On the instability of the flow around rectangular cylinders

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Summary In this work we perform a Floquet stability analysis of the two-dimensional symmetric periodic flow past rectangular cylinders with aspect ratio (AR) between 1 and 5. We show that the occurrence and/or the nature of the secondary instability strongly depends on the aspect ratio of the cylinder. In particular, three ranges of AR can be identified: (i) for $AR \in [1, 2]$, the well known "mode A" three-dimensional synchronous instability is found at a spanwise length scale $\beta \approx 1.6D$ (for $AR = 1$ a further instability of quasi-periodic nature, usually called "mode B", is found for larger Reynolds number at $\beta = 2.6D$); (ii) for $AR \in [3, 4.75]$, the base flow is found to be stable to three-dimensional perturbations, but a secondary two-dimensional instability of the flow occurs; (iii) for $AR \geq 5$, a three-dimensional instability of (almost) subharmonic nature is found at $\beta \approx 2.1D$.

The investigation of the stability of the steady flow around bluff bodies is of great interest to a number of different applications, especially in the field of vortex-induced oscillations [1]. While the circular cylinder is a natural prototype of bluff body, extensively investigated [2], rectangular cylinders are also very interesting both from the viewpoint of applications and that of basic research, since they display flow features that are different with respect to the circular cylinder case: they have sharp edges where flow separation occurs and, depending on the aspect ratio of the rectangle AR, flow reattachment can be present on its lateral sides. In this work we are particularly interested in the effect of the aspect ratio of the cylinder on the secondary instability of the cylinder wake, usually leading to a three-dimensional flow.

The three-dimensional instability of blunt bluff-body wakes has been first investigated by Robichaux et al. [3]. For a square cylinder ($AR = 1$), they reported a three-dimensional instability classified as a subharmonic instability. Blackburn and Lopez [4] then showed this kind of instability to be precluded by the spatio-temporal symmetry of the base flow, and provided new evidence of the quasi-periodic nature of this instability. The subharmonic character of the instability is however recovered when the symmetry of the base flow is broken by slightly rotating the cylinder [5]. The impact of the aspect-ratio on the 3D secondary instability has been investigated in [6] for $0 < AR < 1$, but a comprehensive overview of the effects of the aspect ratio for $AR > 1$ is still lacking. This work addresses the stability of the flow past rectangular cylinders with aspect ratios $AR \in [1, 5]$, investigating both the first instability that leads to a periodic flow as well as the secondary one, by a Floquet stability analysis.

We found several interesting phenomena for both the first and secondary instabilities. In particular, we found that the quantitative and, more importantly, the qualitative nature of the secondary instability of such flows strongly depends on the aspect ratio of the cylinder. In this respect, three subranges of AR can be identified: (i) $AR \in [1, 2]$, (ii) $AR \in [3, 4.75]$, (iii) $AR \geq 4.85$.

For $AR \in [1, 2]$, the first three-dimensional instability is found to occur at $Re = U_\infty D/\nu \approx 200$ (U_∞ is the free-stream velocity, D is the cross-stream size of the cylinder and ν the kinematic viscosity). This a synchronous instability with spanwise length scale $\beta \approx 1.5D$, as shown in the left panel of figure 1 where the Floquet multipliers (μ) for $AR = 1$, $Re = 200$ and $\beta \approx 1.5D$ are plotted: the unstable Floquet multiplier with $|\mu| > 1$ is real and positive. The unstable mode in this case is usually referred to as "mode A" [5]. For the flow past the cylinder with $AR = 1$, a second three-dimensional

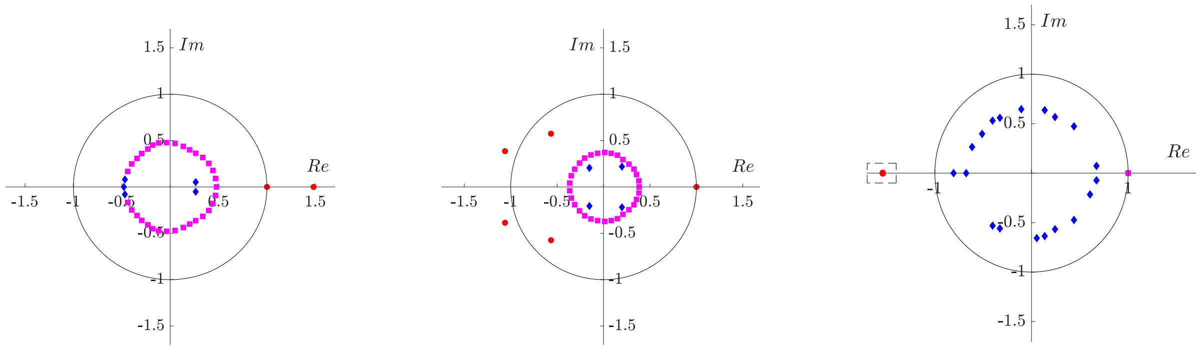

Figure 1: Left: Floquet multipliers for $AR = 1$, $Re = 200$ and $\beta = 1.5D$. Centre: Floquet multipliers for $AR = 1$, $Re = 225$ and $\beta = 2.6D$. Right: Floquet multipliers for $AR = 5$, $Re = 550$ and $\beta = 2.1D$

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*Corresponding author. E-mail: alessandro.chiarini@polimi.it instability is found at larger Reynolds number ($Re \approx 225$). This instability has a spanwise length scale $\beta \approx 2.6D$ and is of quasi-periodic nature, as shown in the central panel of figure 1: the unstable Floquet multipliers form a complex conjugate pair with negative real part. Interestingly enough, this instability is not found for $AR = 2$.

For $AR \in [3, 4.75]$, no 3D instability is found by Floquet linear stability analysis for $Re < 400$, and DNS shows that no three-dimensionality arises up to $Re = 450$. However, a second two-dimensional instability seems to occur in this range of aspect ratio and Reynolds number, which makes the flow loose the spatio-temporal symmetry of the wake. This is shown in figure 2, where a snapshot of the $2D$ flow for $AR = 4$ at $Re = 450$ is plotted.

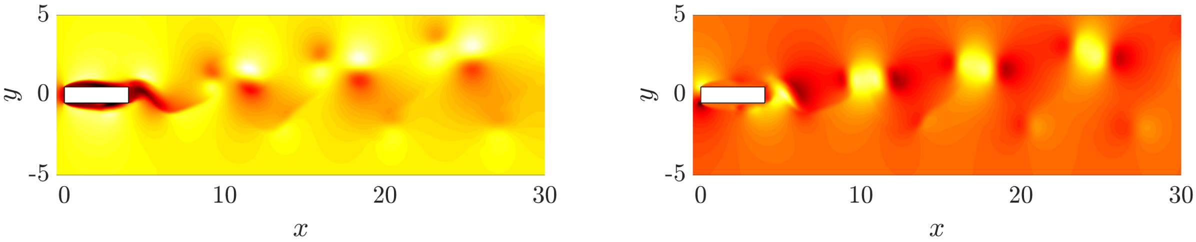

Figure 2: Baseflow for $AR = 4$ at $Re = 450$, without any stabilisation. Left: streamwise of the velocity. Right: cross-stream component of the velocity

For $4.85 \leq AR \leq 5$, a three-dimensional instability is found for $Re \approx 500$. In contrast with what found for $AR \in [1, 2]$, the instability is almost subharmonic and has a spanwise length scale $\beta \approx 2.1D$. This is shown in the right panel of figure 1 where the Floquet multipliers for $AR = 5$, $\beta = 2.1D$ and $Re = 550$ are plotted: the unstable multipliers form a complex conjugate pair with negative real part and imaginary part almost null (in the figure they seem to collapse to the real axis as $\text{Im}(\mu) \approx 10^{-3}$).

Aiming at investigating the nature of such instabilities, we exploit several tools: e.g a detailed investigation of the $2D$ base flows, $3D$ DNSs, linear stability analysis of snapshots of the base flows. An interesting result that further highlights the different nature of the three-dimensional instabilities found for $AR \in [1, 2]$ and $AR \geq 4.85$ is provided by the structural sensitivity [7] that allows one to locate the flow region where the instability is triggered. The structural sensitivity, figure 3, highlights the completely different nature of the instability mechanisms. For $AR \in [1, 2]$, the instability is triggered downstream the trailing edge: this is similar to what found for the flow past a circular cylinder [7]. For $AR \geq 4.85$, instead, the three-dimensional instability is triggered in a region close to the streamwise edges of the cylinder, where the recirculation occurs.

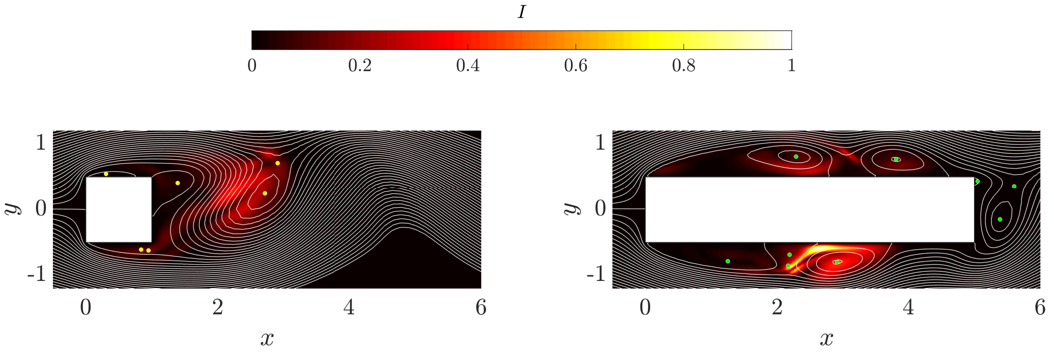

Figure 3: Snapshots of the structural sensitivity at one phase along the period of the base flow for $AR = 1$, $Re = 200$, $\beta/D = 1.5$ (left) and $AR = 5$, $Re = 550$, $\beta/D = 2.1$ (right). Streamlines and stagnation points are show superimposed on the sensitivity map.

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