

# An Optimized Polarization Model for Anode-Supported Solid Oxide Fuel Cells

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## Abstract

A polarization model of an anode-supported Solid Oxide Fuel Cells (SOFCs) has been developed considering activation overpotential (Butler-Volmer equation), ohmic resistance, and concentration overpotential. In this work, the assembly of Ni-YSZ / YSZ / LSM-YSZ as an anode, electrolyte, and cathode, respectively, was considered. An optimization process was also performed in order to verify the accuracy of the polarization model. Three optimum outputs obtained were cell voltage of 0.57 V, current density of 5620 A/m<sup>2</sup>, and power density of 3205 W/m<sup>2</sup>. The model has been tested against another modeling approach and experimental data. It was found that the results of our model compare well with the outcomes of another model. The model validation using literature experimental data was also satisfactory and acceptable. Finally, a parametric analysis has been carried out to highlight a number of parameters, which are most relevant in the overall cell performance as measured by polarization curve. In particular, the following parameters were accounted for in this analytical study: exchange current densities in anode and cathode ( $j_{0a}$  and  $j_{0c}$ ), leakage current density ( $j_{Leak}$ ), and temperature (T).

**Keywords:** SOFCs, Polarization Modeling, Optimization Process, Model Validation, Parametric Analysis.

## 1. Introduction

Fuel cell is an emerging technology that is, to a great extent, promising for the achievement of higher energy efficiency and lower environmental load. In order to meet the requirements of the Kyoto Protocol, fuel cell was considered as an emerging technology [1-2]. SOFCs is one of the high-temperature fuel cells, which convert chemical energy of a fuel to electrical power through electrochemical reactions at high temperatures. These cells have high efficiency, fuel diversity, and lower air pollution potentials. In addition, they can be installed in combined cycle

plants to increase the efficiency level and lower the environmental pollution [3-5]. As result of its operation at high temperatures, SOFCs is often used in power plants and in heavy machineries [6-8].

Considerable research studies have been done on SOFCs in the last two decades [9-12]. Research in this field, including computational modeling, expanded the benefits and reduced the disadvantages, playing an important role in the affirmation of this technology as a viable energy alternative. In general, simulation and modeling are techniques being used since not long ago; however, their application has made a great contribution to scientific and industrial progresses, solving many un-known problems for mankind. Hence, mathematical problem solving techniques have been developed to bring the constructed models to an optimal condition. A summary of the most important findings is found below.

Liu et al. [13] studied anode-supported SOFCs by using Star-CD software, in order to measure the performance analysis parameters, including current density, fuel density, and rate of fuel consumption. Results from this work showed that power density is increased as the fuel consumption is decreased. In addition, the performance of fuel cell was shown to be improved by increasing the current discharge rate. Yakab et al. [14] studied some properties of SOFCs including fuel density, distribution of temperature, and density by using simple electrochemical models within specific mode. Chan et al. [15] offered a 1D model of SOFCs and observed only two types of electrodes for determining diffusion by using Fick's law. Within this model, the assumed temperature, the exchange current density, and the reactants density were fixed. It is important to highlight that the two last parameters are strongly dependent on temperature. Petruzzi et al. [16] investigated a thermo-electric model of SOFCs; the main disadvantages of their study was neglecting the concentration loss and applying the Tafel equation for modeling the activation loss. Autisser et al. [17] calculated the rate of reactions, the distribution of current density, gas current, temperature of the cell, and fuel density by developing infrastructures of fluent software. Zhu et al. [18] established a parametric model for anode-supported SOFCs to explain various polarizations, cell voltage, and power density as a function of current density based on measurements made on cell materials and components. In the research done by Shen et al. [19], a polarization model considering activation and ohmic polarization for SOFCs with a bi-layer electrolyte is presented. Also, the energy conservation equation is used to integrate the electronic conductivity with the local oxygen partial pressure. Based on their outputs, the Open Circuit Voltage (OCV) improves by increasing the cathode exchange current density. In addition, the leakage current density under OCV was found to be unrelated to the cathode exchange current density.

In order to improve the performance of the electrochemical cell by lowering the concentration polarization, Shimada et al. [20] developed a porous microstructure control technique to achieve high power density in Planer SOFCs. Designing a high porosity Ni-YSZ anode increased the power density from 1.95 to 3.09 Wcm<sup>-2</sup> at 800 °C using humidified H<sub>2</sub> fuel. As a key result of this study, porosity and pore size distribution in the electrodes were found to be essential to obtain a high performance electrochemical cell. The anode-cathode supported SOFCs (ACSC) was designed by Su et al. [21] to decrease the concentration and ohmic polarization that exist in the typical design of SOFCs. By using mathematical modeling, the results showed that the ACSC design improved the oxygen concentration distribution and the electric potential distribution in comparison with the anode-supported SOFCs (ASC).

As a summary, different research studies have already been performed using modeling tools to analyze both the polarization losses and their minimization process. Considering the

investigation of the previous studies and the gaps currently existent in the literature, one of the essential goals of this research is to identify a number of parameters, which are most relevant in the overall cell performance by carrying out a parametric analysis. For this purpose, a polarization model of an anode-supported SOFCs has been presented considering total losses including: activation, ohmic, and concentration overpotential. Then, an optimization process was performed to verify the accuracy of the polarization model. The model has been successfully tested against another modeling approach and experimental data. The following parameters were considered for the mentioned parametric study: exchange current densities in anode and cathode ( $j_{0a}$  and  $j_{0c}$ ), leakage current density ( $j_{Leak}$ ), and temperature of the cell ( $T$ ).

## 2. Development of the polarization model

SOFCs is characterized by a solid oxide electrolyte that transports oxygen anions from the cathode to the anode: the most used material is the Ytria-Stabilized Zirconia (YSZ). The electrodes for SOFCs are made with solid oxides: the anode, where the electrochemical oxidation of hydrogen occurs, is a cermet made out of nickel mixed with YSZ (Ni-YSZ), while the cathode, where oxygen reduction takes place, is a composite of Lanthanum Strontium Manganite (LSM) and YSZ (LSM-YSZ). These materials were chosen to be used in our model. SOFCs could have different geometries, but laboratory scale experiments usually adopt anode-supported button cell geometry, which is the one chosen for our research study. This geometry was selected because the anode is the thickest and strongest layer placed on the bottom of the cell, providing mechanical support to the cell [22-25].

The polarization model uses the Butler-Volmer equation, the Ohm's law, and the Fick's law for description of the activation, ohmic, and concentration over-potentials, respectively. Taking into account all the over-potentials effecting SOFCs, the cell effective potential is given by:

$$V_{cell} = E_{cell} - (\eta_{act} + \eta_{ohm} + \eta_{conc} + V_{Leak}) \quad (1)$$

Where  $E_{cell}$  (V) is the equilibrium voltage of the cell,  $\eta_{act}$  (V) is the activation over-potential,  $\eta_{ohm}$  (V) is the ohmic over-potential,  $\eta_{conc}$  (V) is the concentration over-potential, and  $V_{Leak}$  represents the voltage loss related to leakage.

There are several types of mathematical problem-solving techniques among which general linear and non-linear techniques can be mentioned. Usually, technical engineering problems are dynamic and non-linear and their solution is done by non-linear techniques. Eq. (2) shows the general form of a mathematical model [26]:

$$\begin{cases} \text{Min } F(x) \text{ OR } \text{Max } F(x) \text{ [Objective Function]} \\ \text{Sub. } g(x) \geq 0 \text{ OR } \leq 0 \text{ [Cons.]} \\ X \geq 0 \end{cases} \quad (2)$$

In terms of the software used for this research, there are many software related to mathematical problem-solving methods among which Matlab is very well known and reliable. Therefore, Matlab was used for this research.

All operating parameters used in the polarization model can be found in Table. 1.

**Table 1. Operating parameters used in the polarization model [27-31].**

Parameters	Value
Operating pressure, $P$ (atm)	1
Inlet temperature, $T$ (K)	1073
Number of transfer electrons in anode and cathode, $n$	2,4
Charge transfer coefficient range, $a$	0.5
Anode thicknesses $\delta_a$ ( $\mu m$ )	500,500
Cathode thicknesses $\delta_c$ ( $\mu m$ )	50,30
Electrolyte thicknesses $\delta_e$ ( $\mu m$ )	50,25
Current density ranges, $j$ ( $A/m^2$ )	0-8000,0-20000
Limiting Current densities, $j_L$ ( $A/m^2$ )	7500,19500
Anode exchange current density, $j_{0a}$ ( $A/m^2$ )	5300
Cathode exchange current density, $j_{0c}$ ( $A/m^2$ )	2000
Leakage current density ranges, $j_{Leak}$ ( $A/m^2$ )	20-100,0-20

## 2.1. Leakage current density

The leakage current density ( $j_{Leak}$ ) influences the activation and concentration losses, but does not affect the ohm loss. The ohm loss is a function of the cell operating current density.

$$\eta_{act}, \eta_{conc} = f(j_{gross}), \quad j_{gross} = j + j_{Leak} \quad (3)$$

Therefore:

$$\eta_{act}, \eta_{conc} = f(j_{Leak}), \quad \eta_{ohm} = f(j) \quad (4)$$

Where  $j$  is the operating current density and  $j_{gross}$  is the sum of leakage current density and operating current density. Generally, the leakage current density is related to the electronic leakage through the electrolyte and the gas leakage due to the unsealed cell configuration [32]. Leakage current density decreases the OCV of the cell and causes it to be lower than its thermodynamic calculation's value [27].

## 2.2. Activation over-potential

The appearance of activation loss is much more frequent in low- and moderate-temperature fuel cells than in high-temperature cells. In addition, its value is higher in the cathode than in the anode. Instead of using simplified models such as the Tafel equation or the linear current-voltage relationship, which could be applied under different operating conditions, the general Butler-Volmer equation [2, 28] was chosen to calculate the corresponding anodic and cathodic voltage losses. The Butler-Volmer equation utilized in this research is as follows:

$$j = j_0 \left[ \exp(a * n * F * \eta_{act} / RT) - \exp((1 - a) * n * F * \eta_{act} / RT) \right] \quad (5)$$

$$\eta_{act,i} = \frac{RT}{F} \sinh^{-1} \left( \frac{j}{2 * j_{0,i}} \right), \quad i = a, c \quad (6)$$

These formulas would ultimately provide solutions for  $j$  and  $\eta_{act}$ .  $j_0$  ( $A/m^2$ ) stands for exchange current density,  $a$  is charge transfer coefficient,  $n$  is the number of electrons in transfer,  $F$  is the Faraday constant (C/mol),  $R$  is the universal gas constant (J/mol-K),  $T$  (K) is the temperature

of the cell, and  $i$  stands for anode and cathode. The  $j_0$  is the most effective factor when losses occur.

The higher  $j_0$ , the more active an electrode would be. Since  $a$  is responsible for a partial change in the voltage loss and causes the reaction rate constant to change, it is commonly given a value of 0.5 [28]. Hence, the activation losses are defined for this model as:

$$\eta_{act,a} = \frac{RT}{F} * \ln \left[ \frac{j}{2 * j_{0,a}} + \sqrt{\left(\frac{j}{2 * j_{0,a}}\right)^2 + 1} \right] \quad (7)$$

$$\eta_{act,c} = \frac{RT}{F} * \ln \left[ \frac{j}{2 * j_{0,c}} + \sqrt{\left(\frac{j}{2 * j_{0,c}}\right)^2 + 1} \right] \quad (8)$$

$j_0$  is a function of temperature and is known to be related to the charge transfer resistance ( $r_{ct}$ )  $\Omega \text{ m}^2$  as follows [27],

$$r_{ct} = RT / nFj_0 \quad (9)$$

$$j_0 = RT / nFr_{ct} \quad (10)$$

Reducing the activation loss is very important and there are some methods to do it; for example, increasing the operating temperature of fuel cell results in the increase of  $j_0$ ; increasing the electrode surface, which increases the effective level of nominal electrode surface (length and width), increases  $j_0$ ; as well as, increasing the density of reactants, and finally increasing pressure [27].

Activation over-potential exists at both anode and cathode. Nevertheless, whereas the  $j_0$  at this fuel cell is very high, the activation loss is not that considerable. Because of greater exchange current density in the anode than in the cathode, this kind of over-potential is much higher at the cathode. Figures 1a and 1b show the results of the proposed model for activation loss, considering leakage current density of 20 and 100  $\text{A/m}^2$  in both anode and cathode, respectively.

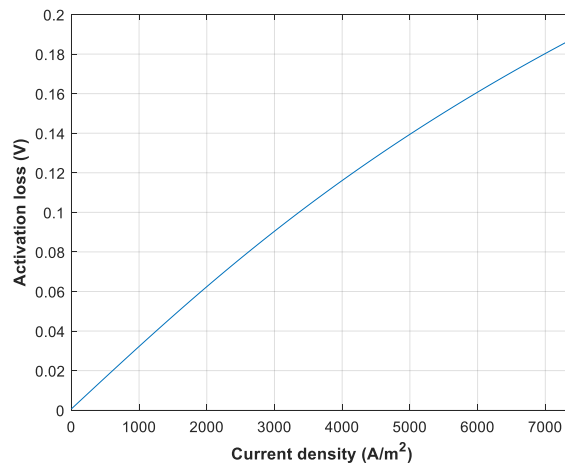


Fig. 1a: Activation loss as a function of current density considering a leakage current density of 20  $\text{A/m}^2$ .

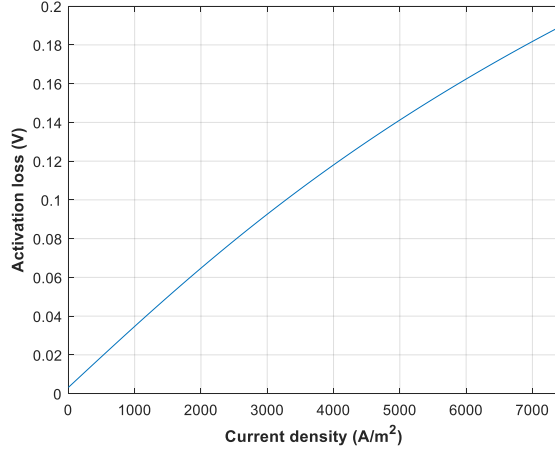


Fig. 1b: Activation loss as a function of current density considering leakage current density of 100 A/m².

### 2.3. Ohmic over-potential

Ohmic loss in which electrolyte and the electrodes of the fuel cell are involved, takes place as result from the resistance to the flow of ions in ionic conductors and the resistance to the flow of electrons in electronic conductors. Since these resistances follow the Ohm's law, the total ohmic voltage loss ( $\eta_{ohm}$ ) would be defined as [29, 32],

$$\eta_{ohm} = R_{ohm} j \quad (11)$$

$$\eta_{ohm} = R_{ohm} j = (\rho_a \delta_a + \rho_c \delta_c + \rho_e \delta_e) j \quad (12)$$

Where  $R_{ohm}$  is the ohmic resistance ( $\Omega$ ),  $\rho$  is resistivity of the medium ( $\Omega \text{ m}$ ), and  $\delta$  is the thickness of the medium (m);  $\rho_a$ ,  $\rho_c$ , and  $\rho_e$  are the electronic and ionic resistivity of the SOFCs components, respectively.  $\delta_a$ ,  $\delta_c$ , and  $\delta_e$  are the thicknesses of anode, cathode, and electrolyte, respectively. The electronic resistivity in the anode and the cathode as well the ionic resistivity in the electrolyte can be calculated by the following equations, valid for the materials considered above for the 3 layers [29],

$$\rho_a = 0.00008114 \exp\left(\frac{600}{T}\right) \quad (13)$$

$$\rho_c = 0.0000298 \exp\left(\frac{-1392}{T}\right) \quad (14)$$

$$\rho_e = 0.0000294 \exp\left(\frac{10350}{T}\right) \quad (15)$$

Decreasing the ohm loss is very important and there are different methods to reduce it; for example: utilizing electrodes and electrolytes with highest electronic and ionic conductance, suitable design for connection of fuel cells, and reduction of electrolyte thickness. This latter method could be problematic in some cases, since sometimes the electrolyte is regarded as the mechanical support for the electrode. Therefore, reducing its thickness decreases the mechanical strength of the cell, which will affect the circulation of current and also be an obstacle for the connection of electrodes [27].

The resistance of the fuel cell components is determined by their material specification and thicknesses. Resistance of anode and electrolyte is reduced by increasing temperature, but resistance at cathode tends to have an inverse effect. However, the resistance of both electrodes

in comparison with the electrolyte is negligible. Figure 2 illustrates the total ohm loss in the cell as a function of current density.

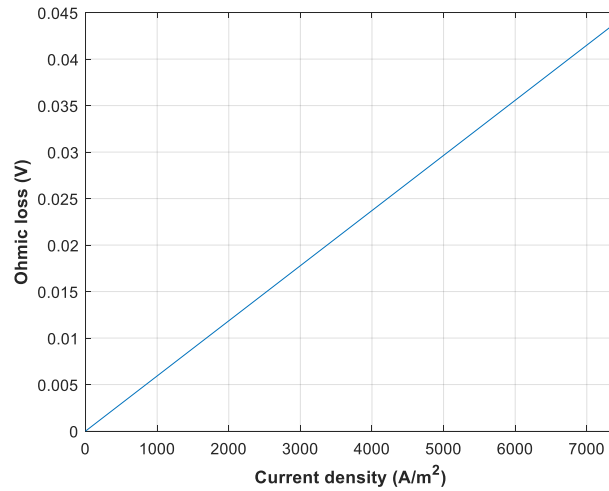


Fig. 2: Ohmic loss as a function of current density.

## 2.4. Concentration over-potential

Concentration loss represents the effects of pressure and concentration on voltage and is considerable when we have a converting system for the fuel or use air instead of oxygen. The presence of nitrogen in the cathode of the fuel cell represents a reduction in oxygen concentration and can result in issues that can increase the concentration over-potential. Under this scenario, when the cell is working, the concentration of the oxygen existing alongside the electrode would be reduced as time goes by. This oxygen concentration reduction depends on such factors as the amount of current taken from the cell, the airflow around the cathode, and the localized oxygen substitution rate. In the other side of the fuel cell, if the hydrogen is injected to the anode as time goes by, the hydrogen pressure would decrease as the hydrogen is consumed, which depends on the amount of the current taken from the cell and parameters related to the hydrogen fuel feeding system [27].

A common equation for total  $\eta_{conc}$ , which has been previously formulated in the literature [27, 29, 33-34] has been used for this study and is shown below,

$$\eta_{conc} = -\frac{RT}{nF} * \ln\left(1 - \frac{j}{j_L}\right) \quad (16)$$

Where  $j_L$  is the limiting current density. The concentration loss is considerable, especially when the cell works at high current density and high utilization factor ranges. If the partial pressure of gases could not be tolerated by the cell, then the total voltage loss will be increased by concentration loss and consequently the cell operation will be stopped.

Ohmic and cathode activation losses are the most important over-potentials within normal operating range of SOFCs, but voltage loss, because of the cathode concentration over-potential, can also be considerable when SOFCs works near to the limiting current density. The cathode has a higher concentration loss in comparison with the anode. Whereas the exchange current density of the cathode directly influences the rate of electrochemical reaction in the cathode, the low rate of the electrochemical reaction in the cathode could cause a high cathode concentration loss at the cell [27]. Figures 3a and 3b demonstrate the results of the proposed

model for concentration loss, considering leakage current density of 20 and 100 A/m<sup>2</sup> in both anode and cathode, respectively.

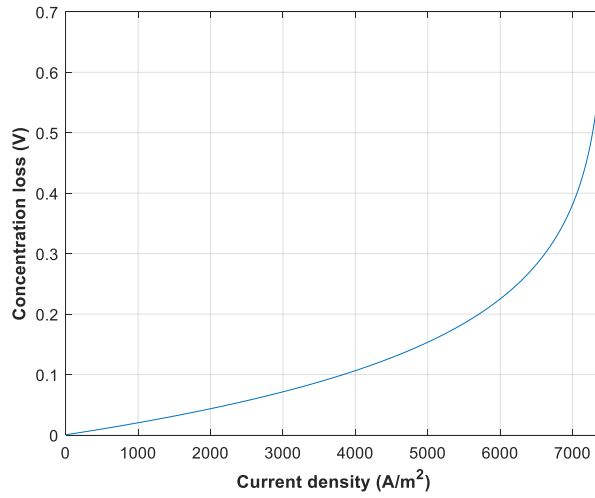


Fig. 3a: Concentration loss as a function of current density considering leakage current density of 20 A/m<sup>2</sup>.

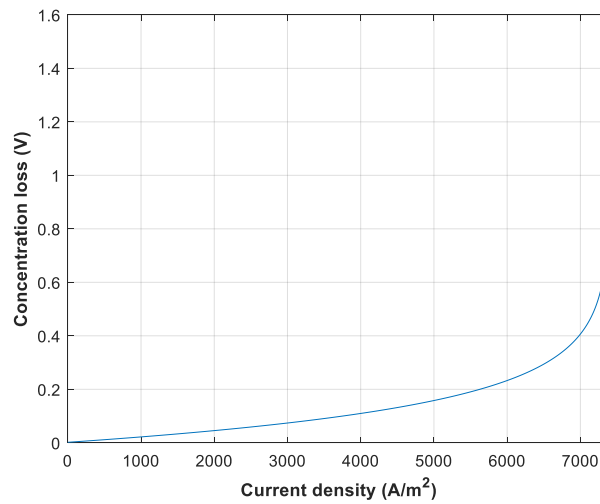


Fig. 3b: Concentration loss as a function of current density considering leakage current density of 100 A/m<sup>2</sup>.

### 3. Results and discussion

#### 3.1. Polarization curves validation

Polarization and power density curves as a function of current density are illustrated in Figures 4a and 4b. The maximum power density for this anode-supported button cell is 3205 W/m<sup>2</sup> which was obtained at an effective cell voltage of 0.57 V, and a current density of 5620 A/m<sup>2</sup>. These results showed good agreement with other modeling results [28], which are illustrated in Figure 5. The details regarding fuel and oxidant inlet condition can be found in [28].



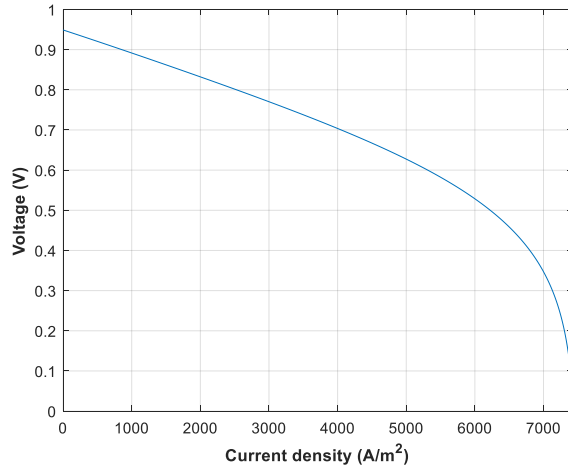


Fig. 4a: Polarization curve as a function of current density (T=1073 K, P=1 atm).

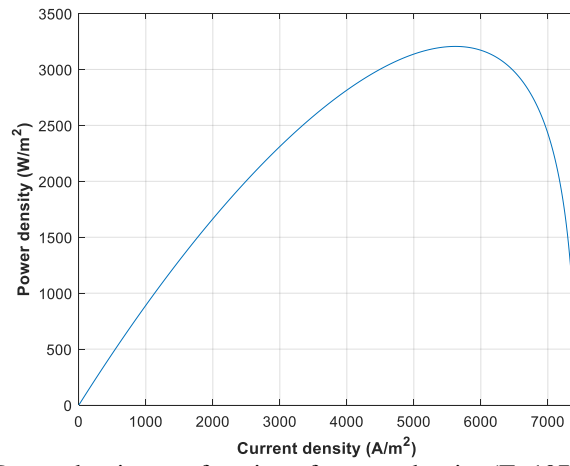


Fig. 4b: Power density as a function of current density (T=1073 K, P=1 atm).

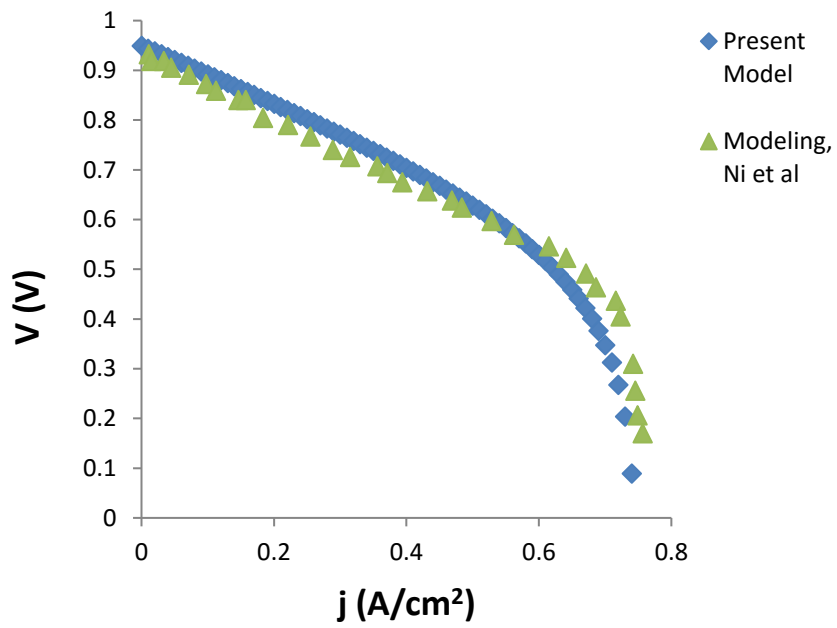


Fig. 5: Comparison of polarization curve as a function of current density for current model and the model by [28].

Also, another simulation was done by changing the thickness of the cell layers, the range of current density, and the limiting current density [30]. The results for this simulation are shown in Figures 6a and 6b. The maximum power density for this new anode-supported button cell is  $8800 \text{ W/m}^2$  which was obtained at an effective cell voltage of  $0.61 \text{ V}$ , and a current density of  $14200 \text{ A/m}^2$ . These results showed an acceptable agreement with experimental data found in the literature [30], which is illustrated in Figure 7. The details about fuel and oxidant inlet condition can be found in [30].

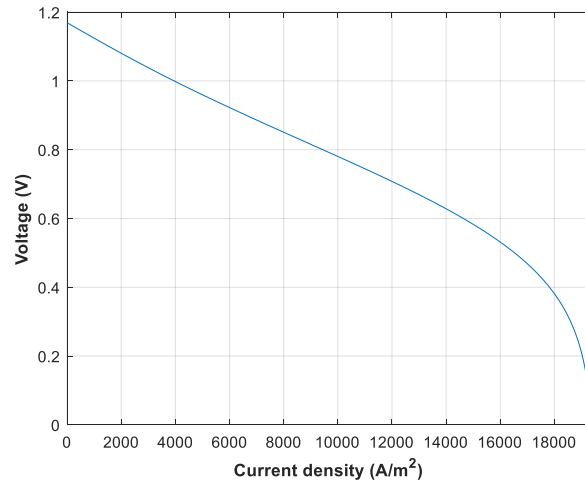


Fig. 6a: Polarization curve as a function of current density ( $T=1073 \text{ K}$ ,  $P=1 \text{ atm}$ ).

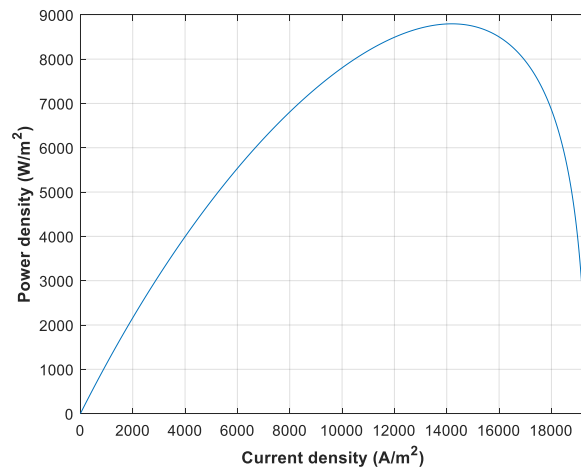


Fig. 6b: Power density as a function of current density ( $T=1073 \text{ K}$ ,  $P=1 \text{ atm}$ ).

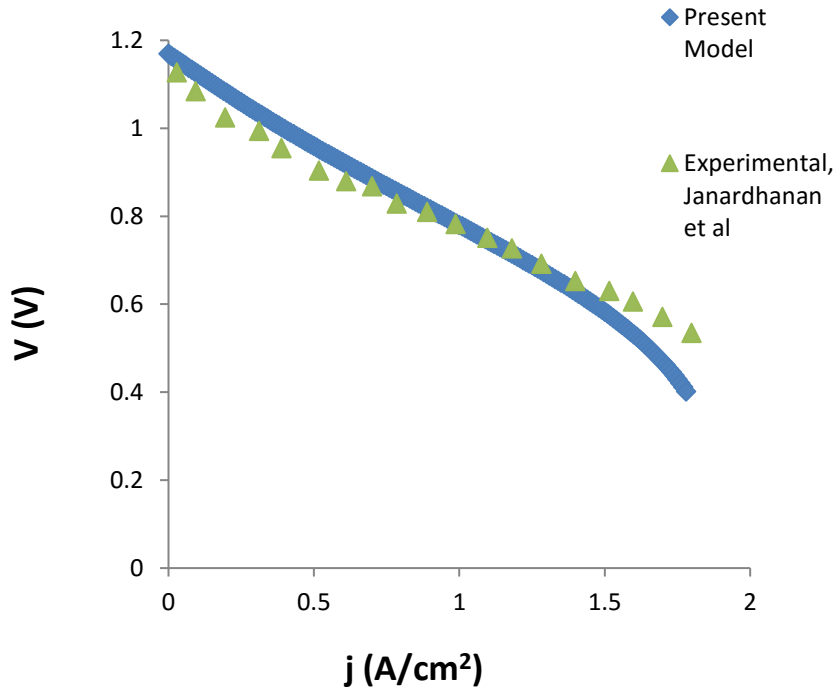


Fig. 7: Validation of polarization curve as a function of current density for current model and the experimental data by [30].

A comparison between the effective results of the present study and some of the previous studies are shown in Table. 2. The quantification of the results in Table. 2 are based on the technical review of the references, which were discussed in the literature review.

**Table. 2. Comparison between the effective results of the present study and some of the previous studies.**

Authors	Year	P (bar)	T (K)	Fuel	$U_f$	$j$ (A/m <sup>2</sup> )	V (Volt)	Specific Power (W/m <sup>2</sup> )	Electrical Efficiency (%)	Ref.
<b>Present study</b>	-	1	1073	CH <sub>4</sub>	0.8	5620	0.57	3205	45.8	-
Riensch et al.	1998	NA	1123	NA*	0.8	1700	0.75	1275	43	[35]
Chan & Ding	2005	1	1073	CH <sub>4</sub>	0.7	NA	NA	NA	NA	[36]
Zhang et al.	2005	1.08	1273	NA	0.85	1780	0.7	1250	52	[37]
Braun et al.	2006	NA	1073	H <sub>2</sub>	0.85	5700	0.75	4300	30.2	[38]
Akkaya et al.	2007	1.11	1203	NA	0.8	2000	0.72	1440	NA	[39]
Chung et al.	2008	NA	913	CH <sub>4</sub>	0.67	NA	0.7	NA	42.5	[40]
Akkaya et al.	2009	1.08	1273	CH <sub>4</sub>	0.75	1800	0.67	1200	55.4	[41]

\*: Not Available

### 3.2. Investigating a number of effective parameters on the polarization model

#### 3.2.1. Effect of the anode and cathode exchange current densities on polarization curve

The effect of anode exchange current density was shown in Figure 8a; the higher exchange current density in the anode leads to decreased in loss and increased performance in the cell. Also in Figure 8b it is shown that increasing the exchange current density in the cathode results in an increased performance, but considering its value, it has a lower effect on the cell performance in comparison with changes in the anode.

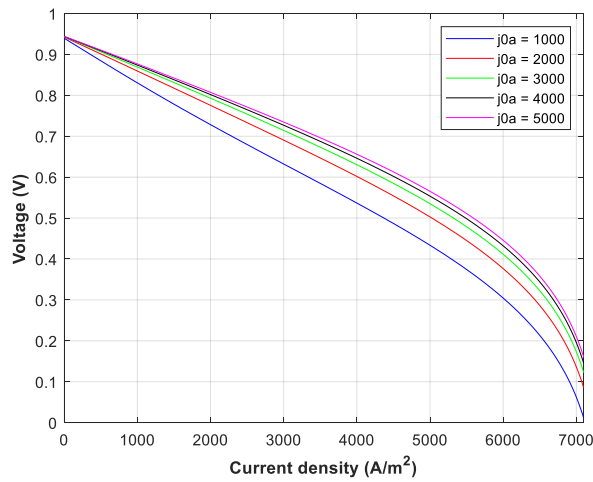


Fig. 8a: Effect of  $j_{0a}$  on polarization curve.

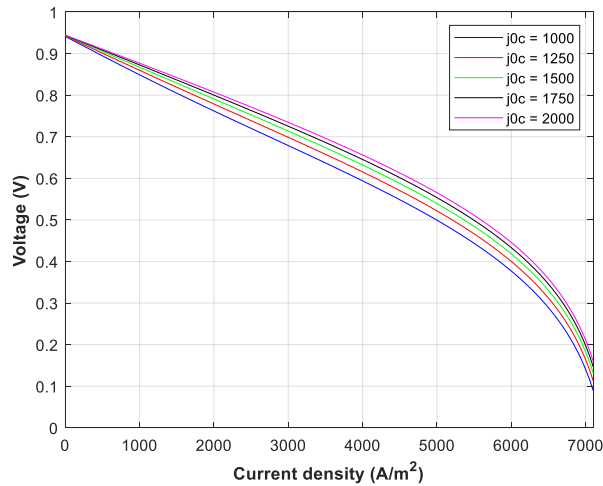


Fig. 8b: Effect of  $j_{0c}$  on polarization curve.

#### 3.2.2. Effect of leakage current density on polarization curve

The effect of leakage current density on the cell performance is shown in Figure 9. The decreasing trend in cell efficiency in the case of a higher leakage current density was found to be about 1% effective on diminishing the cell performance. The leakage current density is

obtained from the non-electrochemical reactions taking place at the two sides of the cell, which do not increase the operating current density.

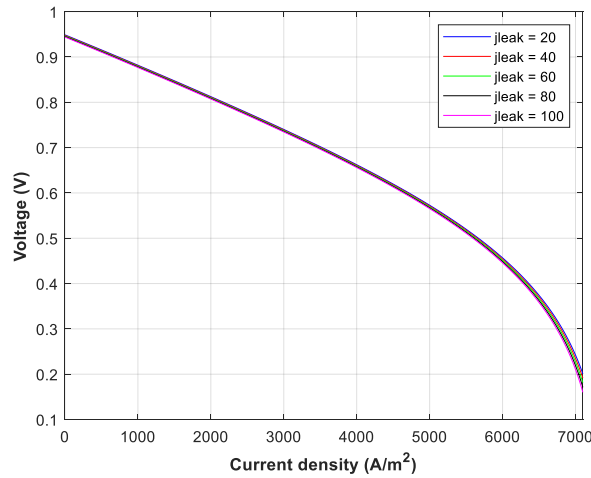


Fig. 9: Effect of  $j_{Leak}$  on polarization curve.

### 3.2.3. Effect of operating temperature on polarization curve

Increasing the operating temperature of the cell resulted in a remarkable decrease on the cathode activation and electrolyte ohmic losses. This comes from the increase of both the exchange current density at the cathode and the ionic conductivity in the electrolyte, causing the improvement of the performance of SOFCs. Figure 10 clearly shows this enhanced efficiency at higher temperatures, which proves that the SOFCs performance is heavily influenced by the cell temperature.

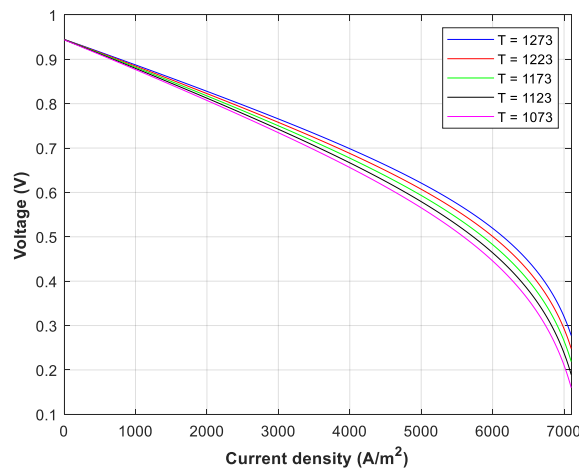


Fig. 10: Effect of temperature on polarization curve.

## 4. Conclusion

Generating a polarization model of an anode-supported SOFCs was the main objective of this research. To achieve this goal, a zero dimension polarization model was developed and then the accuracy of this own-built model was verified by performing an optimization process using a constrained non-linear multivariable function. In the polarization model of the base study, the three optimum outputs were an effective cell voltage of 0.57 V, a current density of 5620 A/m<sup>2</sup>, and a power density of 3205 W/m<sup>2</sup>. Then, the model was successfully tested against another modeling approach and also against experimental data found in the literature within a low and a high range of current densities. The results obtained herein are encouraging as the

modeling approach developed here has the merit of being not excessively computer time consuming, but robust and reliable at the same time. Furthermore, a parametric analysis was carried out to point out which parameters have a significant influence on the overall cell performance. In particular, the following parameters were accounted for this analytical study:  $j_{0a}$ ,  $j_{0c}$ ,  $j_{Leak}$ , and  $T$ .

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**Conflicts of Interest:** The authors declare no conflict of interest.

### Nomenclature:

$V_{cell}$	Cell effective voltage, V
$E_{cell}$	Cell equilibrium voltage, V
$j_0$	Exchange current density, $A\ m^{-2}$
$a$	Charge transfer coefficient
$j_{Leak}$	Leakage current density, $A\ m^{-2}$
$j$	Current density, $A\ m^{-2}$
$j_L$	Limiting current density, $A\ m^{-2}$
$r_{ct}$	Charge transfer resistance, $\Omega\ m^2$
$R_{ohm}$	Ohmic resistance, $\Omega$
$\rho$	Resistivity, $\Omega\ m$
$\delta$	Thickness, m
$\eta_{act}$	Activation loss, V
$\eta_{ohm}$	Ohmic loss, V
$\eta_{Conc}$	Concentration loss, V
$V_{Leak}$	Leakage loss, V
$j_{gross}$	Sum of leakage current density and operating current density, $A\ m^{-2}$
$j_{0a}$	Anode exchange current density, $A\ m^{-2}$
$j_{0c}$	Cathode exchange current density, $A\ m^{-2}$
$N$	Number of transfere electrons in anode and cathode
$T$	Operating pressure, Pa
$F$	Inlet temperature, K
$R$	Faraday Constant, $C\ mol^{-1}$
	Gas constant, $J\ mol^{-1}K^{-1}$

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