A Particle Filter Approach for Identifying Tire Model Parameters From Full-Scale Experimental Tests

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Mathematical models simulating the handling behavior of passenger cars are extensively used at a design stage for evaluating the effects of new structural solutions or control sys-tems. The main source of uncertainty in these type of models lies in tire–road interaction, due to high nonlinearity. Proper estimation of tire model parameters is thus of utter importance to obtain reliable results. This paper presents a methodology aimed at identi-fying the magic formula-tire (MF-Tire) model coefficients of the tires of an axle only based on measurements carried out on board vehicle (vehicle sideslip angle, yaw rate, lateral acceleration, speed, and steer angle) during standard handling maneuvers (step-steers, double lane changes, etc.). The proposed methodology is based on particle filtering (PF) technique. PF may become a serious alternative to classic model-based techniques, such as Kalman filters. Results of the identification procedure were first checked through simulations. Then, PF was applied to experimental data collected using an instrumented passenger car.

Introduction

Mathematical models simulating the handling behavior of vehicles are extensively used at a design stage for evaluating the effects of new structural solutions or control systems.

Contact forces depend on several variables such as longitudinal slip, slip angle, camber angle, vertical load, inflating pressure, wear, and adherence conditions and thus they represent the main source of uncertainty in simulation models.

A widely used semi-empirical tire model to calculate steady-state contact forces in simulations of vehicle handling is the so-called Magic Formula (MF)-Tire model [1,2]. It is a convenient set of analytical formulas that interpolates measured tire data rather than modeling the tire structure itself. Therefore, several laboratory tests are needed to correctly identify MF coefficients [2,3], which however do not account for unavoidable differences between outdoor and indoor conditions [1,4,5].

To overcome these issues, direct identification of tire characteristics from road tests has been considered in the literature [3–9]. In most of the cases, the algorithm for identification employs a very familiar tool within vehicle dynamics, the Kalman filter. In Ref. [6], an extended Kalman filter (EKF) for off-line identification of MF-Tire coefficients of the tires of an axle was proposed. In Ref. [3], a similar EKF algorithm was inserted into a two-step procedure able to identify individual tire parameters including vertical load dependency and implicitly compensating for suspension geometry and compliance. In Ref. [9], a two-stage procedure was developed to estimate vehicle model parameters for a tractor–trailer combination. A dual extended Kalman filter (DEKF) was implemented on purpose. Axles' cornering stiffness and trailer yaw moment of inertia are estimated in the first stage, while trailer center of gravity (cog) position and roll moment of inertia are identified in the second stage. In Ref. [8], an adaptive EKF was designed to estimate in real-time vehicle states (sideslip angle, yaw rate, and roll angle) during handling maneuvers. To compensate for tire force nonlinearities, state vector was augmented including adaptive states, i.e., tire cornering stiffness. An EKF was instead presented in Ref. [7] to identify individual load-dependent tire model parameters.

A different approach is proposed in the present paper, based on particle filtering (PF) technique [10–19]. Particle filter is a sequential Monte Carlo algorithm, i.e., a sampling method for approximating a distribution that makes use of its temporal structure. It can be used to solve the state estimation problem of nonlinear systems with non-Gaussian noise and thus it can represent an alternative to classic model-based techniques, such as Kalman filters. As an example, in Ref. [10], PF technique was applied to estimate vehicle yaw rate and sideslip angle. In this paper, PF technique was employed to identify the MF-Tire model coefficients of the tires of an axle based only on the measurements carried out on board vehicle (vehicle sideslip angle, yaw rate, lateral acceleration, speed, and steer angle) during standard handling maneuvers (step-steer, double lane change, etc.).

Results of the identification procedure were first checked through simulations. Then, PF was applied to the experimental data collected using an instrumented passenger car.

The paper is structured as follows: first, an overview of PF technique is presented. Then, its application to the estimation of the MF-Tire model coefficients is described. Finally, results of the proposed estimation procedure applied to both numerical and experimental data are provided.

Particle Filter Overview

Particle filter is a sequential Monte Carlo algorithm, whose aim is to track a variable of interest as it evolves with time, typically with a nonGaussian and potentiality multimodal probability density function (PDF). The basis of the method is to construct a sample-based representation of the entire PDF.

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On purpose, multiple copies (particles) of the variable of interest are used, each one associated with a weight related to its quality (i.e., its capability to track the variable). The estimate of the variable of interest is obtained by the weighted sum of all particles. The particle filter algorithm is recursive in nature and operates in two phases: prediction and update. As the system moves (and so the variable to be tracked varies), each particle is modified according to a reference model (prediction stage), including the addition of random noise on the variable of interest. Then, each particle weight is evaluated based on available measurements (update stage) and the particles with small weights are eliminated (resampling).

From an analytical point of view, particle filter algorithm can be described as follows.

Consider the following state space model with nonlinear state and measurement functions, $\boldsymbol{f}_k$ and $\boldsymbol{h}_k$, respectively:

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1}) \\ \mathbf{y} = \mathbf{h}_k(\mathbf{x}_k, \mathbf{n}_k) \end{cases} \tag{1}$$

where k is the time index, $\boldsymbol{u}$ is the input vector, $\boldsymbol{x}$ is the state vector, $\boldsymbol{y}$ is the measurement vector, and $\boldsymbol{v}$ and $\boldsymbol{n}$ are state and measurement noise, respectively.

From a Bayesian perspective, the aim of state estimation is to infer the posterior probability function $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ of the system state vector $\mathbf{x}_k$ given the measurement sequence $\mathbf{y}_{1:k}$ ($\mathbf{y}_{1:k} = \{\mathbf{y}_1,..., \mathbf{y}_k\}$). Assuming that the initial condition $p(\mathbf{x}_0|\mathbf{y}_0) = p(\mathbf{x}_0)$ is available, $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ can be obtained sequentially through the prediction

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})d\mathbf{x}_{k-1} \tag{2}$$

and updated as follows:

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_k|\mathbf{x}_{k-1})p(\mathbf{x}_k|\mathbf{y}_{1:k-1})}{p(\mathbf{y}_k|\mathbf{y}_{1:k-1})} \tag{3}$$

where $p(\mathbf{y}_k|\mathbf{y}_{1:k-1})$ is a normalizing factor independent on system state, $\mathbf{x}_k$, while $p(\mathbf{y}_k|\mathbf{x}_{k-1})$ is called likelihood of measurement $\mathbf{y}_k$.

In PF, the unknown posterior probability function $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ is approximated using a set of random samples (particles) $\{\mathbf{x}_k^i, i = 1,\dots,N\}$ with associated weights $\{w_k^i, \; i = 1,\dots,N\}$ where $\sum_{i=1}^{N} w_k^i = 1$

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) \approx \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \tag{4}$$

Function $\delta(\mathbf{x})$ is equal to unity when $\mathbf{x} = 0$, otherwise it is equal to zero. Therefore, the key step is to generate random samples from $p(\mathbf{x}_k|\mathbf{y}_{1:k})$. However, as $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ is not of the conventional form of a probability density function, such as Gaussian or Cauchy, direct sampling is not possible. Therefore, importance sampling is used to obtain the particles and associated weights [14]. The first step in importance sampling is to define an importance density $q(\mathbf{x}_k|\mathbf{y}_{1:k})$ from which samples $\mathbf{x}_k^i$ can be drawn. Thus, the weights are defined as

$$w_k^i \propto \frac{p(\mathbf{x}_k^i|\mathbf{y}_{1:k})}{q(\mathbf{x}_k^i|\mathbf{y}_{1:k})} \tag{5}$$

For the sequential estimation problem, at time k, the particles which approximate $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ are passed through the state function and updated with a new measurement, $\mathbf{y}_k$, to approximate $p(\mathbf{x}_k|\mathbf{y}_{1:k})$. It was shown in Ref. [11] that if the importance density is only dependent on the current measurement, $\mathbf{y}_k$, and the past state, $\mathbf{x}_{k-1}$, the weights can be updated as

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, \mathbf{y}_k)} \tag{6}$$

With these particles and associated weights, the estimated state vector, $\hat{\mathbf{x}}_k$, is the mean of $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ and is calculated as

$$\hat{\mathbf{x}}_k = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \tag{7}$$

An implementation issue often occurring in particle filters is degeneracy of particles. Degeneracy occurs when, after a number of time-steps, only one particle has significant weight and consequently considerable computational effort is spent on updating particles whose contribution to the approximation of $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ is negligible. To avoid/reduce degeneracy of particles the key points to be addressed in particle filter design are [11]:

(a) good choice of importance density;
(b) use of resampling.

For what concerns the first point, to face degeneracy of particles, importance density can be selected so to minimize the variance of particle weights [18]

$$q(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k)_{\text{opt}} = p(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k) = \frac{p(\mathbf{y}_k|\mathbf{x}_k|\mathbf{x}_{k-1}^i)p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)}{p(\mathbf{y}_k|\mathbf{x}_{k-1}^i)} \tag{8}$$

Substitution of Eq. (8) into Eq. (6) leads to

$$w_k^i \propto w_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_{k-1}^i) = w_{k-1}^i \int p(\mathbf{y}_k|\mathbf{x}_k')p(\mathbf{x}_k'|\mathbf{x}_{k-1}^i)d\mathbf{x}_k' \tag{9}$$

This importance density is called optimal since for a given $\mathbf{x}_{k-1}^i$, w_k^i takes the same value, whatever sample is drawn from $q(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k)_{\text{opt}}$. Hence, conditional on $\mathbf{x}_{k-1}^i$, the variance of particle weights is null.

This choice of importance density, however, presents two main issues: it requires the ability to sample from $p(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k)$ and to evaluate the integral over the new state. This is possible only in a very limited number of cases [11]. However, it is possible to find suboptimal approximations of the optimal importance density by using local linearization techniques [18] or using the unscented transform to estimate a Gaussian approximation of $p(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k)$ [19]. The use of optimal importance density, anyway, leads to additional computational cost, which, in many cases, is not justified by a significant increase of performance [11].

Alternatively, to reduce computational effort, importance density can be selected equal to the prior distribution [11]

$$q(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{y}_k) = p(\mathbf{x}_k|\mathbf{x}_{k-1}^i) \tag{10}$$

In this way, a very simple form for uploading particle weights is obtained

$$w_k^i \propto w_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_k^i) \tag{11}$$

Since this latter choice of importance density is intuitive and simple to implement, it is the most common and it was also adopted in the present paper.

The second method to face degeneracy of particles is the use of resampling. Resampling, in fact, can be used to eliminate (probabilistically) those particles with small weights, thereby focusing the analysis on particles with large weights. Resampling can be used at every time instant k according to the sampling importance resampling (SIR) [11,12]. Alternatively, resampling can be performed only when the number of effective particles with large weights falls below a certain threshold number (importance sampling) [11].

Estimation of MF-Tire Coefficients

As already mentioned, the aim of the paper is to estimate the MF-Tire model coefficients for the tires of an axle, using particle filter technique. As previously mentioned, particle filter is a model-based approach relaying on a sequential Monte Carlo algorithm.

The reference model of the particle filter developed in this paper (and described into the details in the section "Particle Filter Implementation") is a two degrees-of-freedom (dof) single-track vehicle model, whose equations of motion are (Fig. 1 and Table 1):

$$\dot{\mathbf{z}} = \left\{ \begin{array}{c} \dot{\beta} \\ \ddot{\psi} \end{array} \right\} = \left\{ \begin{array}{c} \frac{\dot{V}}{V}\beta - \dot{\psi} + \frac{F_{yf}+F_{yr}}{mV} \\ \frac{F_{yf}a - F_{yr}b}{J_z} \end{array} \right\} = \mathbf{l}(\mathbf{z}, \theta, \mathbf{u}) \tag{12}$$

where $\mathbf{z}$ is the state vector of the single-track vehicle model including vehicle sideslip angle, β, and yaw rate, $\dot{\psi}$. $\mathbf{y}$ is the measurement vector, which is coincident with $\mathbf{z}$, while $\mathbf{u}$ is the input vector including the steer angle, δ, and the vehicle speed, V

$$\mathbf{u} = \{\delta \quad V\}^{\mathrm{T}} \tag{13}$$

It is to point out that, at present, the proposed procedure aims at identifying MF-Tire coefficients during on-track handling tests, in which measurement of vehicle sideslip angle is possible using optical sensors (which on the contrary are unsuitable for ordinary passenger-car applications). However, the appearance on the market of recently developed GPS/inertial sensor combination sets claiming 0.1 m/s accuracy in lateral velocity measurements may extend the application of the proposed procedure to ordinary passenger cars.

Axles' cornering forces are determined though the Pacejka's MF-Tire model [1,2]

$$\begin{cases} F_{yf} = D_f \sin\{C_f a\tan[B_f\alpha_f - E_f(B_f\alpha_f - a\tan(B_f\alpha_f))]\} \\ F_{yr} = D_r \sin\{C_r a\tan[B_r\alpha_r - E_r(B_r\alpha_r - a\tan(B_r\alpha_r))]\} \end{cases} \tag{14}$$

where the slip angles of the front (α_f) and the rear axles (α_r) are determined as

$$\begin{cases} \alpha_f = \delta - \frac{\dot{\psi}a}{V} - \beta \\ \alpha_r = \frac{\dot{\psi}b}{V} - \beta \end{cases} \tag{15}$$

The geometric and the inertial parameters m, J_z, a, and b are assumed as known (Table 1); thus, only the coefficients of Eq. (14) must be identified. Since the curvature factor E regulates the

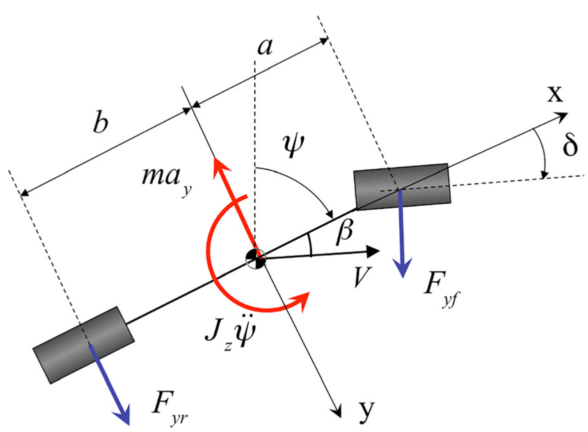

Fig. 1 Single-track vehicle model

slip angle in correspondence to the peak of the force-slip curve, which is hardly reached during road tests, it is not included into the identification set. The value of the curvature factor is thus kept constant during identification of other coefficients and is equal to $E_f = -0.0542$ and $E_r = 1.01$, respectively. Hence, the parameters to be estimated are

$$\boldsymbol{\theta} = \{B_f \;\; C_f \;\; D_f \;\; B_r \;\; C_r \;\; D_r\}^{\mathrm{T}} \tag{16}$$

Assuming that unknown parameters vary slowly with time, Eq. (12) can be rewritten as:

$$\begin{cases} \dot{\mathbf{x}} = \left\{ \begin{array}{c} \dot{\mathbf{z}} \\ \dot{\boldsymbol{\theta}} \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{l}(\mathbf{z}, \boldsymbol{\theta}, \mathbf{u}) \\ 0 \end{array} \right\} + \mathbf{v} \\ \mathbf{y} = \mathbf{z} + \mathbf{n} \end{cases} \tag{17}$$

where the stochastic terms $\mathbf{v}$ and $\mathbf{n}$ have been added to account for process and measurement noise (assumed to be Gaussian white noise), respectively.

Since unknown parameters are introduced as states, while the states of the single-track vehicle model are measured, unknown parameters can be estimated using particle filtering technique.

Particle Filter Implementation

The particle filter for estimating MF-Tire model coefficients developed in this paper is described by the following algorithm (see Algorithm 1 in Appendix).

(1) Initialization: generate $\{\mathbf{x}_0^i,\ i = 1,\ldots,N\}$ drawing it from $p(\mathbf{x}_0)$. In particular, MF-Tire parameters are sampled from uniform distributions with given upper and lower bounds (defined in the section "Numerical Results"). Null initial conditions are instead selected for single-track vehicle model states since maneuvers are assumed to start from straight line condition.
Each sample of the state vector is referred as a particle.

(2) Prediction: draw $\mathbf{v}_k$ from process noise distribution and simulate the model described by Eq. (17). As already mentioned, process noise is assumed to be zero-mean white Gaussian noise.

(3) Measurement update: update weights of each particle by likelihood:

$$w_k^i = w_{k-1}^i p(\mathbf{y}_k | \mathbf{x}_k^i) \tag{18}$$

As already mentioned in the section "Particle Filter Overview," importance density was selected equal to the prior distribution (Eq. (10)) to have a simple form for uploading weights (Eq. (11)).

The following weighting function was used to assign the weight of each particle [17]:

$$p(\mathbf{y}_k | \mathbf{x}_k^i) = \frac{1}{\sqrt{2\pi}\sigma_\beta} e^{-\frac{(\beta - \beta_k^i)^2}{2\sigma_\beta^2}} \frac{1}{\sqrt{2\pi}\sigma_\psi} e^{-\frac{(\dot{\psi} - \dot{\psi}_k^i)^2}{2\sigma_\psi^2}} \tag{19}$$

where σ_β and σ_ψ are the standard deviation of vehicle sideslip angle and yaw rate measurements, respectively. σ_β and σ_ψ are thus representative of measurement noise (which, as already mentioned, is assumed to be zero-mean white Gaussian noise) and can be inferred from experimental data, being a characteristic of the sensors used during the tests.

Table 1 Single-track vehicle model data

Vehicle mass	m	(kg)	1420
Yaw moment of inertia	J_z	(kg m^2)	2124
Front axle-cog distance	a	(m)	0.96
Rear axle-cog distance	b	(m)	1.59

It is worth noting that, through Eq. (19), weights are assigned based on the capability of particles to properly estimate vehicle states, i.e., sideslip angle and yaw rate. In particular, through Eq. (19), particles are weighted based on the error between measured (β, $\dot{\psi}$) and estimated sideslip angle and yaw rate ($\hat{\beta}_k^i$, $\hat{\dot{\psi}}_k^i$), and measurement noise, which accounts for confidence with which measurements can be weighted.

Moreover, since most of MF-Tire coefficients have a physical meaning [1–3], additional constraints relevant to any tire type were introduced to update particle weights:

(a) since the peak factor D is defined as the product between the tire-road friction coefficient and the vertical load [1], it must be verified that [2]:

$$0.1 \leq \frac{D_f(a+b)}{mgb} \leq 1.2; \quad 0.1 \leq \frac{D_r(a+b)}{mga} \leq 1.2 \tag{20a}$$

(b) in order to guarantee the existence of a maximum for the force-slip curve, while avoiding excessive curvatures [1,2], the shape factor C must remain within the range 1–1.8

$$1 \leq C_f \leq 1.8; \qquad 1 \leq C_r \leq 1.8 \tag{20b}$$

(c) the stiffness factor B must be positive to ensure a positive cornering stiffness

$$B_f \geq 0; \qquad B_r \geq 0; \tag{20c}$$

Hence, weight equal to zero is assigned to all particles estimating MF-Tire model coefficients outside the boundaries of Eq. (20).

(4) Normalize*:* normalize the weight of each particle

$$w_k^i = w_k^i \Big/ \sum_{i=1}^{N} w_k^i \tag{21}$$

(5) Estimation*:* estimate the state vector $\mathbf{x}_k$ (including single-track vehicle model states collected into vector $\mathbf{z}$ and MF-Tire coefficients collected into vector $\boldsymbol{\theta}_k$) as the weighted mean of particles at time k

$$\hat{\mathbf{x}}_k = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \tag{22}$$

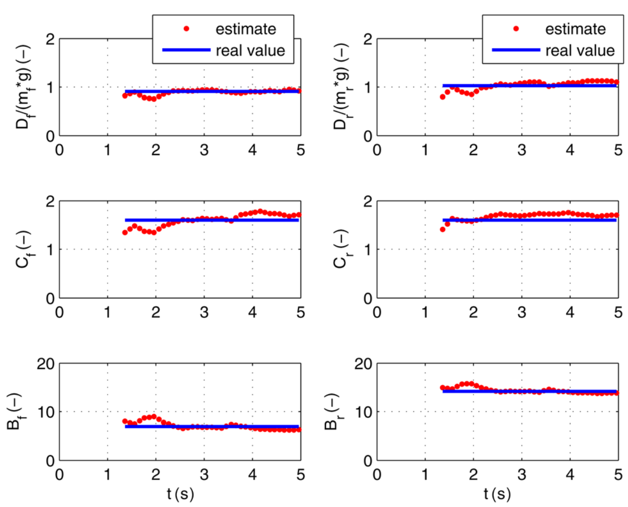

Fig. 2 Step steer maneuver, maximum lateral acceleration 8 m/s². Time histories of coefficients' estimate.

(6) Resampling*:* take N samples with replacement from the set $\{\mathbf{x}_k^i, i = 1,\dots, N\}$, where the probability to take the ith sample is equal to its weight, w_k^i (select with replacement). Algorithm 2 (in Appendix) presents a formal description of select with replacement resampling method.
Let $k = k + 1$ and iterate to item 2).

It must be pointed out that the estimation is performed only after an action takes place, i.e., when a steer angle (δ) larger than zero is imposed.[1] Then, estimation of the state vector $\mathbf{x}$ (including vehicle states and MF-Tire coefficients) occurs at every time-step, employing only one iteration per time-step.[2] Estimation of the PDF is thus achieved as vehicle states evolve with time. Due to the weighting function of Eq. (19), the constraints of Eq. (20) and resampling with replacement, only particles with large weight are propagated in the next time-steps, while avoiding degeneracy.

A formal description of the implemented particle filter is presented in Algorithm 1 (in Appendix).

Numerical Results

Results of the implemented particle filter were first checked through numerical simulations carried out with the previously described single-track vehicle model (Eqs. (12)–(15)). Capability of the proposed identification procedure to estimate MF-Tire model coefficients was verified on a series of handling maneuvers, such as step-steers and double lane changes at different lateral accelerations.

As an example, Figs. 2, 3 and Table 2 refer to the results of the identification procedure for a step-steer maneuver during which a steady-state lateral acceleration of 8 m/s^2 is reached.

The initial value of particles was sampled from uniform distributions within the ranges shown in Table 2 (prior distribution). The number of particles was selected as a trade-off between computation effort and estimation accuracy. In particular, 200 particles ($N = 200$) were used for all the tests presented in the following, so to achieve real-time estimation of MF-Tire coefficients.

As already mentioned, particle filter estimation starts only after the step-steer begins, i.e., when the steer angle exceeds the threshold value of 0.5 deg (i.e., after about 1 s, as it can be seen from Fig. 3, where the time histories of sideslip angle and yaw rate are shown). Measurements input to the particle filter are updated every 0.1 s.

Figure 2 compares the real value of MF-Tire coefficients (solid lines) for the front (left part of the figure) and the rear axle (right part of the figure) with the time histories of the estimates provided by the particle filter (dots). Red dots represent the weighted mean of the N particles at time k. As it can be seen, the particle filter takes only few time-steps to converge to the solution.

Table 2 reports the real value of MF-Tire coefficients and the estimates provided by the particle filter. Estimates reported in the table are the mean value of identified MF-Tire coefficients over the last five time-steps (i.e., over the last 0.5 s of the maneuver, being measurements updated every 0.1 s).

As it can be noticed from Table 2 and Fig. 2, estimation of MF-Tire coefficients is satisfying although the shape factor C is slightly overestimated both for front and rear axles. Maximum differences, however, do not exceed 5%.

As further verification, Fig. 3 compares the time histories of sideslip angle and yaw rate calculated using the previously described single-track vehicle model when real (solid lines) and estimated (dashed lines) MF-Tire coefficients are, respectively, used. As expected, a good agreement can be noticed due to a satisfactory identification of MF-Tire coefficients.

To assess robustness of the developed particle filter, the same maneuver was carried out simulating low adherence conditions.

[1]For practical implementation, estimation takes place when the steer angle is higher than a given threshold $\bar{\delta}$, selected large enough to exceed measurement noise.

[2]Multiple iterations per time-step allow to increase particle filter performance. However improvements were considered small with respect to the increase of computation time.

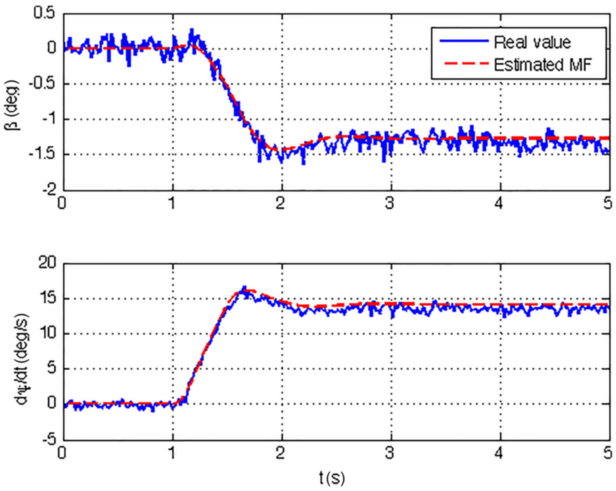

Fig. 3 **Step steer maneuver, maximum lateral acceleration 8 m/s^2. Time histories of sideslip angle and yaw rate.**

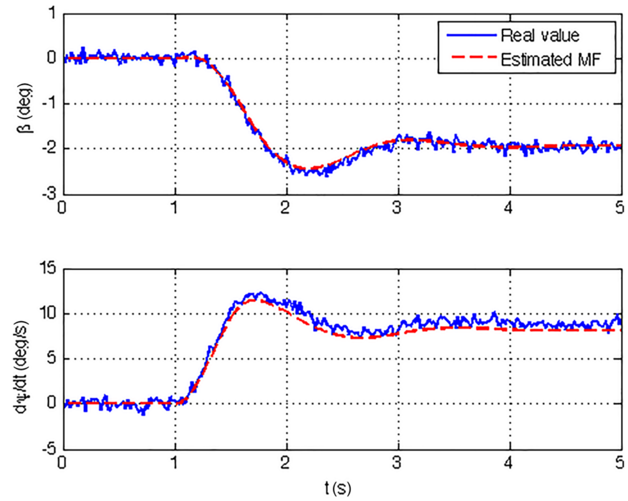

Fig. 4 **Step steer maneuver on a low adherence surface, maximum lateral acceleration 5 m/s^2. Time histories of sideslip angle and yaw rate.**

Table 2 **Results of identification during the maneuver of Fig. 3**

Parameter	$\frac{D_f(a+b)}{mgb}$	$\frac{D_r(a+b)}{mga}$	C_f	C_r	B_f	B_r
Prior distribution	0.5 ~ 1.2	0.5 ~ 1.2	1 ~ 1.8	1 ~ 1.8	5 ~ 20	5 ~ 20
Real value	0.90	1.02	1.60	1.60	7.00	14.10
Estimated value	0.89	1.02	1.67	1.65	7.07	14.24

On purpose, the peak factor (D) of front and the rear axles was multiplied by 0.6, while all the other MF-Tire coefficients were not modified (Table 3). During the maneuver, a steady-state lateral acceleration of 5 m/s^2 is reached.

The initial value of particles was sampled from the previously defined prior distribution (Table 3). Identified MF-Tire coefficients are compared with their real value in Table 3. Even in this case, MF-Tire coefficients reported in Table 3 are the mean value of the estimates calculated over the last five time-steps.

Even in this case, results provided by the particle filter can be considered satisfactory, although the shape factor of front and rear axles is underestimated of about 10%, while the peak factor is overestimated (especially at the rear axle). It must be noted that, however, cornering stiffness (i.e., the product of coefficients B, C, and D) both of the front and the rear axle are estimated with en error of about 5% or lower. As a consequence, response of the single-track vehicle model when real (solid lines) and estimated (dashed lines) MF-Tire coefficients are used, are in good agreement, as it can be clearly seen from Fig. 4, where the time histories of the vehicle sideslip angle an yaw rate are depicted.

Experimental Results

Results of the proposed particle filter were also checked using the experimental data collected with an instrumented passenger car, having the same geometric and inertial characteristics of the previously described single-track vehicle model (Table 1). Table 4 sums up the measured quantities and the corresponding sensors. All the signals were acquired at a frequency of 100 Hz and low-pass filtered at 20 Hz.

Table 4 **Measured quantities and corresponding sensors**

Measure signals	Equipment
Longitudinal acceleration	Inertial gyro platform
Lateral acceleration	Inertial gyro platform
Vertical acceleration	Inertial gyro platform
Pitch rate	Inertial gyro platform
Roll rate	Inertial gyro platform
Yaw rate	Inertial gyro platform
Sideslip angle	Correvit optical sensor
Longitudinal velocity	Correvit optical sensor
Steering wheel angle	CAN bus system

Several handling maneuvers were carried out during the tests (ramp-steer, step-steer, double lane change) on a high adherence surface.

As an example, Table 5 reports identified MF-Tire model coefficients for three different maneuvers: two step-steers at different steady-state lateral acceleration (8.5 m/s^2 and 6 m/s^2) and a double lane change (where a peak lateral acceleration of 7 m/s^2 is reached).

Initial distribution of particles (prior distribution) was selected as in the numerical simulations (and reported in Table 5). Even for experimental tests, 200 particles were used to identify MF-Tire coefficients.

Table 5 reports the value of the MF-Tire coefficients estimated during the three considered tests. Estimates reported in the table are the mean value of the MF-Tire coefficients identified over the

Table 3 **Results of identification during the maneuver of Fig. 4**

Parameter	$\frac{D_f(a+b)}{mgb}$	$\frac{D_r(a+b)}{mga}$	C_f	C_r	B_f	B_r
Prior distribution	0.5 ~ 1.2	0.5 ~ 1.2	1 ~ 1.8	1 ~ 1.8	5 ~ 20	5 ~ 20
Real value	0.55	0.61	1.60	1.60	7.00	14.10
Estimated value	0.57	0.67	1.45	1.48	7.04	13.27

Table 5 Results of identification during three different experimental maneuvers

Parameter	$\frac{D_f(a+b)}{mgb}$	$\frac{D_r(a+b)}{mga}$	C_f	C_r	B_f	B_r
Prior distribution	$0.5 \sim 1.2$	$0.5 \sim 1.2$	$1 \sim 1.8$	$1 \sim 1.8$	$5 \sim 20$	$5 \sim 20$
Step steer ($a_y = 8.5\,\mathrm{m/s^2}$)	0.84	0.98	1.49	1.60	9.55	14.31
Step steer ($a_y = 6\,\mathrm{m/s^2}$)	0.91	1.07	1.46	1.63	8.66	15.81
Double lane change	0.86	1.06	1.48	1.52	8.85	14.87

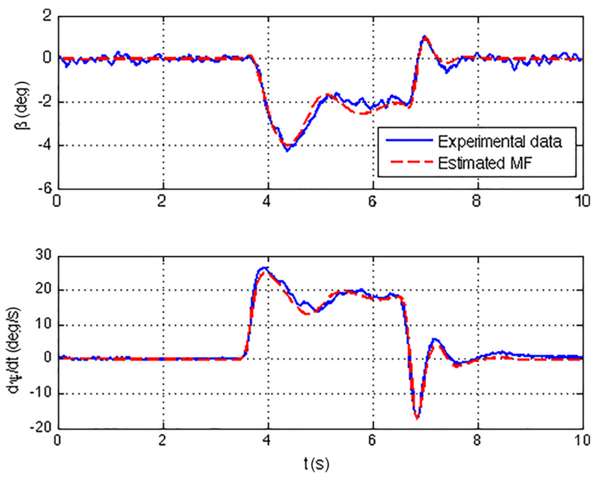

Fig. 5 Experimental–numerical comparison: step-steer maneuver (steady-state lateral acceleration 8.5 m/s²). Time histories of sideslip angle and yaw rate.

last five time-steps of the process. Even in this case, measurements input to the particle filter are updated every 0.1 s.

As it can be seen, although maneuvers differ in frequency content and peak lateral acceleration, MF-Tire model coefficients identified by the developed particle filter appear to be consistent throughout all the tests. Maximum differences are related to the stiffness factor B and are in the order of 10%.

As further verification, Figs. 5 and 6 compare the time histories of measured sideslip angle and yaw rate (solid lines) with the numerical data provided by the previously described single-track vehicle model using the identified MF coefficients (dashed lines) during two of the three maneuvers reported in Table 5. Specifically, Fig. 5 refers to the step-steer reaching the steady-state lateral acceleration of $8.5\,\mathrm{m/s^2}$, while Fig. 6 refers to the double lane change. In both cases, a good agreement between the experimental and the numerical data can be observed, confirming reliability of identified MF-Tire coefficients.

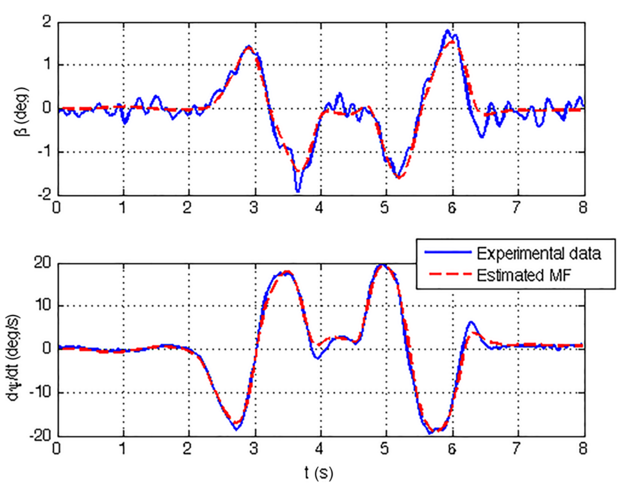

Fig. 6 Experimental–numerical comparison: double lane change maneuver (peak lateral acceleration 6.5 m/s²). Time histories of sideslip angle and yaw rate.

Concluding Remarks

In this work, a method to estimate MF-Tire model coefficients of the tires of an axle based on measurements (vehicle sideslip angle, yaw rate, lateral acceleration, speed, and steer angle) carried out during road handling tests (step-steers, double lane changes, etc.) was proposed. Specifically, the developed method is based on particle filtering technique, which is a sequential Monte Carlo algorithm. Constraints were introduced into the particle filter to account for the physics of tire–road contact and specific weighting functions were designed to track vehicle states (sideslip angle and yaw rate).

Results of the identification procedure were checked first through numerical simulations carried out with a single-track vehicle model. Numerical results revealed a good capability of estimating MF-Tire model coefficients during standard handling maneuvers (such as step steers). Then, the procedure was applied to experimental data collected on an instrumented passenger-car vehicle. Even in this case, the developed procedure was able to provide reliable estimation of MF-Tire model coefficients. Identified coefficients were found to be consistent despite of the maneuver chosen for the identification.

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Nomenclature

a = cog-front axle distance
a_y = lateral acceleration
b = cog-rear axle distance
$B, C, D,$ and E = MF-tire coefficients
$F_{yf/r}$ = front/rear axle cornering force
J_z = vehicle yaw moment of inertia
m = vehicle mass
$\mathbf{u}$ = input vector
V = vehicle speed
w_i = ith particle weight
$\mathbf{x}$ = state vector
$\mathbf{y}$ = measurement vector
$\mathbf{z}$ = state vector of single-track vehicle model
$\alpha_{f/r}$ = front/rear axle slip angle
β = sideslip angle
δ = steer angle
$\boldsymbol{\theta}$ = unknown parameters vector
ψ = yaw angle

Appendix

for $i = 1{:}N$
$\mathbf{x}_0^i = \underline{unifrand}(\mathbf{lb},\mathbf{ub});$ {Initialization}
end for
$k = 1;$
while $(t = k) \leq T$
if $|\delta_k| \geq \bar{\delta}$ {Estimation takes place only if the amplitude of the steer angle δ is larger than the threshold $\bar{\delta}$}
for $i = 1{:}N$
Simulate the model described by Eq. (17) {Prediction}
Assign particle weight, w_k^i, according to Eq. (19)
Update particle weight imposing constraints of Eq. (20) {Update weights}
end for
Calculate total weight: $W = \sum_{i=1}^{N} w_k^i$
for $i = 1{:}N$
Normalize particle weight: $w_k^i = w_k^i / W$ {Normalize weights}
end for
Estimate: $\hat{\mathbf{x}}_k = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i$ {Estimation}
Resample using Algorithm 2 {Resampling}
end if
$k = k + 1;$
end while

Algorithm 1: Particle filter algorithm; functions are noted as underlined text, comments are inside curly brackets. *unifrand* is a random number generator drawing samples from the continuous uniform distribution on the interval from **lb** to **ub**.

$Q = \underline{cumsum}(\mathbf{w} = \{w_k^1, \ldots, w_k^N\})$ {Calculate the running sum of vector **w**, which collects normalized particle weights: $\sum_{i=1}^{N} w_k^i = 1$ }

$s = \underline{rand}(N + 1)$ {s is an array of $N+1$ random numbers}
$S = \underline{sort}(s)$ {Sort elements of vector s}
$S(N + 1) = 1;\ i = 1;\ j = 1;$ {Arrays start at 1}

while $i \leq N$
if $S(i) < Q(j)$
$Index(i) = j;$
$i = i + 1;$
else
$j = j + 1;$
end if
end while

Algorithm 2: Select with replacement resampling algorithm; functions are noted as underlined text, comments are inside curly brackets. *cumsum* is a function calculating the cumulative sum of vector **w**, while *rand* generates a vector of $N+1$ random numbers.

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