© 2020 the authors. This work has been published under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License. 21st IFAC World Congress, vol. 53, no. 2, pp. 12213–12220 DOI: 10.1016/j.ifacol.2020.12.1098.

An Optimization Problem for Day-Ahead Planning of Electrical Energy Aggregators

Francesco Conte, Matteo Saviozzi, Samuele Grillo

Abstract—The widespread diffusion of distributed energy resources, especially those based on renewable energy, and energy storage devices has deeply modified power systems. As a consequence, demand response, the ability of customers to respond to regulating signals, has moved from large high-voltage and medium-voltage end-users to small, low-voltage, customers. In order to be effective, the participation to demand response of such small players must be gathered by aggregators. The role and the business models of these new entities have been studied in literature from a variety of viewpoints. Demand response can be clearly applied by sending a dedicated price signal to customers, but this methodology cannot obtain a diverse, punctual, predictable, and reliable response. These characteristics can be achieved by directly controlling the loads units. This approach involves communication problems and technological readiness. This paper proposes a fully decentralized mixed integer linear programming approach for demand response. In this framework, each load unit performs an optimization, subject to technical and user-based constraints, and gives to the aggregator a desired profile along with a reserve, which is guaranteed to comply with the constraints. In this way, the aggregator can trade the reserve coming from several load units, being the only interface to the market. Upon request, then, the aggregator communicates to the load units the modifications to their desired profiles without either knowing or caring how this modification would be accomplished. The effectiveness is simulated on 200 realistic load units.

Index Terms—demand response, load aggregator, control of renewable energy resources, intelligent control of power systems, optimal operation and control of power systems, smart grids.

I. INTRODUCTION

OWER systems are experiencing an important modification both in their structure and, consequently, in the way they are operated. One of the results of these changes is the fact that customers (passive and irresponsive in the previous paradigm) have become active and potentially effective resources for stability and control of power systems [1].

The way through which this could be accomplished is called demand response (DR). Traditionally, DR has been applied and implemented in the industrial sector [2]. If DR is to be applied to residential customers, an additional player between transmission system operator (TSO) and producers has to be introduced. This new player is the aggregator (AGT), whose role is to gather a (potentially large) number of customers,

collect their availability and trade it into dedicated service markets [3].

There is a vast literature on AGTs and, specifically, on their role in power systems and markets. Their effectiveness and, related to this, the need and convenience to introduce them, is questionable. In fact, DR can naturally be obtained through adequate price schemes, such as real-time pricing (RTP), time of use (TOU), and critical peak pricing (CPP) [4], [5]. However, the usage of such a broadcast signal (*i.e.*, the price of energy) can reduce the ability to obtain a predictable and reliable response [5]. Moreover, these approaches tend to limit the service provision to a load reduction [6].

In [7] the role of AGTs is explored. In this work a DR market scheme is described, along with the interactions among distribution system operators (DSOs), AGTs, and end users. The main objective is to define an overall framework in which all these interactions prove their effectiveness. In [4], [8] a market-based framework for residential temperature controlled loads (TCLs) is developed. Thus, controllable loads are supposed to react to energy prices. In [9] the business models of AGTs are analyzed and an optimization of the participation of AGTs to DR markets is presented. The focus of the work is on the day-ahead operation of AGTs and the maximization of their revenues. The same problem is analyzed in [6], where authors apply a Cournot-based game model, and in [10], where battery degradation estimation is included in the optimization problem.

On the other side, in [5] a control architecture is proposed, under the assumption that loads can be directly controlled. Also in [2] loads are supposed to be controlled by AGTs. Authors introduce the concept of the hourly bounds that each load can provide to DR. However, also in this work the optimization is performed at AGT level. Moreover, since a more detailed knowledge would be unlikely, an hourly demand flexibility ratio is defined based on maximum and minimum powers for each customer without any correlation with past and future behavior.

In [11] both the AGT-level optimization and the real-time control of resources is proposed. The aim of the work is to describe a hierarchical methodology (based on model predictive control) for AGTs to optimize the provision of different market products. Also in this case, resources are supposed to be directly controllable. A similar objective is pursued in [12], where AGTs manage a portfolio of DR products in day-ahead markets.

The present paper proposes a fully-decentralized optimization framework of AGTs for DR. Each load unit (LU),

F. Conte and M. Saviozzi are with Università degli Studi di Genova, DITEN, via all'Opera Pia,11/A, 16145 Genova GE, Italy (e-mail: fr.conte@unige.it, matteo.saviozzi@unige.it).

S. Grillo, is with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, piazza Leonardo da Vinci, 32, I-20133 Milano, Italy (e-mail: samuele.grillo@polimi.it).

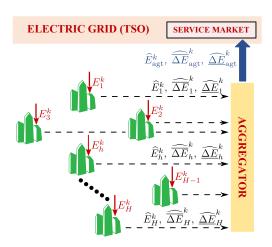


Fig. 1. Aggregate day-ahead planning (DAP) scheme.

equipped with different appliances (e.g., appliances based on phases, battery energy storage systems (BESSs), plug-in electric vehicles (PEVs), TCLs, etc.), performs mixed integer linear programming optimization problem and provides to the AGT a desired load profile, along with a positive and a negative reserve, for the following day. The peculiarity of this approach is that the reserve, apart from being guaranteed, complies with the multiple technical and behavioral (i.e., those imposed by users) constraints. The main contributions of the paper are: i) the decentralized nature of the approach; ii) the intrinsic feasibility of the reserves of each LU; iii) the ability of the approach to compensate through the reserve both forecast and modeling errors; and iv) the detailed description of the models of the equipment installed in LUs. The paper is organized as follows. In Section II day-ahead planning (DAP) strategy and intra-day operation rules for the AGT are presented; Section III is devoted to the description of the models and the constraints of the equipment in each LU; Section IV presents the DAP optimization problem for the single LUs; in Section V the results from the simulations are shown; finally, in Section VI conclusions are drawn.

II. AGGREGATE DAP AND DAY OPERATION

As shown in Fig. 1, we consider an aggregate composed by H LUs managed by an AGT. For each day, the day before, a DAP is computed; then, during the day, the plan is realized by following suitably established rules.

In the next, we illustrate first the DAP strategy and then the rules adopted during the intra-day operations.

A. Day-Ahead Planning

Let us assume that the day-time is partitioned in T sampling intervals lasting Δt [h] $(e.g., 15 \, \mathrm{min} = 0.25 \, \mathrm{h}$). At time step k $(k=0,1\ldots,T-1)$, the kth $(h=1,2,\ldots,H)$ LU exchanges with the grid the energy E_h^k [kWh]. If $E_h^k \geq 0$ the kth LU is importing energy, whereas, if $E_h^k < 0$ it is exporting energy. Notice that the energy export is a modality which can be possible or not depending on the case.

All days, each LU locally solves a DAP optimization problem (LU-DAP) returning, for any time step k, the *optimized* energy exchange \widehat{E}_h^k , and the optimized positive and negative energy reserves $\widehat{\Delta E}_h^k \geq 0$ and $\widehat{\Delta E}_h^k \leq 0$ [kWh]. These two last quantities are computed as the maximal positive and negative deviations from \widehat{E}_h^k that the hth LU declares to be able to guarantee. Notice that the load convention is used; therefore, a positive reserve corresponds to an increase of load and viceversa.

The hth LU pays for the imported energy the price $c^k_{\mathrm{imp},h}$ [\in /kWh] and is (possibly) paid for the exported energy with the price $c^k_{\mathrm{exp},h}$ [\in /kWh]. These prices are negotiated by the single LUs directly with the energy provider. At the same time, the AGT participates to the service market [11]. Specifically, it receives from the LUs their optimized energy profiles and energy reserves and declares to the market the *aggregated base energy profile*, defined as

$$\hat{E}_{\text{agt}}^{k} = \sum_{h=0}^{T-1} \hat{E}_{h}^{k}, \tag{1}$$

and offers the aggregated positive and negative reserves:

$$\widehat{\overline{\Delta E}}_{\mathrm{agt}}^{k} = \sum_{h=1}^{H} \widehat{\overline{\Delta E}}_{h}^{k}, \quad \widehat{\underline{\Delta E}}_{\mathrm{agt}}^{k} = \sum_{h=1}^{H} \widehat{\underline{\Delta E}}_{h}^{k}.$$
 (2)

The AGT is paid for the aggregated reserve with the price ${}^kc_{\rm flx}^{\rm agt}$ [\in /kWh], and pays the single LUs for their reserves ${}^kc_{\rm flx}$ [\in /kWh]. These prices are assumed to be the same for positive and negative reserves, and for each LUs. This assumption is reasonable and coherent with the existing examples of service markets [11].

The resulting programmed income of the AGT is

$$I_{\text{agt}} = \sum_{k=0}^{T} \left({}^{k} c_{\text{flx}}^{\text{agt}} - {}^{k} c_{\text{flx}} \right) \left(\overline{\Delta E}_{\text{agt}}^{k} - \underline{\Delta E}_{\text{agt}}^{k} \right). \tag{3}$$

It is worth remarking that in this paper a fully distributed solution is proposed. Indeed, the DAP optimization is operated only locally by the single LUs. Given a value of the price ${}^kc_{\rm flx}$ paid by the AGT to the LUs for the energy reserves, $I_{\rm agt}$ results to be maximized by the solutions of the distributed DAPs. However, the AGT could influence the decisions of the single LUs by modifying the price ${}^kc_{\rm flx}$, leading them to offer a larger or a lower amount of energy reserve. This will make the problem nonlinear and a centralized or partially distributed solution should be adopted. However, this is beyond the scope of the paper, which is mainly focused on studying how a LU can guarantee an energy reserve. Future works will regard to this kind of developments.

B. Intra-Day operation

During the considered day, the TSO can require an energy profile (DR signal) of the form:

$$E_{\text{agt,ref}}^{k} = \hat{E}_{\text{agt}}^{k} + \Delta E_{\text{agt,ref}}^{k}, \tag{4}$$

where the variation $\Delta E_{\mathrm{agt,ref}}^k$ is such that

$$\widehat{\underline{\Delta E}}_{\mathrm{agt}}^{k} \le \Delta E_{\mathrm{agt,ref}}^{k} \le \widehat{\overline{\Delta E}}_{\mathrm{agt}}^{k}.$$
 (5)

To provide the required energy reserve, the AGT distributes an energy variation reference signal to the H LUs of the form:

$$\Delta E_{\mathrm{ref},h}^{k} = \overline{\gamma}_{h}^{k} \max(0, \Delta E_{\mathrm{agt,ref}}^{k}) + \underline{\gamma}_{h}^{k} \min(0, \Delta E_{\mathrm{agt,ref}}^{k})$$
 (6)

where $h = 1, 2, \dots, H$ and

$$\overline{\gamma}_{h}^{k} = \frac{\widehat{\Delta E}_{h}^{k}}{\widehat{\Delta E}_{\text{agt}}^{k}}, \quad \underline{\gamma}_{h}^{k} = \frac{\widehat{\Delta E}_{h}^{k}}{\widehat{\Delta E}_{\text{agt}}^{k}}.$$
 (7)

In this way, the actual contribution of the hth LU is proportional to the percentage of the positive and negative energy reserves that it has declared to be able to guarantee.

III. LOAD UNIT DEVICES MODELS

Let us focus on the single hth LU. The general assumption is that it is equipped with: a) smart appliances, able to communicate and make decisions; b) smart plugs, able to acquire, transmit measurements, and allow a remote on/off control; c) user interfaces, that allow users to configure the desired behaviour of the controlled devices; d) a local communication network and a gateway, which are necessary to control and monitor all the different devices and to have an interface with the Internet and with the AGT; and e) a local management system which has computational capabilities.

The devices supposed to be installed on the hth LU are:

- 1) $N_{\rm abp}$ appliance based on phases (ABP), such as dish washers, laundries, industrial process appliances;
- 2) N_{pev} **PEVs** for commercial and/or personal use;
- 3) N_{bess} **BESSs**, suitably installed to obtain flexibility;
- 4) N_{tcl} TCLs, such as fridges, air heating/cooling systems, water heaters;
- 5) $N_{\rm res}$ renewable energy sources (RESs), such as photovoltaic (PV) generators;
- 6) $N_{\rm upd}$ user-programmable devices (UPDs), such as lights for industrial and/or commercial use;
- 7) non-controllable device (NCD), such as TVs, PCs, domestic lights, domestic ovens, hair dryers.

In the following, each of these devices type is described and modeled thorough a set of mixed-integer constraints, that are finally included in the LU-DAP optimization problem. The objective of LU-DAP, which will be formalized after in the paper by a cost function, is to minimize the day economical cost of the LU, taking into account the possible provision of energy reserves pad by the AGT. Because of their characteristics, ABPs and PEVs cannot be used to directly provide energy reserves. However, they can offer a time flexibility, which can be exploited to optimize the LU plan. Differently, BESSs and TCLs can directly offer energy reserves, using the storage of electric and thermal energies that they respectively manage. Therefore, the models illustrated in the following for these two types of devices are designed in order to quantify the potential energy reserves they can provide.

It is worth remarking that the proposed models are developments of the ones introduced in [13] for smart houses. In all models the day time is partitioned as done for the aggregate in Section II with the sampling time Δt and time steps are indicated with $k = 0, 1, \dots, T - 1$.

A. Appliances based on phases model

Examples of ABPs are dish washers and washing machines in houses, hotels and restaurants, or process machines in factories. These appliances are characterized by a service to be provided to the user and a corresponding set of ordered working phases necessary to realize such a service. The peculiarity of a working phase is that it cannot be interrupted. Whereas, depending on the case, there is a maximal possible delay between the execution of two consecutive phases. The user is usually interested in obtaining the service up to a certain time. Therefore, the idea is that, the day before, the user indicates the required service and the time preferences for the day-ahead. In the following, we will show how this indication can be included in the optimization model.

Let $j = 1, 2, ..., n_i$ be the index that identify the n_i working phase of the *i*th ABP, $i = 1, 2, ..., N_{abp}$. The following variables defined for any time steps k: p_{ij}^k [kW], power absorbed by jth phase; x_{ij}^k , binary variable which identifies the activation state of jth phase; s_{ij}^k , binary variable equal to 1 if jth phase has been already executed at time step k; t_{ij}^k binary variable equal to 1 if, at time step k, the ABP is passing from (j-1)th to jth phase

The behaviour of the ith ABP can be therefore described by the operation constraints listed in the following:

$$\Delta t \sum_{k=0}^{T-1} p_{ij}^k = E_{ij} \qquad \forall i, j, \tag{8}$$

which sets the total energy used by the jth phase to the parameter E_{ij} [kWh];

$$\sum_{k=1}^{T} x_{ij}^{k} = \overline{T}_{ij} \qquad \forall i, j, \tag{9}$$

which sets the total execution time of the jth phase to the parameter \overline{T}_{ij} [# of time steps];

$$x_{ij}^k P_{ij}^{\min} \le p_{ij}^k \le x_{ij}^k P_{ij}^{\max} \qquad \forall i, j, k, \tag{10} \label{eq:10}$$

which imposes the power limits for the jth phase between the maximum and minimum values P_{ij}^{\max} and P_{ij}^{\min} [kW];

$$x_{i,i}^k + s_{i,i}^k \le 1 \qquad \forall i, j, k \tag{11}$$

$$x_{ij}^{k} + s_{ij}^{k} \le 1 \qquad \forall i, j, k$$

$$x_{ij}^{k-1} - x_{ij}^{k} - s_{ij}^{k} \le 0 \qquad \forall i, j, k = 1, 2, \dots, T - 1$$
(12)

$$s_{ij}^{k-1} - s_{ij}^{k} \le 1$$
 $\forall i, j, k = 1, 2, \dots, T - 1$ (13)

$$x_{ij}^k - s_{i(j-1)}^k \le 0$$
 $\forall i, k, j = 2, 3, \dots, n_i,$ (14)

which set the order of the phases and avoid their interruption;

$$t_{ij}^{k} = s_{i(j-1)}^{k} - (x_{ij}^{k} + s_{ij}^{k}) \quad \forall i, j, k$$
 (15)

$$\sum_{k=1}^{T} t_{ij}^{k} \le D_{ij} \qquad \forall i, j = 2, 3, \dots, n_{i}$$
 (16)

which set to D_{ij} [# of time steps] the maximal delay between the (j-1)th and the jth phase;

$$x_{ij}^k \le \mathrm{UP}_{\mathrm{abp},i}^k \qquad \forall i, j, k$$
 (17)

which assures that the ABP is activated when desired by the user. $UP_{abp,i}^k$ is a binary parameter that defines (when equal to 1) if the ABP can run at time step k, based on the user preferences. For example, assume that the required service must be completed up to a certain hour of the day, corresponding to the time step k^* . Then, $\mathrm{UP}^k_{\mathrm{abp},i}$ =1 for $k=0,1,\ldots,k^*$ and $\mathrm{UP}^k_{\mathrm{abp},i}$ =0 for the remaining entries.

B. PEVs model

Let us consider the ith PEV, $i=1,2,\ldots,N_{\mathrm{pev}}$. To model this type of device, two variables, defined for any time step k, are required: p_i^k [kW], absorbed power; x_i^k , binary variable which identifies the PEV recharge activation state.

The operational constraints are:

$$\Delta t \sum_{k=0}^{T-1} \eta_i p_{ij}^k = \Delta \text{SoC}_i^{\text{p}} E_{\text{p},i}^{\text{nom}} \quad \forall i,$$
 (18)

which sets the final recharge amount to $\Delta SoC_i^p \geq 0$ [p.u.], and where $E_{p,i}^{nom}$ [kWh] is the nominal energy of the battery and $\eta_i \leq 1$ is the recharge efficiency;

$$0 \le p_i^k \le x_i^k P_i^{\text{max}} \qquad \forall i, k, \tag{19}$$

which limits the recharge power under the rated value P_{\cdot}^{\max} [kW]:

$$x_i^k \le \mathrm{UP}_{\mathrm{pev},i}^k \qquad \forall i, k,$$
 (20)

which assures that the PEV is recharged according to the user preferences, represented by the binary parameters $UP_{\text{pev},i}^k$, defined as explained for the ABPs.

C. BESSs model

Let us consider the ith BESS, $i=1,2,\ldots,N_{\mathrm{bess}}$. The basic variables, defined for any time step k are: SoC_i^k [p.u.], battery state-of-charge (SoC); $p_{\mathrm{ch},i}^k$ [kW], charging power; $p_{\mathrm{dsc},i}^k$ [kW], discharging power; $x_{\mathrm{ch},i}^k$, $x_{\mathrm{dsc},i}^k$, binary variables which identifies charging and discharging activation states, respectively.

The SoC time evolution is modeled by

$$SoC_{i}^{k} = SoC_{i}^{k-1} + \frac{\Delta t}{E_{b,i}^{\text{nom}}} \left(\eta_{i}^{\text{ch}} p_{\text{ch},i}^{k-1} + \eta_{i}^{\text{dsc}} p_{\text{dsc},i}^{k-1} \right)$$
(21)

for $k=1,\ldots,T$, given the initial condition SoC_i^0 . In (21): $\eta_i^{\mathrm{ch}} \leq 1$ and $\eta_i^{\mathrm{dsc}} \geq 1$ are the charging and discharging efficiencies, respectively, $E_{\mathrm{b},i}^{\mathrm{nom}}$ [kWh] in the nominal energy of the battery. Model (21) is completed by the following constraints:

$$0 \le p_{\mathrm{ch},i}^k \le x_{\mathrm{ch},i}^k P_{\mathrm{ch},i}^{\mathrm{max}} \qquad \forall i, k, \tag{22}$$

$$x_{\mathrm{dsc},i}^k P_{\mathrm{dsc},i}^{\min} \le p_{\mathrm{dsc},i}^k \le 0 \quad \forall i, k,$$
 (23)

$$x_{\text{ch},i}^k + x_{\text{dsc},i}^k \le 1 \qquad \forall i, k, \tag{24}$$

which limit the power exchange of the BESS between the maximum recharge power $P_{\mathrm{ch},i}^{\mathrm{max}} \geq 0$ and the maximum (in absolute value sense) discharge power $P_{\mathrm{dsc},i}^{\mathrm{min}} \leq 0$, and allow the activation of only one task (charge or discharge) at any time.

In order to quantify the potential energy reserve that can be provided during the day by the BESS, the maximal positive and negative variations of charging and discharging powers are introduced: $\overline{\Delta p}_{\mathrm{ch},i}^k, \overline{\Delta p}_{\mathrm{dsc},i}^k \geq 0$ and $\underline{\Delta p}_{\mathrm{ch},i}^k, \underline{\Delta p}_{\mathrm{dsc},i}^k \leq 0$ [kW]. These variations will imply two maximal deviations from the the base battery power exchange profile $p_{\mathrm{bess},i}^k = p_{\mathrm{ch},i}^k + p_{\mathrm{dsc},i}^k$:

$$\overline{\Delta p}_{\mathrm{bess},i}^k = \overline{\Delta p}_{\mathrm{ch},i}^k + \overline{\Delta p}_{\mathrm{dsc},i}^k \quad \forall i, k, \tag{25}$$

$$\underline{\Delta p_{\text{bess},i}^k} = \underline{\Delta p_{\text{ch},i}^k} + \underline{\Delta p_{\text{dsc},i}^k} \quad \forall i, k, \tag{26}$$

which can therefore be used to provide energy reserves. Given the maximal variation variables, it is also possible to compute the time evolution of the SoC over-bound $\overline{\operatorname{SoC}}_i^k$ and underbound $\underline{\operatorname{SoC}}_i^k$ as it follows:

$$\overline{\operatorname{SoC}}_{i}^{k} = \overline{\operatorname{SoC}}_{i}^{k-1} + \frac{\Delta t}{E_{\mathrm{b},i}^{\mathrm{nom}}} \eta_{i}^{\mathrm{ch}} \left(p_{\mathrm{ch},i}^{k-1} + \overline{\Delta p_{\mathrm{ch},i}^{k-1}} \right) + \frac{\Delta t}{E_{\mathrm{b},i}^{\mathrm{nom}}} \eta_{i}^{\mathrm{dsc}} \left(p_{\mathrm{dsc},i}^{k-1} + \overline{\Delta p_{\mathrm{dsc},i}^{k-1}} \right)$$

$$(27)$$

$$\underline{\operatorname{SoC}}_{i}^{k} = \underline{\operatorname{SoC}}_{i}^{k-1} + \frac{\Delta t}{E_{\mathrm{b},i}^{\mathrm{nom}}} \eta_{i}^{\mathrm{ch}} \left(p_{\mathrm{ch},i}^{k-1} + \underline{\Delta p_{\mathrm{ch},i}^{k-1}} \right) + \frac{\Delta t}{E_{\mathrm{b},i}^{\mathrm{nom}}} \eta_{i}^{\mathrm{dsc}} \left(p_{\mathrm{dsc},i}^{k-1} + \underline{\Delta p_{\mathrm{dsc},i}^{k-1}} \right)$$
(28)

 $\forall i, k = 1, 2, ..., T$, with inital conditions \overline{SoC}_i^0 and \underline{SoC}_i^0 . Models (27) and (28) are completed by:

$$0 \le p_{\mathrm{ch},i}^k + \overline{\Delta p}_{\mathrm{ch},i}^k \le \overline{x}_{\mathrm{ch},i}^k P_{\mathrm{ch},i}^{\mathrm{max}} \qquad \forall i, k, \qquad (29)$$

$$\overline{x}_{\mathrm{dsc},i}^k P_{\mathrm{dsc},i}^{\min} \le p_{\mathrm{dsc},i}^k + \overline{\Delta p}_{\mathrm{dsc},i}^k \le 0 \qquad \forall i, k, \qquad (30)$$

$$\overline{x}_{\text{ch},i}^k + \overline{x}_{\text{dsc},i}^k \le 1 \quad \forall i, k, \quad (31)$$

and

$$0 \le p_{\text{ch},i}^k + \Delta p_{\text{ch},i}^k \le \underline{x}_{\text{ch},i}^k P_{\text{ch},i}^{\text{max}} \qquad \forall i, k, \qquad (32)$$

$$\underline{x}_{\mathrm{dsc},i}^{k} P_{\mathrm{dsc},i}^{\min} \le p_{\mathrm{dsc},i}^{k} + \underline{\Delta p}_{\mathrm{dsc},i}^{k} \le 0 \qquad \forall i, k, \qquad (33)$$

$$\underline{x}_{\mathrm{ch},i}^k + \underline{x}_{\mathrm{dsc},i}^k \le 1 \qquad \forall i, k.$$
 (34)

where $\overline{x}_{\mathrm{ch},i}^k$, $\overline{x}_{\mathrm{dsc},i}^k$, $\underline{x}_{\mathrm{ch},i}^k$, and $\underline{x}_{\mathrm{dsc},i}^k$ are binary variables that identifies the charge and discharge activation states for the two SoC trajectories.

It is easy to show that if

$$\overline{\operatorname{SoC}}_{i}^{k} \leq \operatorname{SoC}_{i}^{\max}, \qquad \operatorname{SoC}_{i}^{\min} \leq \underline{\operatorname{SoC}}_{i}^{k} \quad \forall i, k,$$
 (35)

then $SoC_i^{min} \leq SoC_i^k \leq SoC_i^{max}$, where SoC_i^{max} and SoC_i^{min} are the desired limits for the battery SoC.

Finally, the following constraints are required to guarantee a maximum number of charging and discharging daily cycles equal to the prescribed parameters $\ell_i^{\rm ch}$ and $\ell_i^{\rm dsc}$, in order to limit the BESS ageing:

$$\eta_i^{\text{ch}} \frac{\Delta t}{E_{\text{b,}i}^{\text{nom}}} \sum_{k=0}^{T-1} \left(p_{\text{ch},i}^k + \overline{\Delta p}_{\text{ch},i}^k \right) \le \ell_i^{\text{ch}} \quad \forall i,$$
(36)

$$-\eta_i^{\mathrm{dsc}} \frac{\Delta t}{E_{\mathrm{b},i}^{\mathrm{nom}}} \sum_{k=0}^{T-1} \left(p_{\mathrm{dsc}_i}^k + \underline{\Delta p}_{\mathrm{dsc},i}^k \right) \le \ell_i^{\mathrm{dsc}} \quad \forall i. \tag{37}$$

D. TCLs model

Let us consider the ith TCL, $i=1,2,\ldots,N_{\rm tcl}$. We suppose that this TCL is working only as a cooling unit. However, with trivial changes, the proposed model can be used also for heating units. The basic variables, defined for any time step k are: ϑ_i^k [°C], controlled temperature; $p_{{\rm tcl},i}^k$ [kW], absorbed power by the TCL; $x_{{\rm tcl},i}^k$, binary variable which identifies the activation state of the TCL.

The time evolution of ϑ_i^k is modelled by:

$$\vartheta_i^k = \alpha_i \vartheta_i^{k-1} - \beta_i R_i \eta_i^{c} p_{tel,i}^{k-1} + \beta_i \vartheta_{ex,i}^{k-1}$$
(38)

for $k=1,2,\ldots T-1$, given the initial condition ϑ_i^0 . In (38): η_i^c is the cooling efficiency, R_i [°C/(kW)] is the thermal resistance $\alpha_i = \exp{-\Delta t/(C_iR_i)}$ and $\beta_i = 1-\alpha_i$ are dynamic parameters, where C_i [kWh/°C] is the thermal capacitance of the controlled mass (air or water), and $\vartheta_{\mathrm{ex},i}^k$ [°C] is the external temperature. This last can be external from the LU or be the temperature controlled by another TCL within the LU, i.e., $\hat{\vartheta}_{\mathrm{ex},i}^k = \vartheta_j^k$ for any $j \neq i$. In the first case, a dayahead forecast profile $\{\hat{\vartheta}_{\mathrm{ex},i}^k\}_{k=0}^{T-1}$ is assumed to be available. Forecast errors are modelled by independent Gaussian random variables, i.e.,

$$\vartheta_{\text{ex}\ i}^k \sim \mathcal{N}(\hat{\vartheta}_{\text{ex}}^k, \sigma_{\text{ex}\ i}^k) \quad \forall i, k.$$
 (39)

It is worth remarking that model (39) can be used also to represent the uncertainties of the dynamical model (38) and those introduced by other external disturbances such as windows and doors opening, people occupancy etc..

The model for the standard functioning of the considered TCL is completed by the two following constraints:

$$0 \le p_{\mathrm{tcl},i}^k x_{\mathrm{tcl},i}^k \le P_{\mathrm{tcl},i}^{\max} \quad \forall i, k, \tag{40}$$

which limits the power absorbed by the TCL under the rated value $P_{\mathrm{tcl},i}^{\mathrm{max}}$; and

$$x_{\text{tcl},i}^k \le \text{UP}_{\text{tcl},i}^k \quad \forall i, k,$$
 (41)

which assures that the TCL is activated according to the user preferences, represented by the binary parameters $\mathrm{UP}_{\mathrm{tcl},i}^k$, defined as explained for the ABPs.

Similarly to what done for BESSs in Subsection III-C, in order to quantify the potential energy reserve that can be provided by the TCL during the day, the maximal positive and negative variations of the power consumption are introduced: $\overline{\Delta p}_{\mathrm{tcl},i}^k \geq 0$ and $\underline{\Delta p}_{\mathrm{tcl},i}^k \leq 0$ [kW]. Given these two variations, it is possible to compute the time evolution of the over-bound and the under-bound temperatures $\overline{\vartheta}_i^k$ and $\underline{\vartheta}_i^k$ as it follows:

$$\overline{\vartheta}_{i}^{k} = \alpha_{i} \overline{\vartheta}_{i}^{k-1} - \beta_{i} R_{i} \eta_{i}^{c} \left(p_{\text{tcl},i}^{k-1} + \underline{\Delta p_{\text{tcl},i}^{k-1}} \right) + \beta_{i} \widehat{\vartheta}_{\text{ex},i}^{k-1}$$
 (42)

$$\underline{\vartheta}_{i}^{k} = \alpha_{i}\underline{\vartheta}_{i}^{k-1} - \beta_{i}R_{i}\eta_{i}^{c}\left(p_{\mathrm{tcl},i}^{k-1} + \overline{\Delta p}_{\mathrm{tcl},i}^{k-1}\right) + \beta_{i}\hat{\vartheta}_{\mathrm{ex},i}^{k-1}$$
(43)

for all i and $k=1,2,\ldots,T$, with initial conditions $\overline{\vartheta}_i^0$ and $\underline{\vartheta}_i^0$. The power variations must also satisfy the two following constraints:

$$0 \le p_{\mathrm{tcl},i}^k + \overline{\Delta p}_{\mathrm{tcl}}^k \le x_{\mathrm{tcl},i}^k P_{\mathrm{tcl},i}^{\mathrm{max}} \qquad \forall i, k, \tag{44}$$

$$0 \le p_{\text{tcl},i}^k + \underline{\Delta p}_{\text{tcl}}^k \le x_{\text{tcl},i}^k P_{\text{tcl},i}^{\text{max}} \quad \forall i, k.$$
 (45)

The main task required to a TCL is to keep the controlled temperature within a desired comfort interval, which, for the *i*th TCL, is indicated with $[\vartheta_i^{\min}, \vartheta_i^{\max}]$. Because of the stochastic assumption made for the external temperature in (39), this objective can be assured only in probabilistic sense. In this paper, we use *chance constraints* adopting the *separation constraints approximation* [14]. Thus, the TCL objective is that, for all k,

$$\mathbf{P}\left(\vartheta_{i}^{k} \leq \vartheta_{i}^{\max}\right) \geq 1 - r, \quad \mathbf{P}\left(\vartheta_{i}^{k} \geq \vartheta_{i}^{\min}\right) \geq 1 - r, \quad (46)$$

where 0 < r < 0.5 is the so-called *reliability*. Using (42), (43), and (39), chance constraints in (46) can be rewritten as the following deterministic constraints:

$$\overline{\vartheta}_{i}^{k} \leq \vartheta_{i}^{\max} - \sigma_{\mathrm{ex},i}^{k-1} \sqrt{2} \mathrm{erf}^{-1} (1 - 2r) \quad \forall k, \tag{47}$$

$$\underline{\vartheta}_{i}^{k} \ge \vartheta_{i}^{\min} + \sigma_{\mathrm{ex},i}^{k-1} \sqrt{2} \mathrm{erf}^{-1} (1 - 2r) \quad \forall k, \tag{48}$$

where $\operatorname{erf}^{-1}(\cdot)$ is the inverse Gauss error function.

E. RESs, UPDs, NCDs models

The power generated by the *i*th RES, $i=1,2,\ldots,N_{\rm res}$, at time k, is indicated with $p_{{\rm res},i}^k$ [kW]. A day-ahead forecast profile $\{\hat{p}_{{\rm res},i}^k\}_{k=0}^{T-1}$ is supposed to be available.

The power consumed by the ith UPD, $i=1,2,\ldots,N_{\mathrm{upd}}$, at time k, is indicated with $p_{\mathrm{upd},i}^k$ [kW]. For these devices, it is supposed that the user defines a fixed day plan $\{\hat{p}_{\mathrm{upd},i}^k\}_{k=0}^{T-1}$. This assumption is reasonable for appliances like lights in industrial buildings or in the common spaces of commercial building.

The aggregated power consumed by the NCDs at time k is indicated with $p_{\rm ncd}^k$ [kW]. In order to consider the use of these devices in the LU-DAP optimization problem, a forecast profile $\{\hat{p}_{\rm ncd}^k\}_{k=0}^{T-1}$ is supposed to be available.

Forecast errors are modelled by independent Gaussian random variables. Therefore,

$$p_{\mathrm{res},i}^k \sim \mathcal{N}(\hat{p}_{\mathrm{res},i}^k, \sigma_{\mathrm{res},i}^k) \quad \forall i, k,$$
 (49)

$$p_{\text{ncd}}^k \sim \mathcal{N}(\hat{p}_{\text{ncd}}^k, \sigma_{\text{ncd}}^k) \quad \forall k.$$
 (50)

IV. LOAD UNIT DAY-AHEAD PLANNING

Based on models developed in Section III, the total power expected to be exchanged by the hth LU at any time step k is given by:

$$p_{h}^{k} = \sum_{i=1}^{N_{\text{abp}}} \sum_{j=1}^{n_{i}} p_{ij}^{k} + \sum_{i=1}^{N_{\text{pev}}} p_{i}^{k} + \sum_{i=i}^{N_{\text{bess}}} \left(p_{\text{ch},i}^{k} + p_{\text{dsc},i}^{k} \right) + \sum_{i=1}^{N_{\text{tcl}}} p_{\text{tcl},i}^{k} - \sum_{i=1}^{N_{\text{res}}} \hat{p}_{\text{res},i}^{k} + \sum_{i=1}^{N_{\text{upd}}} \hat{p}_{\text{upd},i}^{k} + \hat{p}_{\text{ncd},h}^{k}.$$
(51)

As for energy, $p_h^k \geq 0$ means power import, and $p_h^k \leq 0$ means power export. Notice that in (51) powers p_{ij}^k , p_i^k , $p_{\mathrm{ch},i}^k$, $p_{\mathrm{dgc},i}^k$, $p_{\mathrm{tcl},i}^k$ are variables which has to be determined, whereas $\hat{p}_{\mathrm{res},i}^k$, $\hat{p}_{\mathrm{upd},i}^k$, and $\hat{p}_{\mathrm{ncd},h}^k$ are given forecast data.

Power p_h^k has to be intended as the *base* power expected to be exchanged by the LU. Indeed, in Subsection III-C and Subsection III-D the time-varying maximal positive and

negative power variations $\overline{\Delta p}_{\mathrm{bess(tcl)},i}^k$, $\underline{\Delta p}_{\mathrm{bess(tcl)},i}^k$ have been introduced in order to dimension the potential provision of energy reserves from BESSs and TCLs. Actually, these power variations are exploited for two different issues: 1) to effectively provide the total energy reserves $\overline{\Delta E}_h^k$ and $\underline{\Delta E}_h^k$ to be communicated to the AGT; and 2) to compensate the uncertainties introduced by the forecast errors of RESs and NCDs. This is realized by setting, for all i, k,

$$\overline{\Delta p}_{\text{bess(tcl)},i}^{k} = \overline{\Delta p}_{\text{bess(tcl)},\text{flx},i}^{k} + \overline{\Delta p}_{\text{bess(tcl)},\text{unc},i}^{k}$$
 (52)

$$\underline{\Delta p_{\text{bess(tcl)},i}^{k}} = \underline{\Delta p_{\text{bess(tcl)},\text{flx},i}^{k}} + \underline{\Delta p_{\text{bess(tcl)},\text{unc},i}^{k}}$$
(53)

where the subscript 'unc' means that the power variation is used to compensate the forecast errors, whereas the subscript 'flx' means that the power variation is used to provide energy reserves to the AGT. Therefore, we can define, for the entire hth LU, the power variations

$$\overline{\Delta p}_{\mathrm{unc},h}^{k} = \sum_{i=1}^{N_{\mathrm{bess}}} \overline{\Delta p}_{\mathrm{bess,unc},i}^{k} + \sum_{i=1}^{N_{\mathrm{tcl}}} \overline{\Delta p}_{\mathrm{tcl,unc},i}^{k} \quad \forall k, \quad (54)$$

$$\underline{\Delta p}_{\mathrm{unc},h}^{k} = \sum_{i=1}^{N_{\mathrm{bess}}} \underline{\Delta p}_{\mathrm{bess,unc},i}^{k} + \sum_{i=1}^{N_{\mathrm{tcl}}} \underline{\Delta p}_{\mathrm{tcl,unc},i}^{k} \quad \forall k, \quad (55)$$

to be used for compensating the forecast errors. The maximal positive and negative energy reserves provided by the hth LU are given by

$$\overline{\Delta E}_{h}^{k} = \Delta t \cdot \overline{\Delta p}_{\mathrm{flx},h}^{k}, \quad \underline{\Delta E}_{h}^{k} = \Delta t \cdot \underline{\Delta p}_{\mathrm{flx},h}^{k} \quad \forall k, \quad (56)$$

$$\overline{\Delta p}_{\mathrm{flx},h}^{k} = \sum_{i=1}^{N_{\mathrm{bess}}} \overline{\Delta p}_{\mathrm{bess},\mathrm{flx},i}^{k} + \sum_{i=1}^{N_{\mathrm{tcl}}} \overline{\Delta p}_{\mathrm{tcl},\mathrm{flx},i}^{k} \quad \forall k,$$
 (57)

$$\underline{\Delta p}_{\mathrm{flx},h}^{k} = \sum_{i=1}^{N_{\mathrm{bess}}} \underline{\Delta p}_{\mathrm{bess},\mathrm{flx},i}^{k} + \sum_{i=1}^{N_{\mathrm{tcl}}} \underline{\Delta p}_{\mathrm{tcl},\mathrm{flx},i}^{k} \quad \forall k.$$
 (58)

Based on (49) and (50) the total forecast error for the power exchanged by the hth LU at time k results to be $\varepsilon_h^k \sim \mathcal{N}(0, \sigma_{\mathrm{unc},h}^k), \text{ where } \sigma_{\mathrm{unc},h}^k = \sqrt{\sum_{i=1}^{N_{\mathrm{res}}} \sigma_{\mathrm{res},i}^k + \sigma_{\mathrm{ncd}}^k}.$ In order to compensate this error, $\overline{\Delta p}_{\mathrm{unc},h}^k$ and $\underline{\Delta p}_{\mathrm{unc},h}^k$ are

set to satisfy:

$$\mathbf{P}\left(\overline{\Delta p}_{\mathrm{unc}}^{k} \ge \varepsilon_{h}\right) \ge 1 - r \quad \forall k, \tag{59}$$

$$\mathbf{P}\left(\underline{\Delta p_{\mathrm{unc}}^k} \le -\varepsilon_h\right) \ge 1 - r \quad \forall k,\tag{60}$$

which correspond to

$$\overline{\Delta p}_{\mathrm{unc},h}^k \ge \sigma_{\mathrm{unc},h}^k \sqrt{2} \mathrm{erf}^{-1} (1 - 2r) \quad \forall k, \tag{61}$$

$$\underline{\Delta p_{\text{unc}\,h}^k} \le -\sigma_{\text{unc},h}^k \sqrt{2} \text{erf}^{-1} (1 - 2r) \quad \forall k.$$
 (62)

The base power exchange and the introduced power variations defined for the hth LU must also satisfy the minimum and maximum power limits $P_h^{\min} \leq 0$ and $P_h^{\max} \geq 0$ [kW]:

$$p_h^k + \overline{\Delta p}_{\text{flx},h} + \overline{\Delta p}_{\text{unc},h} \le P_h^{\text{max}}, \quad \forall k,$$
 (63)

$$p_h^k + \underline{\Delta p}_{\text{flx},h} + \underline{\Delta p}_{\text{unc},h} \ge P_h^{\min}, \quad \forall k.$$
 (64)

Notice that P_h^{\min} is supposed to be negative or null. When it is not null, it means that power export is allowed.

Finally, since the prices paid for importing energy and received for exporting energy are generally different, the base power exchange is partitioned as

$$p_h^k = p_{\text{imp},h}^k + p_{\text{exp},h}^k \quad \forall k, \tag{65}$$

where, by introducing the binary variable x_{imp}^k

$$p_{\text{imp},h}^{k} \le x_{\text{imp}}^{k} P_{h}^{\text{max}} \quad \forall k,$$

$$p_{\text{exp},h}^{k} \ge (1 - x_{\text{imp}}^{k}) P_{h}^{\text{min}} \quad \forall k;$$

$$(66)$$

$$p_{\text{exp},h}^k \ge (1 - x_{\text{imp}}^k) P_h^{\text{min}} \quad \forall k; \tag{67}$$

therefore, the resulting energies imported end exported within the kth sampling interval are:

$$E_{\text{imp},h}^k = \Delta t \cdot p_{\text{imp},h}^k, \quad E_{\text{exp},h}^k = \Delta t \cdot p_{\text{exp},h}^k \quad \forall k.$$
 (68)

A. LU-DAP optimization problem

We have now all the elements for providing the LU-DAP optimization problem for the hth LU, which consists in the minimization of the cost function:

$$J_h = \sum_{k=0}^{T-1} c_{\mathrm{imp},h}^k E_{\mathrm{imp},h}^k - c_{\mathrm{exp},h}^k E_{\mathrm{exp},h}^k - c_{\mathrm{flx}}^k c_{\mathrm{flx}} \left(\overline{\Delta E}_h^k - \underline{\Delta E}_h^k \right)$$

such that the following constraints are satisfied: (8)–(20), (22)– (37), (41)–(45), (47)–(48), (51)–(58), (61)–(68).

This optimization problem is linear mixed-integer. The result of the optimization are the power profiles of all the controllable devices, and the maximal positive and negative power variations of BESSs and TCLs, together with the related aggregated values for the hth LU. In the paper, the optimized variables are indicated with (\cdot) .

If energy export is not enabled, with a fixed price ${}^kc_{\rm flx}$ paid both for positive and negative energy reserves, the LU will choose not to provide any negative reserve to minimize the cost function. Therefore, in this cases, if a minimum amount of negative energy reserve is required, an additive constraint should be added. For example, positive and negative reserves could be forced to be equal:

$$\overline{\Delta E}_h^k = \underline{\Delta E}_h^k, \quad \forall k. \tag{69}$$

B. Intra-Day operation

As illustrated in Section II, once the LU-DAP optimization problem has been executed, the hth LU sends to the AGT the resulting optimized energy exchange $\widehat{E}_h^k = \widehat{E}_{\mathrm{imp},h}^k + \widehat{E}_{\mathrm{exp},h}^k$ and the optimized energy reserves $\widehat{\Delta E}_h^k$ and $\widehat{\Delta E}_h^k$.

During the considered day, the AGT sends to the hth LU the energy reserve reference signal $\Delta E_{\text{ref},h}^k$, defined in (6). At the same time, the power consumption of RESs and NCDs is monitored and the forecasts error ε_h^k is computed. BESSs and TCLs are therefore called to provide the required energy reserves and to compensate the forecasts error proportionally to the percentage of their own (optimized) maximum power variations.

V. SIMULATION RESULTS

To analyze the effectiveness of the proposed approach, the case of an aggregate of H=200 houses has been considered. Simulation parameters are reported in Table I. Each house (i.e., each LU) is equipped with two ABPs (a dish washer and a washing machine), a BESS, an inverter-driven air cooling system (TCL), a PV generator (RES), and a PEV. Table I reports the parameters common to all the houses, which are then differentiated by randomly generating the initial BESSs' SoCs, the initial internal air temperatures, the temperature set-points, the amount of required PEV recharging (ΔSoC_1^p), and the time preferences for the PEV recharge. All TCLs are switched off from 0AM to 8AM and from 8PM to 12PM. RESs have the same rated power and are supposed to have the same generation profile, depicted in Fig. 2. This figure also reports the PV generation and external temperature forecasts used by the LU-DAP optimization.

Notice that the power export is not enabled $(P_h^{\min} = 0, \forall h)$. Moreover, consider that the energy prices are constant (i.e., ${}^k c_{\text{flx}} = c_{\text{flx}}$) and the price paid by the AGT for the energy reserve is five times the cost paid for the imported energy (i.e., ${}^k c_{\text{flx}}^{\text{agt}} = 5 c_{\text{flx}}$). Thus, the LUs have a potential significant economical advantage in providing the energy reserve. The values of these prices have been derived from the Italian Regulation Authority (ARERA). In order to obtain a negative energy reserve, the additional constraint (69) has been included in the optimization problem.

Simulation have been implemented on the MATLAB platform. The LU-DAP opimization problem has been written using the AMPL language and solved with the IBM ILOG Cplex solver.

Figure 3 reports the results obtained for the AGT. It appears clear that until the TCLs are switched on at 8PM, the offered energy reserves are very small, whereas, during the daylight hours, when also RESs are active, the AGT is able to offer a significant amount of energy reserve. Figure 3 also reports an example of a possible required DR signal perfectly realized by the AGT.

Figures 4-6 report the results obtained for one of the 200 houses. In particular, Fig. 4 shows the planned and realized energy profiles and the offered energy reserves. Figure 5 reports the corresponding BESS power exchange and SoC trajectories, whereas Fig. 6 depicts the ones of the TCL power consumption and of the internal temperature. In these two figures, the contribution of the BESS and of the TCL to the energy reserves and the potential consequences on the battery SoC and on the internal air temperature are also shown. It is worth noting that, for the considered house, the PEV recharge is active during the first hours and at the end of the day (the PEV recharge profile is not reported for the sake of brevity). This leads to discharge the BESS in the early morning, without providing significant energy reserves. Then, during the daylight hours, the TCL manages the thermal energy to provide a significant amount of energy reserve. Finally, in Fig. 2(bottom) it is shown how the temperature forecast uncertainty is taken into account in the definition temperature profiles.

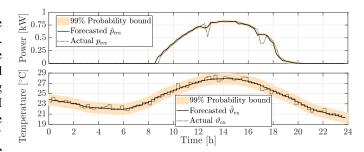


Fig. 2. Forecasts of PV generation (top) and external temperature (bottom).

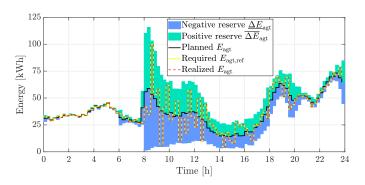


Fig. 3. Resulting day-ahead plan and DR signal realization for the AGT.

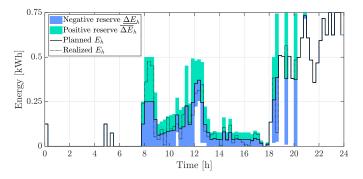


Fig. 4. Energy profiles for one of the 200 houses.

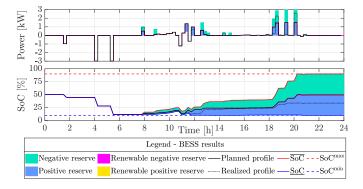


Fig. 5. BESS results for one of the 200 houses.

TABLE I SIMULATION PARAMETERS.

Description	Symbol	Value
ABP		
ABP number	$N_{ m abp}$	2
Phases number	n_i	[4,4]
Energy used by each phase [kWh]	E_{ij}	$\begin{bmatrix} 0.11 & 0.2 & 0.07 & 0.8 \\ 0.11 & 0.2 & 0.07 & 0.8 \end{bmatrix}$
Intervals for each phase	\overline{T}_{ij}	$\begin{bmatrix} 3 & 1 & 2 & 2 \\ 3 & 1 & 2 & 2 \end{bmatrix}$
Max. power for each phase [kW]	P_{ij}^{\max}	$\begin{bmatrix} 0.15 & 1.6 & 0.15 & 1.6 \\ 0.15 & 1.6 & 0.15 & 1.6 \end{bmatrix}$
Min. power for each phase [kW] PEV	P_{ij}^{\min}	$0 \; \forall i, j$
PEV number	A.r	1
	$E_{\text{nom}}^{N_{\text{pev}}}$	15
Nominal energy [kWh]	p,1	
Recharge efficiency [p.u.]	$P_1^{\eta_1}$	0.9
Max. recharge power [kW]	P_1^{max}	3.3
BESS		
BESS number	N hess	1
Nominal energy [kWh]	$E_{\mathrm{b,1}}$	5
Recharge efficiency [p.u.]	η_1^{ch}	0.9
Discharge efficiency [p.u.]	_n dsc	1.1
Max. recharge power [kW]	Pmax	3
	$^{1}_{P}^{\text{ch},1}$	
Max. discharge power [kW]	dsc.1	-3
Min. SoC [%]	SoC ^{max}	90
Max. SoC [%]	SoC ^{min}	10
Max. number of charging cycle	ℓ_1^{ch}	1
Max. number of discharging cycle	$\ell_1^{ m dsc}$	1
TCL number	A.T	1
	$N_{ m tcl}$	_
Thermal resistance [°C/(kW)]	R_1	2.5×10 ⁻⁶
Thermal capacitance [kWs/°C]	c_1	14400 2
Cooling efficiency [p.u.]	$P_{1}^{\eta_{1}^{c}}$	2
Rated power [kW]	$P_{\text{tcl},1}$	
Temperature forecast std. dev. LU	$\sigma_{\mathrm{ex},1}$	0.1
Max. imported power [kW]	P_h^{\max}	3
Max. exported power [kW]	pmin	0
PV rated power [kW]	h h	1
Chance-constraint reliability [p.u.]	r	0.05
DAP	•	0.02
LU number	H	200
Time horizon [h]	T	24
Sampling time [h]	Δt	0.25
Imported energy price [€/kWh]	$c_{h,\text{imp}}$	0.2
AGT reserve price [€/kWh]		30
	$c_{\rm flx}^{\rm agt}$	
LU reserve price [€/kWh]	c_{flx}	1

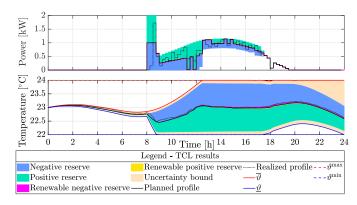


Fig. 6. TCL results for one of the 200 houses.

VI. CONCLUSION

The paper describes a decentralized optimization framework for demand response of the aggregation of load units. The strength of the proposed approach lies in the fact that the optimization is performed by each end user, assuming a complete knowledge of equipment and desires. The outcome of the process is a desired profile, and a reserve, which is, by construction, compliant with all the constraints the end users have designed for their appliances. This reserve can be used not only upon request, but also to compensate modeling and forecast errors.

REFERENCES

- [1] J. Hu, J. Cao, J. M. Guerrero, T. Yong, and J. Yu, "Improving Frequency Stability Based on Distributed Control of Multiple Load Aggregators," *IEEE Trans. Smart Grid*, vol. 8, no. 4, pp. 1553–1567, Jul. 2017.
- [2] X. Ayón, J. K. Gruber, B. P. Hayes, J. Úsaola, and M. Prodanović, "An optimal day-ahead load scheduling approach based on the flexibility of aggregate demands," *App. Energy*, vol. 198, pp. 1–11, 2017.
- [3] B. Zhou, W. Li, K. W. Chan, Y. Cao, Y. Kuang, X. Liu, and X. Wang, "Smart home energy management systems: Concept, configurations, and scheduling strategies," *Renewable and Sustainable Energy Reviews*, vol. 61, pp. 30–40, Aug. 2016.
- [4] S. Li, W. Zhang, J. Lian, and K. Kalsi, "Market-Based Coordination of Thermostatically Controlled Loads—Part I: A Mechanism Design Formulation," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1170–1178, Mar. 2016.
- [5] M. Alizadeh, A. Scaglione, and R. J. Thomas, "From packet to power switching: Digital direct load scheduling," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 6, pp. 1027–1036, Jul. 2012.
- [6] C. Chen, S. Kishore, Z. Wang, M. Alizadeh, and A. Scaglione, "How Will Demand Response Aggregators Affect Electricity Markets? - A Cournot Game Analysis," in 5th International Symposium on Communications Control and Signal Processing, ISCCSP 2012, 2012.
- [7] L. Gkatzikis, I. Koutsopoulos, and T. Salonidis, "The Role of Aggregators in Smart Grid Demand Response Markets," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 7, pp. 1247–1257, Jul. 2013.
- [8] Y. Zhu, J. Wang, D. Tian, and C. Sun, "A bilayer interaction strategy for air-conditioning load aggregators considering market-based demand response," J. Eng., vol. 2019, no. 16, pp. 1241–1246, Mar. 2019.
- [9] M. Parvania, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Optimal Demand Response Aggregation in Wholesale Electricity Markets," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1957–1965, Dec. 2013.
- [10] C. A. Correa-Florez, A. Michiorri, and G. Kariniotakis, "Robust optimization for day-ahead market participation of smart-home aggregators," *App. Energy*, vol. 229, pp. 433–445, Nov. 2018.
- [11] J. Iria and F. Soares, "Real-time provision of multiple electricity market products by an aggregator of prosumers," *App. Energy*, vol. 255, pp. 1–10, 2019
- [12] R. Henriquez, G. Wenzel, D. E. Olivares, and M. Negrete-Pincetic, "Participation of demand response aggregators in electricity markets: Optimal portfolio management," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4861–4871, sep 2018.
- [13] F. Baccino, S. Grillo, S. Massucco, G. Mauri, P. Mora, and F. Silvestro, "An Aggregator for Demand Side Management at Domestic Level Including PEVs," in *IEEE International Electric Vehicle Conference*, (IEVC), 2014.
- [14] F. Conte, S. Massucco, M. Saviozzi, and F. Silvestro, "A Stochastic Optimization Method for Planning and Real-Time Control of Integrated PV-Storage Systems: Design and Experimental Validation," *IEEE Trans. Sustain. Energy*, vol. 9, no. 3, pp. 1188–1197, Jul. 2018.