Cascading-Failure-Resilient Interconnection for Interdependent Power Grid - Optical Network*

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Abstract

The interdependence between communication networks, e.g., an optical backbone network, and power grids is a critical issue to take into account when designing and operating both systems. In fact, failures in one network may cause further failures in the other network and vice versa. This is because nodes in power grids (i.e., power generators, loads or interchange nodes) are controlled and managed by telecommunication equipment, which, in turn, rely on the electricity grid for their power supply. Therefore, failures occurring on a limited portion of one network can cascade multiple times between these two networks, and a robust "interdependency network" (i.e., consisting of the interconnections between nodes in the two networks) is needed. This paper investigates the problem of designing a resilient interconnection against interdependent cascading-failures in interdependent power grid - optical networks. We formalize, using an Integer Linear Program, the new problem of *Power Grid - Optical Network Interconnection* (*PGON-I*), which consists in designing an interconnection between the power grid and the optical network that is resilient to cascading failures, i.e., avoids/reduces cascade. For this problem, we derive an-

*A preliminary version of this work introduced the problem in [1].

alytically upper and lower bounds on the number of interconnection links which ensure resilience against cascading failures initiated from a single node-failure. Starting from the analytical model, we develop a heuristic algorithm to solve large instances of the problem. Our results show that the higher the difference between the number of nodes in the two networks, the more interconnection links are needed to ensure resilience against failures cascade.

Keywords: Network Protection, Interdependent Networks, Cascading Failures, Optical Network, Power Grid.

1. Introduction

Today, optical communication networks and power grids strongly depend on each other to operate [2] [3]. Optical network equipment (e.g., switches) are powered by the power grid. On the other hand, in a power grid, the Supervisory Control And Data Acquisition System (SCADA) uses the communication network to provide monitoring, measurement, and control of remote equipment. Thus, failure of an element in one network may cascade to the other network and vice versa. Such failures may cascade several times between the two networks resulting in widespread failures. These *interdependent cascading failures* (which are different from cascading failures within one network) may lead to network disconnection, even in a highly-connected mesh network. Recently, various occurrences of cascading failures between the power grid and the optical network have led to extensive network disruption. The 2003 Italian blackout was due to such interdependency [3]; during the 2003 U.S. Northeastern power outage, around 3,175 communication networks suffered from connectivity outage [4]; other examples include the 2011 San Diego Blackout [5], 2012 Indian Blackout [6], etc. Given the scale and criticality of these networks, survivability against interdependent cascading failures is a major concern.

Previous works analyzed the robustness of interdependent networks, e.g., determining the fraction of nodes whose removal will cause a complete blackout in the networks [3] [7] [8] [9]. The authors in [10] proposed a load control scheme to mitigate failure due to cascade, though this work does not deal with failures due to disconnection from generators and control centers. On a similar line, authors in [11] provide a mathematical formulation for the joint optimization of the Phasor Measurement Units (PMU) placement and communication infrastructure design which support power grids, minimizing the size of shared risk link groups in order to combat disaster (i.e., multiple-failure) scenarios. The authors in [12] provide a model to assess the effect of geographically-correlated failures on interdependent power grid - communication networks. The effects of the topology of the control network, and the number and reliability of the control devices on the robustness of the power network has been studied in [13]. Many existing papers concentrate on defining proper metrics to evaluate the impact of cascading failures between interdependent networks. For example, authors of [14] extend the considerations in [3] and define performance metrics to evaluate the impact of cascading failures in interdependent networks, which also take into account the capacity degradation (i.e., the amount of affected traffic) in the communication network. Moreover, in [15], the authors evaluate the robustness of interdependent networks upon different attack strategies and considering different coupling strength (i.e., level of interdependency) between the networks. In [16], the problem of interlink optimization with constrained budget is addressed, aiming at maximizing the network robustness against three types of failure propagation, for which different robustness metrics are defined by the authors. In [17], percolation theory is used to study the reliability properties of interdependent networks, also compared with single networks. Thanks to this, the authors conclude that for interdependent networks, Scale Free networks have in general lower reliability than Erdös-Rényi networks. Percolation theory is also used in [18], where the authors propose a greedy heuristic algorithm to identify core structures in the interdependent networks in order to identify most critical nodes. Analysis on critical nodes is also carried out in [19], referring to node load, destructiveness and robustness metrics. The authors compare two strategies, namely, assortative and disassortative coupling, to interconnect critical nodes between the interdependent networks and determine under which topological conditions, i.e., dense vs sparse networks, the two strategies are more effective.

Note that failures cascade between the two networks largely depends on how the equipment in the two networks are dependent on each other, i.e., on the configuration of the interconnection between the two networks. We refer to the "interconnection" between the power grid and the optical network to represent their reciprocal relation: on one hand, nodes in power grids (i.e., power generators, loads or interchange nodes) are controlled and managed by telecommunication equipment (e.g., IP/MPLS routers); in turn, telecommunication devices rely on the electricity grid for their power supply.

Figure 1 shows how an initial failure (of a single node) can damage two interdependent networks through cascading failures. The optical-communication-network nodes and links are shown in green (nodes c1 to c6), whereas the power grid nodes and links are show in blue (nodes p1 to p5). In the power grid, node p3 is a generator, while the other nodes are loads (i.e., they cannot operate if disconnected from p3). Other than communication links and power links, to model an interdependent "power grid" - "optical communication network" system, we also consider the interconnection links between the two networks (represented by the dashed directional links). For example, the presence of link p1 $\rightarrow$ c6 indicates that communication node c6 is powered by power grid node p1, while link c5 $\rightarrow$ p4 indicates power grid node p4 is controlled through communication node c5.

Now, let us suppose power load p1 fails (Fig. 1(a)). Consequently, all the links connected to p1 (both power links and interconnection links) fail. As a result, communication nodes c1 and c6 lose power supply and fail (Fig. 1(b)). Due to c6 failure, node p2 loses its connectivity to the optical network and fails (Fig. 1(c)). In the next step, c5 loses power and fails (Fig. 1(d)), and then p4 loses its control and fails (Fig. 1(e)). At this point, the power grid is fragmented into two islands, i.e., p5 is disconnected from generator p3. Thus, p5 fails, and c3 and c4 fail due to loss of power (Fig. 1(f)). In the final step, p3 and c2 fail as power grid is completely isolated from the optical network (Fig. 1(g)). We call

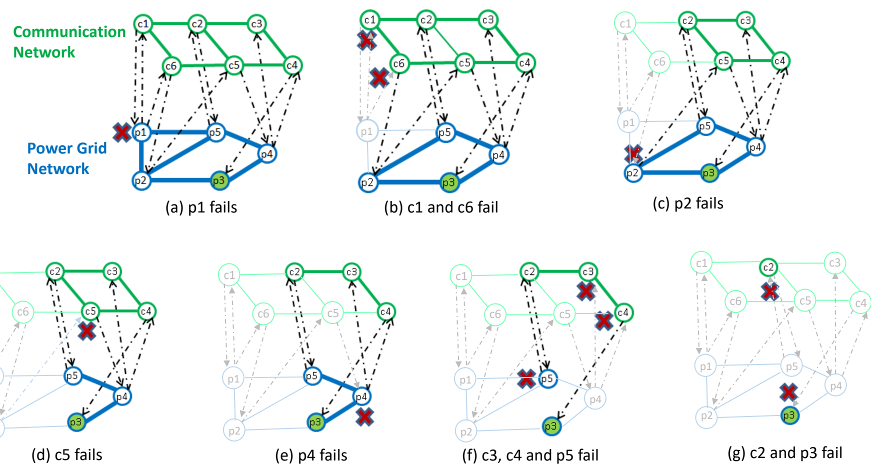


Figure 1: Iterative cascade of failures due to a single failure in interdependent power grid - optical communication network. Power grid and Optical network are shown in blue and green, respectively. Interconnection links are shown by dashed directional links.

the state of the network after the initial failure as *initial failure state* (e.g., Fig. 1(b)), and the state of the network after all cascading failures as final failure state (e.g., the completely damaged network in the previous example).

We observe that an intelligent design of interconnection between the power grid and the optical network can reduce/avoid a failure cascade propagating among the two networks. This study formalizes and investigates the Power Grid - Optical Network Interconnection problem to design the set of interconnection links between interdependent power grid and optical network which is resilient to interdependent cascading failures. By resilience to interdependent failures cascade, we mean that 1) the cascade does not cause network disconnection/fragmentation, and 2) the number of cascading node failures is constrained to a maximum value.

To the best of our knowledge, this research is the first to address this problem. Our contribution in this work can be summarized as follows:

- We introduce and formalize the new problem of designing cascading-failure-resilient interconnection for interdependent power grid - optical communication network, namely *Power Grid - Optical Network Interconnection* (*PGON-I*).

- We provide a mathematical model to solve the problem optimally.
- We provide a heuristic to solve large instances of the problem.
- We analyze the effect of differences in the number of nodes in the power grid and the optical communication network on resiliency against interdependent cascading failures.

The rest of the paper is organized as follows. In Section 2, we present the interdependent power grid - optical network model used in this study. In Section 3, we formally describe the problem. In Section 4, we derive analytically upper and lower bounds on the minimum number of interconnection links that ensure resilience to interdependent cascading failures. Section 5 presents a mathematical model to solve the problem. Our model can be used to design an interconnection from scratch (greenfield) or to augment an existing interconnection (brownfield). We propose a fast and effective heuristic in Section 6 to solve large instances of the problem. In Section 7, we present illustrative results. Section 8 concludes the study.

2. Interdependent Network Model

We consider a power-grid topology, $G_P = (V_P, E_P)$ and an optical communication network topology, $G_C = (V_C, E_C)$, where V and E are the set of nodes and undirected links, respectively. The power grid has three types of nodes: generators, sub-stations, and loads. A load/sub-station can operate only if it is connected to at least one generator through a power grid path. The interconnection links between the two networks are directional and represented by $m \rightarrow n$. Dependency $m \rightarrow n$, with $m \in V_P$ and $n \in V_C$, indicates that node n is dependent on node m, as m provides power to node n. Similarly, dependency $n \rightarrow m$, with $n \in V_C$ and $m \in V_P$, indicates that node m is dependent on node

n due to the fact that node n controls node m. An optical node should be connected to at least one power grid load (not generator or sub-station) for power, and a power grid node (load, sub-station, and generator) should be connected to at least one optical node for operation. A power node can be connected to a backup optical node for protection and vice versa [9].

Our model assumes that a power node can be controlled through only a subset of the optical nodes (e.g., those within some allowed distance of the power node) and vice versa. We use I to represent the set of possible interconnection links, i.e., $I = \{m \to n : m$ can power (control) $n, m \in V_P, n \in V_C (m \in V_C, n \in V_P)\}$. $I = I_{pc} \cup I_{cp}$, where I_{pc} is the set of power links eligible to connect power nodes to optical nodes and I_{cp} is the set of optical links eligible to connect optical nodes to power nodes. Note that, it may be the case that, $m \to n \in I$ but $n \to m \notin I$. This restriction that a power node can be connected to (and, hence, controlled by) only a subset of the optical nodes (typically, the geographically closer), represents realistic reachability constraints. For example, it is not practical that an optical node is powered by a remote power node.

In this model, the first failure (trigger) is a single-node failure in either the optical network or the power grid. Note that even a single initial failure may eventually disrupt the entire network as shown in Fig. 1. For ease of exposition we start the analysis with the case where both power grid and optical network are 2-node-connected, i.e., each of the two networks would remain connected also in case of single-node failure occurring in any node. However, the proposed mathematical model (see Section 5) and the heuristic (see Section 6) are generalized to any connectivity and any size of initial failure.

3. Problem Statement

As described in Section 2, an interdependent power grid - optical network can have four types of links: 1) power links connecting power nodes in the power grid (E_P), 2) optical links connecting optical nodes in the optical network (E_C), 3) directional power-interconnection links through which power nodes provide power to optical nodes (I_{pc}), and 4) directional optical-interconnection links through which optical nodes provide control to power nodes (I_{cp}). In our problem, the power links and the optical links are given, and we have to choose a set of directional interconnection links from the set I (see Section 2).

We formally state the problem of optimized *Power Grid - Optical Network Interconnection* (*PGON-I*) as follows.

Given

- Power grid topology, $G_P = (V_P, E_P)$, $V_P = \{\text{power node}\}$, $E_P = \{\text{power link}\}$. $V_P = \{V_g \cup V_s \cup V_l\}$; V_g, V_s and V_l are the set of generators, sub-stations, and loads, respectively.
- Optical communication network topology, $G_C = (V_C, E_C)$, $V_C = \{\text{communication node}\}$, $E_C = \{\text{communication link}\}$.
- Set of possible interconnection links, $m \to n$, where node m is eligible to power/control node n; $I = \{m \to n : m \text{ can power (control) } n\} = I_{pc} \cup I_{cp}$; $I_{pc} \subseteq \{m \to n : m \in V_l, n \in V_C\}$; $I_{cp} \subseteq \{m \to n : m \in V_C, n \in V_P\}$.
- Set of failures, $Z = \{z : z \text{ is an initial set of failed links and/or nodes}\}$.
- k: Maximum number of nodes that are allowed to fail due to interdependent failures cascade (including the initial failure).

Output

Set of directional interconnection links R selected between the power grid and the optical network, $R \subseteq I$.

Objective

Minimize number of interconnection links, $|R|$.

Constraints

1. In R, every power node is connected to at least one optical node for control and every optical node is connected to at least one power load for power

2. For each initial single-node failure $z \in Z$, in the final failure state[1] (when the failures cascade is terminated), the following must hold:
   (a) every active power node is connected to at least one active optical node and every active optical node is connected to at least one active power load;
   (b) optical network remains connected;
   (c) every active power grid node (load and substation) is connected to at least one active generator through a power grid path[2]; and
   (d) number of cascaded node failures is constrained to a maximum value k.

4. Design of Interconnection for Interdependent Networks

In this section, we first analyze the structure of an interdependent network. We then derive an analytic limit on the number of interconnection links required to ensure resilience to interdependent cascading failures. Such bounds are used to derive solution methods for the *PGON-I* problem in the following sections.

4.1. Power Grid - Optical Network Interconnection and Bipartite Graph Matching

We define a *Cascade Cover* (CC), of size n, as the set of n nodes (including the initial failed node) that fail due to an interdependent failures cascade. Thus, if we constrain the maximum size of a CC to k, then the maximum number of nodes that are allowed to fail due to any initial single-node failure is k (including the initial failed node). Note that a node can fail if all other nodes that it is dependent on (for power or control) fail or if the node gets disconnected from the rest of the network due to, e.g., failures of neighbouring nodes. To reduce/avoid interdependent cascading failures to propagate on too many network nodes, we

[1] We identify a final failure state as the situation in which no more cascaded node failures are possible due to interdependency between power grid and optical network.

[2] This means that power grid might get disconnected.

constrain the maximum size of a CC. As an example, assume we constrain the maximum CC size to 1, so no other node is allowed to fail after the initial node failure. In this case, assuming single-node failure, to satisfy the constraints as mentioned in Section 3, every power node should be connected to at least two optical nodes (primary and backup) for control, and every optical node should be connected to two power load nodes (primary and backup) for power supply. This way, failures do not propagate across the network, as long as both power grid and optical network are 2-node-connected, which is the assumption of this study[3].

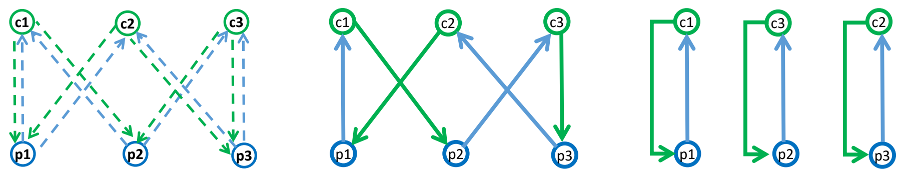
However, providing backup for each node (both power and communication) may not be possible considering the limited set of available interconnection links or feasible considering the cost. Thus, we focus on scenarios where the maximum size of a cascade cover is at least 2, and aim at minimizing the number of interconnection links required to constrain the maximum size of a cascade cover.

Selection of interconnection links between sets V_P and V_C from the set I is similar to a bipartite matching problem [20], in which *i*) interconnection links are directional and *ii*) additional constraints mentioned in Section 3 are added regarding the final failure state.

To show the impact of different selections of interconnecting links between power grid and optical network, consider the example in Fig. 2.

In Fig. 2(a), we see a bipartite power grid - optical communication network. The power grid and the optical network are shown in blue and green, respectively, and all the possible interconnection links are shown in dashed-directional lines. In the example, we consider a relaxed version of our problem, i.e., we do not consider the links within the optical network and within the power grid, and consequently we relax constraints 2(b) and 2(c) described in Sec. 3; moreover,

[3]Note that, in case some nodes in the power grid or communication network have connectivity degree less than 2 (say we have D such nodes), the amount of failed nodes in the CC after an initial failure may increase of at most D.



(a) Bipartite power grid - optical communication network topology

(b) Interconnection that causes large interdependent failures cascade

(c) Interconnection that reduces interdependent failures cascade to maximum two node failures

Figure 2: Interconnection for bipartite power grid - optical communication network.

we assume that all power nodes are loads (though there should have at least one generator in the power grid, we ignore that here).

We show a possible interconnection in Fig. 2(b), where each node is dependent on only one other node. In this particular example, any single node failure will cause failure of all other nodes due to interdependent failures cascade. In Fig. 2(c), we show an interconnection between the power grid - optical network where pairs of nodes i, j are mutually dependent, i.e., if node i is dependent on node j, then node j is dependent on node i and vice versa. In this case, for any single-node failure, only one other node will fail due to cascade, i.e., the maximum size of a CC is 2.

For the particular case where $|V_P| = |V_C|$ as in our example, finding pairs of mutually-dependent nodes reduces to a perfect bipartite matching problem, which can be solved in polynomial time [20]. Moreover, from Hall's theorem [21], a bipartite graph $G = (V, E)$ with bipartition (L, R) such that $|L| = |R|$ has a perfect matching if and only if, for every $A \subseteq L$, we have $|A| \leq |N(A)|$, where $N(A)$ is the neighbourhood of A, i.e., the set of vertices in R that are connected to vertices in A by an edge in E, that is, $N(A) = \{r \in R : \exists a \in A \text{ such that } (a, r) \in E\}$. Therefore, perfect bipartite matching can be found for our relaxed problem (i.e., neglecting constraints 2(b) and 2(c) in Sec. 3) if and only if the following conditions are true:

1. $|V_P| = |V_C|$, and
2. Hall's condition is satisfied considering only interconnection links, i.e., only (m, n) pairs such that both $m \to n \in I$ and $n \to m \in I$.

Note that, in our case, if there is a generator, condition 2) above cannot be satisfied as a generator cannot directly power an optical node. Moreover, when (re)introducing constraints 2(b)-2(c) discussed in Sec. 3, obtaining a feasible solution is not guaranteed even if the above two conditions are true. Although bipartite matching does not directly apply to our problem, we consider it to develop the heuristic in Sec. 6, as the relaxed form of the problem discussed above has similarities with the restricted bipartite matching problem, which has been proven to be NP-complete [22].

4.2. Analytic Limit on Number of Interconnection Links Ensuring Resilience to Interdependent Failures Cascade

We now derive lower and upper bounds on the number of interconnection links that ensure resilience to interdependent failures cascade. As mentioned before, we limit our analysis to the case that the first failure (trigger) is a single-node failure, and both the power grid and the optical network are 2-node-connected.

Lemma 1. *If both power grid and optical communication network are 2-node-connected and have m and n nodes, respectively, then the number of interconnection links required to limit the size of the CC to 2 is bounded between $(m+n)$ and $2(m+n)$.*

Proof. As every power node should be connected to at least one optical node and vice versa, we need at least $(m + n)$ interconnection links. Assuming the ideal case where all power nodes are eligible to power optical nodes (that is, assuming that all the nodes in the power grid are loads and there are no generators and substations), we first show a scenario where $(m + n)$ links can ensure the maximum size of a CC of 2 failures. Later, we discuss the realistic case where the power grid includes generators and substations, which cannot directly power optical nodes.

Without loss of generality, assume that $m = n$ and that we create m pairs of nodes (i, j), where i is a power node, j is an optical node, and both $i \to j \in I$ and $j \to i \in I$. Thus, we create links $i \to j$ and $j \to i$, i.e., power node i powers optical node j, and optical node j controls power node i. Now, node i fails either if it is the initial failure node or if node j fails. Thus, the cascade is constrained to only a single-node failure from each network which would remain connected as we are considering 2-node-connected topologies for both power grid and optical network. Therefore, the total number of interconnection links required to constrain the CC size to 2 is $(m+n)$ in this case. Note that, in the more general case where $i \to j \in I$ and $j \to i \notin I$ (or vice versa), we may not find m pairs of nodes in the two networks that can be mutually interconnected; therefore, the value $(m + n)$ is only a lower bound for the total number of interconnection links required.

Let us now consider the realistic case where the power grid includes generators and possibly substations, which cannot power optical nodes directly. Therefore, in this case, if $m = n$, some power loads may have to power more than one optical node. Thus, to ensure maximum CC size of 2, we need backup power-interconnection links for some optical nodes. As an example, assume that we power each optical node by two power nodes (one primary, one backup); and similarly, that we control a power node by two optical nodes. With this interconnection, as we assume single-node initial failure, even if a power node i fails, this will not affect the nodes that depend on it, as they can be powered by backup nodes in the power grid. In such a scenario, the maximum size of the CC is 1, and the total number of interconnection links used is $2(m + n)$. Note that, in such a scenario, as we are considering that both power grid and optical network are 2-node-connected, if at least two generators are present in the power grid, even if the initial single-node failure affects one generator, all the loads/sub-stations will still have a path towards another generator. Therefore, the cascade will not propagate on the power grid.

Now assume that we have an interconnection with more than $2(m+n)$ links such that the size of the CC is at most 2. This means that there is at least one node (let us assume an optical node) that is dependent on at least three nodes (power nodes in this case) in the other network. Thus, even if two of the three power nodes fail, the optical node survives failure (unless failure of this optical node is the initial failure). As maximum CC size is 2, due to a single node failure in one network, this means that at most one node in the other network fails. Thus, among the three power nodes, at most one node will fail. Consequently, we can remove one of the three interconnection links that connect the optical node with the power nodes, and still the optical node survives any cascade. This way, for each node that is connected to more than two nodes in the other network, we keep two of the links and remove others. The reduced set of interconnection links will then have at most $2(m+n)$ nodes, while still limiting the CC size to at most 2.

□

From Lemma 1, we see that an optimized number of links between $(m+n)$ and $2(m+n)$ will ensure resilience to interdependent failures cascade, which can be calculated using a mathematical model as shown in the next section.

5. Mathematical Model to Design Cascading-Failure-Resilient Interconnection

Below, we present an integer linear program (ILP) formulation to design a cascading-failure-resilient interconnection between power grid and optical communication networks with minimum number of interconnection links. We use the network model as described in Sec. 2 and the *PGON-I* problem statement as presented in Sec. 3. The model shown below is generalized to any number of node and/or link failures as initial failure and to networks which are not 2-node-connected. However, we will show numerical results considering only single-node failure as initial failure.

Variables

- $R_{mn} \in \{0,1\}$: =1 if node $n \in V_C$ (resp., $n \in V_P$) is connected to node $m \in V_P$ (resp., $n \in V_C$) for power/control and $m \to n \in I$
- $S_{nz} \in \{0,1\}$: =1 if node $n \in V_P \cup V_C$ survives after initial failure $z \in Z$ occurs
- $S_{nmz} \in \{0,1\}$: =1 if node $n \in V_C$ (resp., $n \in V_P$) is dependent on node $m \in V_P$ (resp., $n \in V_C$) and node m is not affected by initial failure z
- $f^n_{ijz} \in \{0,1\}$: =1 if link $(i,j) \in E_P$ is used for a path from node $n \in \{V_l \cup V_s\}$ to a generator after initial failure $z \in Z$ occurs
- $\phi^{lm}_{ijz} \in \{0,1\}$: =1 if link $(i,j) \in E_C$ is used for a path from node $l \in V_C$ to node $m \in V_C$ after initial failure $z \in Z$ occurs
- $P^n_{gz} \in \{0,1\}$: =1 if there is a path from $n \in \{V_l \cup V_s\}$ to $g \in V_g$, which does not use nodes in $z \in Z$

Objective function

The objective of the ILP optimization is to minimize the total number of interconnection links, i.e.:

$$minimize|R| = \sum_m \sum_n R_{mn}$$

Constraints

$$\sum_{m \in V_C} R_{mn} \geq 1, \qquad \forall n \in V_P \tag{1}$$

$$\sum_{m \in V_l} R_{mn} \geq 1, \qquad \forall n \in V_C \tag{2}$$

$$S_{nmz} \leq R_{mn},\ \forall n \in V_P\ (n \in V_C),\ m \in V_C\ (m \in V_P),\ z \in Z :\ n \notin z \tag{3}$$

$$S_{nmz} \leq S_{mz},\ \forall n \in V_P\ (n \in V_C),\ m \in V_C\ (m \in V_P),\ z \in Z :\ n \notin z \tag{4}$$

$$S_{nmz} \geq R_{mn} + S_{mz} - 1,\ \forall n \in V_P\ (n \in V_C),\ m \in V_C\ (m \in V_P),\ z \in Z:\ n \notin z \tag{5}$$

$$S_{nz} \geq S_{nmz},\ \forall n \in V_P\ (n \in V_C),\ m \in V_C\ (m \in V_P),\ z \in Z:\ n \notin z \tag{6}$$

$$S_{nz} \leq \sum_m S_{nmz},\ \forall n \in V_P \cup V_C,\ z \in Z:\ n \notin z \tag{7}$$

$$\sum_n S_{nz} \geq x,\ \forall n \in V_P \cup V_C,\ z \in Z : x \text{ is a constant} \tag{8}$$

$$f^n_{ijz} \leq \frac{S_{nz} + S_{iz} + S_{jz}}{3},\ \forall n, i, j \in V_P,\ z \in Z \tag{9}$$

$$P^n_{iz} \leq S_{iz},\ \forall n \in V_P,\ z \in Z,\ i \in V_g \tag{10}$$

$$\sum_j f^n_{ijz} - \sum_j f^n_{jiz} = \begin{cases} S_{nz} & \text{if } i = n \\ -(S_{nz} \wedge P^n_{iz}) & \text{if } i \in V_g \\ 0 & \text{otherwise} \end{cases} \qquad \forall n, i \in V_P,\ z \in Z \tag{11}$$

$$\sum_j \phi^{lm}_{ijz} - \sum_j \phi^{lm}_{jiz} = \begin{cases} (S_{lz} \wedge S_{mz}) & \text{if } i = l \\ -(S_{lz} \wedge S_{mz}) & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall l, m, i \in V_C,\ z \in Z \tag{12}$$

Constraint 1 ensures that every power node is connected to at least one communication node for control and Constraint 2 ensures that every communication node is connected to at least one power load for power. Constraints 3 to 5 are used to set variables S_{nmz} through R_{mn} and S_{mz}, i.e., $S_{nmz} = 1$ if n is dependent on m and m survives z. Constraints 6 and 7 ensure that, if node n is not directly affected by initial failure z, then n survives after z occurs, i.e., n

is not affected by interdependent failures cascade only if n is dependent on at least one node m from the other network that survives z. Constraint 8 restricts the number of failed nodes from both networks, due to initial failure z and the following interdependent failures cascade, to a maximum value. Constraint 9 ensures that power node n can receive power using power link (i, j) if nodes n, i, and j survive after initial failure z and the subsequent failures cascade. Constraint 10 ensures that power grid load n can receive power from generator i if i survives after initial failure z and the subsequent failures cascade. Constraints 11 ensure that, to survive failure z, a power load/sub-station should be connected to at least one generator after z and the subsequent failures cascade occur. We use similar constraints, reported in 12 to ensure communication network connectivity after initial failure z.

6. Heuristic Approach

Now we present our proposed heuristic algorithm to solve the *PGON-I* problem. From our discussion in Sec. 4, we see that, if we find pairs of nodes (i, j) and connect them to each other such that links $i \to j$ and $j \to i$ are selected for interconnection, then that can provide better resilience as this choice will limit the failure cascade (see Fig. 2(c)). The proposed heuristic is designed based on this observation.

In the following, to evaluate the improvement provided by a given interconnection link $w \to v$ in R, we leverage on the definition of the following figure of merit, namely the *benefit* of interconnection link $w \to v$.

Definition 1. *Given an interdependent power grid - optical network, consisting of m and n nodes, respectively, an initial failure set z, and a current set of interconnection links between the two networks, we define the benefit B_{wv} of a new interconnection link $w \to v$ as:*

1) $B_{wv} = m + n$, if adding interconnection link $w \to v$ does not cause neither disconnection in the communication network nor disconnection from all gener-

ators in the power grid;

2) $B_{wv} = m + n - CC$*, otherwise. In this second case, the benefit of interconnection link* $w \to v$ *indicates the number of nodes in the two networks which survive to* z*.*

Following the definition above, we can compare the benefit of interconnection link $w \to v$ with respect to another generic interconnection link $u \to s$ and state that $w \to v$ provides higher *benefit* than $u \to s$ if any of the following is true:

1. Even after including $u \to s$, disconnection in the communication network or disconnection from all generators in the power grid may happen for some initial failures, but after including $w \to v$, disconnection does not happen.
2. None (or both) of the links can avoid disconnection, but the maximum CC size after including $w \to v$ is lower than the maximum CC size after including $u \to s$.

As shown in Alg. 1, we first create a graph V' by including V_P, V_C, and a set of links $B = \{(m, n)\}$ such that both $m \to n \in I$ and $n \to m \in I$ (line 1). We then find a maximum bipartite matching on V' and include the set of links from the matching into R (line 2). Note that a perfect matching is not possible as generators and substations cannot directly power an optical node. Thus, as maximum bipartite matching does not cover all the nodes from both networks, we consider the nodes which are not yet connected to any other node for power/control.

As, in general, the power grid and optical network have different number of nodes, we indicate with V_A and V_B the one with lower and higher number of nodes, respectively. We first take a node $v \in V_A$ such that v is not yet connected to a node in V_B for power or control, and find an interconnection link $w \to v \in I$ such that $w \to v$ provides maximum *benefit* among all possible links. We then include $w \to v$ into R. Note that, if two links have the same *benefit*, we choose randomly among them. We repeat the steps for all such nodes v (lines 3-6).

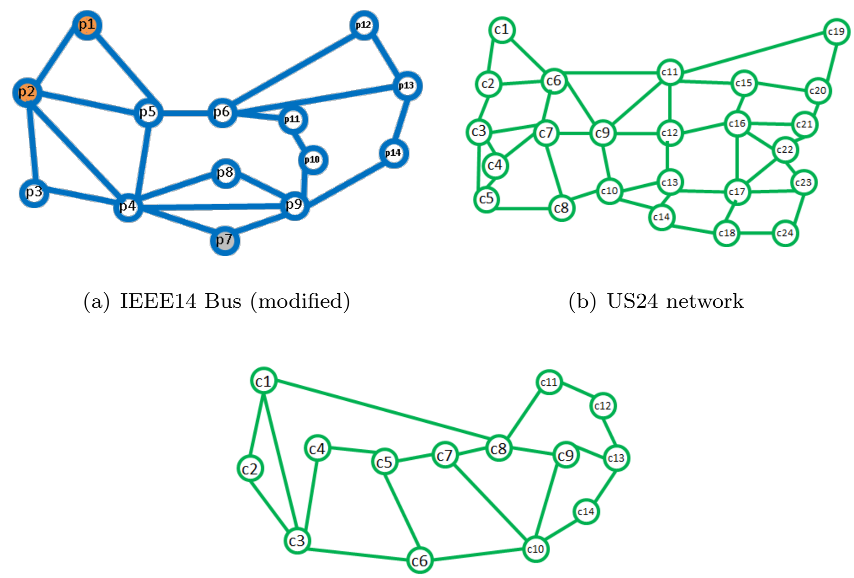
Then, the same steps taken for nodes in V_A are also taken for nodes $v \in V_B$ such that v is not yet connected to a node in V_A for power or control (lines 7-10). After this, each node in both power grid and optical network is connected to one node for power/control, thus $|R| = (m+n)$, which corresponds to the lower bound obtained in Sec. 4.B.

We now consider each node $v \in V_P \cup V_C$ such that the failure of v creates a failures cascade that causes network disconnection. So, let us assume that, after the initial failure of v, the set of nodes $V_F \subseteq V_P \cup V_C$ fails. We consider the set of interconnection links $I_F = \{m \to n : n \in V_F \text{ and } m \to n \notin R\}$. We then choose a link $m \to n \in I_F$ such that the link maximizes *benefit*, and add it into R. This way, a backup link for node n is added to ensure its resilience. We call such maximum-*benefit* link $m \to n$ a *cascade-reduction* link. We keep adding cascade-reduction links into R for nodes in I_F until there is no disconnection after the initial failure of v. In the worst case, we have to add backup links for each node in V_F. In that case, after the failure of v, no other node will be failed. We repeat the above-mentioned steps for all v where failure of v creates a failures cascade (lines 11-16).

Then, we consider nodes $v \in V_P \cup V_C$ such that the failure of v creates a failures cascade cover of size greater than f and new *cascade-reduction links* (similarly to the operations in lines 11-16), with the objective of limiting the maximum size of any CC to k (lines 17-22). Finally, we return the set of all the obtained interconnection links, R.

As both power grid and communication network are 2-node-connected, the heuristic will always find a valid solution by adding backup links. As shown in Sec. 4, in the worst case, the heuristic algorithm provides in output $2(|V_P|+|V_C|)$ interconnection links.

We now derive the complexity of Alg. 1. Line 1 takes $O(|I|)$ time. The maximum bipartite matching in line 2 can be computed in $O(|V_P|^{5/2})$ using Hopcroft-Karp algorithm [23]. Computing *benefit* for one interconnection link takes $O(max(|V_P|^3, |V_C|^3))$, as it includes the cost to compute cascade due to an initial failure, to determine if there is any network disconnection, and number



(a) IEEE14 Bus (modified)

(b) US24 network

(c) NSFnet (modified)

Figure 3: Network topologies used in the study. (a) is a power grid and (b) and (c) are communication networks.

of failed nodes due to failures cascade. Then, computing lines 3 to 6 takes $O(max(|V_P|^4, |V_C|^4) \times |I|)$. Similarly, lines 7 to 10, 11 to 16, and 17 to 22 also take $O(max(|V_P|^4, |V_C|^4) \times |I|)$. Therefore, the complexity of Alg. 1 is $O(max(|V_P|^4, |V_C|^4) \times |I|)$.

7. Illustrative Numerical Examples and Analysis

Figure 3 shows the sample network topologies used in this study: a power grid topology, IEEE14 bus (Fig. 3(a)), and two communication network topologies, namely US24 (Fig. 3(b)) and NSFnet (Fig. 3(c)). We have modified NSFnet to make it planar and IEEE14 bus by replacing a sub-station node with a load. In the power grid, nodes 1 and 2 are generators, node 7 is a sub-station, and all other nodes are loads. As it is unlikely that a node is powered/controlled by remote nodes, here we assume that, based on relative distance, a node in one

Algorithm 1 Design of Interconnection in Power Grid - Optical Network.

INPUT: Power Grid and Optical Network topologies, $G_P(V_P, E_P)$, $G_C(V_C, E_C)$.

Sets of generators, sub-stations, and loads, V_g, V_s, V_l: $V_g \cup V_s \cup V_l = V_P$.

Set of possible interconnection links, $I = I_{cp} \cup I_{pc}$.

Set of initial failures Z.

Maximum number of nodes that are allowed to fail, k.

OUTPUT: Set of directional interconnection links, $R \subseteq I$

Create graph $V' = (V_P \cup V_C, B)$, where $B = \{(m, n), m \in V_P, n \in V_C | m \to n \in I \text{ and } n \to m \in I\}$
$R =$ Maximum Bipartite Matching (V')
for all $v \in V_A$ such that V_A is the set of nodes from the network (either power grid or optical network) with fewer nodes and $\nexists w : w \to v \in R$ **do**
  Choose the link $w \to v$ from I that provides the highest *benefit*
  $R = R \cup \{w \to v\}$
end for
for all $v \in V_B$ such that V_B is the set of nodes from the network (either power grid or optical network) with more nodes and $\nexists w : w \to v \in R$ **do**
  Choose the link $w \to v$ from I that provides the highest *benefit*
  $R = R \cup \{w \to v\}$
end for
for all $v \in V_A \cup V_B$ such that the failures cascade with interconnection R caused by the initial failure of v produces 1) disconnection in the communication network, and/or 2) disconnection of any power load or substation from all generators **do**
  Add *cascade-reduction* links into R to avoid disconnection
  if $|R| = |I|$ **then**
    Return
  end if
end for
for all $v \in V_A \cup V_B$ such that the failures cascade caused by the initial failure of v affects more than k nodes **do**
  Add *cascade-reduction* links into R to reduce size of the CC to k
  if $|R| = |I|$ **then**
    Return
  end if
end for
Return R

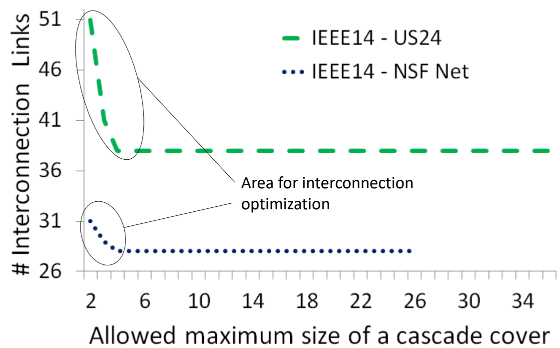
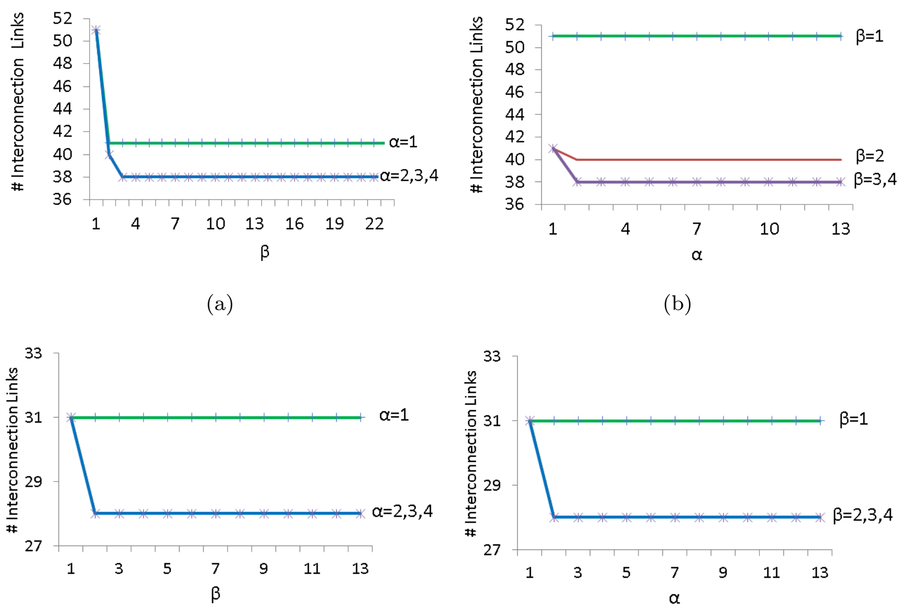


Figure 4: Number of interconnection links required to reduce/avoid interdependent failures cascade.



(a) (b) (c) (d)

Figure 5: Effect of number of nodes in a network on resilience to interdependent failures cascade.

network can be connected to 2 to 4 neighbouring nodes from the other network. As mentioned before, we focus on initial single-node failure. We apply our ILP on these topologies running different optimizations and satisfying the constraints mentioned in Sec. 3.

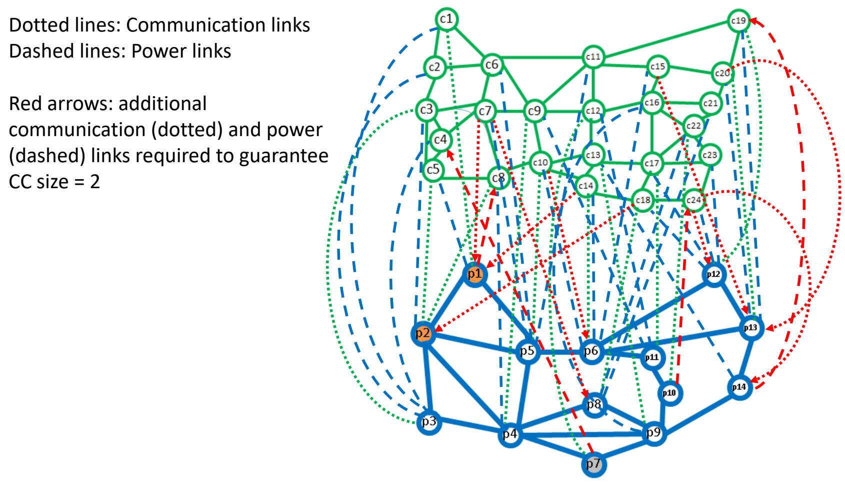


Figure 6: Interconnection links required to guarantee CC size 2 (IEEE14-US24 network topologies).

Figures 4 and 5 show the relation between maximum allowed size of a cascade cover (i.e., maximum number of nodes that are allowed to fail in a cascade) and the required number of interconnection links to guarantee that maximum size. Figure 4 shows the number of interconnection links (on y-axis) required to constrain the maximum size of a CC to a maximum value (on x-axis), while ensuring the constraints mentioned in Sec. 3. In interdependent IEEE14-US24 network, there are a total of 38 (14+24) nodes. Following Lemma 1, the required number of interconnection links should be between 38 and 76 (38×2). We see that, as we increase the maximum number of nodes that are allowed to fail due to cascade (i.e., provide less protection), the required number of interconnection links goes down quickly from 51 to 38[4]. Thus, even with the constraint that a node can only be connected to a neighbouring node in the other network, resilient design is possible using a number of interconnection links close to the

[4]Note that the lower bound $m + n$ provided by Lemma 1 is applicable to *any* CC size, as all power and communication nodes need to have at least one incident link for control or power supply, respectively.

lower bound (38 in this case). As an illustrative example, we show in Fig. 6 the interconnection links required to guarantee CC size equal to 2. In the figure, the dotted (respectively, dashed) lines represent the communication (resp., power) links for the case with CC size 2, among which we indicate with red (either dotted or dashed) arrows, the 13 (=51-38) communication and power links to be added to obtain CC size 2 in comparison to the baseline case with CC size greater than or equal to 4.

We see a similar trend in IEEE14-NSFnet, where the required number of interconnection links is between 28 (14+14) and 56 (28×2). Now, the number of interconnection links goes down from 31 to 28. As in this case, both power grid and optical network have an equal number of nodes (14), there may be a solution with 28 (14 + 14) links which ensures resilience to interdependent failures cascade. We need 31 interconnection links as only 11 out of 14 power nodes are loads which can power neighbouring optical nodes. Consequently, some loads power more than one optical node and so some optical nodes may need backup power links. Two main messages arise from Fig. 4, i.e.: 1) higher the difference between the numbers of nodes in the two networks, the more interconnection links are needed to ensure resilience to interdependent failures cascade, and 2) if we want to ensure a maximum CC size less than 5, then we need a (limited) number of additional links; above a CC size of 5 nodes, the cascade is halted by a number of interconnection links equal to the lower bound.

To analyze further the effect of number of nodes in the networks on resilience to interdependent failures cascade, Fig. 5 shows how the number of interconnection links changes as we change the number of failed nodes allowed in one network, while keeping the number of failed nodes allowed in the other network to be constant. We indicate with α and β the allowed maximum size of a cascade cover in power grid and communication network, respectively. Intuitively, we see that, if the number of failed nodes in IEEE14 is fixed at 1, as the number of failed nodes in US24 network increases, the number of interconnection links decreases (see Fig. 5(a)). Less intuitively, if the number of failed nodes in US24 is fixed at 1, the number of interconnection links does not change even if the

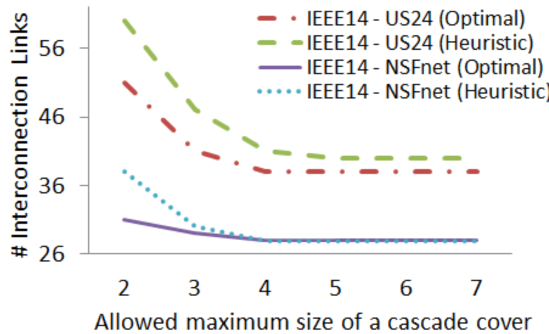


Figure 7: Number of interconnection links required to reduce/avoid interdependent failures cascade.

number of failed nodes in IEEE14 increases (see Fig. 5(b)). This is because, on average, in the network with larger number of nodes, several of such nodes will be dependent on a single node from the other network. Thus, one node failure in the network with fewer nodes may affect multiple nodes in the other network, so some nodes in the network with more nodes may require backup interconnection links.

Similar behaviour is observed for the case of IEEE14-NSFnet when increasing the values of β or α, as shown in Figs. 5(c) and 5(d), respectively. This is because of the equal number of nodes in both networks. We can conclude that the network with more nodes is more critical while providing resilience to interdependent failures cascade.

Finally, Fig. 7 compares the heuristic with the optimal ILP-based solution. For the considered scenarios, the solution time for the ILP and heuristic cases is in the order of minutes and hundreds of ms, respectively. We see that, for IEEE14-US24, the heuristic solution provides a number of interconnection links 17.6% to 5% higher than the optimal solution as the allowed cascade cover size increases from 2 to 5. For IEEE14-NSFnet, the heuristic solution is 22.6% and 3.4% higher than the optimal solution for cascade cover size of 2 and 3, respectively. For, cascade cover size greater than 3, the heuristic is able to reach the optimal solution, showing the effectiveness of the proposed approach.

Although in this paper we focus on initial single-node failures, we observe that the ILP model in Sec. 5 and the heuristic proposed in Sec. 6 can be used for any set of initial failures z, including links and/or nodes. We do expect differences in the amount of interconnection links required with increasing number of initial failures in z, in particular for cases when all the initial failures are concentrated in the same network (i.e., the power grid or the optical network), which increases the probability of creating network disconnection. Specific quantitative evaluations on this aspect are left for future work.

8. Conclusion

We introduced the problem of designing cascading-failure-resilient interconnection for interdependent power grid - optical network. We derived analytic limit on the number of interconnection links that can ensure resilience to interdependent failures cascade. We developed an ILP model and heuristic algorithm to solve the problem. Our results show that it is possible to limit the failure cascade even below a very strict threshold (as no more than 2-3 cascading failures) by selectively placing only few additional interconnecting links using our proposed approach. Results also show that the higher the difference between the numbers of nodes in the two networks, the more interconnection links are needed to ensure resilience to interdependent failures cascade.

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