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ORIGINAL ARTICLE

Development of generalized tool life model for constant and variable speed turning

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Abstract
In this research, a generalized tool life modelling for considering non-stationary cutting conditions was developed . In particular, for the first time in literature, the model was conceived for predicting the life of the tool when spindle speed variation SSV, one of the most effective techniques for suppressing regenerative chatter vibrations, is used. The proposed formulation takes into account the main cutting parameters and the parameters associated to the SSV. A dedicated experimental campaign of turning tests was executed and the data were used for modelling purposes. The model validation was carried out performing additional tool life tests. According to the analyzed technological scenario, it was found that the generalized formulation can be used for predicting the tool life both at constant spindle machining CSM and adopting SSV with the maximum estimating error of 6%.

Keywords Tool life modeling · Spindle speed variation · Non-stationary cutting · Chatter suppression

1 Introduction

The occurrence of high vibrations in machining limits the achievable Material Removal Rate MRR [1], the surface quality and the tool life TL [2]. Vibrations are typically due to regenerative effects that bring cutting process to instability. Spindle speed variation (SSV) is one of the available techniques for suppressing chatter vibrations [3]. It is based on a continuous modulation of the spindle speed that aims at repressing the regenerative effect and thus the growth of undesired vibrations. Over the years, SSV has been developed and tested both in turning and milling applications. First, studies assessed the vibration mitigation properties of the spindle speed variation SSV. Radulescu et al. [4] developed a time domain model for this purpose. Insperger et al. [5, 6] developed an analytical formulation for predicting the stability map when the SSV is adopted. The model was validated through simulations. The same approach was used by Kapoor et al. [7]. Albertelli et al. [8] developed a time domain simulation model for studying SSV in turning. The model was validated through experimental tests. Urbikain et al. [9] studied the effect of SSV on the stability map in a real turning application through the perturbation theory. Yamato et al. [10] studied the stability of variable delay turning through an energy approach. Zhang and Ni [11], adopting a similar method, provided useful indications for selecting the SSV parameters. The approach was tested through numerical simulations. Otto and Radons [12] studied the effectiveness of SSV in turning according to different eigenfrequencies and damping ratios. Wu and Chen [13] extended the application of SSV to non-circular turning. Ding et al. [14] developed a control strategy based on the simultaneous adjusting of the parameters governing the SSV. They demonstrated the validity of the proposed approach through simulations and executing some preliminary cutting tests. The same authors in [15] developed a close-loop implementation that combined monitor and control strategies suitable for turning applications. Meng et al. [16] developed a state feedback control for assuring the steady state stability in SSV turning. They tested the control strategy through

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simulations. Zatarain et al. [17] carried out milling simulations and experimentally validated the numerical findings. Totis et al. [18] developed an analytical formulation for fast estimating the stability maps when the SSV is adopted in milling. A proper experimental validation was not provided. Sinusoidal spindle speed variation $SSSV$ is the most studied approach for modulating the spindle speed. Alternative approaches were developed. For instance, Yilmaz et al. [19] conceived an approach based on a pseudo-random spindle speed variation. Nam et al. [20] recently developed a SSV strategy characterized by a constant acceleration rate that allowed achieving higher MRR than conventional SSV. The same authors in [21] defined novel chatter indices for assessing the chatter growth. Albertelli et al. [22] developed a generalized algorithm based on cyclostationary theory for chatter detection when the spindle speed variation is used.

From the quantitative perspective, it was proven that stability enhancement due to SSV is effective and robust especially in the high-order lobes region of the stability maps, both in turning [8] and milling [17]. The SSV, thanks to the the stabilization capabilities, reduces the risk of tool chipping and too early failures that typically occur when vibrations affect the cutting [23]. Although several research works have been published on SSV so far, few of them focused on potential secondary detrimental effects of the technique. For instance, Albertelli [8] studied the thermal load of the spindle motor due to the spindle modulation and performed a feasibility study with respect to industrial turning applications. Urbikain et al. [9] found that SSV slightly increased (about 20%) the power consumption of the lathe. For what concerns the tool duration, Albertelli et al. [24] found that sinusoidal spindle speed variation $SSSV$ has a negative impact on the life of the tool, fostering the formation of cracks that tend to progressively detach the coating and thus increasing the wear rate. In this study, the achieved results were obtained comparing $SSSV$ machining to constant speed machining CSM, both in stable conditions. Chiappini et al. in [25] used a finite element model FEM for simulating $SSSV$ cutting. It was found that the modulation of the spindle speed is the responsible of an additional mechanical-thermal load on the cutting edge that could bring to the cracks formation. Although both the research works gave an interesting interpretation of the involved phenomena, a quantification of the tool life TL reduction, according to the adopted cutting parameters, was not provided. Being able to quantify the TL reduction would be extremely useful for assessing the potentialities and the limitations of the $SSSV$, especially in terms of industrial applicability. Although variable speed machining VSM was first conceived in the 1970s, its wide diffusion in real applications has not been registered so far. For instance, the use of such technique to stabilize machining operations that otherwise could be regularly carried out with stiffer, but even more expensive machines, needs to be properly analyzed. In such way, a trade off between tooling and machine costs could be investigated. In order to bridge this gap, in this paper a generalization of the Taylor's model for estimating the TL in turning, when both CSM and $SSSV$ are indiscriminately used, was developed.

The literature on the study of the wear of the tool is huge. Some of the works focused on the performance assessment of new tool materials and coatings. For instance, Wojciechowski et al. [26] assessed the performance of Boron Nitride Dispersed Cemented Carbide on a specific spheroidal cast iron focusing on the main wear phenomena. Wojciechowski and Twardowski [27] compared the duration of sintered carbide and cubic boron nitride in hardened steel milling without developing a proper tool life modelling. Even the effects of the adopted cooling lubrication strategy on the tribological behaviour [28] and on energy consumption have been subject of several studies [29]. Cryogenic cutting allows reducing the specific energy involved in cutting. Comparative studies on the life of the tool were carried out. For instance, Albertelli et al. [30] developed a tool life modelling considering conventional and cryogenic Ti6Al4V milling. Cryogenic cutting showed increments in tool duration only if high cutting speeds were adopted. Although Wong et al. [31] put into evidence the limitation of the empirical approaches, Johansson et al. [32], performing an interesting assessment of different tool life modelling formulations, found that Taylor's model assured a tool life estimation error that ranged from 8% to 21% and that the error was not material dependant.

So far, the literature that has been dealt with the tool wear under unstationary cutting conditions is rather poor. For instance, Galante et al. [33] developed a new tool life modelling approach based on Gaussian probability distribution that allows developing more flexible models with respect to the Taylor's model. One of the first studies that presented the difficulties related to the wear modelling when different cutting conditions are set was carried out by Jemielniak et al. in [34]. Lin in [35] and Pálmai in [36] proposed a cumulative wear model for taking into considerations different spindle speed steps. Since, to the authors' knowledge, no specific modelling approaches for estimating the TL when the cutting speed is continuously modulated have been developed, a preliminary formulation is first presented in this study. The paper was structured as follows. In Section 2, a more exhaustive description of the research goals together with the explanation of the conceived approach were provided. The experimental set-up, the designed experimental campaign and the preliminary tests were also presented. In Section 3, the results of the experimental tests, the development of the modelling formulations and their validation were reported and critically analysed. In Section 4, the conclusions were also outlined.

Nomenclature

$\boldsymbol{\beta}$ regression coefficients
$\hat{\boldsymbol{\beta}}$ regression coefficient estimations
$\boldsymbol{L}$ least square function
$\boldsymbol{X}$ regression variables matrix
$\boldsymbol{y}$ response variables vector
χ primary tool lead angle
$\Delta A_{\Omega}\%$ percentage tracking error with respect to the set $A_{\Omega\ set}$
$\Delta RVA\%$ percentage tracking error with respect to the set RVA_{set}
$\hat{TL}_s$ TL estimation carried out with the sth formulation
Ω Spindle speed
Ω_0 Nominal spindle speed
Ω_{0set} nominal imposed spindle speed
$\Omega_{meas}(t)$ measured spindle speed through the spindle encoder
$\Omega_{set}(t)$ imposed spindle speed set-point
a_p radial depth of cut
A_{Ω} amplitude of the sinusoidal modulation of the spindle speed
$A_{B_{ik}}$ area of the wear land on the tool flank
CI confidence interval
$CT_{i(p+1)}$ cutting time associated to the average flank width $VBB_{i(p+1)}$
CT_{ip} cutting time associated to the average flank width VBB_{ip}
DF_j degrees of freedom of the $factor\ j$
f feed per revolution
$F-value_j$ $Fisher$ tests for the $factor\ j$
$freq$ frequency of the sinusoidal spindle variation $SSSV$
H_{0j} null hypothesis for the j^{th} paired $CSM-SSSV$ test
$l_{B_{ik}}$ length of the wear area used for the VBB_{ik} computation
m number of considered factors in the 2^m experimental plans
MS_j Mean Squares of the $factor\ j$
n_{rc} number of replicates of the central points
n_r number of replicates of tests carried out at the corner points
$P-value_j$ P value of the test for the $factor\ j$
R^2 coefficient of determination of the regression
R^2_{adj} adjusted coefficient of determination of the regression
r_ϵ insert corner radius
RVA non-dimensional amplitude variation of the sinusoidal spindle variation $SSSV$: sinusoidal amplitude/Ω_0
SE standard error
$SS_j^{III} \equiv SS_{(j|k,\ldots,r)}$ adjusted sum of squares ($type\ III$) of the $factor\ j$
$SS_{(j,k,\ldots,r)}$ sequential $sum\ of\ square$ ($type\ I$) considering the factors $j, k, \ldots, r$ in the model
t time
TL tool life
TL_i Tool Life TL of the ith tested cutting edge
$TL_{s-error\%}$ percentage errors in the TL estimation adopting the sth formulation
v_c cutting speed
VBB flank wear average width
VBB_t flank wear width threshold
VBB_{ik_j} local measurement of the flank width VBB after the kth stop of the ith wear test
VBB_{ik} is the kth measurement of the average flank width associated to the ith wear test
X_n radial coordinate—distance from the external surface (micro-hardness measurements)

2 Material and methods

According to [8], even in this research experimental tool wear tests (ISO 3685, [37]) were performed both in constant speed CSM and variable speed VSM machining. The definition of the tests was carried out adopting a Design of Experiments DOE approach. More specifically, in both the cases a full factorial scheme was used and properly motivated. Additional information were provided in Section 2.1. The tool life TL data were analyzed and used for the generalized regression model development. A proper validation was even carried out.

2.1 Design of experiments

A steel turning application was selected. Indeed, for the cutting speeds typically adopted in steel machining and considering the limiting eigenmodes generally associated to tool-holder systems (70–160 Hz), it was demonstrated that $SSSV$ can assure effective chatter suppression properties [8]. Moreover, such application is rather widespread in most of the shop-floors.
In this research, two levels full factorial designs 2^m (m is the number of the considered factors) were conceived for the tests performed at CSM and for the tests that involved the sinusoidal spindle speed modulation $SSSV$. Although other design of experiments schemes allow to reduce the experimental effort (i.e. Taguchi, Box-Behnken), it was decided to use a full factorial scheme since it provides a more flexible approach especially if it is necessary to combine and analyze the results of the two separate experimental sessions. Moreover, Tsui [38] found that Taguchi approach

can lead to non-optimal solutions, information loss and efficiency loss. Indeed, Medan et al. [39] obtained better estimation errors adopting a full factorial design instead of using Taguchi. Box-Behnken is a factorial scheme with an incomplete block design that can lead to regions of poor prediction quality (corners), Montgomery [40].

For both the experimental plans, the cutting velocity v_c and the feed per revolution f were the main analyzed factors. Since the $SSSV$ can be described by Eq. 1, two additional factors were considered: RVA and $freq$. RVA is the non-dimensional amplitude variation parameter while $freq$ is the frequency parameter. It is worth noting that, since the spindle modulation makes the chip thickness to continuously vary [8, 25], the f parameter was considered in the experimentation at CSM although, according to the Taylor's theory, the effect of such parameter should be less relevant than v_c.

$$\Omega(t) = \Omega_0 \left(1 + RVA \cdot \sin\left(2\pi \cdot freq \cdot t\right)\right); \quad A_\Omega = RVA \cdot \Omega_0 \tag{1}$$

For what concerns the radial depth of cut a_p, since it is well-known in literature (i.e. Johansson et al. [32] and Hägglund [41]) that its effect on the wear of the tool is less relevant than f and considering the need to limit the experimental resources, this parameter was not varied in the experimentation. For this purpose, a radial depth of cut $a_p = 2\,mm$ was set. This choice simultaneously took into consideration some aspects:

- high a_p values should be used since SSV is typically used for rough machining
- high a_p values should be used in order to avoid the effect of the radius of the tool in the flank wear measurement (see Fig. 5).
- a_p should be lower than the active part of the cutting edge ($6\,mm$)
- should be limited in order to consider the lathe limitations in terms of maximum spindle $torque$ and $power$.
- a_p should be limited to avoid chatter vibrations [1]. Preliminary cutting tests were performed in order verify the absence of any dangerous vibrations (see Section 2.3).

For all the analyzed factors (v_c, f, RVA, $freq$) two levels were considered. Moreover, in order to track possible deviations from the linearity (i.e. curvatures due to second order effects with respect to the considered factors) or to enhance the model adequacy as well, center points were added to the 2^m factorial design scheme. In case of curvature effects (assessment performed in Section 3.1 and Section 3.2) , additional test conditions can be added. To restrict the experimental effort/budget, it was decided to carry out one single test $n_r = 1$ for each corner condition [40] while $n_{rc} = 5$ replicates were set for the intermediate cutting conditions, left side of Fig. 10. Finally, it was decided to use the described approach for the following reasons:

- Since it was found [24, 42] that the wear mechanisms in variable speed machining VSM are rather common in tool wear tests, the expected deviations from the Taylor's model (linear with respect to the factors in logarithmic coordinates) were supposed to be limited
- The Analysis of Variance ($ANOVA$) theoretical framework (Montgomery [40]) provides suitable tools for statistically assessing the curvature effects
- The adequacy of the proposed approach can be further verified selecting, for the model validation phase, different cutting conditions from the ones used for the modelling step (Section 3.4).

The analyzed factors and the corresponding selected values, both for CSM and $SSSV$, were resumed in Tables 1 and 2, respectively. The values of the analyzed factors were chosen according to [42] and considering the technological limitations associated to the adopted tool (v_c and f) and lathe. Indeed, the set parameters allow continuously changing the cutting speed in the $SSSV$ tests. Preliminary turning tests were carried out in order to check the feasibility of both the DOE plans (Section 2.3). Moreover, it was demonstrated in [8] that the selected parameters (RVA and $freq$) assured relevant chatter suppression properties if unstable cut occurred. It is worth noting that all the tests in this experimental campaign were executed in stable conditions (Section 2.3). All the cutting tests (both at CSM and adopting $SSSV$) were completely randomized. More details on the experimental set-up were reported in Section 2.2.

2.2 Experimental set-up

The flank wear average width VBB was monitored during the cutting tests. The flank wear threshold $VBB_t = 0.15\,mm$ was used as the $end-of-tool-life$ criteria. The cutting time TL that corresponds to the considered wear threshold was the main process response for all the tested conditions. Although the selected wear threshold $VBB_t = 0.15\,mm$ is lower than the ones typically used and suggested by the ISO 3685 [37], preliminary wear tests confirmed that the chosen threshold avoided a too high dispersion of the results in terms of insert duration TL. This choice was carried out to limit the experimental effort. Steel bars

Table 1 Design of experiments—CSM, factors and values

Level	v_c (m/min)	f (mm/rev)
High	220	0.3
Center	190	0.2
Low	160	0.1

Table 2 Design of experiments—$SSSV$, factors and values

Level	v_c (m/min)	f (mm/rev)	RVA	$freq$ (Hz)
High	220	0.3	0.3	1.5
Center	190	0.2	0.2	1
Low	160	0.1	0.1	0.5

(material $39NiCrMo3$, with hardness 255 HB, ultimate tensile strength 1145 MPa, Yield strength 1015 MPa and an elongation at break 14.5% (UNI 7845 − 78 [43])) hardened and tempered were used to perform the wear tests. The tool life tests were performed following the standard $ISO3685$ [37]. A Stereomicroscope $Optika\ SZN - T$ with a $Motic\ SMZ - 168T$ was used to measure flank wear width VBB during the cutting tests. More details on the performed wear measurements can be found in Albertelli et al. [42]. A carbide tool with a lead angle $\chi = 95°$ was adopted (ISO code $TNMG220404 - M5$ 5625 (tool radius equal to $r_\epsilon = 0.4\,mm$, rake angle 13° and a relief angle equal to 0° with a $Al2O3 - TiCN$ coating)) and fixed on tool holder, ISO code $MTJNL2525M22$. Cutting fluid (oil-water emulsion with 5% of $HOCUT$ 795 SC) was used in order to reproduce realistic industrial machining conditions. The lubricant was injected to the cutting zone through a flexible and adjustable nozzle, visible also in Fig. 1. It is worth noting that a proper control unit was specifically developed (adopting $National\ Instruments\ NI$ hardware and software) for performing the cutting tests in VSM. Specifically in this research, a $\Omega_{0set} - RVA_{set} - freq_{set}$ parameter combination can be set to the controller to perform cutting tests with the $SSSV$. The conceived solution was integrated with the drives and the numerical controller a $SOMAB\ Unimab$ 400 lathe, refer to Fig. 1. Since it was not possible to use NC build-in functions for modulating the spindle speed, a tailored solution was developed. A circuit allowed to change the operating mode $CSM/SSSV$. If the $SSSV$ mode was selected, the spindle speed set-point $\Omega_{set}\,(t)$ was generated by the external control system. If the CSM mode was selected, the speed set-point was directly generated by the numerical controller NC of the lathe. In both the cases the speed set-point $\Omega_{set}\,(t)$ and the measured spindle speed $\Omega_{meas}\,(t)$ (through the spindle encoder) were acquired. The lathe was equipped with a spindle with a maximum $power$ of 12.5 kW that can rotate up to 3000 rpm. A preliminary version of such control unit described in Albertelli et al. [42]. A $Kistler$ dynamometer ($9265B$) with the associated charge amplifier ($5070A$) was used to measure the cutting force during the preliminary phases of the experimentation (see Section 2.3).

2.3 Preliminary verifications

Before executing the wear tests, some preliminary checks were carried out:

- lathe tracking performance verification
- workpiece hardness analysis
- verification of chatter free cutting conditions
- dispersion of the tool life data according to the selected wear threshold $VBB_t = 0.15\,mm$

The first verification was performed in order to exclude any side effects of the tracking performance of the developed

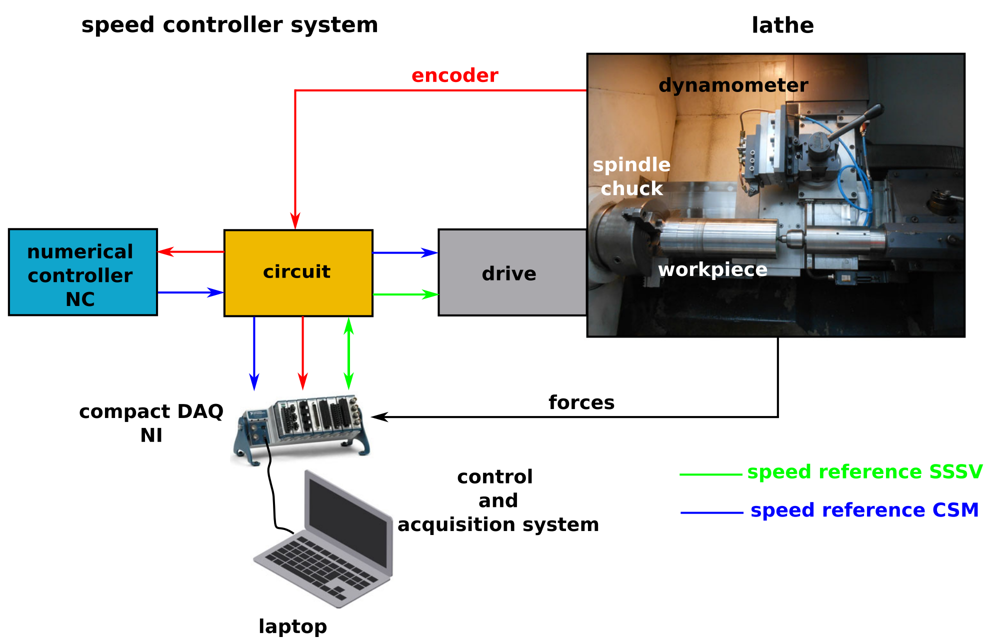

Fig. 1 Experimental set-up: lathe and developed control unit

control unit, especially when the $SSSV$ was adopted, on the results of the experimentation. More specifically, since the spindle available torque is limited, the $SSSV$ cannot be arbitrary implemented. Moreover, the maximum achievable speed modulation (in terms of combination of RVA and $freq$) depends on the nominal cutting speed. Several spindle speed tracking tests were carried out to assess the limitations of the adopted equipment (Fig. 1). All the tracking tests in the $SSSV$ regime were executed with workpiece hold by the spindle in order to reproduce as much as possible the real cutting conditions (workpiece inertia). For each selected combination of $SSSV$ parameters (RVA_{set} and $freq_{set}$), the imposed spindle speed Ω_{0set} was progressively increased. During each run, the actual spindle speed $\Omega_{meas}(t)$ was acquired through the spindle encoder (see Fig. 1) and the developed acquisition system. A $Fast\ Fourier\ Transform\ FFT$ was carried out in order to estimate the average spindle speed $\bar{\Omega}_{0\ meas}$, the resulted modulation $A_{\Omega\ meas}$ and the corresponding RVA_{meas}. In Table 3, some results were reported. The percentage tracking errors ($\Delta A_{\Omega}\%$ and $\Delta RVA\%$) in terms of deviation from the nominal values (respectively $A_{\Omega\ set}$ and RVA_{set}) was computed. Even in Fig. 2, it can be observed that up to $1000\,rpm$ the tracking errors are negligible while just starting from $1200\,rpm$ they become unacceptable. Several additional tests were carried out with different combinations of RVA_{set}-$freq_{set}$.

The second verifications was carried out for investigating if the workpiece hardness changes according to the machined region. Indeed, a non-homogeneous material property could have affected the reliability of the tool life tests introducing a possible bias. In order to exclude such effect, several hardness measurements were performed. Both macro-hardness and micro-hardness measures were carried out on different workpiece locations as reported in Fig. 3. Macro-hardness measurements were executed on the lateral part of the workpiece ($\varnothing = 132\,mm$) both on the external raw surface ($zone\ C$) and on the internal turned part ($zone\ B$ of $\varnothing = 130\,mm$). The macro-hardness data were statistically analyzed and it was confirmed that the hardness measured in the $zone\ C$ cannot be be considered different from the one measured in $zone\ B$. An additional analysis was carried out investigating the dependence of the hardness on the radial coordinate X: three micro-hardness repetitions (durometer $Future-Tech\ FM-700$, $Vickers$, loading $1\,kg$, dwell time $15\,s$) were performed increasing the distance X_n from the external surface $zone\ B$. The micro-hardness measurements were executed on the cross-section $zone\ A$, properly prepared through multiple polishing steps. The obtained results were reported in Fig. 4. It was statistically demonstrated that the hardness cannot be considered affected by the radial coordinate X.

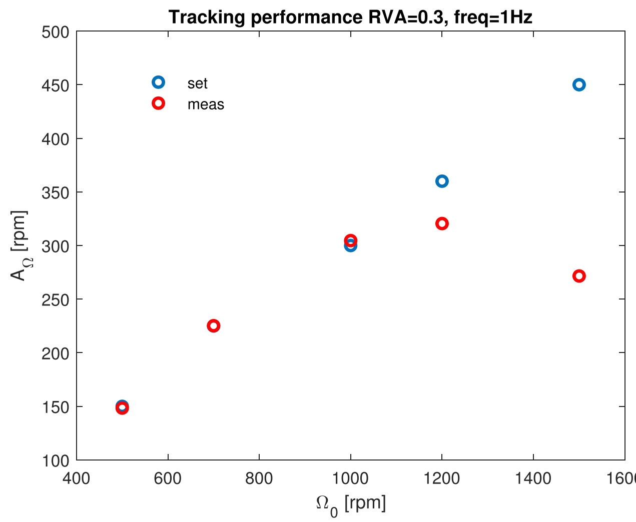

Fig. 2 Tracking errors $RVA = 0.3$ $freq = 1\,Hz$: $A_{\Omega\ set}$ and $A_{\Omega\ meas}$

Both the performed verifications allowed to adequately plan the experimental campaign for the wear tests. Since the experimented tracking limitations and the fact that the bars to be machined can be considered homogeneous it was decide to partially randomize the tool life tests in order to limit the wasted material. Indeed, the cutting tests at high velocity v_c were carried out machining the external parts of the bars as far as the tracking limitations occurred and the remaining part of the workpieces were used for the low cutting speed tests.

Before executing the whole experimentation (tool life tests), all the cutting conditions (combining the cutting parameters as reported in Tables 1 and 2) were tested in order to verify the absence of any undesired vibrations (i.e. due to regenerative chatter) that can negatively affect

Table 3 Tracking performance verification - example ($RVA = 0.3$, $freq = 1\,Hz$)

Ω_{0set} (rpm)	RVA_{set}	$freq_{set}$ (Hz)	$A_{\Omega\ set}$ (rpm)	$A_{\Omega\ meas}$ (rpm)	$\Delta A_{\Omega}\%$	RVA_{meas} (rpm)	$\Delta RVA\%$
500	0.3	1	150	148.4	-1.06	0.2975	-0.85
700	0.3	1	225	225.2	+0.08	0.3001	+0.03
1000	0.3	1	300	304.6	+1.53	0.3047	+1.58
1200	0.3	1	360	320.5	-10.97	0.2707	-9.8
1500	0.3	1	450	271.6	-39.65	0.1892	-36.9

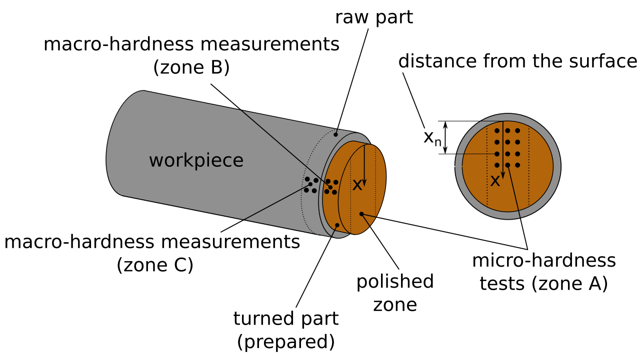

Fig. 3 Workpiece hardness analysis—effects of the position

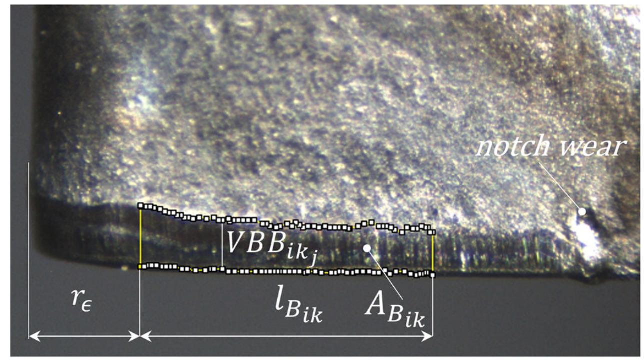

Fig. 5 Flank wear land analysis: average flank width computation VBB_{ik}

the life of the tool. For this purpose, the cutting forces were measured with the Kistler dynamometer and analyzed performing a FFT. No critical frequency components were found in the computed spectra for $a_p = 2\,mm$. The performed tests confirmed that the selected radial depth of cut is far from the stability limit.

As anticipated, preliminary wear tests were executed in order to verify the adequacy of the selected wear threshold $VBB_t = 0.15\,mm$. Repeated tests (wear tests carried out adopting the same cutting parameters) revealed that the wear rate is rather high when the flank shows an average wear land width close to 0.15 mm (see also Figs. 5 and 6). This allowed to proper discriminate the threshold overcoming and, as a consequence, to limit the dispersion of tool life TL data that typically occurs when the wear rate is low. The low dispersion of the tool life data TL can be even appreciated in Fig. 8 that describes the evolution of the flank wear for the repetitions of the tests executed with the center point conditions.

2.4 Tool wear analysis

For the ith tested insert, a set of n VBB measurements is available $\overline{VBB_i} = \{VBB_{i1}, VBB_{i2}, \cdots, VBB_{ik}, \cdots, VBB_{ip}, VBB_{i(p+1)}, \cdots, VBB_{in}\}$ where VBB_{ik} is the generic kth measurement of the average flank width (performed after the kth stop). According to ISO 3685 [37], the VBB_{ik} was computed using Eq. 2 where VBB_{ik_j} is the local measurement of the flank width (see Fig. 5). Integrating the VBB_{ik_j} over the length $l_{B_{ik}}$ of the analyzed region, the area $A_{B_{ik}}$ of the wear land and the VBB_{ik} can be computed. The flank width measurements VBB_{ik_j} were performed, as suggested by ISO 3685 [37], on the rectilinear portion of the insert avoiding the curvilinear part (length r_ϵ) and the portion of the wear land affected by the notch (see Fig. 5). The TL_i can be estimated linearly interpolating two subsequent cutting times (CT_{ip} and $CT_{i(p+1)}$) that were associated respectively at the pth and the $(p+1)$th stops. Moreover, VBB_{ip} and $VBB_{i(p+1)}$ fulfill the relationship reported in Eq. 4. It was verified that the linear assumption between the wear and the tool life assured a proper accuracy since several VBB measurements for each wear test were available and the $VBB_{i(p+1)}$ and

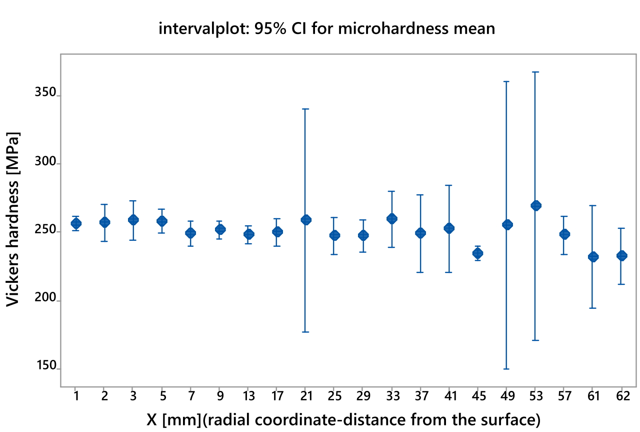

Fig. 4 Micro-hardness measurements

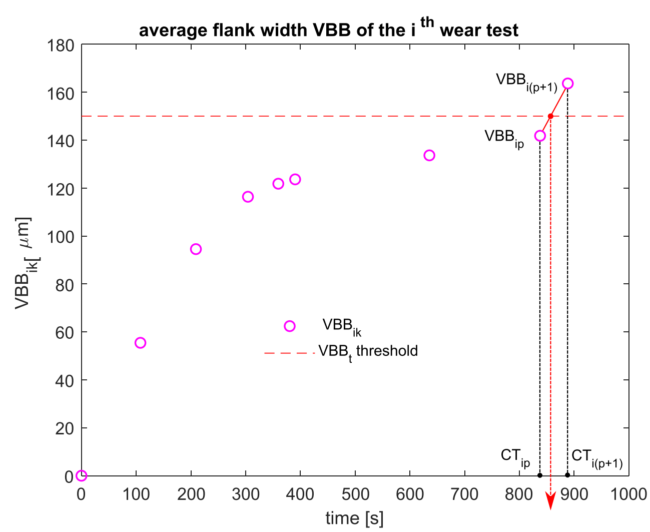

Fig. 6 Example of the evolution of the average flank wear evolution VBB_{ik} of the ith test

VBB_{ip} are rather closed. It was found that a higher order polynomial fitting of the $VBB(TL)$ curve brought to deviations in the estimated tool life TL_i less than 1%.

$$VBB_{ik} = A_{B_{ik}}/l_{B_{ik}} = 1/l_{B_{ik}} \cdot \int_0^{l_{B_{ik}}} VBB_{ik_j} dl \quad (2)$$

$$TL_i = CT_{ip} + \frac{CT_{i(p+1)} - CT_{ip}}{VBB_{i(p+1)} - VBB_{ip}} \cdot \left(VBB_t - VBB_{ip}\right) \quad (3)$$

$$VBB_{ip} < VBB_t < VBB_{i(p+1)} \quad (4)$$

2.5 Tool life modelling and validation

All the acquired data were statistically analyzed through the Analysis of Variance $ANOVA$. For what concerns the modelling approach, a multiple linear regression was adopted [40]. In the present research, the TL data or a transformation of them, were considered in the *response variables* vector $\boldsymbol{y} = [y_1, y_2, \cdots, y_n]$ while the main analyzed factors, a sub-combinations or a transformation of them (j, k, $\cdots$, r), were considered as the *regression variables* matrix, (i.e. $\boldsymbol{X} = [\mathbf{1}, \boldsymbol{x}_j, \boldsymbol{x}_k, \cdots, \boldsymbol{x}_r]$). $\boldsymbol{\beta} = [\beta_0, \beta_j, \beta_k, \cdots, \beta_r]$ is the vector of the *regression coefficients*. The matrix notation was reported in Eq. 5.

$$\boldsymbol{y} = \boldsymbol{X} \cdot \boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (5)$$

The estimation of the unknown $regression\ coefficients$ $\hat{\boldsymbol{\beta}}$ was carried out through the least squares minimization. Indeed, Eq. 6 brings to Eq. 7.

$$L = \boldsymbol{\epsilon}' \cdot \boldsymbol{\epsilon} = (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})' \cdot (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \quad (6)$$

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1} \boldsymbol{X}'\boldsymbol{y} \quad (7)$$

The developed models were also validated assessing the predicting capabilities on additional wear tests, executed for this specific purpose.

3 Results and discussion

In this section, the analysis of the performed wear tests was presented. The achieved tool life TL data were reported in Table 4. For sake of completeness, 30 total wear tests were carried out. 9 tests were executed at constant speed machining CSM: 4 tests considering the parameters $v_c - f$ combinations according to Table 1 and 5 central point tests. 21 tests were carried out in the sinusoidal spindle speed variation $SSSV$ regime. 16 test conditions were originated combining the $SSSV$ factors (v_c, f, RVA, $freq$, see Table 2) while 5 tests were performed considering intermediate parameter values. A graphical representation of the initial experimental scheme is reported in Fig. 10 (left side). As already explained in the previous paper section, at the beginning, the results of the cutting tests performed at CSM and adopting the $SSSV$ were separately analysed. The idea underpinning the conceived experimental campaigns was based on the suspicion that not all the parameters describing the speed modulating law really affect the life of the tool TL. If it is the case, the data coming from each plan could be grouped and can be analysed allowing the development of a general modelling. Before doing the analysis, a natural logarithmic transformation was performed on the following factors: TL (expressed in min), v_c, f, RVA, $freq$.

3.1 *CSM* modelling

The transformed tool life data $lnTL$, obtained in the constant speed machining CSM experimental session, were analyzed through the $ANOVA$ [44]. For sake of generality, according to Eq. 8, the Fisher's statistics $F - value_j$ and the associated $P - value_j$ were computed for each generic $factor\ j$ considered in the model. SS_j^{III} is the adjusted ($type\ III$) $sum\ of\ squares$ for the $factor\ j$, obtained removing any possible confounding due to the the remaining factors added to the model, Eq. 9. $SS_{(j,k,\ldots,r)}$ is the sequential $sum\ of\ squares$ ($type\ I$) of the model considering the $j, k, \ldots, r$ factors while $SS_{(k,\ldots,r)}$ is the $sum\ of\ squares$ ($type\ I$) considering a model with a subset of factors $(k, \ldots, r)$, thus excluding the one under investigation [44]. MS_j is the resultant $mean\ of\ squares$ ($MS_j = SS_j^{III}/DF_j$) while DF_j is the correspondent degrees of freedom. As expected, the main affecting factors were lnv_c and lnf while their interaction is not statistically relevant. The $ANOVA$ results considering the main factors were reported in Table 5. For sake of completeness, the analyses performed on the residuals were summarize in Fig. 7. In addition, the regression equation (Eq. 10) was even outlined through the $least\ square$ L_{CSM} minimization, Eq. 6. All the estimated regression coefficients ($\hat{\beta}_{j-CSM}, \cdots, \hat{\beta}_{r-CSM}$) and the corresponding 95% confidence intervals CI were reported in Table 6. As suggested in [44], the $\boldsymbol{CI}$ were computed estimating the $standard\ error$ $\boldsymbol{SE_{CSM}} = \sqrt{diag\left(Cov\left(\hat{\boldsymbol{\beta}}_{CSM}\right)\right)}$. The last two columns in Table 6 reported the $t - student$ $t - test$ statistic values and the corresponding $P - values$ that measure how the model parameters are usefulness in the proposed formulation. The coefficient of determination $R^2_{CSM} = 99.26\%$ and the adjusted statistic $R^2_{adj-CSM} = 96.92\%$. It is worth noting that the statistical test on $curvature$ (associated to the added center point) showed a

Table 4 Tool life test results - CSM and $SSSV$ factorial designs

Cutting edge number	Run order	Center point	Mode	$v_c\ (m/min)$	$f\ (mm/tooth)$	RVA	$freq$	$TL\ (s)$
15	1	0	$SSSV$	160	0.3	0.3	1.5	767.3
2	2	0	CSM	220	0.1	0	0	765.6
3	3	0	$SSSV$	160	0.3	0.1	0.5	1011.1
4	4	0	CSM	220	0.3	0	0	345.6
14	5	0	$SSSV$	220	0.1	0.3	1.5	574.6
5	6	1	CSM	190	0.2	0	0	798.2
8	7	0	$SSSV$	220	0.3	0.3	0.5	310.0
1	8	0	CSM	160	0.1	0	0	2736.5
13	9	0	$SSSV$	160	0.1	0.3	1.5	2471.9
23	10	0	CSM	160	0.3	0	0	1062.1
24	11	0	$SSSV$	220	0.3	0.1	0.5	337.8
17	12	1	$SSSV$	190	0.2	0.1	1	641.4
11	13	0	$SSSV$	160	0.3	0.1	1.5	896.8
6	14	0	$SSSV$	220	0.1	0.3	0.5	661.8
12	15	0	$SSSV$	220	0.3	0.1	1.5	348.1
16	16	0	$SSSV$	220	0.3	0.3	1.5	305.7
10	17	0	$SSSV$	220	0.1	0.1	1.5	816.1
25	18	0	$SSSV$	160	0.1	0.3	0.5	1830.9
21	19	0	$SSSV$	160	0.1	0.1	0.5	2331.5
07	20	0	$SSSV$	160	0.3	0.3	0.5	764.9
22	21	0	$SSSV$	220	0.1	0.1	0.5	856.0
09	22	0	$SSSV$	220	0.1	0.1	1.5	2891.3
20	23	1	CSM	190	0.2	0	0	856.9
18	24	1	$SSSV$	190	0.2	0.2	1	727.4
26	25	1	CSM	190	0.2	0	0	853.1
19	26	1	$SSSV$	190	0.2	0.2	1	791.0
30	27	1	CSM	190	0.2	0	0	804.4
29	28	1	$SSSV$	190	0.2	0.2	1	739.8
28	29	1	CSM	190	0.2	0	0	867.4
27	30	1	$SSSV$	190	0.2	0.2	1	723.4

quite high $P - value$, this means that the linearity in the factors effect cannot be confuted.

$$F - value_j = \frac{SS_j^{III}/DF_j}{SS_{error}/DF_{error}} \tag{8}$$

$$SS_j^{III} = SS_{(j|k,\ldots,r)} = SS_{(j,k,\ldots,r)} - SS_{(k,\ldots,r)} \tag{9}$$

$$y_{CSM} \equiv \ln TL_{CSM} = \hat{\beta}_{0-CSM} + \hat{\beta}_{\ln v_c-CSM} \cdot \ln v_c + \hat{\beta}_{\ln f-CSM} \cdot \ln f \tag{10}$$

3.2 *SSSV* modelling

The described methodology (Section 3.1) was adopted even for analysing the transformed $\ln TL$ data obtained from the experimental campaign performed in the sinusoidal spindle speed $SSSV$ regime. The $ANOVA$ results, considering the main factors ($\ln v_c$, $\ln f$, $\ln RVA$ and $\ln freq$) and the $2 - ways$ interactions, were reported in Table 7.

As can be observed, the relevant factors to be considered in the $SSSV$ formulation are respectively $\ln v_c$, $\ln f$, $\ln RVA$ while $\ln freq$ seems not affecting the life of the tool. The effect of $SSSV$ cutting can be also observed in Fig. 8 where the VBB_{ik} associated to the tests carried out in both the center points (Fig. 10, left side) were reported. Five repetitions for each cutting condition (CSM or $SSSV$) were performed. Although the results were affected by the process variability, it is quite evident that the tools adopted for the $SSSV$ cutting showed a reduced TL. The $ANOVA\ residuals$ analysis was reported in Fig. 9. Even in this case, the regression equation was obtained (Eq. 11) as well as the $R^2_{SSSV} = 97.93\%$ and the adjusted statistic $R^2_{adj-SSSV} = 97.69\%$. For sake of simplicity, it was

Table 5 CSM: $ANOVA$ results

Source	DF	SS_j^{III}	MS_j	$F-value$	$P-value$
Model	2	2.21858	1.10929	403.05	0.000
Linear	2	2.21858	1.10929	403.05	0.000
lnv_c	1	1.43110	1.43110	519.97	0.000
lnf	1	0.76357	0.76357	277.43	0.000
Error	6	0.01651	0.00275		
Curvature	1	0.00481	0.00481	2.06	0.211
$Lack\ of\ Fit$	1	0.00570	0.00570	3.80	0.123
$Pure\ Error$	4	0.00600	0.00150		
Total	8	2.23509			

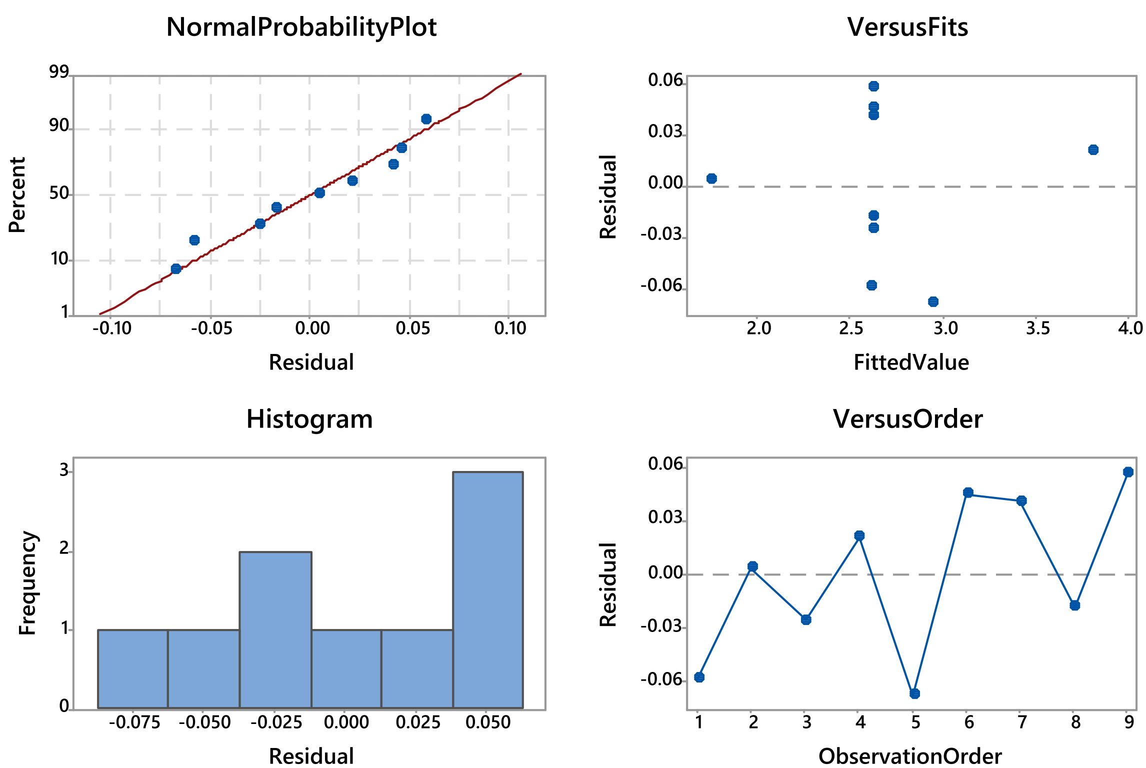

Fig. 7 CSM: residuals analysis

Table 6 CSM: regression results

Model term CSM	$\hat{\beta}_{CSM}$	SE_{CSM}	$\%95\ CI$	$t-value$	$P-value$
Constant	$\hat{\beta}_{0-CSM} = 21.035$	0.867	(18.914; 23.155)	24.27	0.000
lnv_c	$\hat{\beta}_{lnv_c-CSM} = -3.750$	0.164	$(-4.153; -3.348)$	-22.8	0.000
lnf	$\hat{\beta}_{lnf-CSM} = -0.7807$	0.0469	$(-0.8954; -0.6660)$	-16.66	0.000

Table 7 $SSSV$: $ANOVA$ results

Source	DF	SS_j^{III}	MS_j	$F - value$	$P - value$
Model	11	8.22306	0.74755	77.97	0.000
Linear	4	8.06582	2.01646	210.31	0.000
$\ln v_c$	1	4.63283	4.63283	483.19	0.000
$\ln f$	1	3.25812	3.25812	339.81	0.000
$\ln RVA$	1	0.17168	0.17168	17.91	0.002
$\ln freq$	1	0.00319	0.00319	0.33	0.578
$2\ way\ inter.$	6	0.08812	0.01469	1.15	0.271
$\ln v_c \cdot \ln f$	1	0.04813	0.04813	5.02	0.052
$\ln v_c \cdot \ln RVA$	1	0.00001	0.00001	0.00	0.982
$\ln v_c \cdot \ln freq$	1	0.02038	0.02038	2.13	0.179
$\ln f \cdot \ln RVA$	1	0.00792	0.00792	0.83	0.387
$\ln f \cdot \ln freq$	1	0.01137	0.01137	1.19	0.304
$\ln RVA \cdot \ln freq$	1	0.00031	0.00031	0.03	0.861
Curvature	1	0.00873	0.00873	0.91	0.365
Error	9	0.08629	0.00959		
$Lack\ of\ Fit$	5	0.06329	0.01266	2.20	0.232
$Pure\ Error$	4	0.02300	0.00575		
Total	20	8.30936			

decided to consider in the model formulation the amplitude modulation RVA instead of its logarithmic transformation. Even in this case the linearity in the factors effect cannot be confuted (Table 7). Similarly to CSM modelling, the relevance of the model parameter was reported in Table 8.

$$\begin{aligned} y_{SSSV} &\equiv \ln TL_{SSSV} \\ &= \hat{\beta}_{0-SSSV} + \hat{\beta}_{\ln v_c - SSSV} \cdot \ln v_c \\ &\quad + \hat{\beta}_{\ln f - SSSV} \cdot \ln f + \hat{\beta}_{RVA-SSSV} \cdot RVA \end{aligned} \tag{11}$$

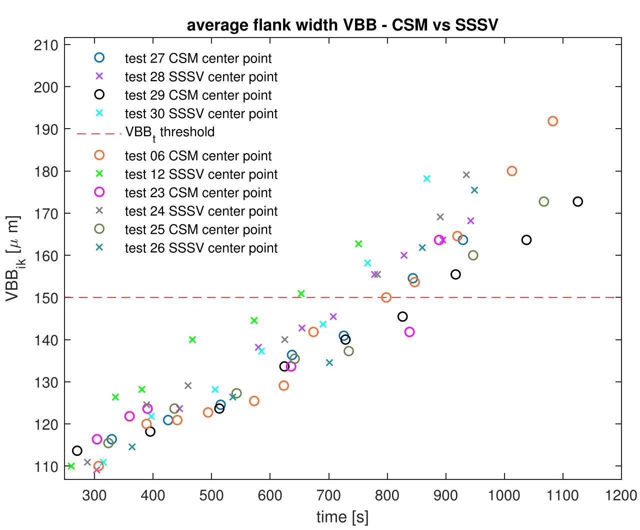

Fig. 8 VBB_{ik} evolution: effect of $SSSV$ on TL in the intermediate cutting conditions

3.3 General model development

Since the frequency of the speed modulation $freq$ resulted not to be a significant factor, therefore the $unreplicated$ design in four factors conceived for performing the $SSSV$ campaign can be considered a two times replicated plan with three factors: $\ln v_c$, $\ln f$ and RVA (Fig. 10). According to the CSM and $SSSV$ models, the velocity and feed were found to be significant factors in both the cases, while for the $SSSV$ case even RVA resulted to be significant. Since the experimentation performed at CSM can be considered a specific realization of the $SSSV$ formulation (setting $RVA = 0$), it is interesting to develop a general model considering the whole tool life data-set. For sake of clarity, the resultant graphical representation was reported in Fig. 10. Moreover, the test conditions used for the model formulations validation (see Section 3.4) were put into evidence in Fig. 10, right side.

Before doing that, the test of equality of means was performed on the coefficients present in by both CSM (Section 3.1) and $SSSV$ (Section 3.2) formulations: $H_{0-constant}$: $\hat{\beta}_{0-CSM} = \hat{\beta}_{0-SSSV}$, $H_{0-\ln v_c}$: $\hat{\beta}_{lnv_c-CSM} = \hat{\beta}_{\ln v_c-SSSV}$, $H_{0-\ln f}$: $\hat{\beta}_{\ln f-CSM} = \hat{\beta}_{\ln f-SSSV}$. All the tests confirmed that the two proposed formulations shared the fitted parameters (the generic null hypotheses H_{0k} cannot be refused) and this confirmed the adequacy of a $general$ model for interpreting all the experimental data. For sake of completeness, the $ANOVA$ results and the residual analysis were respectively reported in Table 9 and Fig. 11.

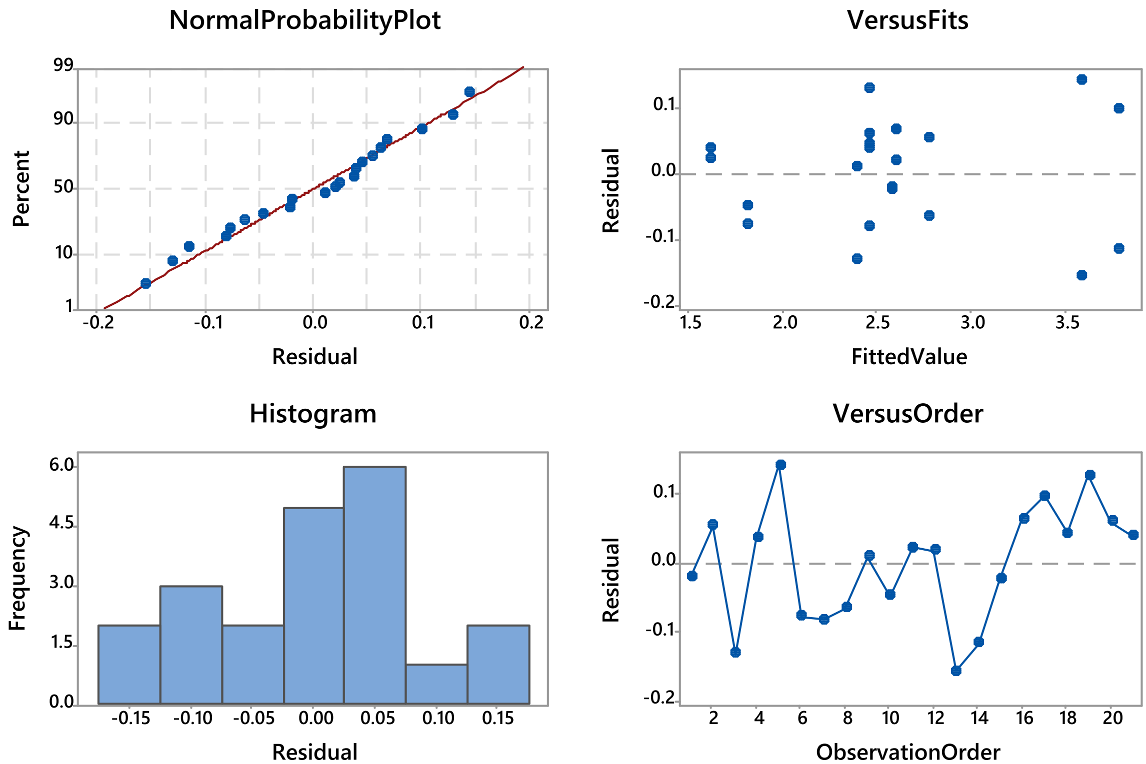

Fig. 9 $SSSV$ formulation: residuals analysis

The general model equation and the corresponding identified terms were reported respectively in Eq. 13 and Table 10.

$$y_{gen} \equiv \ln TL_{gen} \quad (12)$$
$$= \hat{\beta}_{0-gen} + \hat{\beta}_{\ln v_c-gen} \cdot \ln v_c + \hat{\beta}_{\ln f-gen} \cdot \ln f + \hat{\beta}_{RVA-gen} \cdot RVA \quad (13)$$

3.4 Model validation and discussion

In order to validate the model, additional experimental tests were carried out. More specifically, new cutting conditions (both in CSM and $SSSV$) were tested (Fig. 10). The adopted parameters and the new obtained tool life TL data were reported in Table 11.

In Table 12, the percentage errors $TL_{s-error_{\%}}$ (Eq. 14) obtained adopting the sth formulation for the tool life estimation $\hat{TL}_s$ were reported. For sake of clarity, as an example, $TL_{gen-error_{\%}}$ is the percentage estimation error that results if the tool life is estimated through the *general* model formulation $\hat{TL}_{gen}$ (Section 3.3). Other percentage errors can be analogously computed. It can be observed that the developed general model shows limited errors $TL_{gen-error_{\%}}$ (less than 6%) and a lower standard error SE_{k-gen} for all the shared $\hat{\beta}_k$ with respect to the other formulations maybe because it was developed exploiting the full data set. $SSSV$ formulation shows similar performances in terms of estimating errors (less than 5%) while CSM modelling exhibits a worse performance (although the error is limited to 10%) if compared to the other modelling approaches. The estimation errors would increased (ranging from -9.4 to -16%, see the values reported among brackets in Table 12 in the column $TL_{CSM-error_{\%}}$) if the

Table 8 $SSSV$: regression results

Model Term $SSSV$	$\hat{\beta}_{SSSV}$	SE_{SSSV}	$\%95\ CI$	$t-value$	$P-value$
Constant	$\hat{\beta}_{0-SSSV} = 19.075$	0.852	(17.278; 20.873)	22.39	0.000
$\ln v_c$	$\hat{\beta}_{\ln v_c-SSSV} = -3.376$	0.162	$(-3.717; -3.036)$	-20.9	0.000
$\ln f$	$\hat{\beta}_{\ln f-SSSV} = -0.8186$	0.0465	$(-0.8882; -0.7205)$	-17.61	0.000
RVA	$\hat{\beta}_{RVA-SSSV} = -1.036$	0.257	$(-1.136; -0.493)$	-4.02	0.001

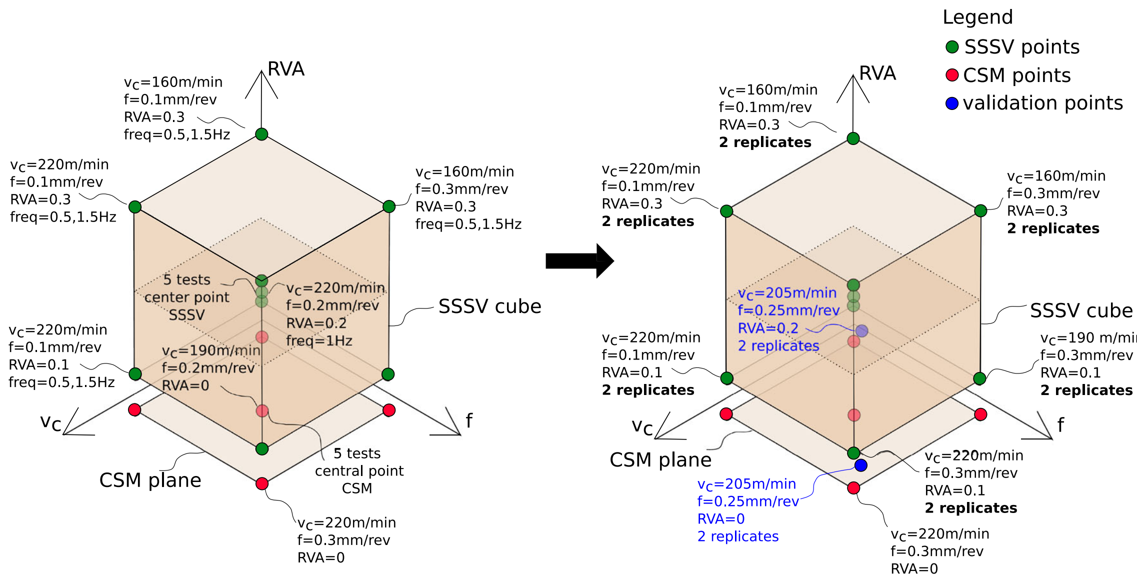

Fig. 10 General model scheme: initial experimental scheme (left) - resulted test representation and validation test conditions (right)

formulation developed for CSM was used for predicting the tool duration performed with the sinusoidal spindle speed variation $SSSV$. As expected, the CSM model overestimates the tool life TL since it does not consider the detrimental effect of the $SSSV$. On the contrary, the $SSSV$ formulation would behave well (error less than 5%) if it was used for estimating the TL of cutting tests performed at CSM. Indeed, although the $SSSV$ formulation was developed just considering turning tests performed with $RVA \neq 0$ the model can be used for extrapolation (CSM). For sake of completeness, the values reported among brackets in Table 12 refer to a not proper exploitation of the developed model: for instance, the CSM model for predicting the tool duration in $SSSV$ cutting or vice-versa. Moreover, referring to the specific literature, the maximum observed estimation error adopting the developed $general$ formulation is in accordance with other advanced models suitable only for CSM [32]. This further confirms the adequacy of the proposed modelling approach. Since the limited prediction errors observed for the validation points and the statistical tests on $curvature$, the linear dependence of the tool life TL on the considered factors can be further confirmed. To the authors' knowledge, no one developed specific formulation for modelling the effect of $SSSV$ on the TL. Previous works of the same research group focused mainly on the phenomenological aspects and not on the quantification of the detrimental effects of $SSSV$ when the cutting is stable.

$$TL_{s-error_{\%}} = 100 \cdot \frac{TL - \hat{TL}_s}{\hat{TL}_s} \tag{14}$$

Although the limitations of empirical approaches were underlined by Wong et al. [31], the authors believe that the proposed $general$ formulations, in terms of the structure of the model, being developed from Taylor's basic model, is suitable for considering the effects of $SSSV$ on different materials, especially for other steels. The study carried by Johansson et al. [32] supports the previous considerations

Table 9 $general$: $ANOVA$ results

Source	DF	SS_j^{III}	MS_j	$F-value$	$P-value$
Model	3	10.3793	3.45977	410.21	0.000
Linear	3	10.3793	3.45977	410.21	0.000
ln v_c	1	6.0553	6.05527	717.95	0.000
ln f	1	4.0659	4.06592	482.08	0.000
ln RVA	1	0.2913	0.29135	34.54	0.000
Error	26	0.2913	0.00843		
Curvature	1	0.0037	0.00374	0.43	0.516
$Lack\ of\ Fit$	9	0.0995	0.01105	1.52	0.221
$Pure\ Error$	16	0.1161	0.00725		
Total	29	10.5986			

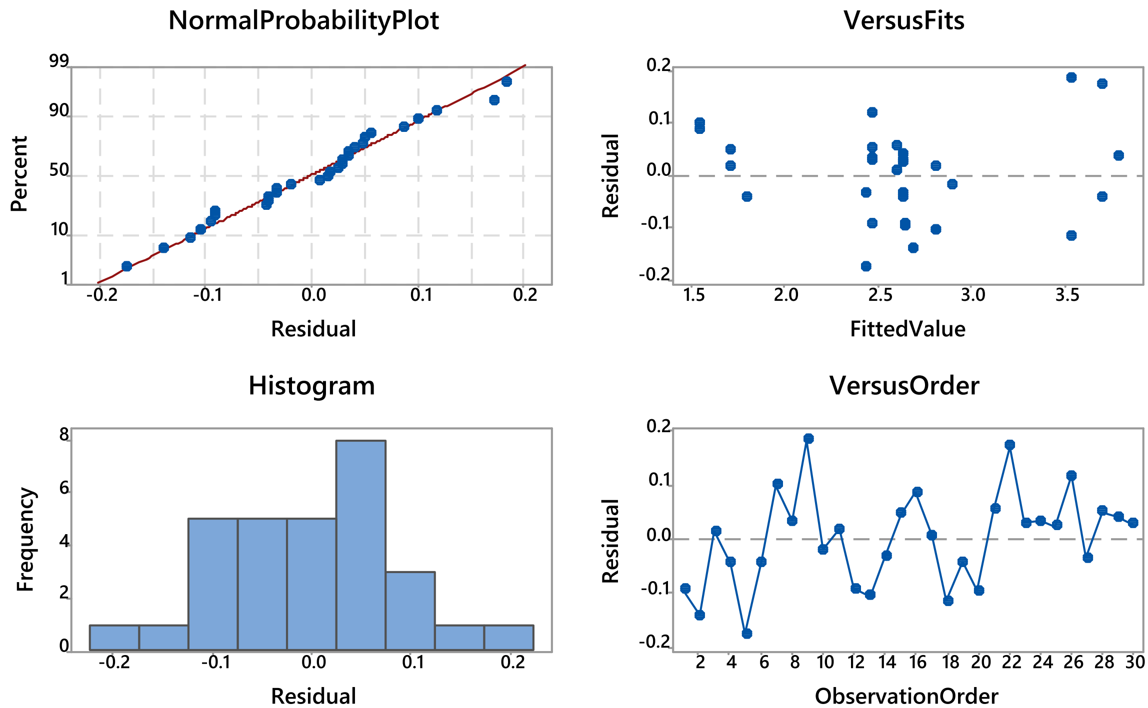

Fig. 11 *General* residuals analysis

Table 10 General model: regression results

Model Term *gen*	$\hat{\beta}_{gen}$	SE_{gen}	$\%95\ CI$	$t-value$	$P-value$
Constant	$\hat{\beta}_{0-gen} = 19.438$	0.679	(18.042; 20.834)	28.63	0.000
$\ln v_c$	$\hat{\beta}_{\ln v_c-gen} = -3.452$	0.129	$(-3.717; -3.188)$	-26.79	0.000
$\ln f$	$\hat{\beta}_{\ln f-gen} = -0.8121$	0.0370	$(-0.8882; -0.7361)$	-21.96	0.000
RVA	$\hat{\beta}_{RVA-gen} = -0.842$	0.143	$(-1.136; -0.547)$	-5.88	0.000

Table 11 Tool life: tested conditions for the model validation

Cutter	Test	Mode	$v_c\ [m/min]$	$f_z\ [mm/tooth]$	RVA	$freq$	$TL\ [s]$
33	31	$SSSV$	205	0.25	0.2	1	435.1
34	32	CSM	205	0.25	0	0	559.2
31	33	CSM	205	0.25	0	0	566.7
32	34	$SSSV$	205	0.25	0.2	1	469.1

Table 12 predicting modelling errors

Cutter	Test	Mode	$\hat{TL}_{CSM}\ [s]$	$TL_{CSM-error\%}$	$\hat{TL}_{SSSV}\ [s]$	$TL_{SSSV-error\%}$	$\hat{TL}_{gen}\ [s]$	$TL_{gen-error\%}$
33	31	$SSSV$	(518.2)	(-16)	457.8	-4.96	452.5	-3.84
34	32	CSM	518.2	7.92	(563.23)	(-0.71)	535.5	4.44
31	33	CSM	518.2	9.37	(563.23)	(0.62)	535.5	5.84
32	34	$SSSV$	(518.2)	(-9.4)	457.8	2.46	452.5	3.67

since they found that Taylor's model works fine within a quite broad range of materials (steels, cast irons and stainless steels). Previous research works that focused on the phenomenological aspects that make the TL of $SSSV$ cutting shorter than CSM can help understanding the potentialities of the developed formulation. Albertelli et al. [42] found that during $SSSV$ turning a crack on the insert fosters the coating delamination and therefore a faster wear of the tool. This was observed considering the same insert-working material of the present study. Chiappini et al. [25] studied the mechanics of chip formation in $SSSV$ turning of Ti6Al4V (heat resistant alloy HRA) and adopting a different insert geometry. It was found that, in view of the fact that the modulation of the cutting speed origins a fluctuation of the maximum cutting temperature and a peak of pressure on the inserts, thermal gradients and a subsequent thermal fatigue can affect the tool. According to other research works (i.e. Evans and Hutchinson [45]) that found that thermal gradients and thermal fatigue are the main relevant causes of coating delamination, it can be concluded that this was the cause of the observed detrimental effect of $SSSV$ on the wear of the tool. Since the thermal fatigue is mainly associated to the speed modulation [25], it can be stated that an analogous TL reduction can be expected regardless the processed material. These considerations bring to conclude that the proposed formulation could be used for a wide range of materials especially if coated inserts are used. Moreover, since spindle speed variation is typically used for suppressing unstable cutting, its application is particularly suitable for roughing operations in which high depths of cut are involved.

4 Conclusions

A generalized model for the prediction of the tool duration in steel turning when the sinusoidal spindle speed $SSSV$ is used was developed and presented. The proposed formulation is even suitable for modelling CSM constant speed machining. The model was outlined exploiting tool wear tests performed in different cutting conditions. Since the conceived tool life model takes into account the detrimental effects of spindle speed on tool life TL, it could be used to widely analyse the economic feasibility of the technique in different cutting scenarios and applications. The following results can therefore be summarised:

- The statistical analysis performed on the experimental data confirmed that the continuous modulation of the spindle speed, while keeping the feed velocity constant, negatively affects the achievable useful life of the tool.
- From the experimental session carried out with the sinusoidal spindle speed modulation it was found that the modulating frequency $freq$ does not affect the tool duration.
- A specific tool life model formulation for $SSSV$ was developed exploiting the experimental test results. The normalized amplitude of the spindle modulation RVA, together with the other affecting parameters (cutting speed v_c and feed f), was included in the model.
- According to the previous reported findings, it was possible to develop a generalized tool life model formulation that can be indifferently used both in regular cutting regime CSM and when the $SSSV$ is adopted.
- The generalized formulation was validated performing additional tool life tests. It was found that it capable of predicting the useful tool life within a maximum estimating error of 6%

Future research efforts will be surely focused on a broader validation and on a generalization of the developed model. Although there are some hints that bring to consider the proposed *general* formulation robust to changes in workpiece material and tool geometry, a proper validation would be extremely valuable. Moreover, even the model generalization considering the effect of the depth of cut could be developed.

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Author contribution Paolo Albertelli conceived the research, performed the tests, made the analysis, wrote the paper. Valerio Mussi: performed the tests; Michele Monno carried out the proof-reading.

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Availability of data and materials Data are reported in the paper

Declarations

Ethics approval This research follows ethical standards

Consent for publication The authors consent for publication.

Consent for participate The authors agree.

Conflict of interest The authors declare no competing interests.

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