

## BEHAVIOUR FACTOR ASSESSMENT OF ANCIENT MASONRY TOWERS THROUGH AN INNOVATIVE SIMPLIFIED PUSHOVER METHOD

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### Abstract

*The seismic vulnerability of masonry towers is a critical research field and it has an important role in the preservation of worldwide conservation of such kind of masonry building. Indeed, masonry towers have a very high seismic vulnerability and are often located in high seismic hazard zones. Researches about this topic have become even more frequent in the last decades. However, the results about the seismic behaviour of the masonry towers are still limited and further researches are needed. This paper aims to study the value of the behaviour factor (usually called  $q$ -factor or  $R$ -factor) of ancient masonry tower-like structures. This parameter provides quantitative information about the ductility of the tower and it can be computed by the capacity curve of the structure. In particular, the pushover curve can be bilinearized, and the  $q$ -factor can be computed according to the equal displacement or equal energy rule. The bilinearization of the capacity curve implies the knowledge of the ultimate displacement, whose computation is not a trivial task for this kind of structures because the capacity curves usually computed for such structures are apparently infinite ductility curves and the ultimate displacement cannot be directly computed. Therefore, an innovative pushover method (called “manual” pushover) has been formulated and implemented in a Matlab® code to achieve the  $q$ -factor computation goal. The tower is modelled by a vertical cantilever beam with lumped plasticity and it is loaded by self-weight and a user-defined horizontal distributed load profile. The non-linearities are modelled by the Moment-Curvature diagram of the cross-section where the plasticity is lumped. These cross-sections are positioned along the height of the tower splitting it into parts as uniform as possible in material and geometry characteristics. It allows considering the influence of openings and irregularities. The capacity curve is built by curvature control; it is similar to a displacement control and it allows to*

*capture the softening branch and, consequently, to compute the ultimate displacement. This method is quick, computationally light and it has been properly benchmarked with FEM results. The obtained capacity curve fits the request of the q-factor computation method presented above. It is worth noting that the Moment-Curvature diagrams have been computed by a Matlab code based on digital image processing. In particular, it can read the geometry of a generic cross-section through its digital image representation and it automatically computes the diagram according to equilibrium, compatibility and energy considerations. Finally, the q-factor value has been computed for various real case-studies highlighting its strong variability on the tower geometry (e.g. slenderness and shear area). The results have been compared with the suggested behaviour factor given by the Italian Guidelines for Built Heritage (2011) and it has been found that the code suggestions overestimate the tower ductility and they are too limited.*

**Keywords:** masonry towers; earthquake; simplified models; pushover; Matlab; behaviour factor.

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## 1 INTRODUCTION

The seismic vulnerability of masonry towers is a critical research field and it has an important role in the preservation of worldwide conservation of such kind of masonry building. Indeed, masonry towers have a very high seismic vulnerability and are often located in high seismic hazard zones. Researches about this topic have become even more frequent in the last decades. However, the results about the seismic behaviour of the masonry towers are still limited and further researches are needed.

Masonry towers have usually shown high seismic vulnerability [1–6] and their seismic response is complex and depends on various parameters, both mechanical and geometrical. In [7,8], the authors have demonstrated that five collapse mechanisms (Figure 1) are the most likely to occur and they depend on both slenderness and shear area.

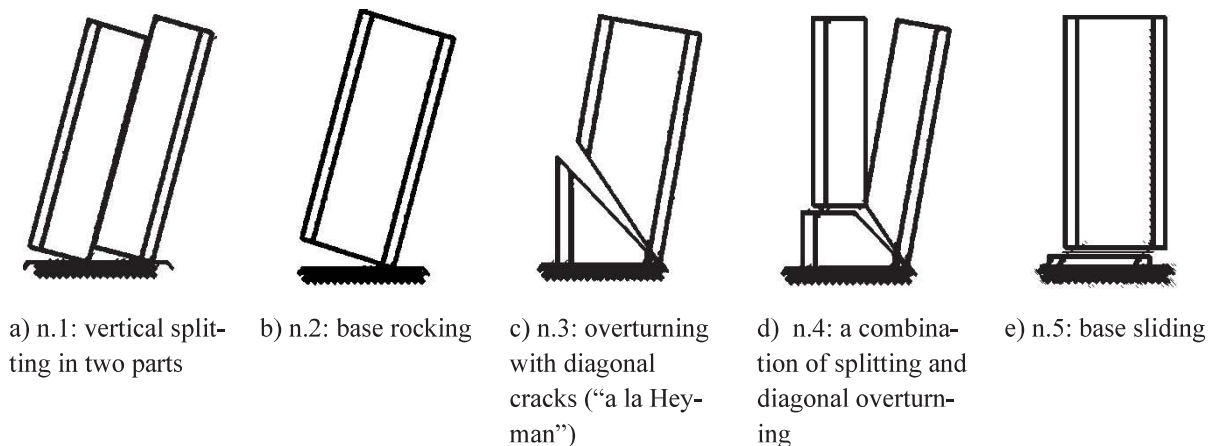


Figure 1 Collapse mechanism for masonry towers selected in [7] (images adapted from [7]).

Various analysis methods have been usually applied. Non-linear static and dynamic analysis are the most diffused analysis tools and they have been fully and successfully used in [1,2,9–12]. Also, simplified methods have been introduced, like the kinematic limit analysis has shown its accuracy and efficiency in various studies (e.g. in [7,8,13,14]). More sophisticated methods have been recently discovered, like the DEM modelling [7,15] or the Non-Smooth Contact Dynamic method [16].

Towers specific features have been analysed too. In [17–19] it has been demonstrated the importance of the tower inclination in their seismic assessment, while the influence of the surrounding buildings has been studied in [20].

This paper aims to study the value of the behaviour factor (usually called  $q$ -factor or  $R$ -factor) of ancient masonry tower-like structures. This parameter provides quantitative information about the ductility of the tower and it can be computed by the capacity curve of the structure. The capacity curve can be bi-linearized, and the behaviour factor may be computed according to the equal displacement rule or equal energy rule.

The bilinearization of the capacity curve implies the knowledge of the ultimate displacement, whose computation is not a trivial task for this kind of structures because the capacity curves usually computed by FE simulations for such structures are apparently infinite ductility curves and the ultimate displacement cannot be directly computed.

To achieve this goal, an innovative pushover method has been developed by the authors and it has been called “manual” pushover analysis. It is based on the modelling of the tower as a cantilever beam with lumped plasticity and it is loaded by self-weight and a user-defined horizontal distributed load profile.

The non-linearities are modelled by the Moment-Curvature diagram of the cross-section where the plasticity is lumped. These cross-sections are positioned along the height of the tower splitting it into parts as uniform as possible in material and geometry characteristics. It allows considering the influence of openings and irregularities.

The capacity curve is built by *curvature* control; it is similar to a displacement control and it allows to capture the softening branch and, consequently, to compute the ultimate displacement. This method is quick, computationally light and it has been properly benchmarked with FEM results. The obtained capacity curve fits the request of the q-factor computation method presented above. The Moment-Curvature diagrams have been computed by a Matlab code based on digital image processing. In particular, it can read the geometry of a generic cross-section through its digital image representation and it automatically computes the diagram according to equilibrium, compatibility and energy considerations.

Finally, the q-factor value has been computed for various real case-studies highlighting its strong variability on the tower geometry (e.g. slenderness and shear area). The results have been compared with the suggested behaviour factor given by the Italian Guidelines for Built Heritage (2011) and it has been found that the code suggestions overestimate the tower ductility and they are too limited. In particular, the suggestion made by the Italian Guidelines provides a double choice for the behaviour factor value: equal to 3.6 for regular towers and 2.8 for irregular ones. This value is used in the LV1 procedure proposed by guidelines, which is a method to quickly study the seismic vulnerability of such structures. The behaviour factor value is inversely proportional to the applied inertia horizontal forces; therefore its overestimation may lead to unsafe results. Similar final considerations have been produced also in [21].

A real case-study will be considered: a Chinese masonry pagoda (the *Longhu Pagoda*). It is the case study presented in [22], in which several FE pushover analyses have been carried out. These data will be used in this paper.

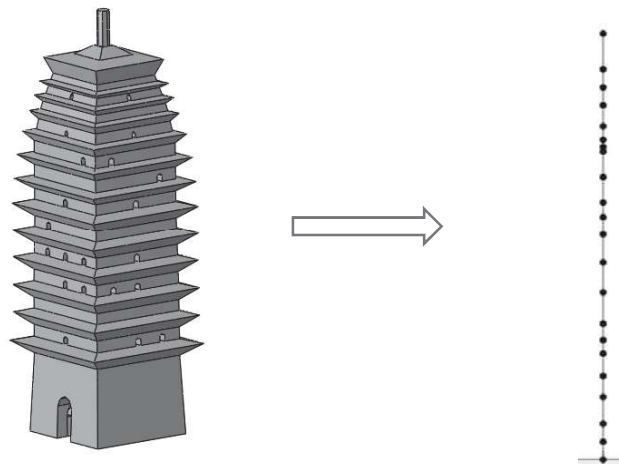


Figure 2 Lumped plasticity model for the "manual" PO applied to the Longhu Pagoda.

The structure of this paper is as follow. Section 2 will briefly present the “manual” pushover method and its validation. Finally, in Section 3 the assessment of the behaviour factor value is reported. Discussions and conclusions are drawn in Section 4.

## 2 THE “MANUAL” PUSHOVER ANALYSIS

Unfortunately, pushover analysis often needs sophisticated FE tools. Thus, a simplified pushover method is presented for masonry towers seismic assessment. The computational cost

is lower, and no FE software are needed. A similar method but with different algorithms and analysis tools has been proposed in [23].

The tower is modelled as a cantilever beam with lumped plasticity. The plastic properties are considered through the Moment-Curvature ( $M-\chi$ ) diagram where the plasticity is lumped. The tower body is split into  $n$  parts, posing the horizontal cutting, particularly at openings and irregularities. Each part has to be as uniform as possible in geometry and materials, similarly to what is suggested by Italian guidelines [24] to apply the LV1 analysis. An example is represented in Figure 2.

The  $M-\chi$  diagram has been computed by a Matlab subroutine the author has created that can compute specific cross-sections properties based on a digital image processing of cross-section itself and equilibrium equations (a specific study will be published on this algorithm by the authors, an example is shown in Figure 3). The diagram is computed by considering a compressive axial force equal to the weight of the tower above the cross-section itself. The material model is implemented through a non-linear stress-strain relationship with softening both in tension and compression. Figure 4 shows the implemented stress-strain relationship, while Table 1 reports the main mechanical parameters. The material properties have been supposed according to the usual values of masonry models.

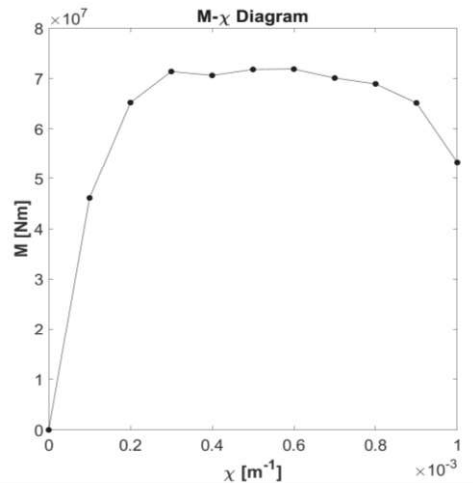


Figure 3 Example of the implemented  $M-\chi$  diagrams.

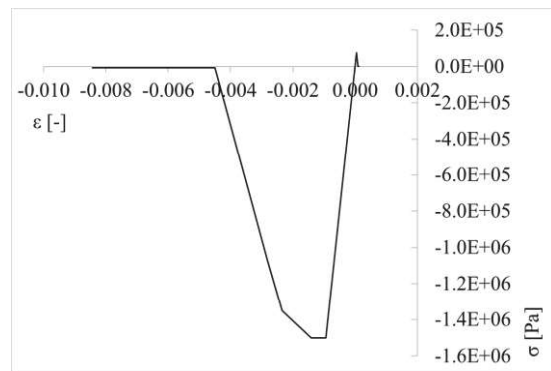


Figure 4 The implemented stress-strain relationship.

Density $\rho$ [kg/m <sup>3</sup> ]	Young's Modulus $E$ [MPa]	Poisson's Coeff. $\nu$ [-]	$\sigma_c$ [MPa]	$\sigma_t$ [MPa]
2000	1600	0.2	1.5	0.075

Table 1 Main mechanical parameters.

The proposed “Manual” pushover analysis is performed in *curvature* control. It is a sort of *displacement* control method that allows capturing the softening branch. The curvature is progressively increased in a chosen cross-section (i.e. the first one that would fail, named *controlled* cross-section). Both G1 and G2 (Figure 5) load distributions have been used. The structure is statically determined, thus the bending moment diagram depends only on equilibrium considerations and its expression will never lose its validity during the whole analysis procedure. The usual formulas used to compute the bending moment distribution on a cantilever beam have been used. The external horizontal load is progressively incremented



and the equilibrium at each step is achieved. Through the bending moment at each cross-section and the related  $M-\chi$  diagrams, the curvature distribution is computed along the height and, subsequently the top displacement. Finally, the capacity curve is obtained by the base shear and the top displacement at each step.

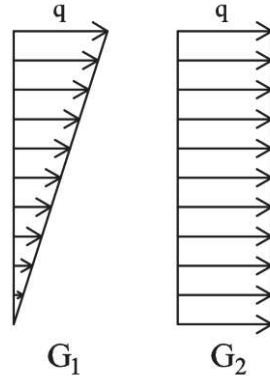


Figure 5  $G_1$  and  $G_2$  load distributions.

The horizontal load value decrease after that the structure strength reaches its pick and the same happen to the bending moment distribution. However, while in the *controlled* cross-section the *curvature* is continuously increased, the other cross-sections experience a decreasing in bending moment and the same happens to the curvature: linear unloading is implemented for them.

The method has been applied to real case-studies and its results have been compared to capacity curves obtained by FE simulations. It is the *Longhu Pagoda* which is a Chinese masonry pagoda and it is the case study presented in [22]. The data collected in this latter study are presented here to show the validity of the “manual” pushover method.

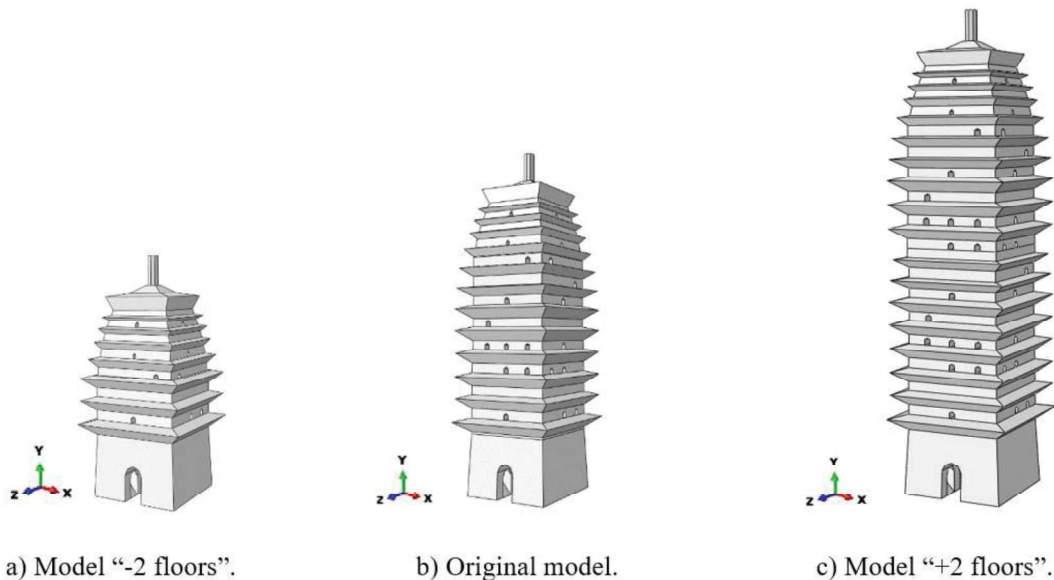


Figure 6 The pagoda models.

In [22] peculiar considerations have been made on the slenderness influence by building two fictitious models of the pagoda adding and removing two floors. In this way, the interiors

and mass distribution are still plausible. The three models are shown in Figure 6 and the interested reader is remanded to [22] for further details about them. Therefore, three case-studies will be presented in the following. The material model is reported in Figure 4 and Table 1. Both G1 and G2 load cases have been considered and the pushovers have been carried out along the  $z^+$  direction only (Figure 6).

The “manual” pushover results have been compared with the FEM pushover capacity curves presented in [22] and carried in the Abaqus environment. The material models are similar in terms of a stress-strain relationship. Figures 7-9 shows the comparison between the capacity curves for the three models. The “Load Factor” is the ratio between the total base shear and the total weight of the structure. A new phenomenon is here present: the *snap-back* of the top displacement in the last phases of the analysis. This novelty has been possible to be caught only thanks to the special pushover control parameter used (i.e. the *curvature* control in the worst cross-section). Apart from the “+2 Model”, in which the predictions of the two methods differ, in the other cases good agreement is achieved. These results will be used to compute the behaviour factor of the pagoda.

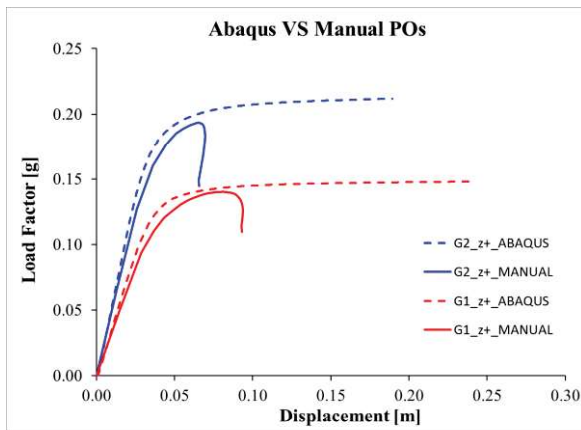


Figure 7 Abaqus vs Manual POs: original model.

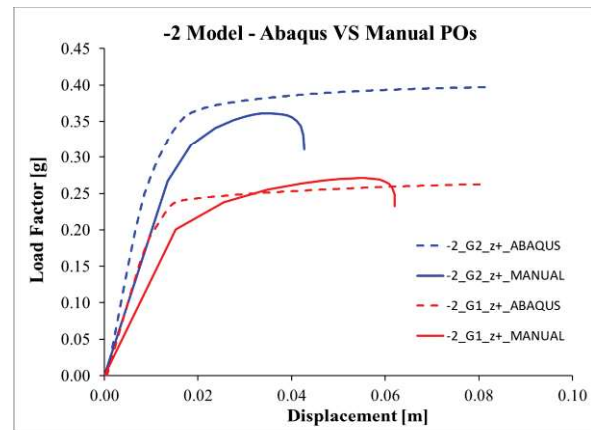


Figure 8 Abaqus vs Manual POs: “-2 Model”.

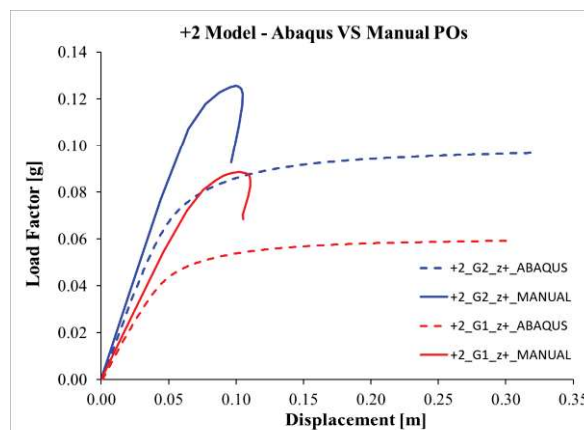


Figure 9 Abaqus vs Manual POs: “+2 Model”.

### 3 THE BEHAVIOUR FACTOR OF THE TOWER

To estimate the  $q$ -factor it has been decided to use the bi-linearization of the capacity curve and then to apply the equal displacement rule or the equal energy rule. The bi-linearization

procedure is taken from the Italian building code [25] and its schematization is shown in Figure 10. It is a function of the frequency content of the seismic action (its elastic-response spectrum) and the natural period of the structure.

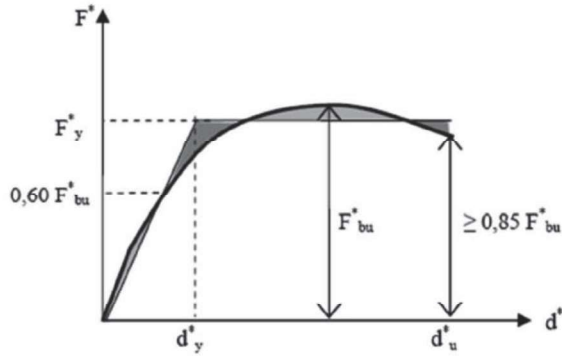


Figure 10 Equivalent bi-linear curve (adapted from [25]).

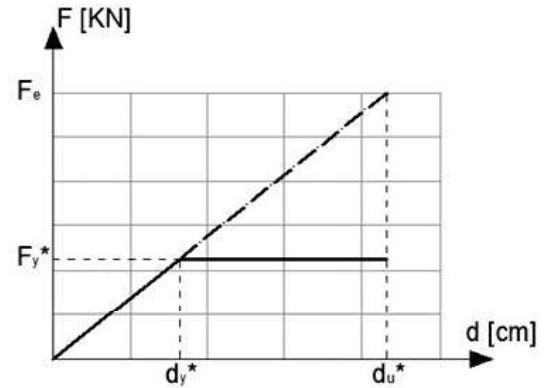


Figure 11 Equal displacement criterion (adapted from [26]).

In particular, considering the elastic system and the elastic-plastic system (Figure 11), they are related between each other according to the value of the natural period of the structure and the period  $T_c$ , which is the upper limit of the period of the constant spectral acceleration branch in the elastic response spectrum that represents the seismic demand (as defined in the Eurocode 8 [27]). In this case  $T_c = 0.4 \text{ s}$  (according to the spectrum defined by the Chinese Building Code [28] for the site of the pagoda and presented in [22]). Two cases can be defined:

- $T \geq T_c$ : the ultimate displacement is the same between the elastic and the elastic-plastic system (Figure 11) (“equal displacement” criterion).
- $T < T_c$ : the deformation energy is the same between the elastic and the elastic-plastic system (“equal energy” criterion).

The q-factor can be computed by Eq. (1).

$$\begin{aligned} q &= \mu & T &\geq T_c \\ q &= 1 + (\mu - 1) \frac{T}{T_c} & T &< T_c \end{aligned} \quad (1)$$

where  $\mu = \frac{d_u^*}{d_y^*}$ . Note that  $\frac{F_e}{F_y^*} = \frac{d_u^*}{d_y^*}$  if  $T \geq T_c$ .

This method has been applied to the capacity curves obtained from the “manual” pushovers. As an example, the computations made on the capacity curve related to the “manual” pushover performed on the original model along the z+ direction with a G1 load distribution is shown in detail (Table 2 and Figure 12). Table 3 report the behaviour factors computed for each case. It is worth noting that the behaviour factor changes as the applied load and the height change. Moreover, the value computed for the “+2 Model” might be furtherly studied due to the discrepancy between the “manual” pushover and the FE capacity curves.

It is noticeable that the behaviour factor values presented in Table 3 are lower than the values proposed by the Italian guidelines [24] and they are different from each other. This means that what is suggested in [24] may lead to unsafe and unrealistic results in terms of seismic vulnerability. Indeed, the Italian guidelines [24] propose two values of the behaviour factor:



3.6 for regular towers and 2.8 for irregular ones and these suggestions are not confirmed by this research.

$F_{MAX}$	$F_Y$	$d_Y$	$d_U$	$T$	$T_C$	$\mu$	$q$
[g]	[g]	[m]	[m]	[s]	[s]	[-]	[-]
0.14	0.131	0.039	0.09	0.6537	0.4	2.398	2.398

Table 2 q-factor assessment for the original model case (G1 load).

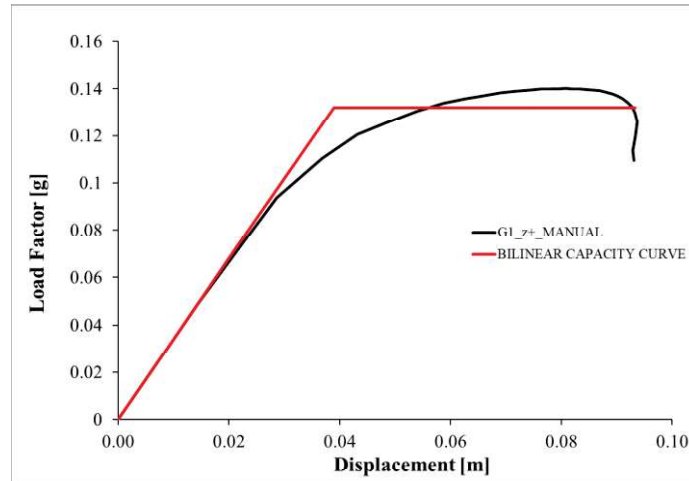


Figure 12 Capacity curve and the relative equivalent bilinear curve for the original model case (G1 load).

Model	Load Case	$q$
		[-]
original	G1	2.398
original	G2	1.883
-2	G1	2.578
-2	G2	2.095
+2	G1	1.503
+2	G2	1.485

Table 3 Behaviour factor's values

## 4 CONCLUSIONS

This paper has studied the value of the behaviour factor of masonry towers through an innovative pushover method that allows to capture the softening branch and thus, to apply the bi-linearization procedure to the capacity curve to compute the behaviour factor.

The behaviour factor values have been computed for some case-studies: three Chinese masonry pagodas with different slenderness. The behaviour factor has been computed by the capacity curves of these structures obtained from the application of the “manual” pushover method using both a G1 and G2 load distribution.

The computed values of the behaviour factors show that the suggestions of the Italian guidelines may underestimate it. Indeed, the Italian guidelines [24] propose two values of the

behaviour factor: 3.6 for regular towers and 2.8 for irregular ones and these suggestions are not confirmed by this research.

Therefore, further analysis of this topic may be useful to better quantify the behaviour factor of masonry towers and to propose new guidelines. Indeed, the simple and reliable formula may be very useful for practitioners for the safety assessment of the masonry towers. However, this study has shown that the behaviour factor may depend on several factors, among them the geometry of the structures. To assess these dependencies, further researches will be needed.

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