Optimal Sensors Positioning for Condition-based Risk Assessment by Particle Swarm Optimization

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Value of Information (VoI) has been proposed for optimal sensors positioning on Systems, Structure, and Components (SSCs). The non-sub-modularity property of VoI calls for non-greedy optimization methods for the solution of the positioning optimization problem. In this work, we use Particle Swarm Optimization (PSO) for solving the optimal sensors positioning problem and condition-based risk assessment. A practical case study is considered regarding the positioning of sensors measuring the thickness of a manifold of a Steam Generator (SG) for a Prototype Fast Breeder Reactor (PFBR) that can fail due to creep. Results show that the PSO-optimized sensors positions provide results that enable more accurate risk estimates than using greedy optimization methods.

Keywords: Condition-based Risk Assessment, Sensors Positioning, Value of Information (VoI), Greedy Optimization, Particle Swarm Optimization (PSO), Fast Breeder Reactor, Steam Generator.

Nomenclature

F	Multivariate random field
$f(\bar{x})$	Physical property at each location $\bar{x}$
$\bar{f}(\bar{x})$	Multivariate Physical properties at each location $\bar{x}$
$\bar{x}$	Location
$\bar{y}(\bar{x})$	Set of measurement values at location $\bar{x}$
$\bar{Y}$	Set of all feasible measurements
$\bar{y}\,(\bar{x}^*)$	Optimal set of measurement
$\bar{\varepsilon}$	Noise vector
n	Number of optimal sensors
j	Number of all candidate sensors locations
$\bar{\bar{R}}$	Matrix of measured sensors
$\bar{p}_F$	Prior distribution
$\bar{p}_{F\|\bar{y}}$	Posterior distribution
$\bar{P}_F$	Prior probability of failure
$\bar{P}_{F\|\bar{y}}$	Posterior probability of failure
$\mathbb{E}L(\emptyset)$	Prior expected loss
$\mathbb{E}L(\bar{y}(\bar{x}))$	Posterior expected loss
VoI	Value of information

Acronyms

PFBR	Prototype Fast Breeder Reactor
PSO	Particle Swarm Optimization
SG	Steam Generator
SSC	Systems, Structures, and Components
VoI	Value of Information

1. Introduction

Positioning of the sensors on safety-critical Systems, Structures and Components (SSCs) for monitoring their condition is a fundamental problem for condition-based risk assessment (Zio, (2018)). One way to address it is by maximizing the Value of Information (VoI), which amounts to solving a VoI-based optimization problem (Malings & Pozzi, (2016a); Malings and Pozzi, (2016b); Agusta, et al., (2017)). VoI, a mathematical concept based on Bayesian statistical decision theory (Raiffa and Schlaifer, (1961)), is a metric to quantify the benefits of acquiring information from sensors (Straub, (2014)). The best sensors positions have largest VoI.
Greedy optimization has been recently used to address the optimization problem (Khan, et al., (2014)); (Hoseyni, et al., (2019)). However, this method cannot guarantee an optimal solution for VoI-based optimization problems, because of the

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non-sub-modularity property of VoI (Malings and Pozzi, (2019)).
To overcome this issue, two approaches can be followed: i) a sub-modular metric can be used instead of VoI; ii) a non-greedy optimization approach can be used to solve the VoI-based optimization problem (Hoseyni, et al., (2019)).
In the present work, Particle Swarm Optimization (PSO), a non-greedy optimization method, is used to overcome the non-sub-modularity property of VoI. A practical case study of a manifold of a Steam Generator (SG) for a Prototype Fast Breeder Reactor (PFBR) is considered. The positions of the sensors obtained by both VoI-based greedy and non-greedy (i.e., PSO) approaches are compared. Then, the risk is estimated with the two sensors positioning strategies and compared.
The remainder of the paper is organized as follows: in Section 2, a brief description of the VoI-based greedy and non-greedy optimization approaches is provided; in Section 3, the case study is worked out with both greedy optimization and PSO; in Section 4, the condition-based risk estimates are estimated and compared; finally, Section 5 draws some conclusions.

2. Greedy and Non-greedy Optimization based on VoI

Sensors are positioned at locations $\bar{x}$ on SSCs to measure physical parameters $\bar{f}(\bar{x})$ (e.g., temperature, pressure, stress, thickness, etc.) (Bisdikian, et al., (2013)). The set of measurement values collected at at the locations $\bar{x}$ is $\bar{y}(\bar{x})$. Let $\bar{p}_F$ be the prior belief distribution of the multivariate random field F and $\bar{p}_{F|\bar{y}}$ the posterior distribution which is derived, by Bayesian inference method, as a physical characteristic of the SSC is measured with after $\bar{y}(\bar{x})$.
Consider the loss $L(\bar{f}(\bar{x}), \bar{a})$ as the negative utility, in monetary terms, derived by the managerial actions $\bar{a}$ aimed at maintaining the SSC available and inhibiting the SSC degradation progression.

The decision maker can take a prior decision, based on the prior belief distribution $\bar{p}_F$ when measurements are not yet available ($\emptyset$), by minimizing the prior expected loss:

$$\mathbb{E}L(\emptyset) = min\{\mathbb{E}_F L(\bar{f}(\bar{x}), \bar{a})\} \quad (1)$$

If the measurement $\bar{y}(\bar{x})$ of the physical parameters $\bar{f}(\bar{x})$ were available, the decision maker could take a posterior decision, based on the posterior belief distribution $\bar{p}_{F|\bar{y}}$, by minimizing the posterior expected loss:

$$\mathbb{E}L(\bar{y}(\bar{x})) = \mathbb{E}_Y min\{\mathbb{E}_{F|\bar{y}} L(\bar{f}(\bar{x}), \bar{a})\} \quad (2)$$

The benefits of acquiring information from measurements $\bar{y}(\bar{x})$ can be evaluated by subtracting the posterior expected loss $\mathbb{E}L(\bar{y}(\bar{x}))$ from the prior expected loss $\mathbb{E}L(\emptyset)$, as:

$$\mathrm{VoI}\,(\bar{y}(\bar{x})) = \mathbb{E}\mathrm{L}(\emptyset) - \mathbb{E}\mathrm{L}(\bar{y}(\bar{x})) \quad (3)$$

where VoI $(\bar{y}(\bar{x}))$ is the value of information of $\bar{y}(\bar{x})$.
To find the optimal sensors locations $\bar{x}^*$, VoIs can be compared. This requires solving the combinatorial optimization problem:

$$\bar{y}(\bar{x}^*) = argmax_{\,\bar{Y}\subseteq\Omega_Y}\,(VoI(\bar{Y})) \quad (4)$$

where Ω_Y is the measurement spatial domain and $\bar{Y}$ the set of all feasible candidate sensors locations.
Greedy optimization and non-greedy optimization methods can be used to solve the problem of Eq. (4).

Consider the case of feasible sensors location and $n<j$ sensors locations to be located. Greedy optimization consists in an iterative and progressive addition of an individual sensor location from j potential candidate locations to a set that finally will contain n sensors locations. In each iteration, an individual sensor is identified in a field of previously identified n-1 sensors assuming the current iteration's prior field $(\bar{p}_F)_n$ is equal to the previous iteration's posterior $(\bar{p}_{F|\bar{y}})_{n-1}$. The iterations continue until a set of n sensors that yields the max VoI is found (i.e., finding $\bar{y}(\bar{x}^*)$.

The greedy optimization is very efficient since the number of VoI computations is dramatically less than other optimization methods. Metrics that have the characteristics of sub-modularity (i.e., they provide a diminishing return when new measurements are added to an already identified set) are suitable to be used within a greedy optimization problem. However, since, VoI is not a sub-modular metric, there is no guarantee that the obtained optimal set of sensors positioning is the real optimal one when relying on a greedy approach.
To overcome this issue, a non-greedy optimization method, i.e., Particle Swarm Optimization (PSO), is here used, because it is not sensitive to the non-sub-modularity of the VoI (Hoseyni, et al., (2019)).

Instead of building up the set of n sensors by iteratively adding single sensor locations for n iterations, PSO searches among all possible sets of sensors locations to find $\bar{y}(\bar{x}^*)$.

3. Case Study

The case study regards a manifold of a SG of a Prototype Fast Breeder Reactor (PFBR) (Fig. 1), whose thickness is monitored by ultrasonic thickness gauges (hereafter referred to as sensors) to prevent creep (Di Maio, et al., (2018)).

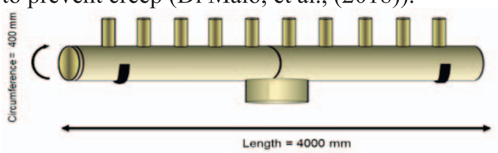

Fig. 1. The manifold of the SG of PFBR

For the purpose of our analysis, the manifold is modeled as a plate-like surface discretized into a spatial domain Ω_x with 160 squares of 100×100 mm, corresponding to j=160 candidates for sensors mounting (Fig. 2).

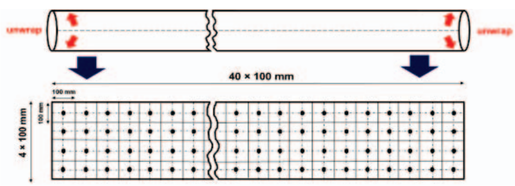

Fig. 2. The spatial model Ω_x of the manifold discritized into 160 squares

It is assumed that the thickness of the manifold $f(\bar{x})$ at each $\bar{x}$ location is a random variable that follows a Gaussian distribution $\mathbb{N}$~(20mm, 1mm), except the area of Welding and Heat Affected Zones (WHAZ) that is assumed to have larger thickness uncertainty, distributed as $\mathbb{N}$~(20mm, 2mm) (Fig.3).

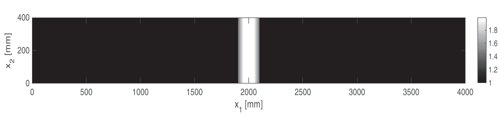

Fig. 3. Contour of the standard deviation of thickness of the manifold

Based on the prior distribution $\bar{p}_F$, and assuming the thickness threshold to be equal to 16.9 mm (Hoseyni, et al., (2019)), the prior probability of failure $\bar{P}_F(\bar{x})$ of the manifold is calculated using the limit state method (Zio, (2007)). The resulting probability of failure in Ω_x is shown in Fig. 4: in the WHAZ, where larger uncertainty on the thickness is expected, the largest prior probability of failure can be found.

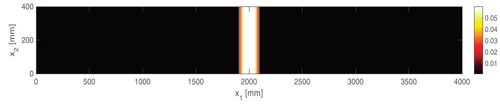

Fig. 4. Prior probability of failure of the manifold

The measurement set $\bar{y}(\bar{x})$ is the set composed of measurements at location $\bar{x}$:

$$\bar{y}(\bar{x}) = \bar{\bar{R}}\bar{f}(\bar{x}) + \bar{\varepsilon} \qquad (5)$$

where $\bar{\bar{R}}$ is a row matrix that gives the discrete locations in which measurements are taken and $\bar{\varepsilon}$ is the noise vector. When measurements are taken, the distribution of the random vector $\bar{f}(\bar{x})$ can be updated by Bayesian inference from the prior distributions $\bar{p}_F$ to the posterior distribution $\bar{p}_{F|\bar{y}}$. The posterior probability of failure $\bar{P}_{F|\bar{y}}(\bar{x})$, can, then, be obtained from $\bar{p}_{F|\bar{y}}$ using, again, the limit state method (Zio, (2007)).

The decision-maker can take two actions $\bar{a}$ which are: 1. do nothing (a = 0) or 2. mitigation (a = 1) to counteract creep evolution (e.g. by weld repair or reduction of operational load, etc. (Sposito, et al., (2010))). The true state $\bar{s}$ of the manifold, which the decision-maker is unaware of, is failed (s = 0) or operational (s = 1).

The defined loss function is:

$$L(\bar{f},\bar{a}) = \begin{Bmatrix} 0 & if\ s = 1\ and\ a = 0 \\ C_f = 200000\,€ & if\ s = 0\ and\ a = 0 \\ C_m = 5000\,€ & if\ a = 1 \end{Bmatrix} \quad (6)$$

where C_f is the cost of failure for a wrong decision, and C_m is the mitigation cost.

Then, using $\bar{p}_F$ and its consequent prior probability of failure $\bar{P}_F(\bar{x})$, the prior expected loss $\mathbb{E}L(\emptyset)$ of Eq. 1 is quantified by the loss function of Eq. 6. When a measurement set $\bar{y}(\bar{x})$ is considered, the posterior random field $\bar{p}_{F|\bar{y}}$ is updated and the posterior probability of failure $\bar{P}_{F|\bar{y}}(\bar{x})$ is quantified. Then, the posterior expected loss $\mathbb{E}L(\bar{Y})$ is quantified by Eq.2, and finally the VoI is quantified by Eq.3.

3.1 *Greedy optimization*

There are j=160 candidate locations where sensors can be positioned. A trivial solution is reached when all n=160 sensors are positioned in the j=160 candidate locations since this would give the maximum VoI because it is assumed that there is no cost for sensors positioning. We, on the other hand, search for the $\bar{x}^*$ composed of n sensors that guarantees achieving a reasonable level of VoI. More specifically, we are searching for the n sensors set $\bar{x}^*$ where adding $(n+1)_{th}$ sensor to it would cause a negligible improvement in the VoI (here assumed, arbitrarily as 80% of mitigation cost C_m, equal to 4000 €).

First, a single sensor location is measured in all of the j=160 candidate locations and the location with maximum VoI is selected. Then, the 2nd sensor location which yields the maximum VoI, conditional to the positioning of the 1st sensor (found at the previous step) is searched and found. Then, iteratively all n sensors locations are found and added to the set $\bar{y}(\bar{x}^*)$ until $VoI_{n+1} - VoI_n$ <4000 €. Table 1 shows the results: the optimal set $\bar{y}^*(\bar{x})$ is the set obtained at the 5th iteration.

Table 1. VoI obtained at each step of greedy optimization

Iteration	VoI	VoI_{n+1} - VoI_n	Sensor locations ID number
1	6203	-	82
2	12463	6260	82,80
3	17745	5282	82,80,77
4	22464	4719	82,80,77,83
5	26495	4031	82,80,77,83,78
6	30043	3548	82,80,77,83,78,79

Fig. 5 shows the positions of $\bar{y}(\bar{x}^*)$ obtained with greedy optimization (star shapes, with sensor ID written beside). It can be seen, as explained that all 5 sensors locations are located in WHAZ area that is highlighted with the grey boxes.

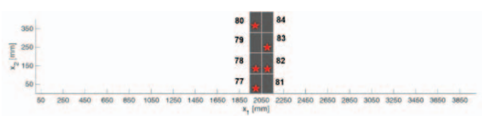

Fig. 5. Optimal set of sensor locations $\bar{y}(\bar{x}^*)$obtained from greedy optimization

3.2 *Non-greedy optimization*

In this Section, PSO is used to find $\bar{y}(\bar{x}^*)$ constrained to n=5 for comparison to the results obtained in Section 3.1. In other words, PSO searches all possible combinations of n=5 sensors locations in the j=160 candidate locations to find the set that has the maximum VoI. The results in Table 2 show that PSO yields different optimal set of n=5 sensors, with a larger VoI than the greedy approach.

Table 2. Comparison of the $\bar{y}(\bar{x}^*)$ and VoI obtained with greedy optimization and PSO

	Greedy optimization	PSO
$\bar{y}(\bar{x}^*)$	82,80,77,83,78	82,80,77,79,84
VoI	26495 €	26926 €

Fig. 6 compares the sensors locations that are obtained with greedy optimization and PSO: in both cases, the WHAZ area is targeted by sensoring, because of the larger uncertainty; 3 (out of 5) sensors locations identified by both methods are identical, whereas the 2 different sensors locations identified by the PSO increase the VoI of greedy optimization by 2%.

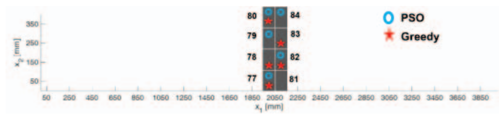

Fig. 6. Schematic view and comparison of the optimal set of sensor locations $\bar{y}(\bar{x}^*)$ obtained from greedy optimization and PSO

4. The Risk Estimates

In this Section, we show the benefits of using $\bar{y}(\bar{x}^*)$ obtained by PSO for assessing the risk of the manifold failure due to creep.
The risk metric at each location $\bar{x}$ is the probability of failure $Pr(\bar{x})$ at that specific location. The mean probability of failure E_{Pr}, the standard deviation σ_{Pr} and the maximum probability of failure Pr_{max} among all 160 $Pr(\bar{x})$ are given in Table 3, for three different measurement conditions: 1) without any measurement (∅); 2) measurement taken in $\bar{y}(\bar{x}^*)$ obtained by greedy approach; 3) measurement taken in $\bar{y}(\bar{x}^*)$ obtained by PSO.

Table 3. Mean probability of failure E_{Pr}, standard deviation σ_{Pr} and maximum probability of failure Pr_{max} for three different measurement conditions

Measurement locations	∅	$\bar{y}(\bar{x}^*)$ of greedy optimization	$\bar{y}(\bar{x}^*)$ of PSO
E_{Pr}	0.002169	0.001341	0.001328
σ_{Pr}	2.76e-5	6.68e-6	6.18e-6
Pr_{max}	0.0250	0.0206	0.0201

As can be seen in Table 3, when no measurement is taken, the mean and maximum probability of failure, respectively E_{Pr} and Pr_{max}, are the largest. Taking measurements, reduces the risk of failure, as E_{Pr} and Pr_{max} decrease by using $\bar{y}(\bar{x}^*)$ obtained by either greedy approach or PSO. Finally, PSO yields the smallest E_{Pr}, Pr_{max}

and σ_{Pr} (i.e., PSO provides the most accurate risk estimate).

5. Conclusion

In this work, the challenges of using greedy optimization for the VoI maximizing problems is discussed since VoI lacks the characteristic of sub-modularity. To overcome this, we used PSO, a non-greedy optimization approach, that is not sensitive to the non-sub-modularity of the VoI. A case study of the manifold of a SG is presented whose thickness can be measured by the sensors. The optimal sensors positioning of the case study is identified by both VoI-based greedy optimization and PSO. Results show that using the sensors positioning obtained by PSO not only yields better VoI, but also provides more accurate risk estimates.

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