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Decentralized Sliding Mode Control of Islanded AC Microgrids with Arbitrary Topology

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***Abstract*—The present paper deals with modelling of complex microgrids and the design of advanced control strategies of sliding mode type to control them in a decentralized way. More specifically, the model of a microgrid including several distributed generation units (DGus), connected according to an arbitrary complex and meshed topology, and working in islanded operation mode (IOM), is proposed. Moreover, it takes into account all the connection line parameters and it is affected by unknown load dynamics, nonlinearities and unavoidable modelling uncertainties, which make sliding mode control algorithms suitable to solve the considered control problem. Then, a decentralized second order sliding mode (SOSM) control scheme, based on the Suboptimal algorithm is designed for each DGu. The overall control scheme is theoretically analyzed, proving the asymptotic stability of the whole microgrid system. Simulation results confirm the effectiveness of the proposed control approach.**

***Index Terms*—Microgrids, Sliding mode control, Uncertain Systems.**

I. INTRODUCTION

IN recent industrial research, one of the most relevant key challenges deals with the evolution of the power grid and electricity generation and distribution networks [1]. In this context, the terms “microgrid” and “Smart Grid” indicate the realization of a resilient and sustainable power network in which the local distributed consumers play a vital role.

In a typical Smart Grid, the presence of new technologies and tools for the smart metering of the processes is mandatory, above all because of the increasing presence of Renewable Energy Sources (RES), such as photovoltaic panels or wind turbines. The latter, by nature, are characterized by unpredictable behaviors which make the adoption of suitable robust control strategies essential to regulate the electrical signals of the so-called Distributed Generation units (DGus) with respect to the nominal ones [2]. A microgrid is an electrically connected cluster of DGus equipped with its own control and Energy Management System (EMS) [3], in order to allow the so-called Islanded Operation Mode (IOM). Technologies used for power systems have to include protections, data acquisition units and robust control equipments. Automation is widely spread in this systems so that, in the last decades, several works have been published to cope with the aforementioned problem.

The controllers introduced in the literature are of several types, including PI control algorithms [4], [5], $\mathcal{H}_\infty$ controllers [6], [7], or Model Predictive Control [8], [9], formulated both in the so-called Grid Connected Operation Mode (GCOM) and in IOM. The last case is particularly interesting since islanded microgrids offer several promising advantages in reducing power losses, smoothly integrating RES and increasing the network resiliency.

Among the control strategies, also Sliding Mode Control (SMC) methodology has been applied to power systems [10]–[18]. In fact, SMC is very appreciated for its robustness properties against a wide class of uncertainties and perfectly fits the control problem to solve [19], [20]. Moreover, in a typical DGu, the interface medium between the grid and the energy source is a voltage source converter (VSC). This element provides an alternate voltage signal to the load, given the direct current voltage energy source.

The VSC can be very critical, because it can be responsible of disturbances affecting the system. On the other hand, the control signal generated by the VSC is discontinuous by construction. SMC belongs to the class of Variable Structure Control Systems so that it seems perfectly adequate to control the VSC. In fact, power electronic systems represent a typical example in which the discontinuous control is intrinsically provided. The so-called chattering phenomenon is already attenuated by construction thanks to the presence of the VSC output filter [21], [22]. In this paper, the Suboptimal Second Order Sliding Mode (SSOSM) control algorithm [23]–[25] is proposed to regulate the microgrid voltage. The choice of a SSOSM control algorithm is motivated by the fact that it is a very easy-to-implement solution even in practical cases. In fact, it is intrinsically a bilevel control law as required to act directly on the switches of the VSC; it does not require the knowledge of the time derivatives of the so-called sliding variable; moreover it has only one control parameter, which makes the tuning procedure quite easy. Note however that, in order to obtain more regular modulating signals, Higher Order Sliding Mode controllers can be applied by increasing the natural relative degree of the auxiliary system, as shown in [12].

The proposed control approach is decentralized and each controller uses only voltage local information. This implies that communication networks among DGus are not necessary. The asymptotic stability of the overall controlled system has been

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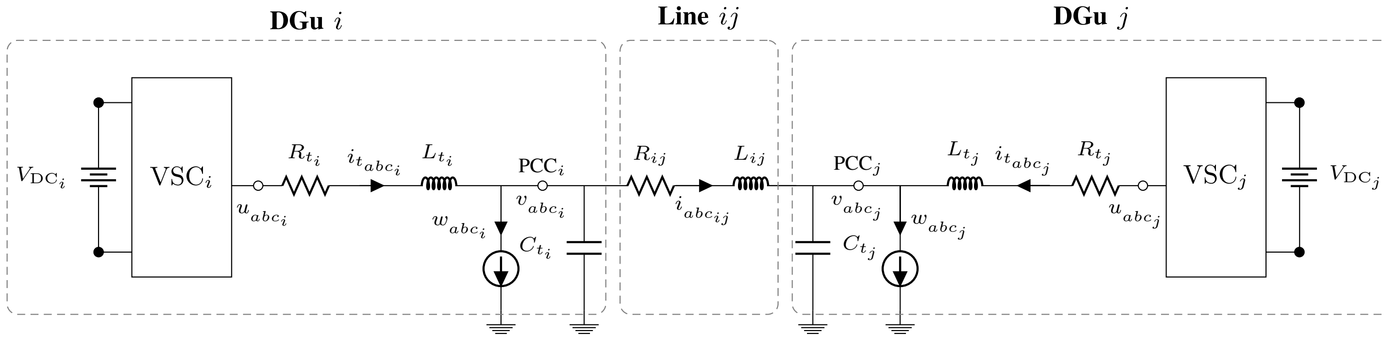

Figure 1. The considered electrical single-line diagram of a typical AC microgrid composed of two DGUs

analytically verified. Note that, differently from [12], in this paper the stability properties have been proved not neglecting the dynamics of the interconnecting distribution lines.

II. Preliminary Issues

Consider the schematic electrical single-line diagram of a typical microgrid composed of two DGUs in Figure I. The renewable energy source of a DGu can be represented by a direct current (DC) voltage source V_{DC}, and it is interfaced with the electrical network through the so-called Voltage-Source-Converter (VSC). Usually, the VSC is equipped with a resistive-inductive filter (R_tL_t) able to extract the fundamental frequency of the VSC output voltage. The electrical connection point of the DGu to the rest of the power network is the so-called Point of Common Coupling (PCC) where a three-phase parallel resistive-inductive-capacitive load (RLC) is located. The DGu$_i$ can exchange power with the DGu$_j$ through the resistive-inductive line ($R_{ij}L_{ij}$).

In IOM the PCC voltage and frequency could deviate significantly from the desired values, due to the power mismatch between the DGu and the load. Therefore, in IOM the DGu has to provide the voltage and frequency control in order to keep the load voltage magnitude and frequency constant with respect to the reference values. Specifically, in this paper the frequency is controlled in open-loop by equipping each DGu in the microgrid with an internal oscillator which provides the Park's transformation angle $\theta(t) = \int_{t_0}^{t} \omega_0 d\tau$, with $\omega_0 = 2\pi f_0$, f_0 being the nominal frequency.

Note that, in the dq-coordinates, the generated active and reactive powers can be expressed as

$$P = \frac{3}{2}(V_d I_{t_d} + V_q I_{t_q}), \quad Q = \frac{3}{2}(V_q I_{t_d} - V_d I_{t_q}), \tag{1}$$

with V_d and V_q being the direct and quadrature components of the load voltage v_{abc}, I_{t_d} and I_{t_q} being the direct and quadrature components of the generated current $i_{t_{abc}}$. Usually, in order to decouple the active and reactive power control, the PCC quadrature voltage component V_q is steered to zero, such that the active and reactive powers in (1) become

$$P = \frac{3}{2} V_d I_{t_d}, \quad Q = -\frac{3}{2} V_d I_{t_q}, \tag{2}$$

which depend only on the direct and quadrature current component, respectively.

III. Problem Formulation

Consider a microgrid of n DGus. The network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V} = \{1, ..., n\}$, represent the DGus and the edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V} = \{1, ..., m\}$ represent the distribution lines interconnecting the DGus. The network structure can be represented by its corresponding incidence matrix $D \in \mathbb{R}^{n\times m}$. The ends of edge k are arbitrary labeled with a '+' and a '-'. Then, one has that

$$d_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise .} \end{cases}$$

Consider the scheme reported in Figure I and assume the system to be symmetric and balanced. For the sake of simplicity, the dependence of the variables on time t is omitted throughout this paper. In the stationary abc-frame, by applying the Kirchhoff's current (KCL) and voltage (KVL) laws, the dynamics equations of the microgrid in IOM are expressed as follow,

$$\begin{cases} \frac{d}{dt} v_{_{abc}} = [C_t]^{-1} i_{t_{abc}} + [C_t]^{-1}[D] i_{_{abc}} - [C_t]^{-1} w_{_{abc}} \\ \frac{d}{dt} i_{t_{abc}} = -[L_t]^{-1}[R_t] i_{t_{abc}} - [L_t]^{-1} v_{_{abc}} + [L_t]^{-1} u_{_{abc}} \\ \frac{d}{dt} i_{_{abc}} = -[L]^{-1}[D^T] v_{_{abc}} - [L]^{-1}[R] i_{_{abc}} \end{cases}, \tag{3}$$

where $s_{_{abc}} = [s_{_a}^T, s_{_b}^T, s_{_c}^T]^T \in \mathbb{R}^{3n}$, $s_{_\pi} = [s_{_{\pi_1}}, \ldots, s_{_{\pi_n}}]^T \in \mathbb{R}^n$, with $\pi = a, b, c$ and $s \in \{v, i_t, w, u\}$, while $i_{_{abc}} = [i_{_a}^T, i_{_b}^T, i_{_c}^T]^T \in \mathbb{R}^{3m}$, $i_{_\pi} = [i_{_{\pi_1}}, \ldots, i_{_{\pi_m}}]^T \in \mathbb{R}^m$. In (3) $v_{_{abc}}$, $i_{t_{abc}}$, $i_{_{abc}}$, $w_{_{abc}}$, and $u_{_{abc}}$ represent the following three-phase signals: the loads voltages, the currents generated by the DGus, the currents along the interconnecting lines, the currents demanded by the loads, and the VSCs output voltages. Moreover, in system (3) we used $[M]$ to denote the following block diagonal matrix

$$[M] = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix},$$

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$$A = \begin{bmatrix} 0 & \omega_0 I_{n\times n} & C_t^{-1} & 0 & C_t^{-1}D & 0 \\ -\omega_0 I_{n\times n} & 0 & 0 & C_t^{-1} & 0 & C_t^{-1}D \\ -L_t^{-1} & 0 & -L_t^{-1}R_t & \omega_0 I_{n\times n} & 0 & 0 \\ 0 & -L_t^{-1} & -\omega_0 I_{n\times n} & -L_t^{-1}R_t & 0 & 0 \\ -L^{-1}D^T & 0 & 0 & 0 & -L^{-1}R & \omega_0 I_{m\times m} \\ 0 & -L^{-1}D^T & 0 & 0 & -\omega_0 I_{m\times m} & -L^{-1}R \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_t^{-1} & 0 \\ 0 & L_t^{-1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_w = \begin{bmatrix} -C_t^{-1} & 0 \\ 0 & -C_t^{-1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} I_{n\times n} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{n\times n} & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $M \in \{C_t,\, L_t,\, R_t,\, L,\, R\}$, with $C_t,\, L_t,\, R_t$ being $n \times n$ diagonal matrices and $L,\, R$ being $m\times m$ diagonal matrices, e.g., $R_t = \mathrm{diag}\{R_{t_1}, \ldots, R_{t_n}\}$ and $R = \mathrm{diag}\{R_1, \ldots, R_m\}$, with $R_k = R_{ij}$. Each three-phase variable of (3) can be transferred to the rotating dq-frame by applying the Clarke's and Park's transformations. In the following we use $x_{[S]}$ to denote the vector $[S_1, \ldots, S_n]^T$ with $S \in \{V_d, V_q, I_{t_d}, I_{t_q}\}$, and $x_{[Z]}$ to denote the vector $[Z_1, \ldots, Z_m]^T$, with $Z_k = Z_{ij}$ and $Z \in \{I_d, I_q\}$. Then, the so-called state-space representation of the whole system (3) can be expressed as

$$\begin{cases} \dot{x}_{[V_d]} = \omega_0 x_{[V_q]} + C_t^{-1} x_{[I_{t_d}]} + C_t^{-1} D x_{[I_d]} - C_t^{-1} w_d \\ \dot{x}_{[V_q]} = -\omega_0 x_{[V_d]} + C_t^{-1} x_{[I_{t_q}]} + C_t^{-1} D x_{[I_q]} - C_t^{-1} w_q \\ \dot{x}_{[I_{t_d}]} = -L_t^{-1} x_{[V_d]} - L_t^{-1} R_t x_{[I_{t_d}]} + \omega_0 x_{[I_{t_q}]} + L_t^{-1} u_d \\ \dot{x}_{[I_{t_q}]} = -L_t^{-1} x_{[V_q]} - \omega_0 x_{[I_{t_d}]} - L_t^{-1} R_t x_{[I_{t_q}]} + L_t^{-1} u_q \\ \dot{x}_{[I_d]} = -L^{-1} D^T x_{[V_d]} - L^{-1} R x_{[I_d]} + \omega_0 x_{[I_q]} \\ \dot{x}_{[I_q]} = -L^{-1} D^T x_{[V_q]} - \omega_0 x_{[I_d]} - L^{-1} R x_{[I_q]} \\ y_d = x_{[V_d]} \\ y_q = x_{[V_q]} \end{cases}, \tag{4}$$

where $x = \left[x_{[V_d]}^T\, x_{[V_q]}^T\, x_{[I_{t_d}]}^T\, x_{[I_{t_q}]}^T\, x_{[I_d]}^T\, x_{[I_q]}^T\right]^T \in \mathbb{R}^{4n+2m}$ is the state variables vector, $u = [u_d^T\, u_q^T]^T \in \mathbb{R}^{2n}$ is the input vector, $w = [w_d^T\, w_q^T]^T \in \mathbb{R}^{2n}$ is the disturbance vector, and $y = [x_{[V_d]}^T\, x_{[V_q]}^T]^T \in \mathbb{R}^{2n}$ is the output vector. Then, the previous system can be written as

$$\begin{cases} \dot{x} = Ax + Bu + B_w w \\ y = Cx \end{cases}, \tag{5}$$

where $A \in \mathbb{R}^{(4n+2m)\times(4n+2m)}$ is the dynamics matrix of the microgrid, $B \in \mathbb{R}^{(4n+2m)\times(2n)}$, $B_w \in \mathbb{R}^{(4n+2m)\times(2n)}$, and $C \in \mathbb{R}^{2n\times(4n+2m)}$, defined as reported above, with I being the identity matrix.

To permit the controller design in the next section, the following assumption is required on the state and the disturbance.

Assumption 1 *The load voltages V_{d_i} and V_{q_i} are locally available at DGu$_i$, $i = 1, \ldots, n$. The disturbances w_{d_i} and w_{q_i} are unknown but bounded, of class* C *and Lipschitz continuous.*

Remark 1 *Note that Assumption 1 requires only the local measurement of the load voltage that is used only by the controller of DGu$_i$.*

Now we are in a position to formulate the control problem. *Let Assumption 1 hold. Given system* (3)-(5)*, design a decentralized control scheme capable of guaranteeing that the tracking error between any controlled variable and the corresponding reference is steered to zero in a finite time in spite of the uncertainties, such that the overall system is asymptotically stable.*

IV. The Proposed Decentralized Higher Order Sliding Mode Control Scheme

In this section, the decentralized SSOSM control algorithm, used to solve the aforementioned control problem, is designed. Moreover, a Third Order Sliding Mode (3-SM) control algorithm is proposed in order to obtain continuous control signals.

A. Suboptimal Second Order Sliding Mode (SSOSM) Control Algorithm

Consider the state-space model (5) and select the so-called sliding variables vector as

$$\begin{aligned} \sigma &= y - y^\star \\ &= Cx - y^\star, \end{aligned} \tag{6}$$

where $y^\star = [x_{[V_d]}^{\star T}\, x_{[V_q]}^{\star T}]^T$ is the vector of reference values.

To permit the controller design, the following assumption is required on the generation of reference values.

Assumption 2 *The load voltage references $V_{d_i}^\star$ and $V_{q_i}^\star$ are of class* C^2 *and with first time derivatives Lipschitz continuous.*

Let r be the relative degree of the system, i.e., the minimum order r of the time derivative $\sigma^{(r)}$ of the sliding variable in which the control u explicitly appears. With reference to (6), it appears that r is equal to 2, so that a Second Order Sliding Mode (SOSM) control naturally applies [23], [25]. According to the SOSM control theory, the so-called auxiliary variables $\xi_{1_\nu} = \sigma_\nu$ and $\xi_{2_\nu} = \dot{\sigma}_\nu$, with the subscript $\nu = d, q$, have

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to be defined and the corresponding auxiliary systems can be expressed as

$$\begin{cases} \dot{\xi}_{1_\nu} = \xi_{2_\nu} \\ \dot{\xi}_{2_\nu} = f_\nu(x,w) + g_\nu u_\nu \end{cases}, \tag{7}$$

where u_ν are the control inputs previously defined, and ξ_{2_ν} is assumed to be unmeasurable. More specifically, one has that

$$\begin{aligned} f_d(x,w) &= -\left(\omega_0^2 I_{n\times n} + C_t^{-1}L_t^{-1} + C_t^{-1}DL^{-1}D^T\right)x_{[V_d]} \\ &\quad -C_t^{-1}L_t^{-1}R_t x_{[I_{t_d}]} + 2\omega_0 C_t^{-1}x_{[I_{t_q}]} \\ &\quad -C_t^{-1}DL^{-1}Rx_{[I_d]} + 2\omega_0 C_t^{-1}Dx_{[I_q]} \\ &\quad -C_t^{-1}\dot{w}_d - \omega_0 C_t^{-1}w_q - \ddot{x}^\star_{[V_d]} \\ f_q(x,w) &= -\left(\omega_0^2 I_{n\times n} + C_t^{-1}L_t^{-1} + C_t^{-1}DL^{-1}D^T\right)x_{[V_q]} \\ &\quad -2\omega_0 C_t^{-1}x_{[I_{t_d}]} - C_t^{-1}L_t^{-1}R_t x_{[I_{t_q}]} \\ &\quad -2\omega_0 C_t^{-1}Dx_{[I_d]} - C_t^{-1}DL^{-1}Rx_{[I_q]} \\ &\quad +\omega_0 C_t^{-1}w_d - C_t^{-1}\dot{w}_q - \ddot{x}^\star_{[V_q]} \\ g_d = g_q &= C_t^{-1}L_t^{-1}, \end{aligned} \tag{8}$$

are allowed to be uncertain with known bounds for each entry as follows

$$|f_{\nu_i}(\cdot)| \le F_{\nu_i}, \quad G_{\min_{\nu_i}} \le g_{\nu_{ii}} \le G_{\max_{\nu_i}}, \quad i = 1,\ldots,n, \tag{9}$$

with F_{ν_i}, $G_{\min_{\nu_i}}$ and $G_{\max_{\nu_i}}$, $\nu = d,q$, being positive constants. Note that, it is reasonable to assume that such bounds exist. In fact, the functions f_ν depend on electrical signals related to the finite power of the system, while $g_{\nu_{ii}}$ are uncertain constant values. In practical cases, these bounds can be estimated relying on data analysis and engineering understanding.

The i-th control law, u_{ν_i} that we propose to steer $\xi_{1_{\nu_i}}$ and $\xi_{2_{\nu_i}}$, $i = 1,\ldots,n$, to zero in a finite time in spite of the uncertainties, in analogy with [25], can be expressed as follows

$$u_{\nu_i} = -\alpha_{\nu_i} U_{\max_{\nu_i}} \operatorname{sgn}\left(\xi_{1_{\nu_i}} - \tfrac{1}{2}\xi_{1_{\max_{\nu_i}}}\right), \tag{10}$$

with bounds

$$U_{\max_{\nu_i}} > \max\left(\frac{F_{\nu_i}}{\alpha^*_{\nu_i} G_{\min_{\nu_i}}}; \frac{4F_{\nu_i}}{3G_{\min_{\nu_i}} - \alpha^*_{\nu_i} G_{\max_{\nu_i}}}\right) \tag{11}$$

$$\alpha^*_{\nu_i} \in (0,1] \cap \left(0, \frac{3G_{\min_{\nu_i}}}{G_{\max_{\nu_i}}}\right). \tag{12}$$

B. An Alternative Solution: Third Order Sliding Mode (3-SM) Control Algorithm

Usually, to control inverters, the Pulse Width Modulation (PWM) technique is used. To do this, the VSC requires continuous control signals u_{d_i} and u_{q_i}, that can be transferred back to the stationary abc-frame and used to generate the gating signals through the comparison with a triangular carrier. In order to obtain continuous control signals, as suggested in [23], the system relative degree can be artificially increased. As proposed in [12], a Third Order Sliding Mode (3-SM) control law to solve the microgrid voltage control problem in question, can be introduced. Defining the auxiliary variables $\xi_{1_\nu} = \sigma_\nu$, $\xi_{2_\nu} = \dot{\sigma}_\nu$ and $\xi_{3_\nu} = \ddot{\sigma}_\nu$, then the auxiliary system can be expressed as

$$\begin{cases} \dot{\xi}_{1_\nu} = \xi_{2_\nu} \\ \dot{\xi}_{2_\nu} = \xi_{3_\nu} \\ \dot{\xi}_{3_\nu} = \phi_\nu(x,w,u) + \gamma_\nu \mu_\nu \\ \dot{u}_\nu = \mu_\nu \end{cases}, \tag{13}$$

where μ_ν is an auxiliary control variable, ξ_{2_ν}, ξ_{3_ν} are assumed to be unmeasurable, while $\phi_\nu = \dot{f}_\nu$ and $\gamma_\nu = g_\nu$ are uncertain smooth bounded functions, such that for each entry

$$|\phi_{\nu_i}(\cdot)| \le \Phi_{\nu_i}, \quad \Gamma_{\min_{\nu_i}} \le \gamma_{\nu_{ii}} \le \Gamma_{\max_{\nu_i}}, \quad i = 1,\ldots,n, \tag{14}$$

with Φ_{ν_i}, $\Gamma_{\min_{\nu_i}}$ and $\Gamma_{\max_{\nu_i}}$, $\nu = d,q$ being positive constants. In this case, the 3-SM algorithm requires that the discontinuous controls only affect $\sigma_\nu^{(3)}$, but not $\ddot{\sigma}_\nu$, so that the controls fed into the plant are continuous.

Let $\bar{\sigma}_{\nu_i} = [\sigma_{\nu_i}, \dot{\sigma}_{\nu_i}, \ddot{\sigma}_{\nu_i}]^T$, then the i-th discontinuous control law μ_{ν_i} can be expressed as follows

$$\mu_{\nu_i} = -\alpha_{\nu_i}\begin{cases} \mu_{\nu_i,1} = \operatorname{sgn}(\ddot{\sigma}_{\nu_i}), & \bar{\sigma}_{\nu_i} \in \mathcal{M}_{\nu_i,1}/\mathcal{M}_{\nu_i,0} \\ \mu_{\nu_i,2} = \operatorname{sgn}\left(\dot{\sigma}_{\nu_i} + \frac{\ddot{\sigma}^2_{\nu_i}\mu_{\nu_i,1}}{2\alpha_{\nu_i,\mathrm{r}}}\right), & \bar{\sigma}_{\nu_i} \in \mathcal{M}_{\nu_i,2}/\mathcal{M}_{\nu_i,1} \\ \mu_{\nu_i,3} = \operatorname{sgn}(s_{\nu_i}), & \text{else} \end{cases} \tag{15}$$

where

$$s_{\nu_i} = \sigma_{\nu_i} + \frac{\ddot{\sigma}^3_{\nu_i}}{3\alpha^2_{\nu_i,\mathrm{r}}} + \mu_{\nu_i,2}\left[\frac{1}{\sqrt{\alpha_{\nu_i,\mathrm{r}}}}\left(\mu_{\nu_i,2}\dot{\sigma}_{\nu_i} + \frac{\ddot{\sigma}^2_{\nu_i}}{2\alpha_{\nu_i,\mathrm{r}}}\right)^{\frac{3}{2}} + \frac{\dot{\sigma}_{\nu_i}\ddot{\sigma}_{\nu_i}}{\alpha_{\nu_i,\mathrm{r}}}\right], \tag{16}$$

with $\alpha_{\nu_i,\mathrm{r}}$ being the reduced control amplitude, such that

$$\alpha_{\nu_i,\mathrm{r}} = \alpha_{\nu_i}\Gamma_{\min_{\nu_i}} - \Phi_{\nu_i} > 0. \tag{17}$$

In (15), (17) there are not parameters to be tuned, except for the control amplitudes α_{ν_i}, $\nu = d,q$. In (15) the manifolds $\mathcal{M}_{\nu_i,0}$, $\mathcal{M}_{\nu_i,1}$, $\mathcal{M}_{\nu_i,2}$ are defined as

$$\begin{aligned} \mathcal{M}_{\nu_i,0} &= \{\bar{\sigma}_{\nu_i} \in \mathbb{R}^3 : \sigma_{\nu_i} = \dot{\sigma}_{\nu_i} = \ddot{\sigma}_{\nu_i} = 0\} \\ \mathcal{M}_{\nu_i,1} &= \{\bar{\sigma}_{\nu_i} \in \mathbb{R}^3 : \sigma_{\nu_i} - \tfrac{\ddot{\sigma}^3_{\nu_i}}{6\alpha^2_{\nu_i,\mathrm{r}}} = 0,\ \dot{\sigma}_{\nu_i} + \tfrac{\ddot{\sigma}_{\nu_i}|\ddot{\sigma}_{\nu_i}|}{2\alpha_{\nu_i,\mathrm{r}}} = 0\} \\ \mathcal{M}_{\nu_i,2} &= \{\bar{\sigma}_{\nu_i} \in \mathbb{R}^3 : s_{\nu_i} = 0\}. \end{aligned} \tag{18}$$

From (15), one can observe that the controller of DGu$_i$ requires not only σ_{ν_i}, but also $\dot{\sigma}_{\nu_i}$ and $\ddot{\sigma}_{\nu_i}$. Yet, according to Assumption 1, only the load voltages V_{d_i} and V_{q_i} are measurable at DGu$_i$. Then, one can rely on Levant's second-order differentiator [26] to retrieve $\dot{\sigma}_{\nu_i}$ and $\ddot{\sigma}_{\nu_i}$ in a finite time. With reference to system (13), for $\nu = d,q$, and $i = 1,\ldots,n$, one has

$$\begin{cases} \dot{\hat{\xi}}_{1_{\nu_i}} = -\lambda_{0_{\nu_i}}\left|\hat{\xi}_{1_{\nu_i}} - \xi_{1_{\nu_i}}\right|^{\frac{2}{3}}\operatorname{sgn}\left(\hat{\xi}_{1_{\nu_i}} - \xi_{1_{\nu_i}}\right) + \hat{\xi}_{2_{\nu_i}} \\ \dot{\hat{\xi}}_{2_{\nu_i}} = -\lambda_{1_{\nu_i}}\left|\hat{\xi}_{2_{\nu_i}} - \dot{\hat{\xi}}_{1_{\nu_i}}\right|^{\frac{1}{2}}\operatorname{sgn}\left(\hat{\xi}_{2_{\nu_i}} - \dot{\hat{\xi}}_{1_{\nu_i}}\right) + \hat{\xi}_{3_{\nu_i}} \\ \dot{\hat{\xi}}_{3_{\nu_i}} = -\lambda_{2_{\nu_i}}\operatorname{sgn}\left(\hat{\xi}_{3_{\nu_i}} - \dot{\hat{\xi}}_{2_{\nu_i}}\right), \end{cases} \tag{19}$$

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where $\hat{\xi}_{1_{\nu_i}}, \hat{\xi}_{2_{\nu_i}}, \hat{\xi}_{3_{\nu_i}}$ are estimates of $\xi_{1_{\nu_i}}, \xi_{2_{\nu_i}}, \xi_{3_{\nu_i}}$, respectively, and $\lambda_{0_{\nu_i}} = 3\Lambda_{\nu_i}^{1/3}$, $\lambda_{1_{\nu_i}} = 1.5\Lambda_{\nu_i}^{1/2}$, $\lambda_{2_{\nu_i}} = 1.1\Lambda_{\nu_i}$, $\Lambda_{\nu_i} > 0$, is a possible choice of the differentiator parameters suggested in [26].

V. Stability Analysis

With reference to the proposed decentralized sliding mode control approach, the following results can be proved.

Lemma 1 *Let Assumptions 1 and 2 hold. Given the auxiliary system* (7) *controlled via the SSOSM algorithm* (10)-(12)*, then the sliding variables* (6) *and their first time derivatives are steered to zero in a finite time t_r, in spite of the uncertainties.*

Proof: This result directly follows from [25, Theorem 1]. ■

Let $\tilde{x}$ be the error given by the difference between the state and the equilibrium point associated to the desired value of voltages $y^\star$ when w is constant, and let $\tilde{u}$ be the corresponding control input. Hence, the error system is defined as

$$\begin{cases} \dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \\ \sigma = C\tilde{x} \end{cases} . \tag{20}$$

Theorem 1 *Let Assumptions 1 and 2 hold. Consider system* (3)-(5) *controlled via the SSOSM control algorithm* (10)-(12)*. Then, given constant reference $y^\star$ and constant disturbance w, $\forall\, t \geq t_\mathrm{r}, \forall\, x(t_\mathrm{r}) \in \mathbb{R}^{4n+2m}$, the origin of the error system* (20) *is a robust asymptotically stable equilibrium point.*

Proof: Consider the d component of the sliding variable $\sigma_d = \tilde{x}_{[V_d]}$. Compute now the first time derivative and the second time derivative of σ_d, i.e.,

$$\begin{aligned} \dot{\sigma}_d = \dot{\tilde{x}}_{[V_d]} = \;& \omega_0\tilde{x}_{[V_q]} + C_t^{-1}\tilde{x}_{[I_{t_d}]} + C_t^{-1}D\tilde{x}_{[I_d]} \\ \ddot{\sigma}_d = \ddot{\tilde{x}}_{[V_d]} = \;& -\big(\omega_0^2 I_{n\times n} + C_t^{-1}L_t^{-1} \\ & + C_t^{-1}DL^{-1}D^T\big)\tilde{x}_{[V_d]} - C_t^{-1}L_t^{-1}R_t\tilde{x}_{[I_{t_d}]} \\ & + 2\omega_0 C_t^{-1}\tilde{x}_{[I_{t_q}]} - C_t^{-1}DL^{-1}R\tilde{x}_{[I_d]} \\ & + 2\omega_0 C_t^{-1}D\tilde{x}_{[I_q]} + C_t^{-1}L_t^{-1}\tilde{u}_d \; . \end{aligned} \tag{21}$$

According to the equivalent control concept [19], by posing $\ddot{\sigma}_d = 0$, one obtains

$$\begin{aligned} \tilde{u}_{d_{eq}} = \;& R_t\tilde{x}_{[I_{t_d}]} - 2\omega_0 L_t\tilde{x}_{[I_{t_q}]} \\ & + L_t DL^{-1}R\tilde{x}_{[I_d]} - 2\omega_0 L_t D\tilde{x}_{[I_q]} \; . \end{aligned} \tag{22}$$

Analogously, the q component of the sliding variable $\sigma_q = \tilde{x}_{[V_q]}$ and its time derivatives can be computed as

$$\begin{aligned} \dot{\sigma}_q = \dot{\tilde{x}}_{[V_q]} = \;& -\omega_0\tilde{x}_{[V_d]} + C_t^{-1}\tilde{x}_{[I_{t_q}]} + C_t^{-1}D\tilde{x}_{[I_q]} \\ \ddot{\sigma}_q = \ddot{\tilde{x}}_{[V_q]} = \;& -\big(\omega_0^2 I_{n\times n} + C_t^{-1}L_t^{-1} \\ & + C_t^{-1}DL^{-1}D^T\big)\tilde{x}_{[V_q]} - 2\omega_0 C_t^{-1}\tilde{x}_{[I_{t_d}]} \\ & - C_t^{-1}L_t^{-1}R_t\tilde{x}_{[I_{t_q}]} - 2\omega_0 C_t^{-1}D\tilde{x}_{[I_d]} \\ & - C_t^{-1}DL^{-1}R\tilde{x}_{[I_q]} + C_t^{-1}L_t^{-1}\tilde{u}_q \; . \end{aligned} \tag{23}$$

The corresponding equivalent control, obtained by posing $\ddot{\sigma}_q = 0$ is

$$\begin{aligned} \tilde{u}_{q_{eq}} = \;& 2\omega_0 L_t\tilde{x}_{[I_{t_d}]} + R_t\tilde{x}_{[I_{t_q}]} \\ & + 2\omega_0 L_t D\tilde{x}_{[I_d]} + L_t DL^{-1}R\tilde{x}_{[I_q]} . \end{aligned} \tag{24}$$

Considering that, after t_r, $\sigma_\nu = \dot{\sigma}_\nu = 0$, that is $\tilde{x}_{[V_d]} = \tilde{x}_{[V_q]} = \dot{\tilde{x}}_{[V_d]} = \dot{\tilde{x}}_{[V_q]} = 0$, one obtains the following set of algebraic equations

$$\begin{cases} 0 = C_t^{-1}\tilde{x}_{[I_{t_d}]} + C_t^{-1}D\tilde{x}_{[I_d]} \\ 0 = C_t^{-1}\tilde{x}_{[I_{t_q}]} + C_t^{-1}D\tilde{x}_{[I_q]} \end{cases} . \tag{25}$$

Then, by using the relations in (25) and by substituting (22) and (24) into system (20), the residual dynamics results in being

$$\begin{cases} \dot{\tilde{x}}_{[I_{t_d}]} = DL^{-1}R\tilde{x}_{[I_d]} - \omega_0 D\tilde{x}_{[I_q]} \\ \dot{\tilde{x}}_{[I_{t_q}]} = \omega_0 D\tilde{x}_{[I_d]} + DL^{-1}R\tilde{x}_{[I_q]} \\ \dot{\tilde{x}}_{[I_d]} = -L^{-1}R\tilde{x}_{[I_d]} + \omega_0\tilde{x}_{[I_q]} \\ \dot{\tilde{x}}_{[I_q]} = -\omega_0\tilde{x}_{[I_d]} - L^{-1}R\tilde{x}_{[I_q]} \end{cases} . \tag{26}$$

Remark 2 *Note that, since the relative degree of the system is $r = 2$, the original system with 4n+2m dynamic independent equations, $\forall\, t \geq t_\mathrm{r}$, can be described by the $4n$ sliding constraints $\sigma_d = \sigma_q = \dot{\sigma}_d = \dot{\sigma}_q = 0$, and by $2m$ independent dynamic equations.*

More specifically, the resulting reduced order dynamics can be represented by the last two equations related to the distribution lines dynamics. Moreover, these latter, according to the sliding mode control theory [20], are the zero dynamics of the system which can be written in a matrix form as

$$\begin{bmatrix} \dot{\tilde{x}}_{[I_d]} \\ \dot{\tilde{x}}_{[I_q]} \end{bmatrix} = \tilde{A} \begin{bmatrix} \tilde{x}_{[I_d]} \\ \tilde{x}_{[I_q]} \end{bmatrix} = \begin{bmatrix} -L^{-1}R & \omega_0 I_{m\times m} \\ -\omega_0 I_{m\times m} & -L^{-1}R \end{bmatrix} \begin{bmatrix} \tilde{x}_{[I_d]} \\ \tilde{x}_{[I_q]} \end{bmatrix}, \tag{27}$$

where I is the identity matrix, and the matrix $\tilde{A}$ is Hurwitz so that $\tilde{x}_{[I_d]}$ and $\tilde{x}_{[I_q]}$ asymptotically converge to zero. Then, from the algebraic equations (25), one can observe that also $\tilde{x}_{[I_{td}]}$ and $\tilde{x}_{[I_{tq}]}$ asymptotically converge to zero, which concludes the proof. ■

Remark 3 *Note also that the eigenvalues λ_i, $i = 1, \ldots, 2m$, of matrix $\tilde{A}$ are complex conjugates. Yet, considering realistic values of the parameters R and L, one has that $|\mathrm{Re}\{\lambda_i\}| \gg |\mathrm{Im}\{\lambda_i\}|$.*

Lemma 2 *Let Assumptions 1 and 2 hold. Let assume $t_0 \geq t_\mathrm{Ld}$, t_0, t_Ld being the initial time instant and the finite time necessary for the convergence of the Levant's differentiator* (19)*, respectively. Given the auxiliary system* (13) *controlled via the 3-SM control law* (15)-(18)*, then the sliding variables* (6) *and their first and second time derivatives are steered to zero in a finite time t_r, in spite of the uncertainties.*

Proof: This result directly follows from [27, Theorem 2]. ■

Theorem 2 *Let Assumptions 1 and 2 hold. Consider system* (3)-(5) *controlled via the 3-SM control law* (15)-(18)*. Then,*

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Table I
ELECTRICAL PARAMETERS OF THE MICROGRID

DGus	Filter Parameters		Shunt capacitance	Load Currents		Reference Voltages	
	R_{t_i} [mΩ]	L_{t_i} [mH]	C_{t_i} [µF]	W_{d_i} [A]	W_{q_i} [A]	$V^{\star}_{d_i}$ [V]	$V^{\star}_{q_i}$ [V]
DGu_1	40.2	9.5	62.86	50	-20	$120.0\sqrt{2}$	0
DGu_2	38.7	9.2	62.86	100	-15	$120.0\sqrt{2}$	0
DGu_3	34.6	8.7	62.86	40	-10	$122.4\sqrt{2}$	0
DGu_4	31.8	8.3	62.86	80	-18	$117.6\sqrt{2}$	0

Table II
ELECTRICAL PARAMETERS OF THE DISTRIBUTION LINES

Line impedance Z_{ij}	R_{ij} [Ω]	L_{ij} [µH]
Z_{12}	0.25	1.2
Z_{23}	0.27	1.3
Z_{34}	0.24	1.8
Z_{14}	0.26	2.1

Table III
PI CONTROL PARAMETERS

Parameter	Value	Description
$K_{\mathrm{P}_d} = K_{\mathrm{P}_q}$	10	Proportional gain of voltage loop
$K_{\mathrm{I}_d} = K_{\mathrm{I}_q}$	400	Integral gain of voltage loop
$K_{\mathrm{P}_d} = K_{\mathrm{P}_q}$	20	Proportional gain of current loop
$K_{\mathrm{I}_d} = K_{\mathrm{I}_q}$	400	Integral gain of current loop

given constant reference $y^\star$ *and constant disturbance* w, $\forall\, t \geq t_{\mathrm{r}} \geq t_0 \geq t_{\mathrm{Ld}}$, $\forall\, x(t_{\mathrm{r}}) \in \mathbb{R}^{4n+2m}$, *the origin of the error system* (20) *is a robust asymptotically stable equilibrium point.*

Proof: The proof is analogous to that of Theorem 1. ■

VI. SIMULATION RESULTS

In this section, the proposed control solution is assessed in simulation by implementing an AC islanded microgrid with nominal frequency $f_0 = 60\,\mathrm{Hz}$, and composed of four DGus (n =4). The DGus are in a ring topology (m =4), as depicted in Figure 2. The electrical parameters of the single DGus and of the interconnecting distribution lines are reported in Table I and in Table II, respectively.

We choose the control amplitude $U_{\max}$, for all the decentralized controllers, equal to 1000. Traditional PI controllers are also used in the same test for the sake of comparison. They are tuned by using Ziegler-Nichols method, and the control parameters are reported in Table III. Note that the PI control scheme works as voltage regulation through current compensation. More precisely, the (outer) voltage controllers generate current references for the (inner) current regulation.

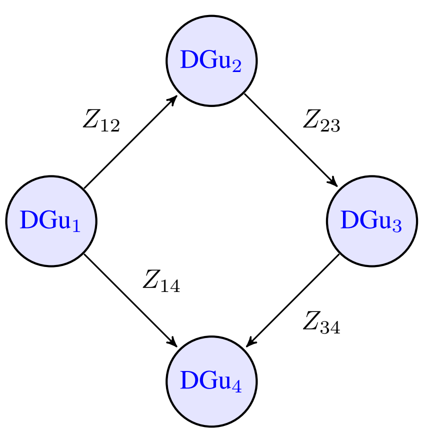

Figure 2. Scheme of the considered microgrid composed of 4 DGus. The arrows indicate the positive direction of the currents through the power network.

The dynamic performances of the controlled microgrid system in Figure 2 are validated considering unknown load dynamics and voltage reference changes. In particular, at $t = 0.04\,\mathrm{s}$, $V^\star_{d_2}$ becomes $114\sqrt{2}\mathrm{V}$, i.e., it is reduced by 5%, and at $t = 0.06\,\mathrm{s}$, the power demanded by the local load of DGu_4 increases by 25%, i.e., W_{d_4} becomes $100\,\mathrm{A}$.

In Figure 3 the time evolution of the dq-components of the load voltages is represented. One can observe the robustness of the proposed decentralized SSOSM control approach (solid lines) with respect to both reference and load variations. In particular, the voltage dynamics of the neighbouring DGus are not affected neither by load nor by reference variations and faster voltage tracking performance than that by using the PI control (dashed lines) is guaranteed.

In the same figure, also the time evolution of the d-component of the generated currents and the exchanged currents through the distribution lines interconnecting the DGus is illustrated. In particular, by using the SSOSM control, one can observe that when the voltage reference $V^\star_{d_2}$ becomes lower than the d-component of the voltage at PCC_1 and PCC_3, respectively, then DGu_1 and DGu_3 (i.e., the neighbours of DGu_2) increase the generated current, and deliver, through the distribution lines, the extra power to the DGu_2, which, instead, decreases its own generation. On the other hand, when the local load of DGu_4 requires more power, only DGu_4 increases its own generation.

Finally, the three-phase signals of the DGu_2, i.e., the load voltages (dashed lines) and the generated currents (solid lines) are also shown in Figure 3, together with the a-phase of load voltages. From this latter, one can observe that all the load voltages are synchronized with frequency equal to the nominal one $f_0 = 60\,\mathrm{Hz}$.

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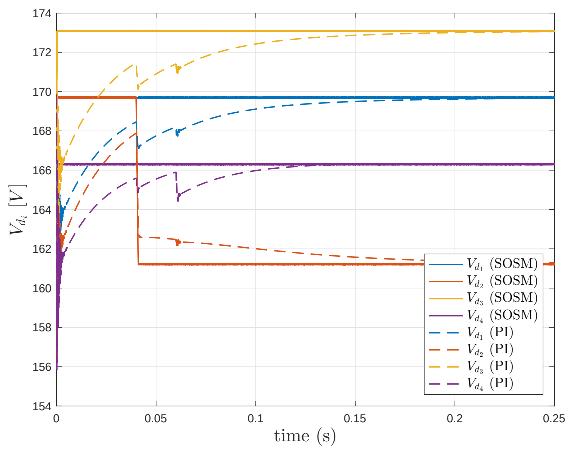
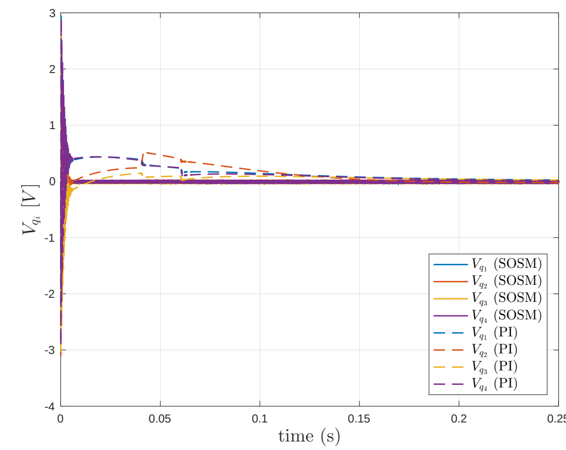
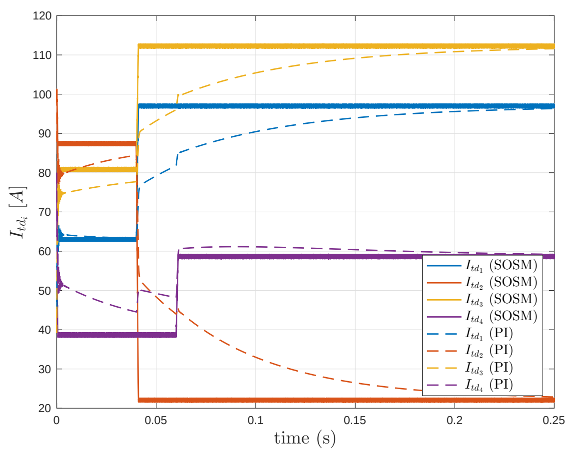


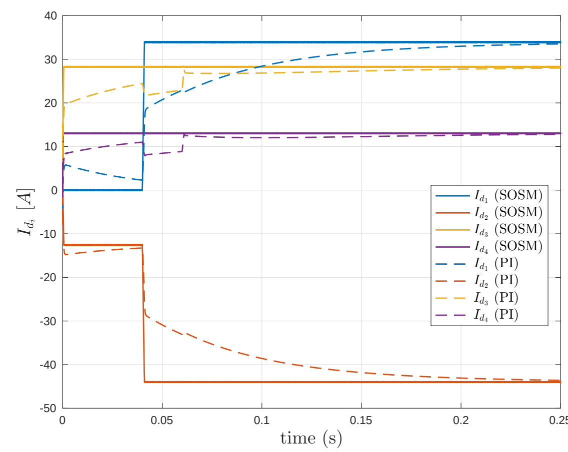
(a) d-component of the load voltages



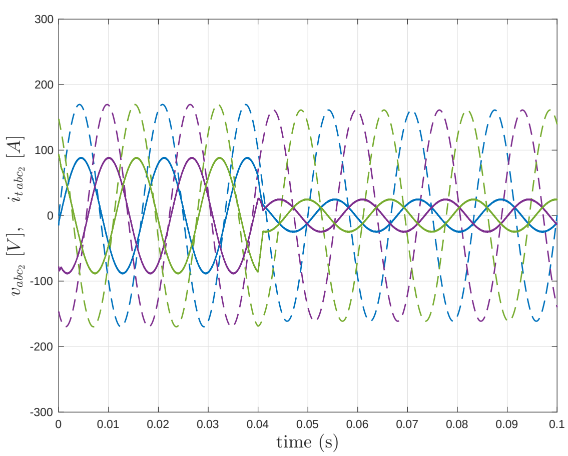
(b) q-component of the load voltages



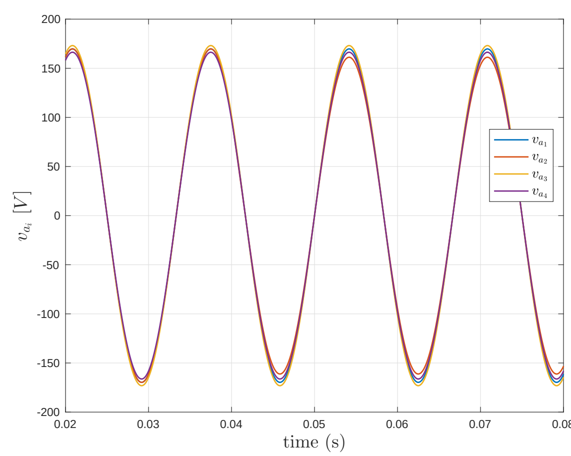
(c) d-component of the generated currents



(d) d-component of the line currents



(e) three-phase signals of DGu_2



(f) a-phase load voltages

Figure 3. Comparison between SSOSM and PI controllers in presence of reference and load variation. **a)** Time evolution of the d-component of the load voltages. **b)** Time evolution of the q-component of the load voltages. **c)** Time evolution of the d-component of the generated currents. **d)** Time evolution of the d-component of the currents exchanged among the DGus through interconnecting power lines. **e)** Time evolution of the three-phase signals (load voltage and generated current) of DGu_2. **f)** Time evolution of the a-phase of the load voltages.

VII. Conclusions

In this paper a decentralized SSOSM control scheme is designed for an AC microgrid with arbitrary topology, affected by unknown load dynamics and model uncertainties, operating in IOM. The system has been modelled by introducing an incidence matrix and the controller has been suitably designed on the basis of the proposed model. The asymptotical stability of the whole microgrid has been proved and the performance of the proposed algorithm have been evaluated in simulation considering a microgrid with four DGus in a ring topology.

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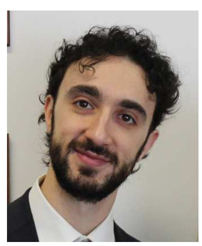

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