

A NURBS-BASED NUMERICAL APPROACH FOR THE ASSESSMENT OF MASONRY VAULTS UNDERGOING DIFFERENTIAL SETTLEMENTS

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Abstract

In this paper, we propose a NURBS-based adaptive approach to the three-dimensional analysis of masonry vaults undergoing foundation settlements. A given masonry vault of arbitrary geometry is described through its NURBS (Non-Uniform Rational B-Spline) parametric representation in the three-dimensional Euclidean space. The vault surface is then discretized into an initial set of rigid elements. Such discretization is obtained by suitably subdividing the NURBS parameters space. Jumps of displacement are allowed solely at the interfaces. Given a known displacement on the external boundary, which comprises possible settlements, the resulting displacement field is computed by minimizing the total potential energy of the system by means of a linear programming optimization algorithm. Moreover, a mesh adaptation scheme based on a suitable Genetic Algorithm (GA) is used to determine the crack pattern yielding the mechanism actually induced by the settlement. The procedure is here demonstrated through a numerical example.

Keywords: Masonry, Foundation Settlements, NURBS, Genetic Algorithm

1 INTRODUCTION

The interest in studying the crack patterns induced by foundation settlements in masonry structures is quite recent. In fact, differential settlements are one of the most frequent causes of cracks in historical masonry structures. When undergoing foundation settlements, masonry structures deform like mechanisms in order to be able to accommodate such settlements, accompanied by a pattern of cracks. The strong nonlinearities of the masonry material, which have been summarized by Heyman [1] by means of the no-tension material assumption, render masonry constructions quite unsuited to resist differential settlements. In the last decades, the problem of settlements on masonry structures has been studied by means of nonlinear analyses within the Finite Element Method (FEM) [2–4], the Discrete Element Method (DEM) [5], or, rigid blocks analyses under the hypothesis of unilateral contact and simple linear programming techniques [6–9].

The analysis of the response to settlements response using systems of rigid blocks entails procedures similar to the ones typical of limit analysis. Under the action of settlements, the above structure reaches a new configuration of equilibrium exhibiting small deformation [10]. However, since masonry cannot withstand deformations of any kind, cracks appear and propagate within the structure, transforming the initially continuous construction into an assembly of relatively big and approximately rigid blocks. The new configuration of equilibrium can be described through a discontinuous displacement field. As a result, the structure behaves like a mechanism and can be studied by using limit analysis techniques. The analogy between limit analysis and analysis of settlements for rigid blocks structures has been analyzed in detail in a recent publication [11]. Here, it is shown that the discontinuous displacement field can be obtained by solving a unilateral contact problem. The discrete formulation can be written in terms of a linear programming problem, in which the external work is maximized. The formulation by rigid blocks and linear programming allows a quick assessment of crack patterns with a sensibly reduced computational effort with respect to standard finite element nonlinear techniques and iterative procedures.

In this contribution, we present the study of masonry vaults undergoing differential settlements through a new adaptive GA-NURBS based approach. A masonry vault is modeled through NURBS rigid shell elements, which allow accurate reproduction of the actual geometry even with a small number of elements is used. Given an initial mesh of NURBS rigid element, the displacement field is found by maximizing the work of external loads. A Genetic Algorithm mesh adaptation search scheme is devised in order to find the actual displacement field induced by a given settlement. The procedure is illustrated by solving the historical vault previously analyzed in [13-14].

2 DISCRETE VARIATIONAL FORMULATION

The behavior of a masonry structures undergoing foundation settlements can be studied by idealizing it as an assembly of rigid blocks and applying unilateral contact conditions at the interfaces [11]. The solution \mathbf{u} of the variational contact problem, at the continuum level, is given by

$$\mathbf{u} = \arg \inf_{\mathbf{v} \in M^m} \Pi_r(\mathbf{v}), \quad (1)$$

where

$$\Pi_r(\mathbf{v}) = - \sum_{i=1}^{N_b} \ell_i(\mathbf{v}) \quad (2)$$

is the potential energy of external loads acting on each block and, M^{*m} is the convex set of rigid motions and $\ell(\cdot)$ is the linear form:

$$\ell(\mathbf{v}) = \int_{\Gamma_q} \mathbf{q}\mathbf{v}d\gamma. \quad (3)$$

In other words, a displacement field solution of the boundary value problem at hand, can be found by minimizing the potential energy of the external loads in the set M^{*m} of all the mechanisms satisfying the unilateral constraints in the normal direction and the no-sliding condition.

At the discrete level, a generic piecewise two-dimensional rigid displacement $\mathbf{v} \in M^{*m}$ is represented by a vector \mathbf{d} composed by $3N_b$ elements being the three barycentric components of rigid body motion (two translations and a rotation) for each block. Obviously, it is possible to define the potential energy associated to the rigid body motion \mathbf{v} represented by the vector \mathbf{d} as a linear function of \mathbf{d} :

$$\Pi_r(\mathbf{v}) = -\hat{\mathbf{q}}^T \mathbf{d}, \quad (4)$$

where $\hat{\mathbf{q}} \in \mathbb{R}^{3N_b}$ is a vector containing, for each body \mathcal{B}_i , the resultants – in terms of two translational forces and one moment – of the external load distribution $\mathbf{q}(\mathbf{x})$ on Γ_q . Therefore, the discrete linear programming problem associated to (1) can be written as follows:

$$\text{find } \mathbf{d} \text{ s.t. } \{\hat{\mathbf{q}}^T \mathbf{d}\} \text{ is maximum,} \quad (5)$$

under the following linear constraints:

$$\mathbf{A}\mathbf{d} \leq \mathbf{0} \quad (6)$$

$$\mathbf{B}\mathbf{d} = \mathbf{0} \quad (7)$$

$$\mathbf{C}\mathbf{d} = \hat{\mathbf{u}}_0, \quad (8)$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{N_{col,p} \times 3N_b}$, $\mathbf{C} \in \mathbb{R}^{N_{col,r} \times 3N_b}$ and $\hat{\mathbf{u}}_0$ is a vector whose elements are the values of the function $\mathbf{u}_0(\mathbf{x})$, representing the non-homogeneous essential boundary conditions, evaluated at collocation points on Γ . Constraints (6) and (7) represent the requirement that $v_n \leq 0$ on Γ_c and $v_t = 0$ on Γ_c respectively, Γ_c being the union of all the possible contact curves among the blocks. On the other hand, constraint (8) represents the non-homogeneous essential boundary conditions (i.e. foundation settlements) on Γ_u . It should be noticed that the objective function in (5) is the work of the assigned external loads.

3 ADAPTIVE GA-NURBS SCHEME

The structural response is described by roto-translations of the rigid blocks and jumps of displacement on the contact points. The overall displacement field can be obtained by solving problem (5), in which the external work is maximized under the constraints imposed by unilateral contact conditions, by means of an efficient linear programming.

In this work, a discretization through few NURBS shell rigid elements is adopted. The NURBS geometry description of the mesh allows to preserve the actual geometry of the structure even when using a small number of elements, avoiding the need for fine discretizations which would be otherwise required for masonry vaults. Thus, in place of modeling every single brick composing the structure, NURBS elements reproduce the curved macro-blocks

which determine the mechanism induced by the applied settlement, provided that the crack pattern is suitably adjusted by means of a suitable meta-heuristic optimization algorithm. On each interface, the unilateral contact conditions are applied. In case a rigid plastic behavior is to be accounted for, an associative flow rule can be adopted. As already discussed, the overall disposition of interfaces, i.e. the mesh adopted, represents only a possible crack pattern but, since the real crack pattern is not a-priori known, mesh adaptation is required. For this reason, in order to find the absolute maximum of the external work, we allow a Genetic Algorithm (GA) to adjust the initial mesh until a good approximation of the actual crack pattern is obtained. The procedure is very similar to the one used in the GA-NURBS limit analysis of masonry vaults proposed in [12], which was proven to be effective in the study of several typologies of masonry constructions [15–22].

4 NUMERICAL EXAMPLE

As an example, we study a reproduction of the historical barrel vault which constituting the roofing of the central nave of the Bothwell Parish Church (Glasgow, United Kingdom, see Fig. 1(a)). An in-depth geometrical survey revealed that the vault is slightly skewed, with cracks that has opened over the years due to settlement across the less braced South edge. In [14], this response of this vault under foundation settlements has been investigated by means of finite element analyses and several experimental test (also carried out in [13]) on a 1/12 scaled model (Fig. 1(b)). The actual vault geometry features an interior span of 6.1 m, a rise of 3.8 m, and a length of 16.8 m, and an average thickness of 36 cm. We assume a 520 kg/m³ density for masonry material. Finally, in order to better reproduce sliding deformations observed in the experimental test performed in [14], a 3D Mohr-Coulomb failure surface with tension cut-off and compression linear cap is assumed, with ultimate tensile and compression stresses equal to 0.03 MPa and 2.6 MPa respectively. Shear failures are controlled by a cohesion of 0.02 MPa and a tangent of the friction angle of 0.5. To replicate the experimental test, in the first numerical simulation, a vertical linear settlement has been applied (see Fig. 2), with a 1 cm maximum drop along the external edge.



Figure 1: (a) Bothwell Parish Church (Glasgow, UK) barrel vault [14] and (b) crack pattern observed in the experimental test [13].

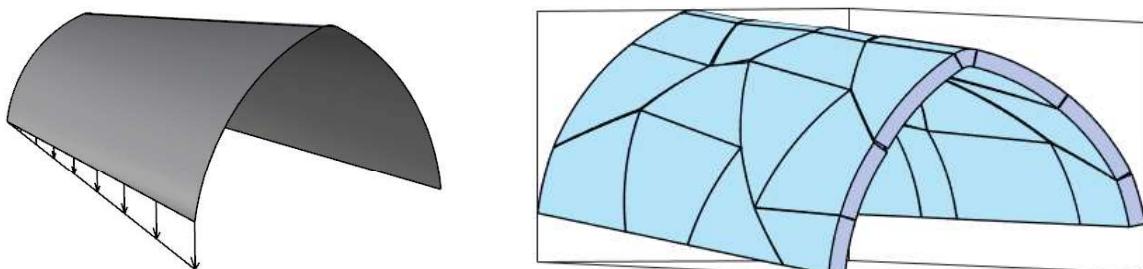


Figure 2: Analysis of the displacement field under the action of non-uniform vertical settlement.

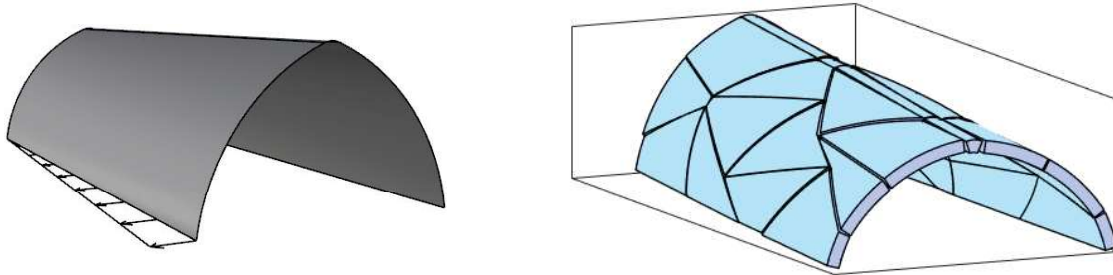


Figure 3: Analysis of the displacement field under the action of non-uniform horizontal settlement.

Each of the two NURBS surfaces describing half of the vault is discretized with 3×4 shell elements. Fig. 2 depicts the obtained displacement field. It can be noted that the crack pattern close to the external arch reproduces the typical hinges of the masonry arch undergoing a vertical differential settlement [11]. The shape of the crack pattern long the longitudinal direction of the vault is mainly due to settlements inducing torsional effects. The crack obtained with the proposed procedure is in good agreement with the results previously obtained both experimentally and numerically [13].

This barrel vault has been also studied under a horizontal linear settlement. Differential horizontal settlements are common in vaults used as roof elements in historical constructions, in which collapse during seismic events often occurs because of the differential movement of their supports, rather than because of the horizontal seismic actions applied to the vault. Fig. 3, represents the applied settlements and the obtained results. As in the previous case, it can be noted that the extremal part of the vault behaves like an arch. Also in this configuration, the proposed adaptive GA-NURBS scheme allows to accurately predict the 3D behavior of the vault.

5 CONCLUSIONS

In this paper, a NURBS-based adaptive approach to the three-dimensional analysis of masonry vaults undergoing foundation settlements is presented. A given masonry vault of arbitrary geometry is described through its NURBS parametric representation in the three-dimensional Euclidean space. The vault surface is then discretized into an initial set of rigid elements. Such discretization is obtained by suitably subdividing the NURBS parameters space. Jumps of displacement are allowed at the interfaces only. Given a known displacement on the external boundary, which comprises possible settlements, the resulting displacement field is computed by minimizing the total potential energy of the system by means of a linear programming optimization algorithm. Moreover, a GA-based mesh adaptation scheme is used to determine the crack pattern yielding the mechanism actually induced by the settlement. The example

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