^a\geq 0$	Costs of MNO $i\in\mathcal{O}$ in area $a\in\mathcal{A}$
$r_i^a\geq 0$	Revenues of MNO $i\in\mathcal{O}$ in area $a\in\mathcal{A}$

assigned, divided among the coalition's members proportionally to their number of users:

$$c_i^a = \sum_{s\in\mathcal{S}_i} g \frac{\sigma_i}{\sum_{j\in s}\sigma_j} u_s^a, \quad \forall\ i \in O,\ \forall\ a \in \mathcal{A}. \tag{12}$$

Although the *socially optimal* infrastructure sharing configurations provide the optimal service level for users, MNOs cannot be forced to undertake lossy investments. Therefore, Constraints (13) make sure that each MNO obtains a non-negative profit:

$$\sum_{a\in\mathcal{A}} (r_i^a - c_i^a) \geq 0, \quad \forall\ i \in O. \tag{13}$$

We consider two candidate objective functions to be maximized to determine the *socially optimal* sharing configurations:

$$\sum_{i\in O, a\in\mathcal{A}} q_i^a, \tag{14a}$$

$$\min_{i\in O, a\in\mathcal{A}} q_i^a. \tag{14b}$$

Objective (14a) favors efficiency by maximizing the sum of user rate over all MNOs and areas, whereas (14b) maximizes the smallest user rate (over all areas and MNOs), so as to privilege users' fairness. We denote Objectives (14a) and (14b) by TOT_Q and MIN_Q, respectively and use this notation throughout Section V. Sets and parameters describing the instances are recapped in Table I whereas variables in Table II. In Appendix B, we prove that the decision version of the problem with objective MIN_Q is NP-complete.

B. Stable Coalitional Structures - A Non Transferable Utility Cooperative Game Model

We now describe the problem of determining *stable* infrastructure sharing configurations. We assume that MNOs in a coalition will share their cost while each MNO will keep its individual revenue since the latter is incurred from its own share of users. As a result, the coalition worth, that is, the difference between the coalition global revenues and cost, cannot be redistributed among its members: therefore we adopt solution concepts of NTU cooperative games [32].

The game is formalized as a pair (O, V), where the player set O coincides with the set of MNOs and V is a function that associates to each non-empty coalition $s \in \mathcal{S}$ a subset of payoff allocation vectors $(\pi_i)_{i\in O}$, i.e.,

$$V(s) = \{(\pi_i)_{i\in O} :\ \pi_i \leq p_s^i \quad \forall\ i \in s\},$$

where p_s^i is the optimal payoff of player i in coalition s.

Since each MNO is a self-interested entity that aims to maximize its individual profits from the investment, we define its optimal payoff p_s^i from a given coalition as the largest profit (difference between total revenues and total cost) it can achieve if it becomes part of that coalition. Such payoffs are calculated in the following fashion: given a coalition $s \in \mathcal{S}$, we determine the optimal number of BSs ($\tilde{u}_s^a$) activated in each area $a \in \mathcal{A}$, calculate each member's revenues and costs for each area and therefore calculate the MNO total profit.

The optimal number $\tilde{u}_s^a$ of BSs coalition s can deploy in area a is obtained solving the following problem[7]:

$$\max \sum_{i\in s} r_i^a - c_i^a \tag{15}$$

$$r_i^a = \delta D \sigma_i N_a \rho_s^a, \qquad \forall\ i \in s, \tag{16}$$

$$c_i^a = \frac{\sigma_i}{\sum_{j\in s}\sigma_j} g u_s^a, \quad \forall\ i \in s, \tag{17}$$

$$u_s^a \leq U_{max}, \tag{18}$$

$$z_s^a \leq u_s^a, \tag{19}$$

$$\rho_s^a \leq R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l}) + M(1 - z_s^a), \tag{20}$$

$$\forall\ l \in \{0, \dots, L-1\},$$

$$\rho_s^a \leq R_s^{a,L} z_s^a, \tag{21}$$

$$u_s^a \in \mathbb{Z}_+,\ \rho_s^a \geq 0,\ z_s^a \in \{0,1\}. \tag{22}$$

The objective function (15) can be rewritten as

$$\sum_{i\in s} \left(\delta D \sigma_i N_a \rho_s^a - \frac{\sigma_i}{\sum_{j\in s}\sigma_j} g u_s^a \right)$$

$$= \left(\sum_{i\in s} \sigma_i \right) \left(\delta D N_a \rho_s^a - \frac{1}{\sum_{j\in s}\sigma_j} g u_s^a \right), \tag{23}$$

where ρ_s^a depends on u_s^a. As $\delta D N_a \rho_s^a - \frac{1}{\sum_{j\in s}\sigma_j} g u_s^a$ is independent of the MNOs, the optimal number $\tilde{u}_s^a$ of BSs is the

[7]We remark that, in the problem we upper bound the number of BSs activated by each coalition in the area to U_{max} (Constraint (18)) since, for the considered instances (see Section V), the total number of BSs activated by any partition of MNOs in the set O does not exceed U_{max}, that is, the more stringent Constraint (6) which limits the number of BSs activated by all coalitions in the area to U_{max} is never tight.

same for all the players and can be easily computed solving the above problem.

Therefore, the optimal payoff p_s^i of each MNO $i \in s$ is

$$p_s^i = \sum_{a\in\mathcal{A}} \left(\delta D \sigma_i N_a \rho_s^a(\widetilde{u}_s^a) - \frac{\sigma_i}{\sum\limits_{j\in s}\sigma_j} g\widetilde{u}_s^a \right)$$

$$= \frac{\sigma_i}{\sum_{j\in s}\sigma_j}\sum_{a\in\mathcal{A}}\left(\delta D N_a \rho_s^a(\widetilde{u}_s^a)\sum_{j\in s}\sigma_j - g\widetilde{u}_s^a\right). \quad (24)$$

In other words, the optimal payoff allocations p_s^i correspond to dividing the optimal worth of coalition s, i.e., $\sum_{a\in\mathcal{A}}\left(\delta D N_a \rho_s^a(\widetilde{u}_s^a)\sum_{j\in s}\sigma_j - g\widetilde{u}_s^a\right)$, among its members according to their relative market shares, i.e., $\sigma_i / \sum_{j\in s}\sigma_j$.

In the following we look for *stable* infrastructure sharing configurations. We define a sharing configuration as a partition $(s_1, \ldots, s_p)$ of the MNOs set $\mathcal{O}$, where coalitions $s_1, \ldots, s_p \in \mathcal{S}$. A configuration $(s_1, \ldots, s_p)$ is said *stable* if for any $j = 1, \ldots, p$ there is no nonempty subset $s'_j \subset s_j$ such that

$$p^i_{s'_j} > p^i_{s_j}, \quad \forall\ i \in s'_j,$$

that is, for any coalition s_j no subset of MNOs has incentive to leave it.

IV. Experimental Settings

We run several tests to evaluate how the coalitional structure, the level of investment, and therefore the performance indicators of both the *socially optimal* and *stable* configurations are affected by the user economic standpoint.

The MILP model (Section III-A) and problem (15)–(22) for any $s \in \mathcal{S}$ and $a \in \mathcal{A}$ (Section III-B) have been implemented in AMPL [33]. We have used Gurobi 6.0 [34] as a MILP solver. All tests were run on an Intel(R) Core(TM) i5-3230M CPU @2.6 Ghz. To keep the computational time limited, for some of the instances the acceptable relative MIP gap of Gurobi was set equal to 1e-6. When optimizing MIN_Q, several equivalent optimal solutions may be found, which may not provide consistent values for the user rate of the non-bottleneck areas and MNOs. When needed, they have been computed in post-processing.

A. BS Deployment Simulation

A simulation environment was set up to derive the coalition user rate per area ρ_s^a as a function of each possible number u_s^a of BSs that coalition s can activate in area a, *i.e.*, from 1 up to U_{max}. In details, the entire set of U_{max} BSs is uniformly distributed in a pseudo-random fashion on the considered square areas; 10 sample users are also randomly distributed over each area a. The downlink SINR of each sample user in a for each coalition s ($SINR_s^a$) is calculated for each possible value of u_s^a as a function of: the signal power P_k the sample user receives from its serving BS k (*i.e.*, the BS from which receives the strongest signal), the signal power $\sum_{j\neq k} P_j$ received from the interfering (non-serving) BSs and the white Gaussian noise signal power[8] P_{noise}. Since users are characterized by an activity factor η, the captured interference is scaled down by the load of coalition s in area a, i.e., $l_s^a = 1-(1-\eta)^{\frac{\sum_{i\in O_s}\sigma_i^a N_a}{u_s^a}}$. $SINR_s^a$ is therefore calculated as follows:

$$SINR_s^a = \frac{P_k}{l_s^a\left(\sum\limits_{j\neq k} P_j\right) + P_{noise}}, \quad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}. \quad (25)$$

The received signal power $P_{rx}[dBm]$ has been calculated according to a three-parameter path loss model (transmitted signal power P_{tx}, fixed path loss C_{pl} and path loss exponent Γ) defined within the GreenTouch Consortium [35]:

$$P_{rx}[dBm] = P_{tx}[dBm] - C_{pl}[dB] - 10\Gamma log(d[km]), \quad (26)$$

where d is the sample user–BS distance. The calculated SINR is finally mapped to LTE nominal rate ($\rho_s^{a,nom}$) according to a multilevel SINR–to–rate scheme [35]. A single value for $\rho_s^{a,nom}$ is obtained by averaging over the 10 sample users. An additional averaging is obtained by applying 100 iterations for each value of u_s^a; ρ_s^a is then calculated analytically as the product $\rho_s^{a,nom}(1-\eta)^{\frac{\sum_{i\in s}\sigma_i N_a}{u_s^a}}$, according to the definition in Section III-A.

TABLE III
CHARACTERISTICS OF THE SET OF AREAS

Area	Number of users	Size
Z1	$N_1 = 20000$	$A_1 = 4\ km^2$
Z2	$N_2 = 20000$	$A_2 = 0.5\ km^2$
Z3	$N_3 = 10000$	$A_3 = 1\ km^2$

B. Instances

We consider three square dense areas (their size and number of users are provided in Table III) and three MNOs (A, B and C) which is quite reasonable for the Italian (also European) telecom playground [16]. Assuming the dense urban areas belong to the same city, we consider the same distribution of users among MNOs in all of them. We report the results obtained for two such user distributions: M_1, MNOs have equal market shares ($\sigma_A = \sigma_B = \sigma_C = 1/3$) and M_2, for which the market shares of A, B and C are 10%, 30% and 60%, respectively ($\sigma_A = 0.1$, $\sigma_B = 0.3$, $\sigma_C = 0.6$).

The values of the user's willingness to pay for 1 Mbps of service on a monthly basis δ were deduced from current data tariff-plans applied by different Italian MNOs. We have considered 100 values in the range [0.02, 2] €/Mbps which were obtained discretizing the range uniformly with a 0.02 step.

The number of available sites for installing small cell BSs in a given geographical area is finite and most likely different for each area. We set U_{max} to 4000 for all the considered areas; such number of BSs it at least one order of magnitude larger than the minimum needed for coverage[9] whereas deploying

[8] The white Gaussian noise signal power accounts for the considered system bandwidth.

[9] If we consider small cells of 50 m range, the minimum number of small cell BSs for coverage would be roughly 500 for the largest area (Z1).

TABLE IV

VALUES OF δ FOR WHICH A COALITIONAL STRUCTURE IS *socially optimal* – USER DISTRIBUTION M_1

Coalitional structure	δ
ABC	[0.02, 0.04]
A/BC, B/AC, C/AB	[0.04, 0.1], [0.14, 2]
A/B/C	[0.06, 2]

(a) TOT_Q

Coalitional structure	δ
ABC	[0.02, 0.06]
A/BC, B/AC, C/AB	0.06, 0.1, [0.14, 0.22]
A/B/C	[0.06, 2]

(b) MIN_Q

TABLE V

VALUES OF δ FOR WHICH A COALITIONAL STRUCTURE IS *socially optimal*–USER DISTRIBUTION M_2

Coalitional structure	δ
ABC	[0.02, 0.06]
A/BC	—
B/AC	0.04, 0.08, [0.12, 0.26]
C/AB	[0.04, 2]
A/B/C	[0.28, 2]

(a) TOT_Q

Coalitional structure	δ
ABC	[0.02, 0.06], 0.1
A/BC	—
B/AC	[0.1, 0.16], [0.2, 2]
C/AB	[0.06, 2]
A/B/C	[0.26, 2]

(b) MIN_Q

TABLE VI

Socially optimal COALITIONAL STRUCTURES AND CORRESPONDING NUMBER OF ACTIVATED BSs – USER DISTRIBUTION M_1

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 157	ABC 443	A/BC 1007/1928	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000
Z2	ABC 161	ABC 448	A/BC 1000/2000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000
Z3	ABC 115	ABC 274	A/BC 706/1496	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000

(a) TOT_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 177	ABC 467	A/B/C 1091/1091/1091	A/B/C 1257/1486/1257	A/B/C 1257/1257/1257	A/B/C 1257/1257/1257
Z2	ABC 164	ABC 462	A/B/C 1141/1141/1141	A/B/C 1333/1334/1333	A/B/C 1334/1333/1333	A/B/C 1334/1333/1333
Z3	ABC 91	ABC 232	A/B/C 488/488/488	A/B/C 1080/2288/632	A/B/C 2288/633/633	A/B/C 2288/633/633

(b) MIN_Q

more BSs would result in only a marginal increase of the average user rate ρ_s for the considered instances (see Figure 3).

The investment lifetime period D is set to 120 months (see, e.g., [28], [30]) for all instances.

For the two user distributions we generate a scenario for each value of δ, while the rest of parameters ($\mathcal{O}$, $\mathcal{A}$, N_a, A_a, g, U_{max}, η, D) are fixed to the values provided in Table I.

V. RESULTS

In this section, we examine the impact of the user economic standpoint (different values of δ) and of the user distribution among MNOs (σ_i) on the coalitional structures and the level of investment first of the *socially optimal* configurations (Subsection V-A) and then of the *stable* configurations (Subsection V-B). The two configurations are then compared in Subsection V-C.

We recall that the user rate as a function of the number of deployed BSs for the different sharing configurations was obtained by means of simulation (Subsection IV-A) and that it behaves nonlinearly in the number of BSs; to obtain a MILP formulation of the problem, we have approximated the user rate functions with piecewise linear ones (see Subsection III-A, Appendix A). In order to account for the error introduced by the approximation, we investigate multiple configurations which perform very similarly. This allows us to identify general trends concerning the size and composition of the selected coalitional structures as we vary δ and the user distribution. For each value of δ, we consider as *socially optimal* sharing configurations the ones selected by the optimal solution of problem (2)-(13), solved either under objective TOT_Q (14a) or MIN_Q (14b), and all configurations for which the objective function value is at most 0.5% smaller with respect to the optimal one. Similarly, for *stable* sharing configurations, we relax the stability condition as follows: we consider a configuration $(s_1, \ldots, s_p)$ to be *stable* if for any $j = 1, \ldots, p$ there is no nonempty subset $s'_j \subset s_j$ such that

$$\frac{p^i_{s'_j} - p^i_{s_j}}{p^i_{s_j}} > 0.5\%, \qquad \forall\, i \in s'_j.$$

The different outcomes are denoted by the following notation: ABC represents the grand coalition, coalitional structures that consist of a singleton (i.e., a MNO investing alone) and a coalition of two MNOs are denoted by A/BC, B/AC and C/AB,[10] whereas the case when no sharing takes place, that is, when each MNO invests by itself, is denoted by A/B/C.

For each possible outcome, we report the values of δ for which the outcome is *socially optimal* under objectives TOT_Q and MIN_Q in Tables IV and V for user distributions M_1 and M_2, respectively. The results concerning the *stable* configurations are reported in Tables VIIIa and VIIIb for user distributions M_1 and M_2, respectively.

Concerning the level of investment, we report the number of BSs deployed by the sharing configurations only for a subset of the considered values of δ (i.e., {0.02, 0.04, 0.2, 0.4, 1, 2}) due to space limitations. For all values of δ for which we have identified multiple configurations (as illustrated in Tables IV, V, VIIIa and VIIIb), we report the results of the configuration selected by the optimal solution of the MILP model for the *socially optimal* configurations in Tables VI and VII, for user distributions M_1 and M_2, respectively. Similarly, when multiple configurations are *stable*, only one of them is reported in Tables IXa and IXb, for user distributions M_1 and M_2

[10] We remark that outcomes A/BC, B/AC and C/AB are equivalent for user distribution M_1 since MNOs have equal market shares.

TABLE VII
Socially optimal COALITIONAL STRUCTURES AND CORRESPONDING NUMBER OF ACTIVATED BSs – USER DISTRIBUTION M_2

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 169	ABC 398	C/AB 2000/1364	A/B/C 700/1300/2000	A/B/C 700/1300/2000	A/B/C 700/1300/2000
Z2	ABC 156	ABC 472	C/AB 1700/1200	A/B/C 617/1383/2000	A/B/C 700/1300/2000	A/B/C 700/1300/2000
Z3	ABC 110	ABC 287	C/AB 1200/716	A/B/C 395/1200/2000	A/B/C 554/1200/2000	A/B/C 554/1200/2000

(a) TOT_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 174	ABC 469	C/AB 1776/1191	A/B/C 401/1031/2568	A/B/C 401/1031/2568	A/B/C 401/1031/2568
Z2	ABC 172	ABC 459	C/AB 2185/1472	A/B/C 381/1055/2564	A/B/C 381/1055/2564	A/B/C 381/1055/2564
Z3	ABC 88	ABC 230	C/AB 925/595	A/B/C 309/480/2662	A/B/C 858/480/2662	A/B/C 309/1029/2662

(b) MIN_Q

TABLE VIII
VALUES OF δ FOR WHICH A COALITIONAL STRUCTURE IS *stable*

Coalitional structure	δ
ABC	[0.02, 0.1], [0.16, 0.22], 0.28
A/BC, B/AC, C/AB	[0.02, 0.52], 0.6, [0.98, 2]

(a) User distribution M_1

Coalitional structure	δ
ABC	[0.02, 0.04], [0.1, 0.12], [0.18, 0.30]
A/BC	0.02, 0.06, [0.1, 0.14], [0.18, 0.36], [0.52, 0.54]
B/AC	[0.02, 0.08], [0.12, 0.16], [0.22, 0.52], [0.6, 2]
C/AB	[0.04, 0.06], [0.1, 2]

(b) User distribution M_2

TABLE IX
Stable COALITIONAL STRUCTURES AND CORRESPONDING NUMBER OF ACTIVATED BSs

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 67	ABC 157	ABC 443	A/BC 349/686	A/BC 606/1500	A/BC 1000/2000
Z2	ABC 65	ABC 163	ABC 471	A/BC 357/628	A/BC 558/1500	A/BC 1000/2000
Z3	ABC 54	ABC 54	ABC 272	A/BC 178/274	A/BC 298/678	A/BC 490/1000

(a) User distribution M_1

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 74	ABC 169	ABC 700	C/AB 700/491	C/AB 1200/700	C/AB 2000/1200
Z2	ABC 69	ABC 156	ABC 472	C/AB 700/465	C/AB 1200/700	C/AB 1700/1200
Z3	ABC 12	ABC 66	ABC 287	C/AB 273/237	C/AB 700/476	C/AB 700/700

(b) User distribution M_2

respectively. The notation concerning the number of deployed BSs in Tables VI, VII, IX is the following: for outcome ABC, the reported number represents the number of BSs deployed by the grand coalition, for outcomes A/BC, C/AB and B/AC, the first number represents the number of BSs deployed by the singleton whereas the second represents the number of BSs deployed by the coalition of two, whereas for outcome A/B/C, the number of BSs deployed by each MNO are reported in order (i.e., the first number corresponds to A, the second to B and third to C).

A. Socially Optimal Configurations

As a general rule, results show that as users are willing to pay more (i.e., for higher values of δ) and, as a result, MNOs can afford a larger network cost, the *socially optimal* configurations consist of smaller and less congested coalitions in order to provide the best service level. Regarding the level of investment, the higher the value of δ, the denser the network deployment as larger revenues make up for increasing network cost.

For very low and high values of δ, results are very similar for both user distribution scenarios (M_1 and M_2). The grand coalition (ABC) outperforms the other configurations for $\delta = 0.02$ for TOT_Q and for $\delta \leq 0.04$ for MIN_Q for both M_1 and M_2. Although ABC is selected also for few other low values of δ for both objectives and user distributions, it performs similarly to other outcomes (Tables IV and V): e.g., for M_2, ABC is selected by TOT_Q also for $\delta = 0.06$ but performs similarly to C/AB. Instead, A/B/C, which represents the case when no sharing takes place, is always among the selected outcomes for $\delta \geq 0.06$ for both TOT_Q and MIN_Q for M_1 (Table IV) and for $\delta \geq 0.28$ for TOT_Q and for $\delta \geq 0.26$ for MIN_Q for M_2 (Table V).

However, for intermediate values of δ, results seem more sensitive to the user distribution. For M_1, the equivalent outcomes A/BC, B/AC and C/AB are selected for almost all values of δ in [0.06, 2] for TOT_Q and for some values of δ in [0.06, 0.22] for MIN_Q (Table IV). However, since they are always selected alongside A/B/C, that is, they perform very similarly to the case when there is no sharing, there is practically no incentive for sharing also for intermediate values of δ for M_1. Instead for M_2, for δ in [0.08, 0.26], the only *socially optimal* configurations selected by TOT_Q are C/AB and, for a subset of the values of δ in this range, also B/AC (Table Va); similarly for MIN_Q for δ in [0.12, 0.24] (Table Vb). In C/AB and B/AC, both coalitions of two MNOs, AB and AC, involve A which has the smallest market share (10%) and therefore introduces the minimum level of interference to a coalition. Moreover, for low values of δ, A benefits from being in a coalition since it cannot afford to invest sufficiently by itself given its small market share.[11] For these values, C/AB is more persistent than B/AC (i.e., it is selected for all δ in [0.04, 2] by TOT_Q and all δ in [0.06, 2] by MIN_Q) since C and AB are smaller (less congested) than AC. In turn A/BC, which involves the largest coalition of two MNOs (BC) and the smallest MNO (A) investing alone, is never selected.

[11] For instance, if all MNOs were to invest by themselves, for $\delta \leq 0.26$ users of MNO A would perceive the worst service level (user rate) due to A's low level of investment. Instead, for $\delta \geq 0.28$, as A is able to densify its network, users of C perceive the lowest user rate since C is the largest/most congested MNO.

Concerning the level of investment, in Tables VIa and VIb, we report the number of small cell BSs deployed in each area for the *socially optimal* sharing configuration selected by the optimal solution under TOT_Q and MIN_Q, respectively, for a subset of the considered values of δ ({0.02, 0.04, 0.2, 0.4, 1, 2}) and user distribution M_1. Results concerning user distribution M_2 are reported in Tables VIIa and VIIb.

For most instances, both objectives TOT_Q and MIN_Q provide the same coalitional structures but slightly different number of deployed BSs. For instance, for user distribution M_2 and $\delta = 0.02$ (see Tables VIIa and VIIb), the grand coalition deploys 5 more BSs under MIN_Q compared to TOT_Q in the largest area (Z1), 16 more in the most congested/dense area (Z2), and 22 BSs less in area Z3 (smaller than Z1 and less congested than Z2). Since the overall profit of each MNO has to be non-negative, objective MIN_Q achieves fairness by "redistributing" BSs across the areas so that the user rate of the worst served ones (Z1 & Z2) is increased at the expense of sacrificing the user rate of the better served one (Z3) (see also Figure 2 and observation (iv) in Section V-D).

Similar observations can be made for both user distribution scenarios concerning the impact of δ on the number of deployed BSs (Tables VI–VII). A little incentive from users (small δ) forces MNOs to deploy only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution M_2, $\delta = 0.02$, under objective TOT_Q the grand coalition deploys 169 BSs in area Z1, 156 BSs in area Z2 and 110 BSs in area Z3 (Table VIIa). However, as users are willing to pay more (larger values of δ), more BSs are deployed since higher revenues compensate the costs of deploying more BSs. In particular, all available sites per area (U_{max}) are used up in all the areas for user distribution M_1 under objective TOT_Q when $\delta \geq 0.4$ (Tables VIa); instead, for M_2, the U_{max} BSs are exhausted only in areas Z1 and Z2 when $\delta \geq 0.4$ whereas in Z3 the rate saturation is achieved by deploying less than U_{max} BSs when $\delta \geq 0.46$ (Table VIIa).

B. Stable Configurations

Also for *stable* configurations, the higher the value of δ, the smaller and less congested are the selected coalitions. For low values of δ, MNOs prefer to collaborate with a larger number of MNOs so as to minimize the network cost. Instead, for higher δ, i.e., higher revenues per unit of service provided, MNOs prefer to increase the service level, which in turn requires building less congested networks, i.e., either shared networks with fewer and smaller MNOs or individual ones.

For user distribution M_1 (see Table VIIIa), when $\delta \leq 0.52$ there is always incentive for sharing, i.e., each MNO is better off building a shared network with at least one other MNO than investing alone. The grand coalition (ABC) is *stable* for all values of δ in [0.02, 0.1] and a subset of values in [0.16, 0.28] but it ceases to be the *stable* when $\delta \geq 0.3$. The equivalent outcomes A/BC, B/AC and C/AB are *stable* for all δ in [0.02, 0.52] but they become unstable for a subset of values of δ in [0.54, 2] which in turn means that in such cases no sharing will take place and MNOs will build individual networks. However, for $\delta \geq 0.3$, A/BC, B/AC and C/AB perform very similarly to A/B/C.

For user distribution M_2 (see Table VIIIb), as δ increases only configurations containing the least congested coalitions of two MNOs remain *stable*. The grand coalition (ABC) and outcome A/BC (which involves the largest coalition of two MNOs) are never *stable* for $\delta \geq 0.32$ and $\delta \geq 0.56$, respectively. For $\delta \geq 0.56$, C/AB and, for a subset of values of δ, also B/AC are *stable*. In particular, outcome C/AB, in which the largest MNO C invests by itself whereas the smaller MNOs A and B collaborate, is always *stable* for $\delta \geq 0.1$.

Concerning the number of BSs deployed by the *stable* configurations (Tables IX), a little incentive from users (small δ) forces MNOs to activate only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution M_2 and $\delta = 0.02$, the grand coalition is *stable* and it activates 74 BSs in area Z1, 69 BSs in area Z2 and 12 BSs in area Z3 (Table IXb). However, as users are willing to pay more (larger values of δ), more BSs are activated since higher revenues compensate the costs of activating more BSs.

C. Comparison

We now compare the behavior of the *socially optimal* and *stable* configurations. The impact of δ on the two configurations is overall very similar. However, there is incentive for sharing for a larger range of the values of δ in order to maximize the MNOs profits (i.e., for *stable* configurations) compared to maximizing the global/minimum user rate (i.e. for the *socially optimal* configurations). In other words, shared networks can be more beneficial from the MNOs perspective as sharing the network cost allows for larger profits but less beneficial from the user perspective due to the service level degradation experienced in more congested networks. Consider for instance user distribution M_1. The grand coalition ABC is socially optimal for $\delta \in [0.02, 0.04]$ for TOT_Q and for $\delta \in [0.02, 0.06]$ for MIN_Q, but it is stable for a larger number of values of δ between 0.02 and 0.28. In general, under MIN_Q sharing is selected as optimal strategy only for $\delta \leq 0.22$, while sharing configurations are *stable* for a wider range of values (up to $\delta = 2$), which means that for higher values of δ no sharing should takes place in order to provide the best service level, while there is incentive to share in order to maximize the MNOs' profit.

Regarding the level of investment, the higher the value of δ, the denser the network deployment for both configurations as larger revenues make up for increasing network cost. Nevertheless, for the same value of δ more BSs are deployed by the *socially optimal* configurations compared to the *stable* ones, as the former focus on the user rate whereas the latter, focusing on the profit, reflect the trade-off between increased revenues and cost. For instance, for M_1 and $\delta = 0.04$, the grand coalition is selected by TOT_Q and it is *stable*; however, it deploys 443 BSs in area Z1, 448 in Z2 and 274 in Z3 under objective TOT_Q (Tables VIa) whereas in order to maximize the MNOs profit, 157 BSs are deployed in area Z1, 163 in Z2 and 54 in Z3 (Table IXa).

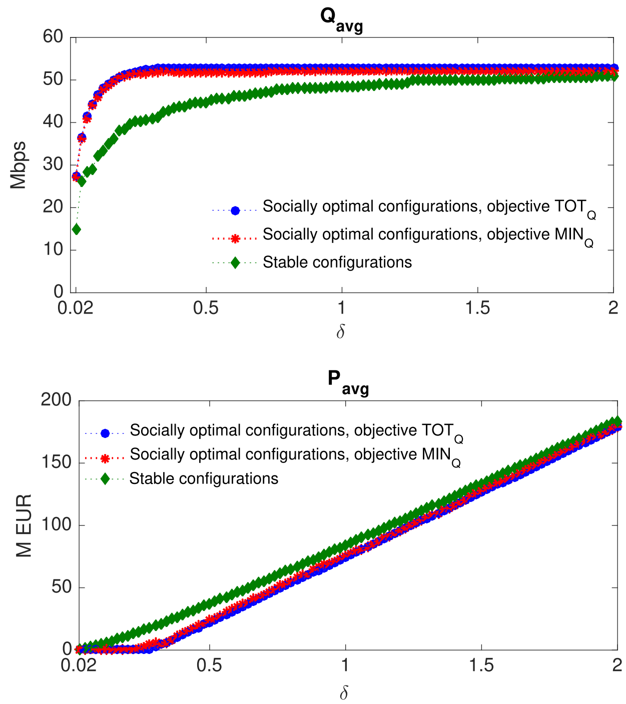

Fig. 1. Average user rate (Q_{avg}) and average profit (P_{avg}) *vs.* δ–user distribution M_2.

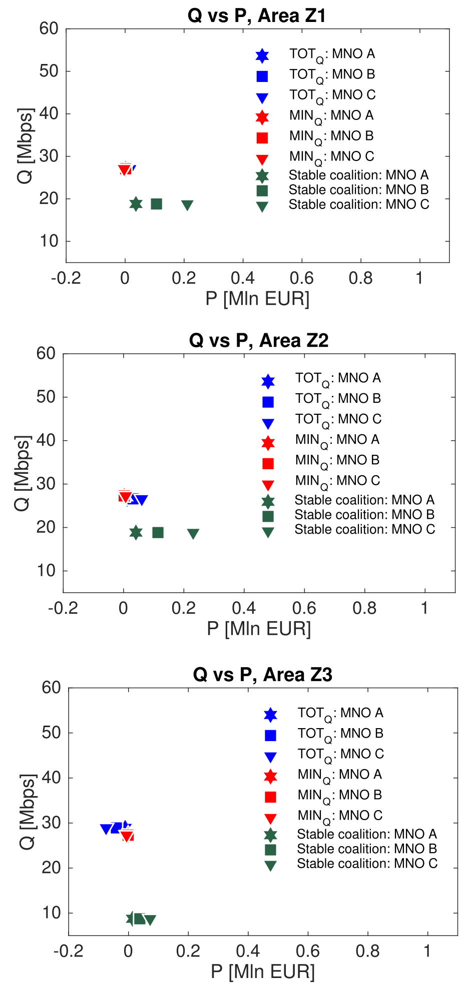

Fig. 2. User rate (Q) *vs.* profit (P) for each area and MNO – user distribution M_2, $\delta = 0.02$.

D. Performance Indicators Analysis

We now analyze how different values of δ impact two key performance indicators for the users and the MNOs: the average user rate, $Q_{avg} = \frac{\sum_{i\in\mathcal{O},a\in\mathcal{A}} q_i^a}{|\mathcal{O}|\times|\mathcal{A}|}$, and the average global profit, $P_{avg} = \frac{\sum_{i\in\mathcal{O}}\sum_{a\in\mathcal{A}}(r_i^a - c_i^a)}{|\mathcal{O}|}$; when multiple configurations are selected for the same value of δ (as reported in Tables IV, V, VII), we average also over the different configurations. In particular, we analyze the "price" of imposing a fair coalitional structure (objective MIN_Q).

Results show that the *socially optimal* infrastructure sharing configurations outperform *stable* ones in terms Q_{avg} and vice versa for P_{avg}. However, as users are willing to pay more, the two configuration types tend to provide very similar values of Q_{avg} and P_{avg}.

As similar observations regarding the behavior of Q_{avg} and P_{avg} as a function of δ can be drawn for both user distributions M_1 and M_2, we report results concerning only M_2 in Figure 1.

As pointed out in Section V-A, the *socially optimal* configurations obtained applying objectives TOT_Q and MIN_Q are the same for most instances and they also provide very similar Q_{avg} (the largest difference across all values of δ is approximately 1.1 Mbps) which can be observed by the overlap of their corresponding plots (see Figure 1). Therefore, solutions that are fair to all users in all the areas are also efficient.

More BSs are activated by the *socially optimal* configurations than by *stable* ones (see Subsection V-C) which is reflected in their corresponding Q_{avg} and P_{avg}. The difference in the Q_{avg} provided by the *socially optimal* configurations and *stable* ones for $\delta = 0.02$ is nearly 12.6 Mbps (45.8% gap); it goes down to 4.3 Mbps (8.1%) for $\delta = 1$ and eventually becomes nearly 1.8 Mbps (3.3%) for $\delta = 2$. Thus, for high δ, the two types of configurations provide roughly the same quality of service to the users if they are very interested in the new service.

As far as P_{avg} is concerned, for low values of δ, the difference in the P_{avg} provided by the two types of configurations is significantly different (see Figure 1). For $\delta = 0.02$, the configuration selected by TOT_Q provides on the average only 55.2 € per MNO, whereas the stable configurations provide 262306.3 €. This suggests that solutions obtained from objectives TOT_Q and MIN_Q merely satisfy the constraint on having a positive profit while providing, on the average, a 12.6 Mbps higher user rate. However, with the increase of δ, the difference in rate between the two types of configurations becomes negligible, and so does the difference in profit (only 2.8% for $\delta = 2$).

So far we have investigated the average performance indicators (Q_{avg} and P_{avg}). We now analyze how the user rate per area and MNO (Q) and profit per area and MNO (P) are affected by the characteristics of MNOs (market share) and by the characteristics of the areas (size and population, reported in Table III) for both configurations.

Figure 2 illustrates the behavior of Q with respect to P in each area, for each MNO for the user distribution

M$_2$ when $\delta = 0.02$. We recall that, when $\delta = 0.02$, the grand coalition (ABC) is *socially optimal* (for both TOT_Q and MIN_Q objectives) and *stable*. For this scenario we can observe that: (i) the *socially optimal* configurations provide in every area higher user rates than the *stable* one, which in turn guarantee higher revenues, (ii) the grand coalition results in all MNOs providing the same user rate to users of the same area, while their profit follows their market shares (see Equations (12), (24)), (iii) in area Z3, MNOs obtain a negative profit under objective TOT_Q, while the global profit for each MNO is positive, which indicates that a negative balance between costs and revenues can be accepted in some areas by the *socially optimal* configurations, (iv) the objective that favors fairness (MIN_Q) improves the quality of service of the users of the largest area (Z1) and most congested area (Z2) at the cost of lowering the user rate of area Z3 and (v) since the user rate provided by a given coalitional structure in an area depends on the user density, on the size of the area and on the number of BSs activated in that area, a slightly higher user rate is achieved for the small, low user density area (Z3) by the *socially optimal* configurations as the LTE nominal rate is divided among less users and on the average users are closer to their serving BSs.

VI. Conclusions

This work analyzes the strategic situation in which MNOs have to decide whether to invest in LTE small cells in dense urban areas and whether to share the investment with other MNOs. A mathematical framework is proposed to address the problem of infrastructure sharing for the considered scenario. This framework accounts for techno-economic parameters such as the achievable throughput and a general pricing model for the LTE service. The problem has been tackled from two perspectives: the one of a regulatory entity which imposes infrastructure sharing configurations that optimize the quality of service perceived by all users and the MNOs perspective, which captures their competitive and profit-maximizing nature. We propose an MILP formulation to determine socially optimal configurations (regulator perspective) and adopt concepts of cooperative game theory to determine stable configurations (MNOs perspective).

Results show that sharing configurations obtained under both perspectives are strongly affected by how much users are willing to pay for the new services but they also depend on the user distribution (MNOs market shares). Sharing is appealing from both perspectives when users are willing to pay little, regardless of the MNOs market shares as they all struggle with high infrastructure cost. Instead, if users were willing to pay more, there is generally more incentive to share from the MNO perspective and in particular when MNOs have significantly different market shares. For both perspectives, the selected configurations involve less congested coalitions, that is, coalitions of fewer and smaller MNOs, when the market shares are significantly different. When the focus is on the quality of service, such configurations behave very similarly to the case when no sharing takes place, that is, users are best served either by less congested coalitions or when all MNOs build individual networks.

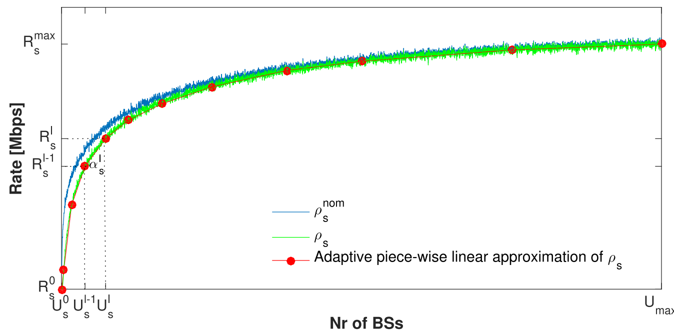

Fig. 3. Simulated nominal user rate ($\rho_s^{a,nom}$), average user rate (ρ_s^a) and adaptive piece-wise linearization for coalition ABC in area Z1 (20000 users, 4 km^2).

The proposed mathematical framework has proved to be a flexible instrument of limited complexity to analyze in detail the possible strategies for different infrastructure sharing configurations under different techno-economic conditions. It can be further extended to incorporate spectrum management issues and therefore more elaborated game theory models, as well as different classes of users and heterogeneous technologies.

Appendix A

We recall that the nominal user rate ($\rho_s^{a,nom}$) is computed by means of the simulation described in Subsection IV-A whereas the average user rate (ρ_s^a) is derived from $\rho_s^{a,nom}$ according to Equations (27). ρ_s^a is then approximated by a concave piecewise linear function in order to formulate the problem as a MILP.

$$\rho_s^a = \rho_s^{a,nom}(1-\eta)^{\frac{\sum_{i\in O_s}\sigma_i N_a}{u_s^a}}, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}. \quad (27)$$

Figure 3 illustrates the simulated nominal user rate $\rho_s^{a,nom}$, the average user rate ρ_s^a and the piece-wise linear function approximating ρ_s^a for coalition ABC in area Z1 (similarly for all the other considered areas and coalitions). In the following, we explain how the approximation was modeled in the MILP formulation.

As mentioned, L denotes the number of linear pieces (intervals) that approximate ρ_s^a. We have considered equal values of L for all the coalitions $s \in \mathcal{S}$ and all the areas $a \in \mathcal{A}$. L was set to 11 for user distribution M_1 and to 10 for M_2. For each interval $l \in \{1,\ldots,L\}$, coalition s and area a, $[U_s^{a,l-1},\ U_s^{a,l}]$ represents the range of the number of BSs that characterize the l^{th} interval, $R_s^{a,l}$ is the average user rate when s activates $U_s^{a,l}$ BSs in a and $\alpha_s^{a,l}$ is the slope associated with the l^{th} interval. The average user rate ρ_s^a obtained by activating u_s^a BSs, with $u_s^a \in [U_s^{a,l-1}, U_s^{a,l}]$, is therefore equal to $R_s^{a,l-1} + \alpha_s^{a,l}(u_s^a - U_s^{a,l-1})$. Equations (28) show how these parameters are related with one another.

$$\begin{aligned} R_s^{a,0} &= \rho_s^a(1), \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}, \\ R_s^{a,l} &= R_s^{a,l-1} + \alpha_s^{a,l}(U_s^{a,l} - U_s^{a,l-1}), \\ &\quad \forall s \in \mathcal{S}, \quad \forall\, a \in \mathcal{A},\ \forall\, l \in \{1,\ldots,L\}. \qquad (28) \end{aligned}$$

In particular, $U_s^{a,0}$ is equal to 1, whereas $U_s^{a,L}$ is equal to U_{max}, $\forall s \in \mathcal{S}$ and $\forall a \in \mathcal{A}$. Thus, the average user rate ρ_s^a

obtained by activating u_s^a BSs can be reformulated as:

$$\rho_s^a = \min_{l\in\{0,\dots,L-1\}} \{R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l})\},$$
$$\forall\ s \in \mathcal{S}, \quad \forall\ a \in \mathcal{A}. \tag{29}$$

As ρ_s^a is maximized by any of the considered objective functions, Equations (29) can be replaced by Constraints (8). Notice that, the auxiliary binary variables z_s^a equal zero when either no BSs are activated by s in a ($u_s^a = 0$ and therefore $z_s^a = 0$ due to Constraint (7)) or s is not active ($y_s = 0$ and therefore $z_s^a = 0, \forall a \in \mathcal{A}$ due to Constraints (4) and (7)). In turn, when $z_s^a = 0$, we should also have $\rho_s^a = 0$, which is guaranteed by Constraints (9) while Constraints (8) are made redundant by the term $M(1 - z_s^a)$, where $M = 1000$.

Appendix B

The optimization problem with objective MIN_Q and Constraints (2)–(13) will be denoted by *Infrastructure Sharing Problem* (ISP).

Theorem: The decision version of (ISP) is NP-complete.

Proof: The decision version of (ISP) can be formulated as:

Given a threshold $\bar{Q} > 0$ on the quality, are there variables y_s, z_s^a, u_s^a and ρ_s^a, with $s \in \mathcal{S}$ and $a \in \mathcal{A}$, such that Constraints (2)–(13) *are satisfied and $MIN_Q \geq \bar{Q}$?*

We will prove that the decision version of (ISP) is NP-complete by reduction from the *Set Partitioning Problem* (SPP) which is a well-known NP-complete problem (see, e.g., [36]). We recall the decision version of (SPP):

Given a universe $\mathcal{U}$, a family $\mathcal{C}$ of subsets of $\mathcal{U}$ and a positive integer K, is there a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that $|\mathcal{C}'| \leq K$ and each element of the universe $\mathcal{U}$ belongs to exactly one member of $\mathcal{C}'$?

The proof is carried out in 3 steps.

1) The decision version of (ISP) is a NP problem because verifying that a given solution is a YES one requires $O(|\mathcal{O}| + L|\mathcal{S}|)$ number of operations.
2) It is possible to make a polynomial time transformation of any instance I_{SPP} of the decision version of (SPP) into an instance I_{ISP} of the decision version of (ISP). Given I_{SPP}=($\mathcal{U}$, $\mathcal{C}$, K), we build I_{ISP}=($\mathcal{O}$, $\mathcal{S}$, $\mathcal{A}$, N_a, $\{\sigma_i\}_{i\in\mathcal{O}}$, U_{max}, L, $\{U_s^l\}_{l=0}^L$, $\{R_s^l\}_{l=0}^L$, $\{\alpha_s^l\}_{l=1}^L$, δ, D, g, $\bar{Q}$) as follows:
   - $\mathcal{O} = \mathcal{U}$, $\mathcal{S} = \mathcal{C}$, $|\mathcal{A}| = 1$, $N_a = |\mathcal{U}|$, $\sigma_i = 1/|\mathcal{U}|$ for any $i \in \mathcal{O}$, $U_{max} = K$, $L = K - 1$.
   - For any coalition $s \in \mathcal{S}$, we set $U_s^0 = 1, U_s^1 = 2, \dots, U_s^{K-1} = K$.
   - Given an arbitrary coalition $\bar{s} \in \mathcal{S}$, we set:
     $R_s^0 = |\bar{s}|/|s|$ for any $s \in \mathcal{S}$,
     $R_s^l = R_s^{l-1} + \alpha_s^l$ for any $s \in \mathcal{S}$ and $l = 1, \dots, K-1$,
   - For any coalition $s \in \mathcal{S}$, we set $R_s^0 > \alpha_s^1 > \alpha_s^2 > \dots > \alpha_s^{K-1} > 0$.
   - $\delta = 1$, $D = 1$, $g = |\bar{s}|$, $\bar{Q} = \min_{s\in\mathcal{S}} R_s^0$.

   It is clear that such transformation can be done in polynomial time with respect to size of I_{ISP}.
3) I_{ISP} is a YES instance if and only if I_{SPP} is a YES instance.

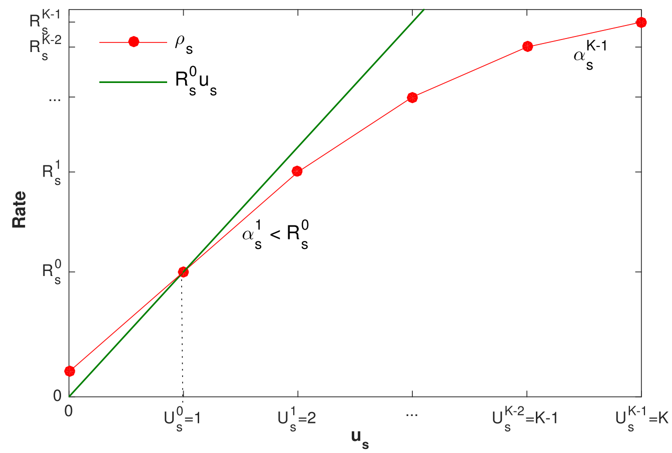

Fig. 4. Graphical illustration of the number of BSs activated by coalitions.

First, we prove the *if* part. Since I_{SPP} is a YES instance, there is a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that $|\mathcal{C}'| \leq K$ and each element of the universe $\mathcal{U}$ belongs to exactly one member of $\mathcal{C}'$. We define the variables

$$y_s = z_s^a = u_s^a = \begin{cases} 1 & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise,} \end{cases} \quad \rho_s^a = \begin{cases} R_s^0 & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to check that Constraints (2)–(5) are satisfied. Constraint (6) is fulfilled since

$$\sum_{s\in\mathcal{S}} u_s^a = |\mathcal{C}'| \leq K = U_{max}.$$

The values of variables z_s^a, u_s^a and ρ_s^a guarantee that Constraints (7)–(9) hold. Furthermore, Constraints (13) on the nonnegative profit of MNOs hold because

$$\sum_{a\in\mathcal{A}} (r_i^a - c_i^a) = \delta D \sigma_i N_a q_i^a - \sum_{s\in\mathcal{S}_i} g \frac{\sigma_i}{\sum_{j\in s}\sigma_j} u_s^a$$
$$= \sum_{s\in\mathcal{S}_i} R_s^0 u_s^a - \sum_{s\in\mathcal{S}_i} \frac{|\bar{s}|}{|s|} u_s^a = 0.$$

Finally, since $\mathcal{C}'$ is a partition of $\mathcal{O}$, any MNO i belongs to a unique coalition $s_i \in \mathcal{C}'$ and $q_i^a = \rho_{s_i}^a = R_{s_i}^0 \geq \bar{Q}$ for any $i \in \mathcal{O}$, that is $MIN_Q \geq \bar{Q}$. Therefore, I_{ISP} is a YES instance.

Now, we prove the *only if* part. Assume that I_{ISP} is a YES instance, i.e., there are variables y_s, z_s^a, u_s^a and ρ_s^a, with $s \in \mathcal{S}$ and $a \in \mathcal{A}$, such that all the Constraints (2)–(13) are satisfied and $MIN_Q \geq \bar{Q}$. For any $i \in \mathcal{O}$ we have $q_i^a \geq \bar{Q} > 0$, hence we get from Constraints (4), (7) and (9) that for any $i \in \mathcal{O}$ there exists a unique coalition $s_i \in \mathcal{S}_i$ such that $y_{s_i} = 1$. Thus, $u_{s_i}^a \geq 1$ by Constraint (5). On the other hand, $r_i^a = \delta D \sigma_i N_a q_i^a = \rho_{s_i}^a$ and

$$c_i^a = \sum_{s\in\mathcal{S}_i} g \frac{\sigma_i}{\sum_{j\in s}\sigma_j} u_s^a = g \frac{\sigma_i}{\sum_{j\in s_i}\sigma_j} u_{s_i}^a$$
$$= \frac{|\bar{s}|}{|s_i|} u_{s_i}^a = R_{s_i}^0 u_{s_i}^a.$$

Since $0 \leq r_i^a - c_i^a = \rho_{s_i}^a - R_{s_i}^0 u_{s_i}^a$, we obtain from Figure 4 that $u_{s_i}^a \leq 1$. Thus, for any activated coalition s

(i.e., $y_s = 1$) the number of deployed BSs is $u_s^a = 1$. If we define

$$\mathcal{C}' = \{s \in \mathcal{S} : \; y_s = 1\},$$

then $\mathcal{C}'$ is a partition of $\mathcal{U}$ and $|\mathcal{C}'| = \sum_{s \in \mathcal{S}} u_s^a \leq U_{max} = K$, therefore I_{SPP} is a YES instance.

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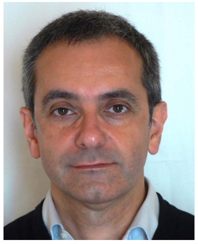

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