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PERIODIC MATERIALS WITH RESONANT CAVITIES FOR ENERGY LOCALIZATION

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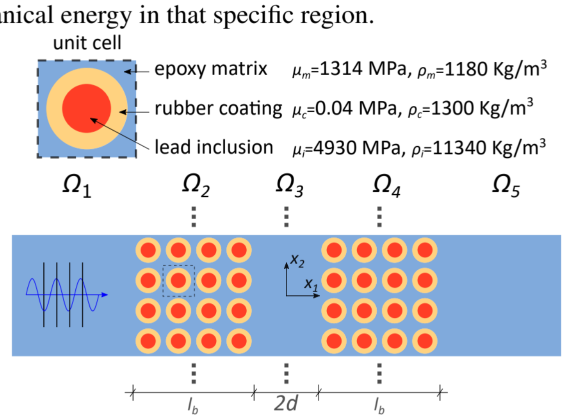
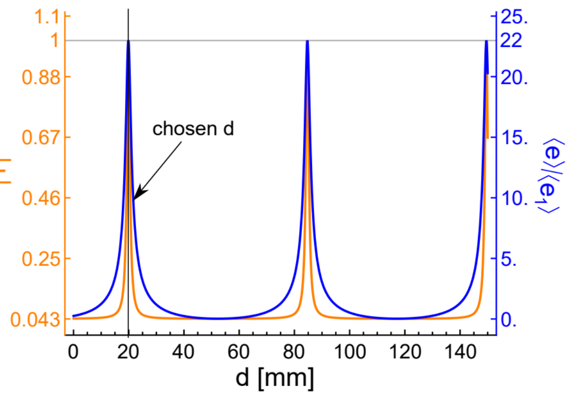
Summary Composite periodic materials, made by a regular repetition of a unit cell with two or more components, can exhibit an "unusual" dynamic behavior. In particular, the propagation of elastic and/or acoustic waves of frequency within specific intervals can be attenuated. This peculiarity makes them feasible for controlling the mechanical energy flow carried by the traveling waves, guiding it into some demarcated regions where it can be localized.

RESONANT CAVITY IN THREE COMPONENT PERIODIC MATERIALS

In this contribution, we consider periodic continua whose unit cell contains an heavy and stiff inclusion with a soft coating, embedded in a stiff matrix, see Figure 1a . This particular internal structure is typical of the so-called "Locally Resonant Materials" (LRMs): the very compliant material used for the coating allows for the presence of local resonances inside each cell generating the so-called band-gaps, i.e. intervals of frequencies in the dispersion plot corresponding to a complex wavenumber and, thus, to attenuated propagating waves, as shown for instance in [1]. Here we analyze the possibility of focusing the mechanical energy of anti-plane elastic waves inside an homogeneous cavity (part Ω_3 in Figure 1a) placed between two barriers (Ω_2 and Ω_4) made of three-components LRMs. The described system resembles a Fabry-Pérot interferometer [2]; the dimensions along axis x_2 and x_3 (out-of-plane axis) are comparable and both very large with respect to the size of the unit cells composing the LRMs. The incoming wave, traveling through the homogeneous part Ω_1 with a fixed angular frequency ω inside a band-gap of the LRM and a wave front perpendicular to the x_1 axis, reaches the first barrier Ω_2 where it experiences both reflection and attenuation; once inside the cavity Ω_3, due to the presence of the two barriers, the wave is rebounded back and forward several times before leaving that region and reaching part Ω_5. The propagating elastic wave can hence be trapped inside the cavity, obtaining an accumulation and a focusing of its mechanical energy in that specific region.



(a)



(b)

Figure 1: (a) Unit cell of the LRM with material properties and general scheme of the analyzed system: composite with resonant cavity; (b) modulus of wave amplitude $|T| = 1$ (Orange, left axis) and cavity energy density $\langle e_3 \rangle$ normalized with input energy density $\langle e_1 \rangle$ (Blue, right axis) versus cavity width d.

Homogenized properties and bandgaps of LRM

The problem is characterized by two different length scales: a scale corresponding to the size a of the unit cell of the LRM and a scale L of the wave length of the considered wave propagating in the matrix. When $a/L = \epsilon << 1$, a two-scale asymptotic analysis could be applied for obtaining a macroscopic equivalent description of the behavior of the heterogeneous medium (see e.g. [3]). The unknown displacement field u can be expressed by asymptotic expansion, in powers of ϵ. By approximating the solution at the first order of the expansion, both an effective shear modulus μ_{eff} and an effective mass density ρ_{eff} can be derived. More in details, μ_{eff} is a function only of the elastic properties and the geometry of the LRM unit cells, whereas ρ_{eff} depends also on the frequency and can become negative. This latter peculiarity may occur only if $\mu_c/\mu_m = \mathcal{O}\left(\epsilon^2\right)$, where μ_c and μ_m are the shear moduli respectively of the coating and of the matrix materials. As shown in [4], the effective mass is negative for frequencies corresponding to a band-gap and becomes unbounded at the resonance frequencies of the internal inclusions. Considering the geometry of the unit cell shown in figure 1a, ρ_{eff} can be analytically derived, hence the effective motion of the two barriers becomes completely known in closed form, with the only exception of μ_{eff}, which can be computed numerically and remains constant for all the frequencies.

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Optimization of the cavity

For a specific chosen LRM, the wave motion in the system of Figure 1a depends only on 3 parameters, namely the frequency (ω), the width of the barriers (l_b) and the width of the internal cavity ($2d$). For any fixed value of ω inside the band-gap and any fixed number of unit cells (i.e. for any l_b) the mechanical energy which is trapped inside the cavity is averaged in time over the period and optimized with respect to d. The problem can be entirely solved analytically and a sequence of optimal dimensions of the cavity are obtained. The maximization of $\langle e_3 \rangle$ (average energy density inside part Ω_3) provides the same optimal d of the maximization of the amplitude T of the transmitted wave propagating in part Ω_5. When $|\mathrm{T}| = 1$ (complete transmission), the energy reaches its maximum value, see Figure 1b.

ANALYTICAL AND NUMERICAL RESULTS

We have applied the analytical procedure previously described to an example problem, comparing the results with a finite element analysis. The analyzed composite is constituted by square unit cells,1mm$\times$1mm, as specified in Figure 1a. Notice that the ratio $\mu_c/\mu_m << 1$, respecting the condition which guarantees the presence of band-gaps. The two barriers are made up of forty unit cells in the x_1 direction. The energy $\langle e_3 \rangle$ has been maximized considering a traveling wave with a frequency of $\omega = 4068$ Hz (value taken in the middle of the lowest band-gap). From the set of optimal cavity widths, the first value $d = 19.9$mm has been chosen, see Figure 1b. Note that a change of the frequency or a scaling of the unit cells geometry would imply a new optimal set of d.

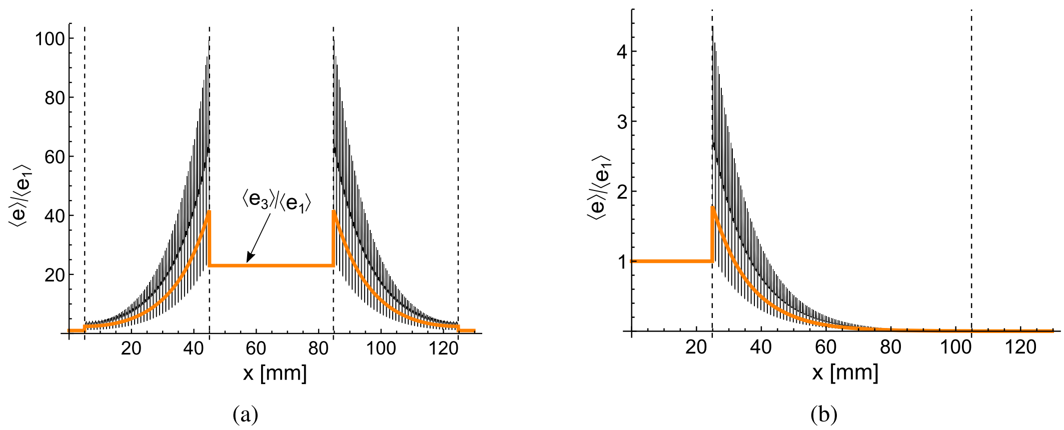

Figure 2: Normalized mechanical energy density for ω=4087 Hz in the different parts of the analysed systems: (a) system with the optimal cavity (d = 19.9 mm), (b) system without the cavity. Orange curves: analytical results, black curves: numerical results.

In Figure 2a we report the average mechanical energy density generated by the motion caused by the propagating wave, normalized with respect to the energy of the incoming wave $\langle e_1 \rangle$. The analytical results (in orange) are in good agreement with the numerical one (in black), attaining for both cases a value of energy $\langle e_3 \rangle$ inside the cavity twenty-two times bigger than the value of the incoming wave in part Ω_1. In the LRM barriers the analytical solution, based on the homogenized material, does not describe the oscillations due to the inclusions which are evidenced by the numerical analysis. For comparison, Figure 2b shows the results obtained in a system without a resonant cavity (i.e. with d=0). One can see that in this latter case the LRM as expected attenuates the energy of the incoming wave and no energy localization occurs.

CONCLUSIONS

With the aim of focusing the vibration mechanical energy in a confined region, we analysed a system made with a composite material with a resonant cavity. By applying a two-scale asymptotic technique we computed the band-gap of the composite material and we derived the optimized cavity dimension to maximize the energy localization. Since $|\mathrm{T}| = 1$ when the energy inside the cavity is maximum, the system could also be employed for selecting some specific frequencies within a packet of waves which are traveling at a frequency situated inside a band-gap.

References

[1] Moscatelli M., Ardito R., Driemeier L., Comi C. Band-gap structure in two- and three-dimensional cellular locally resonant materials. *J. Sound and Vibration*, **454**: 73-84, 2019.

[2] C. Fabry, A. Pérot, Théorie et applications d'une nouvelle méthode de spectroscopie interférentielle. *Annales de Chimie et de Physique*, **16**(7), 115, 1899.

[3] Auriault J.-L., Boutin C., Geindreau C. Homogenization of Coupled Phenomena in Heterogenous Media. ISTE Ltd and John Wiley & Sons, GB 2009.

[4] Comi C., Marigo J-J. Homogenization approach and Bloch-Floquet theory for band-gap prediction in 2D locally resonant metamaterials. *J. Elast.* 2019 https://doi.org/10.1007/s10659-019-09743-x.