POLITECNICO
MILANO 1863

VALIDATION OF A WIND FARM’S WAKE MODEL

Authors:

Asghar Ali Raza
Cioffi Antonio
Schito Paolo
Zasso Alberto

Overview of the workflow

1. Literature Review
- HAWT modelling
- Wake models

2. Modelling
- GWM applicability
- GWM modifications
- Turbine model validation

3. Simulations
- Framework
- Setup

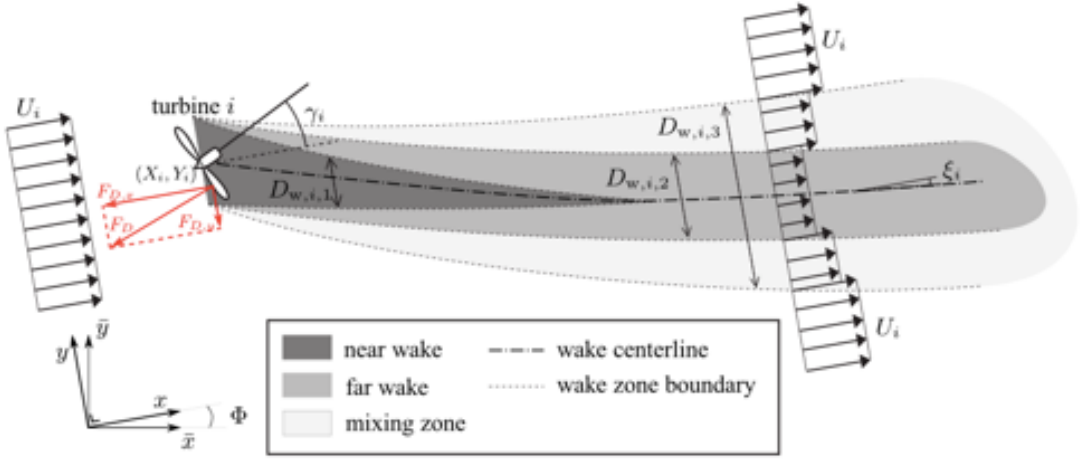
4. Results
- Single turbine
- Simple wind farm

5. Conclusions
- Key outcomes
- Future developments

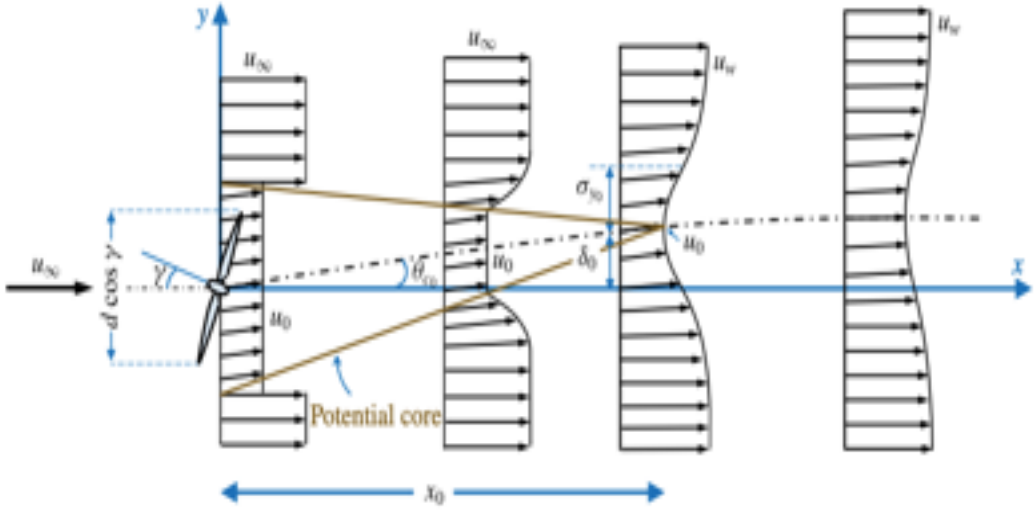
Wake Models

- High-fidelity models (SOWFA)

- Control-oriented models
- Jensen: 1D model that assumes top-hat velocity shape, derived from kinetic energy assumptions
- Frandsen: top hat velocity shape, derived from self similarity assumptions
- Zones: three different wake regions, each with different expansion rate
- Porté-Agel: gaussian velocity deficit, based on self similarity assumption



(a) Top view



Wake Models

Gaussian Wake Model

$$\frac{\Delta U(x,y,x)}{U_0} = \left(1-\sqrt{1-\frac{C_T\cos\gamma}{8(\sigma_y\sigma_z/{D_0}^2)}}\right)\exp\left(-0.5\left(\frac{y-\delta}{\sigma_y}\right)^2\right)\exp\left(-0.5\left(\frac{z-z_h}{\sigma_z}\right)^2\right),$$

$$\frac{\sigma_y(x,\gamma)}{D_0} = k_y\frac{(x-x_0)}{D_0}+\frac{\cos\gamma}{\sqrt{8}} \quad \text{and} \quad \frac{\sigma_z(x)}{D_0} = k_z\frac{(x-x_0)}{D_0}+\frac{1}{\sqrt{8}},$$

$$k_y = k_z = \boldsymbol{k_a} I_0 + \boldsymbol{k_b},$$

$$\frac{x_0(I_0,\gamma)}{D_0} = \frac{\cos\gamma(1+\sqrt{1-C_T})}{\sqrt{2}(\boldsymbol{\alpha^*} I_0 + \boldsymbol{\beta^*}(1-\sqrt{1-C_T}))},$$

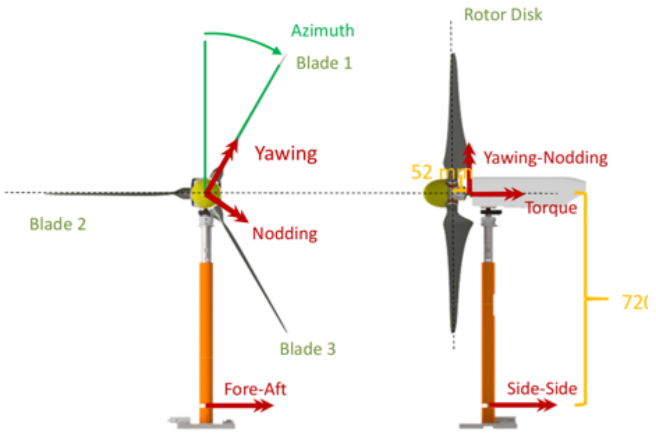
$$\frac{\delta}{D_0} = \theta_{C_0}\frac{x_0}{D_0} + \frac{\theta_{C_0}}{14.7}\sqrt{\frac{\cos\gamma}{k_y k_z C_T}}(2.9+1.3\sqrt{1-C_T}-C_T)\ln\left[\frac{(1.6+\sqrt{C_T})\left(1.6\sqrt{\frac{8\sigma_y\sigma_z}{{D_0}^2\cos\gamma}}-\sqrt{C_T}\right)}{(1.6-\sqrt{C_T})\left(1.6\sqrt{\frac{8\sigma_y\sigma_z}{{D_0}^2\cos\gamma}}+\sqrt{C_T}\right)}\right]$$

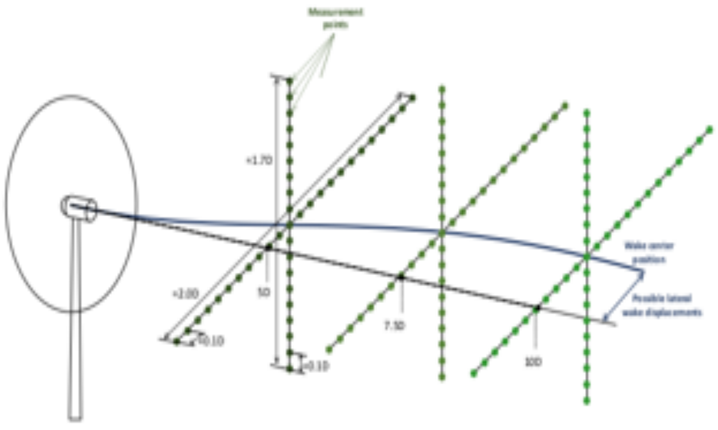
$$\theta_{C_0}(\gamma) = \frac{0.3\gamma}{\cos\gamma}(1-\sqrt{1-C_T\cos\gamma}),$$

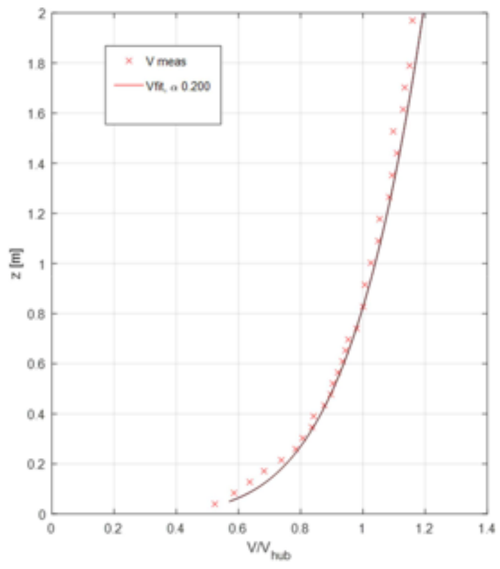
$$I_{wake}^2 = \sqrt{I_0^2+I_+^2}, \quad \text{with } I_+(I_0,x) = \boldsymbol{TI_a} a^{\boldsymbol{TI_b}} I_0^{\boldsymbol{TI_c}}(x/D_0)^{\boldsymbol{TI_d}}.$$

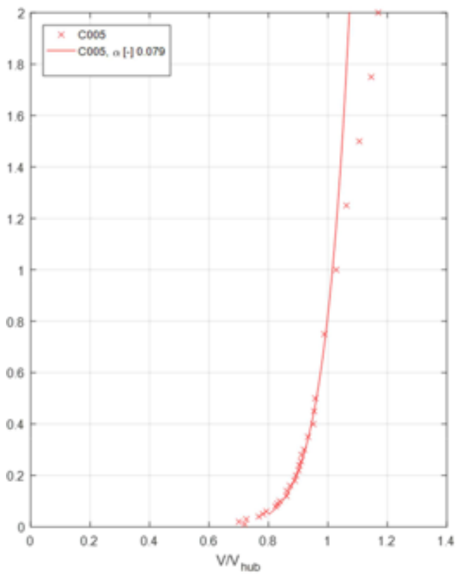
Experimental setup

- ➢ Data collected within CL-Windcon project
- ➢ GVPM and the G1 turbine
- ➢ HT conditions
- U_∞ = 5.46 m/s
- TI = 11 %
- ➢ LT conditions
- U_∞ = 5.6 m/s
- TI = 6.1 %



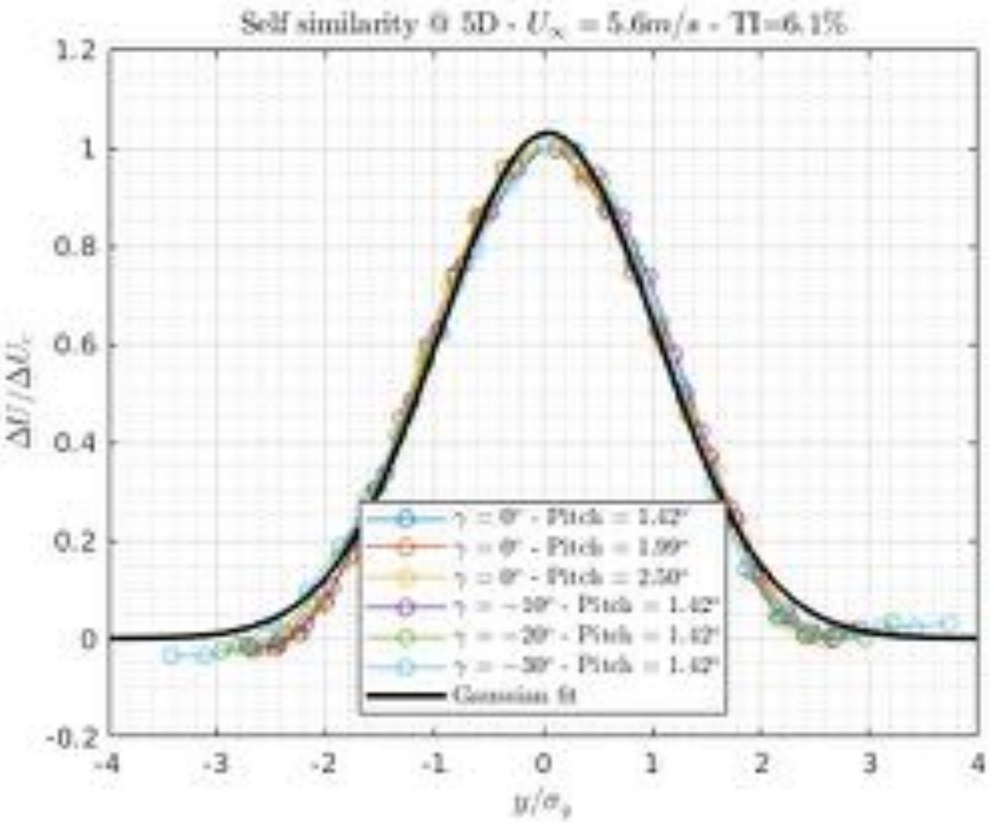


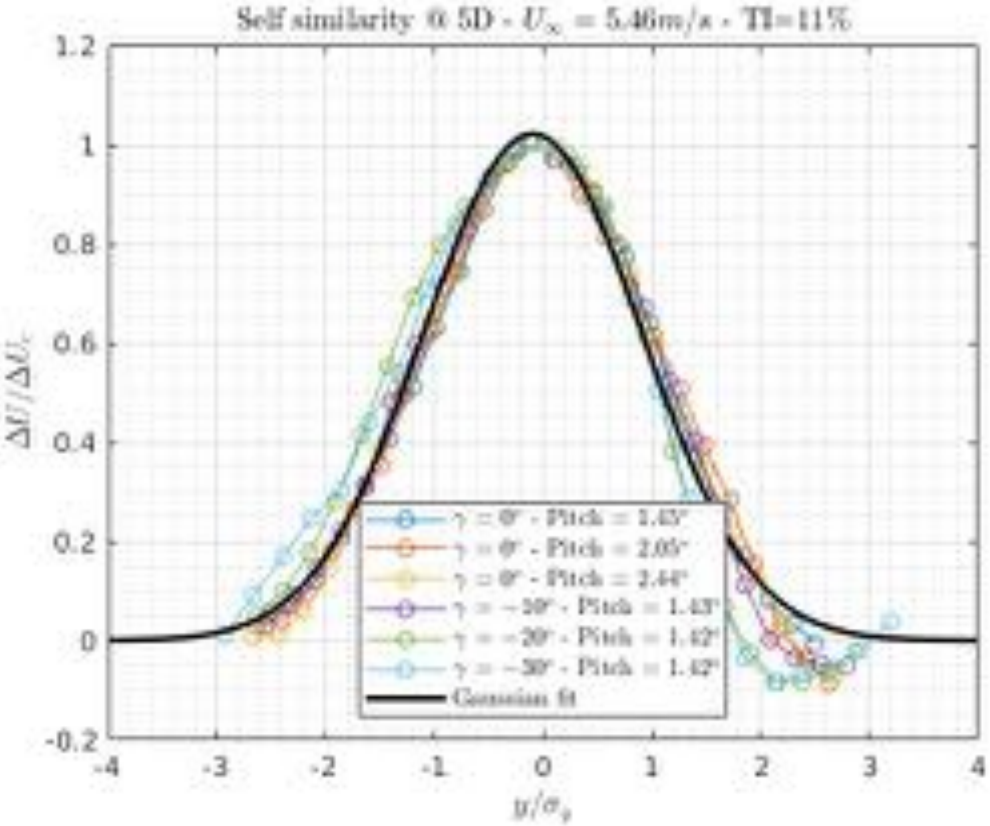




GWM applicability

Self similarity

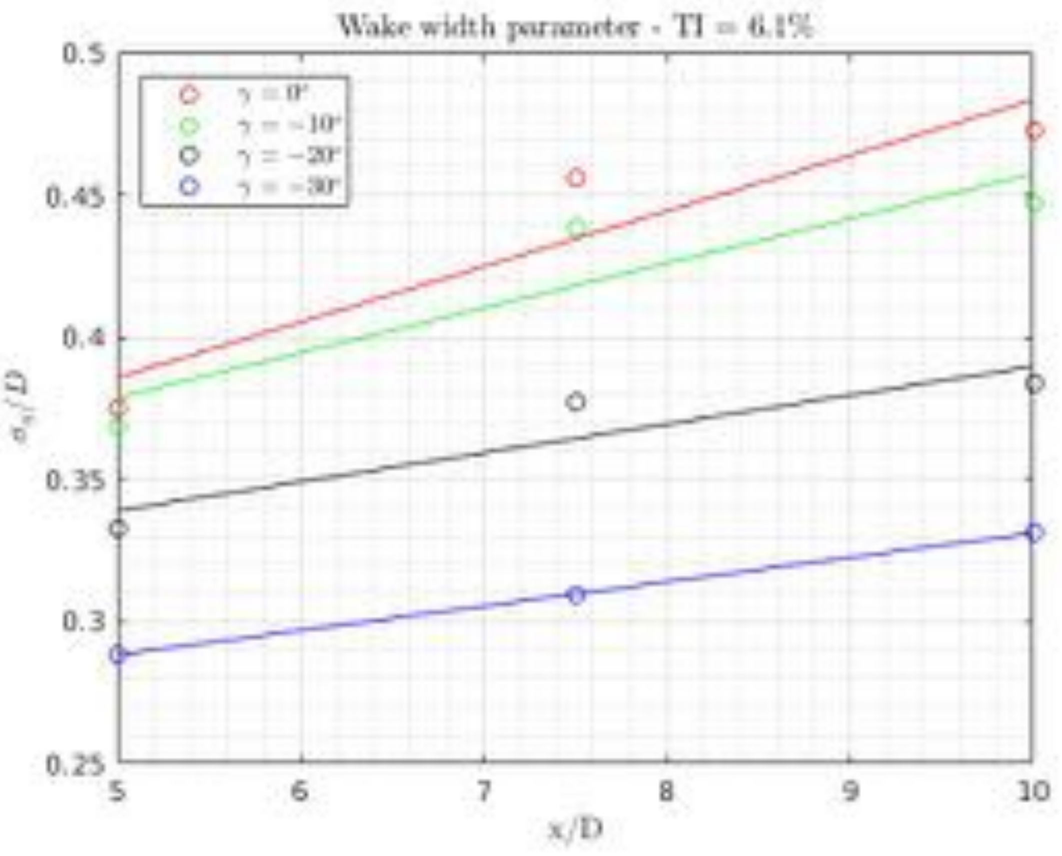


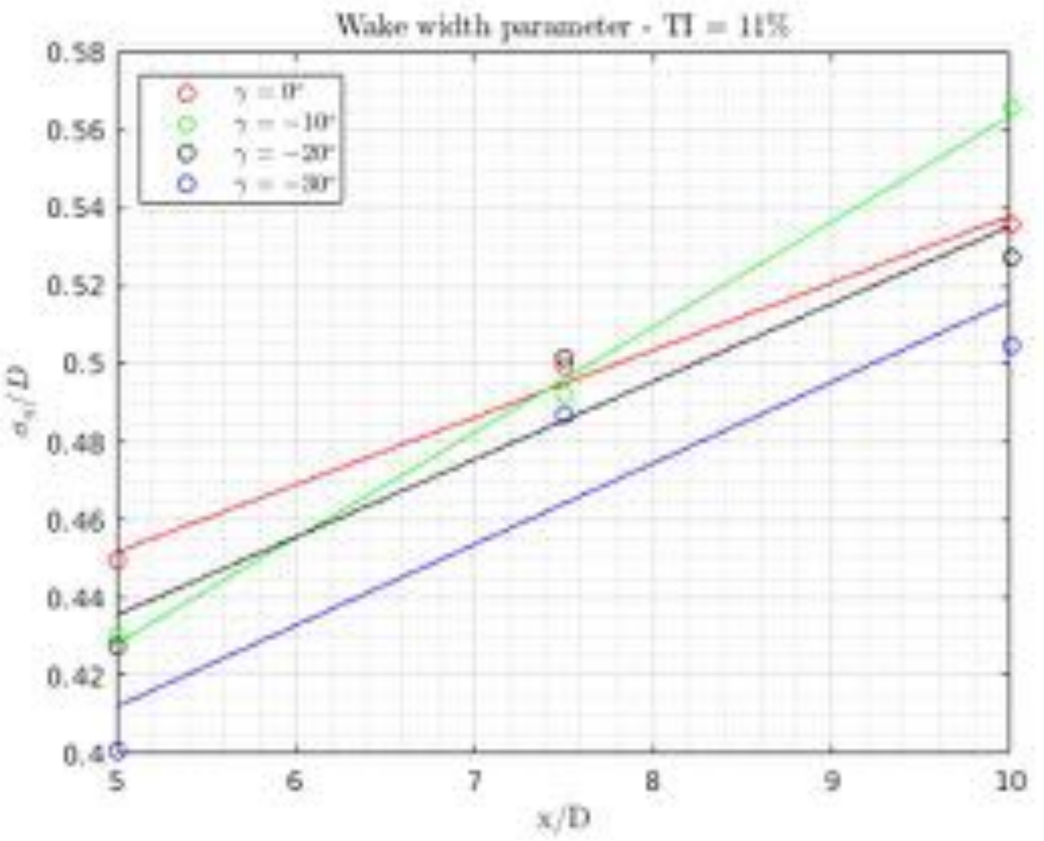


- $R^2 = 0.98$
- Self-similarity verified

GWM applicability

Wake width





- Linear wake expansion assumption is valid

Integration of the GWM in OpenFAST

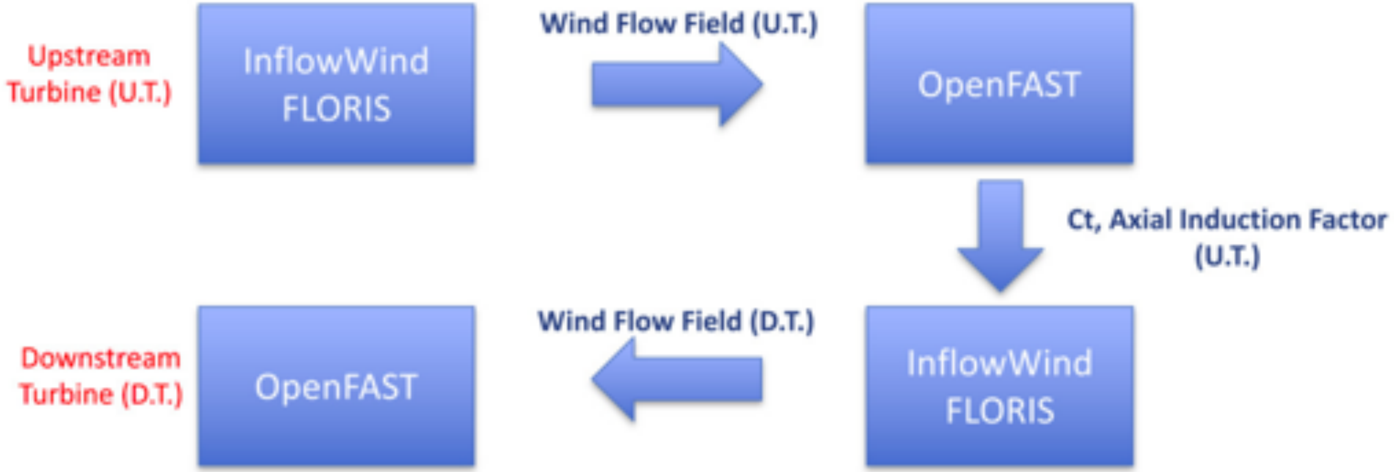
- ➢ Structural dynamic effects are kept into account
- ➢ No approximations needed for the axial induction factor
- ➢ Computation of loads is possible (for control purposes)
- ➢ No approximations needed for the power and thrust coefficient

- ➢ Corrections for the velocity and turbulence in a wind farm configuration

$$a = \frac{1}{3R}\sum_{j=1}^{3}\sum_{i=1}^{N_p} a_{local;j,i}r_{j,i}$$

$$\sigma_{z0} = \frac{D}{2}\sqrt{\frac{1-a}{1+\sqrt{1-C_T}}}$$

$$\theta = 0.6a\gamma$$



Upstream Turbine (U.T.)

InflowWind FLORIS

Wind Flow Field (U.T.)

OpenFAST

Ct, Axial Induction Factor (U.T.)

Downstream Turbine (D.T.)

OpenFAST

Wind Flow Field (D.T.)

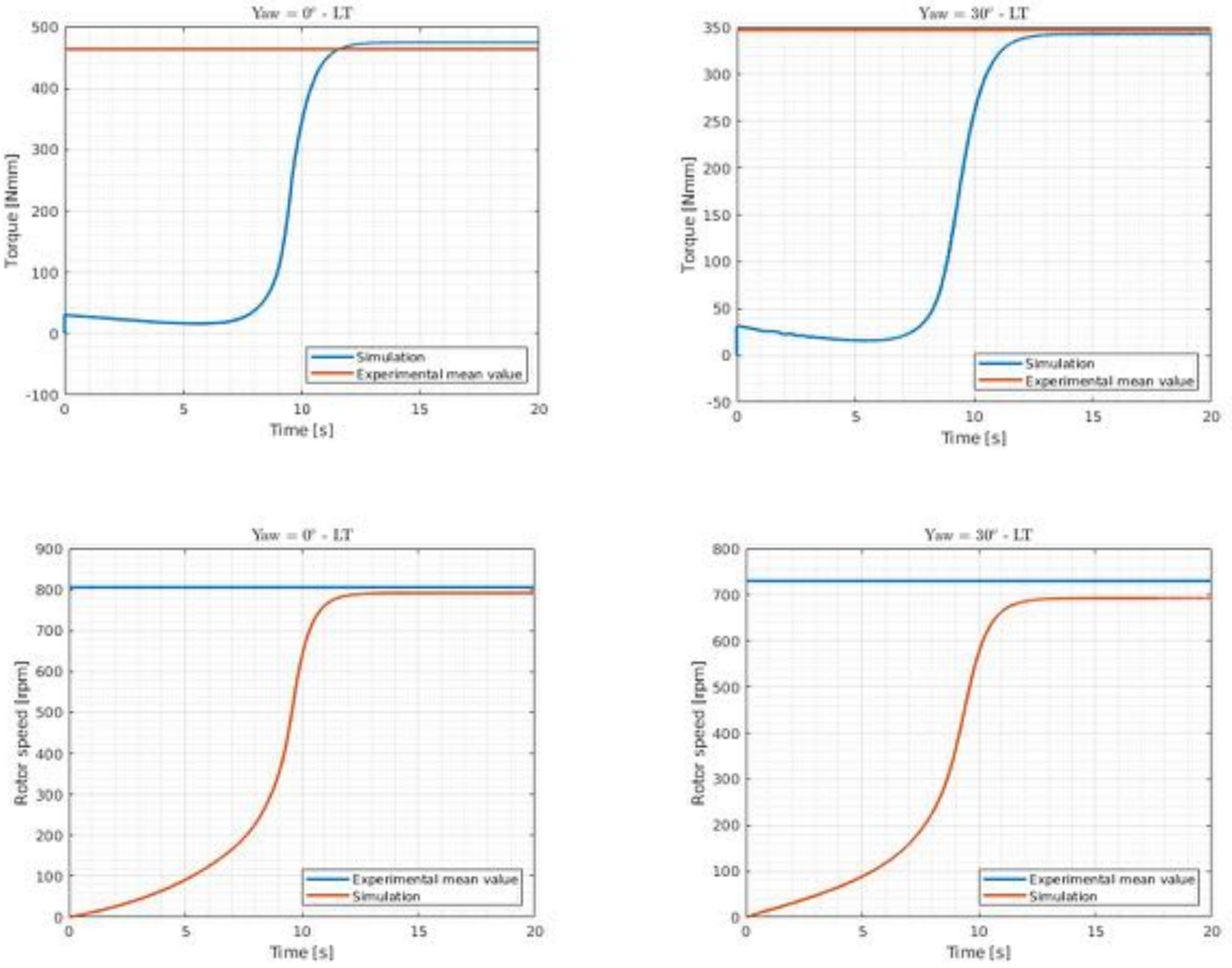
InflowWind FLORIS

$$x_0 = D\frac{\cos\gamma\sqrt{(1-a)(1+\sqrt{1-C_T})}}{4\alpha I + 2\beta(1-\sqrt{1-C_T})}$$

$$I = \sqrt{\sum_{i=1}^{n} I_{+,i}^2 + I_0^2}$$

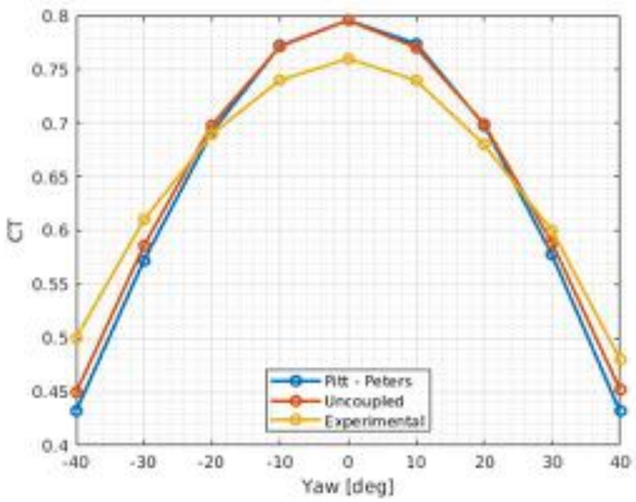
$$\Delta U_{n+1} = \sqrt{\sum_{i=1}^{n}\Delta U_i^2}$$

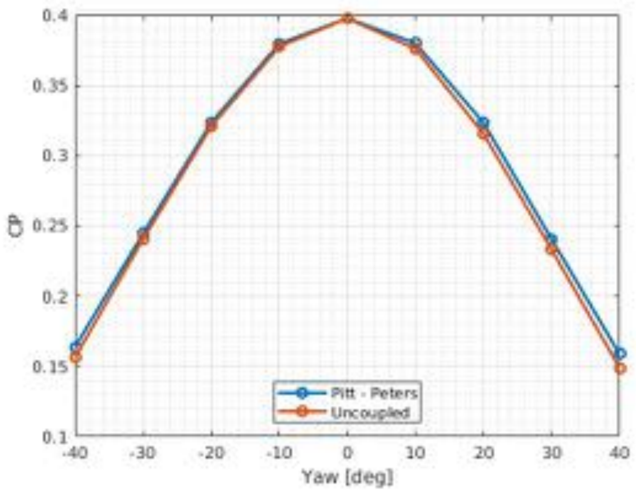
G1 model validation

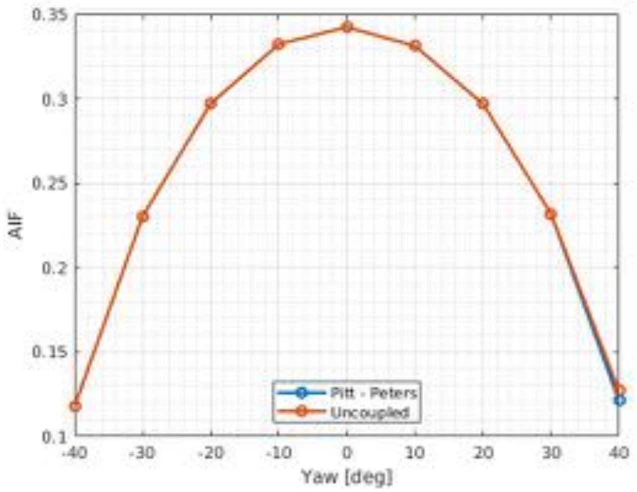


Yaw [deg]	Quantity	Relative error [%]
0	Rotor speed	1.80
0	Torque	2.33
30	Rotor speed	5.13
30	Torque	1.18

G1 model validation







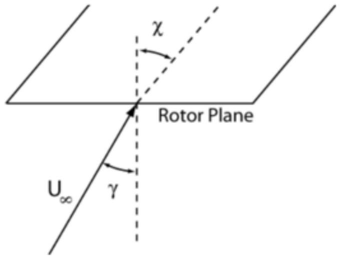
Uncoupled BEM

$$a = \left[\frac{4F\sin^2\phi}{\sigma'(C_L\cos\phi + C_D\sin\phi)} + 1\right]^{-1}$$

$$a' = \left[\frac{4F\sin\phi\cos\phi}{\sigma'(C_L\sin\phi - C_D\cos\phi)} - 1\right]^{-1}$$

Pitt-Peters

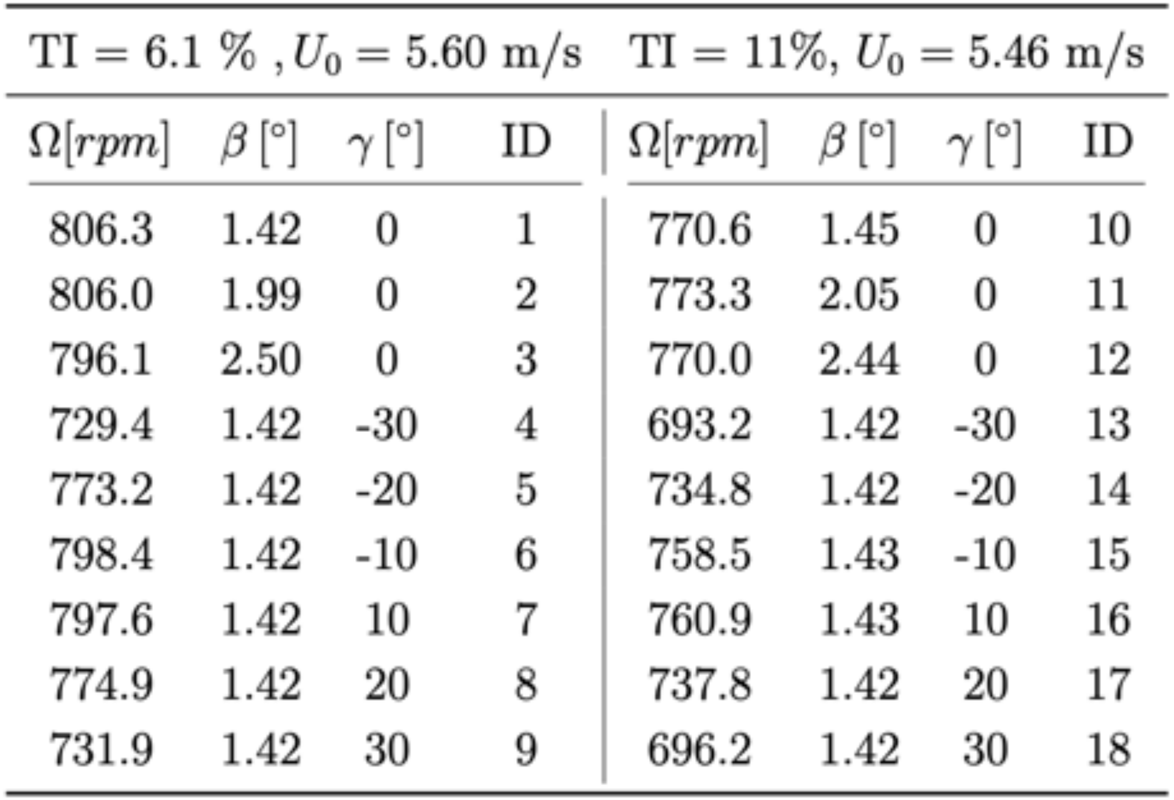
$$a_{yaw} = a\left(1 + \frac{15\pi}{64}\tan\left(\frac{\chi}{2}\right)\frac{r}{R}\sin\psi\right)$$



Results

Single turbine – Analysed cases and cost function

Analysed cases



TI = 6.1 %, $U_0 = 5.60$ m/s				TI = 11%, $U_0 = 5.46$ m/s			
$\Omega [rpm]$	β [°]	γ [°]	ID	$\Omega [rpm]$	β [°]	γ [°]	ID
806.3	1.42	0	1	770.6	1.45	0	10
806.0	1.99	0	2	773.3	2.05	0	11
796.1	2.50	0	3	770.0	2.44	0	12
729.4	1.42	-30	4	693.2	1.42	-30	13
773.2	1.42	-20	5	734.8	1.42	-20	14
798.4	1.42	-10	6	758.5	1.43	-10	15
797.6	1.42	10	7	760.9	1.43	10	16
774.9	1.42	20	8	737.8	1.42	20	17
731.9	1.42	30	9	696.2	1.42	30	18

Cost functions

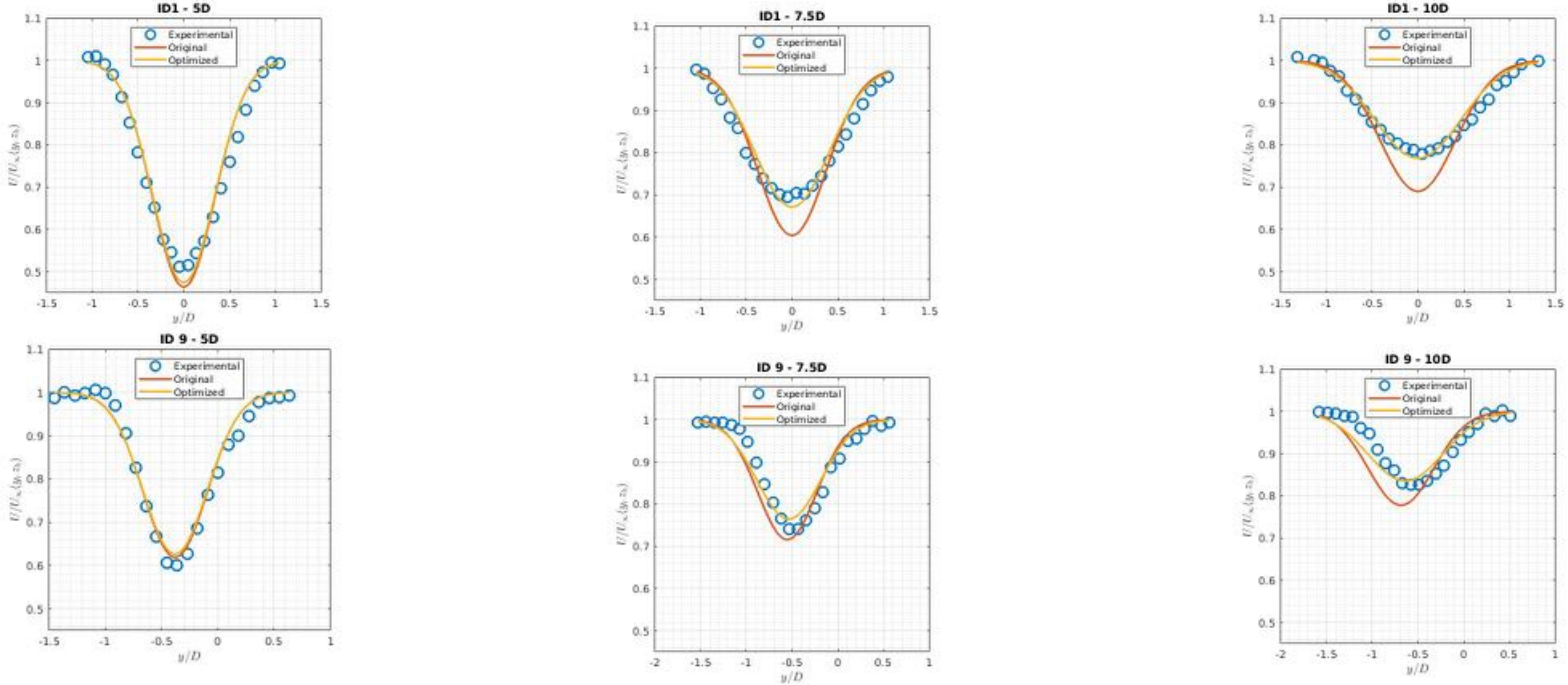
$$f_{vel} = \sum_{i}^{M} \sum_{d} \sum_{j=1}^{N^{i,d}} \left(\frac{\hat{v}_j^{i,d} - \tilde{v}_j^{i,d}}{\tilde{v}_j^{i,d}} \right)^2$$

$$f_{turb} = \sum_{i}^{M} \sum_{d} \sum_{j=1}^{N^{i,d}} \left(\frac{\hat{t}_j^{i,d} - \tilde{t}_j^{i,d}}{\tilde{t}_j^{i,d}} \right)^2$$

Minimized using Matlab's fminsearch

Results

Single turbine - Velocity

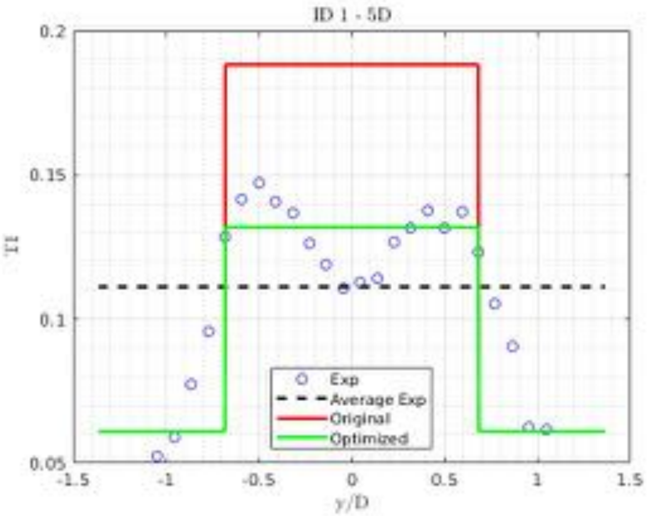


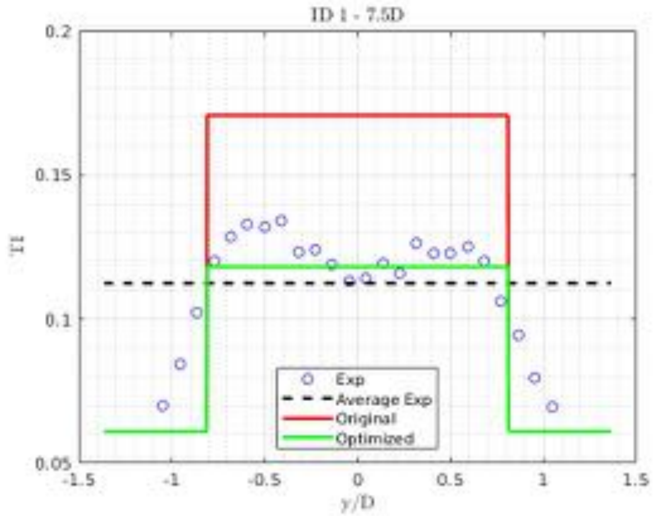
- Velocity and turbulence parameters are decoupled
- 32% overall reduction of velocity cost function
- Accurate wake reproduction up to yaw = 30°
- Model not too-sensitive to parameters (as expected)

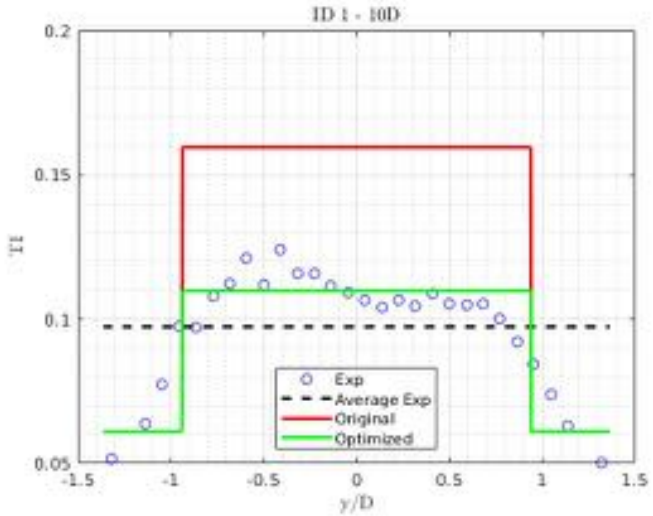
Results

Single turbine - Turbulence

- Only non-yawed cases are considered
- Wake deflection and width are given by velocity model







- 88% overall reduction of the cost function
- Great discrepancies wrt to experimental data using original parameters

$$[ka,\ kb,\ \alpha,\ \beta]_{init} = [0.2,\ 0.003,\ 0.48,\ 0.077]$$

$$[TIa,\ TIb,\ TIc,\ TId]_{init} = [0.8,\ 0.73,\ 0.1,\ -0.273]$$

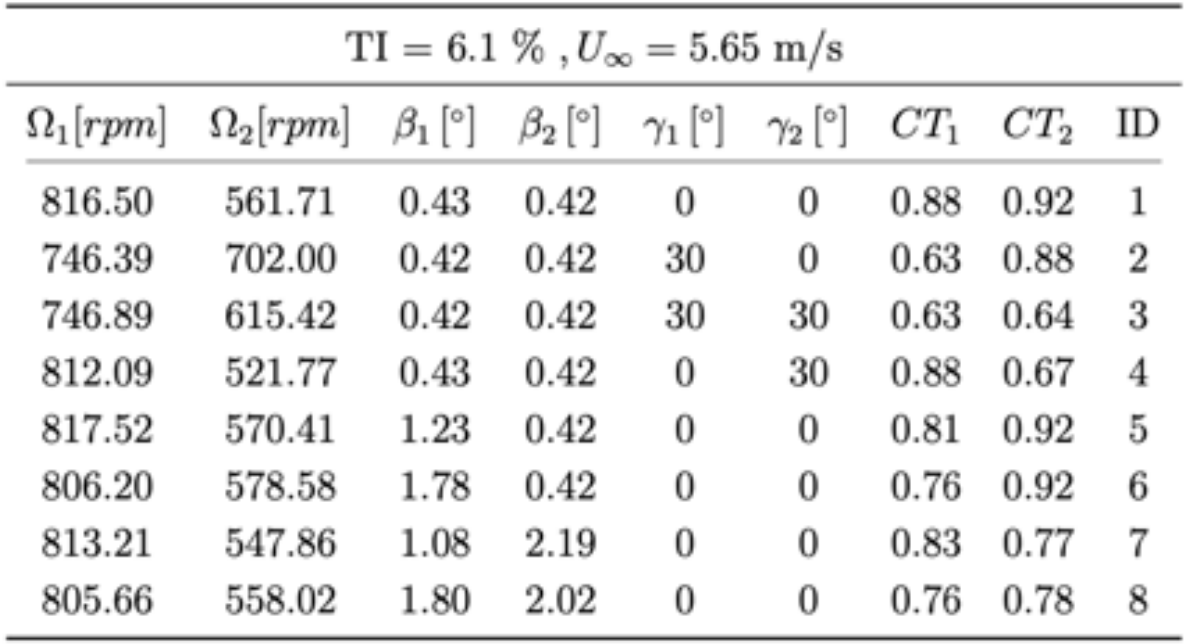
$$[ka,\ kb,\ \alpha,\ \beta]_{fin} = [-0.0074,\ 0.0261,\ 0.6057,\ 0.05]$$

$$[TIa,\ TIb,\ TIc,\ TId]_{fin} = [0.1342,\ 0.3801,\ -0.3007,\ -0.3549]$$

Results

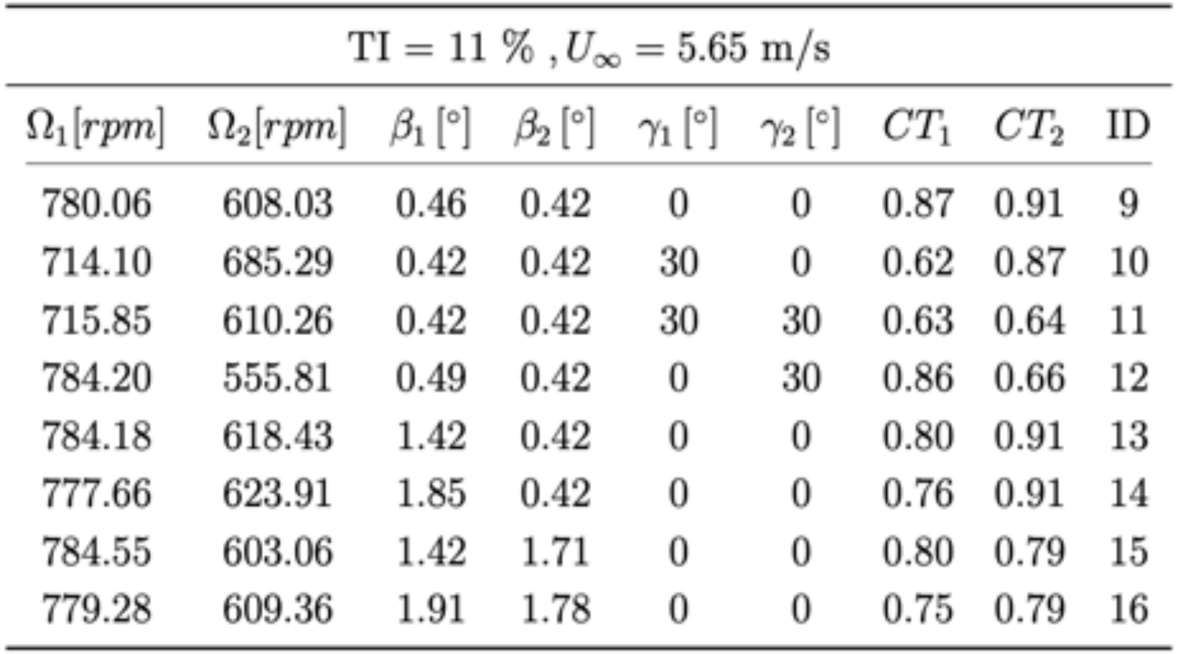
Simple wind farm – Analysed cases

- Two turbines at 5D distance
- Model includes wake superimposition effects



TI = 6.1 % , $U_\infty = 5.65$ m/s

$\Omega_1[rpm]$	$\Omega_2[rpm]$	$\beta_1\,[°]$	$\beta_2\,[°]$	$\gamma_1\,[°]$	$\gamma_2\,[°]$	CT_1	CT_2	ID
816.50	561.71	0.43	0.42	0	0	0.88	0.92	1
746.39	702.00	0.42	0.42	30	0	0.63	0.88	2
746.89	615.42	0.42	0.42	30	30	0.63	0.64	3
812.09	521.77	0.43	0.42	0	30	0.88	0.67	4
817.52	570.41	1.23	0.42	0	0	0.81	0.92	5
806.20	578.58	1.78	0.42	0	0	0.76	0.92	6
813.21	547.86	1.08	2.19	0	0	0.83	0.77	7
805.66	558.02	1.80	2.02	0	0	0.76	0.78	8



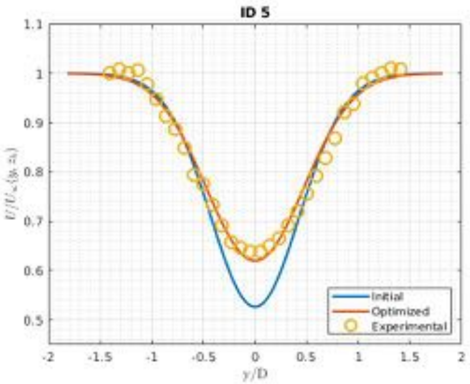
TI = 11 % , $U_\infty = 5.65$ m/s

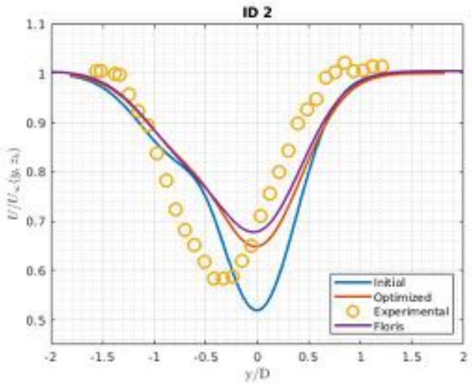
$\Omega_1[rpm]$	$\Omega_2[rpm]$	$\beta_1\,[°]$	$\beta_2\,[°]$	$\gamma_1\,[°]$	$\gamma_2\,[°]$	CT_1	CT_2	ID
780.06	608.03	0.46	0.42	0	0	0.87	0.91	9
714.10	685.29	0.42	0.42	30	0	0.62	0.87	10
715.85	610.26	0.42	0.42	30	30	0.63	0.64	11
784.20	555.81	0.49	0.42	0	30	0.86	0.66	12
784.18	618.43	1.42	0.42	0	0	0.80	0.91	13
777.66	623.91	1.85	0.42	0	0	0.76	0.91	14
784.55	603.06	1.42	1.71	0	0	0.80	0.79	15
779.28	609.36	1.91	1.78	0	0	0.75	0.79	16

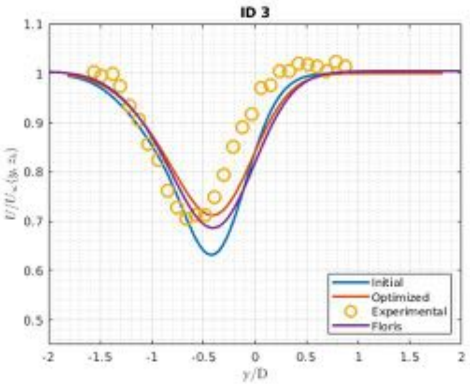
- Only measurements at 10D distance are considered (far wake region)
- Need to iteratively run OpenFAST (change of parameters -> change of inflow of second turbine)
- Old parameters for turbulence (convergence issues)
- Cost function similar to single turbine case

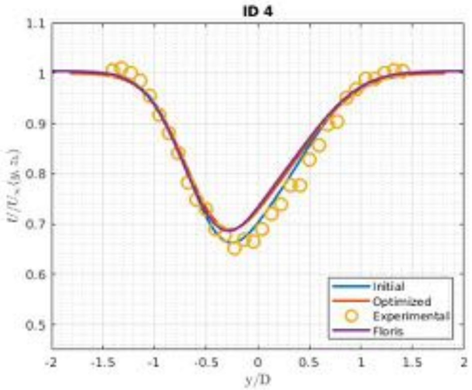
Results

Simple wind farm – Velocity









- ➢ Initial guess: optimized parameters for the single turbine case

$$[ka,\ kb,\ \alpha,\ \beta]_{init} = [-0.0074,\ 0.0261,\ 0.6057,\ 0.05]$$

$$[ka,\ kb,\ \alpha,\ \beta]_{fin} = [0.1784,\ 0.0101,\ 0.0529,\ 0.3590]$$

- ➢ Accurate results when:
- Both turbines have yaw = 0°
- Only downstream turbine is yawed

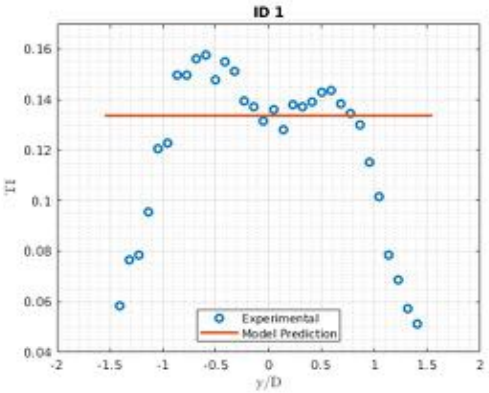
- ➢ Inaccurate results when:
- Both turbines are yawed - wake centre displacement not accurate
- Only upstream turbine is yawed – velocity summation model inadequate

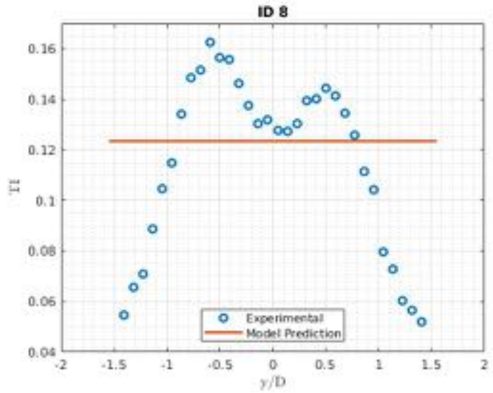
- ➢ Comparison with original floris:
- Implemented model is consistent with original
- Upgrades in velocity deficit summation model are needed

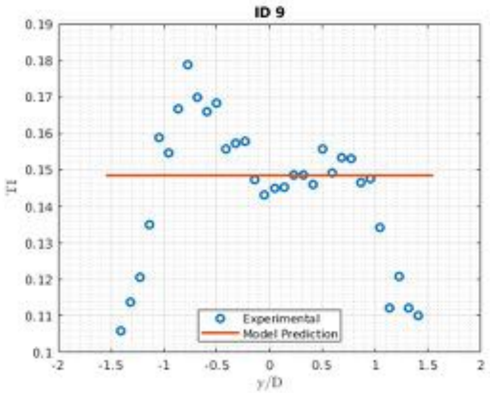
Results

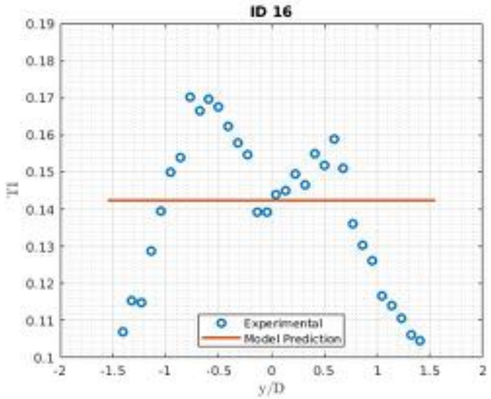
Simple wind farm – A posteriori analysis of turbulence

- ➢ Mean turbulence at the rotor of an (hypothetical) turbine at distance 10D and yaw = 0°









- ➢ Parameters computed with single turbine case give accurate results
- ➢ Possible two-step optimization

Conclusions and future work

Modelling

- Quasi-steady wake model implementation
- Accurate reproduction of loads
- No approximations for AIF, CT and CP

Results

- Wake model validated against experimental data and Floris
- Steady simulations’ results similar to Floris in most cases
- Superimposition models aren’t accurate enough

What’s next?

- Dynamic wake model implementation and validation in OpenFAST
- Modification of the wake summation models (different parameters for each turbine or use weighing factors)
- Modification of the GWM to take into account local AIF
- Obtain computationally cheap and accurate dynamic wake model

Closing

Thank you for your attention!

Ali Raza Asghar